Parameter: p = % green

1. **Prior**: $p \sim \text{Beta}(a, b)$ ← we will get a & b by matching mean, Var

i.e. pdf $\pi(p) \propto p^{a-1}(1-p)^{b-1}$, $p \in (0,1)$

2. **Data**: $L(\underset{\sim}{x}|p) = p^{S}(1-p)^{n-S}$ where $S = x_1 + x_2 + \cdots + x_n$ = # greens in sample

3. **Posterior distribution for p**

$$\pi(p|\underset{\sim}{x}) \propto L(\underset{\sim}{x}|p) \cdot \pi(p) \qquad p \in (0,1)$$

$$\propto \frac{p^{S}(1-p)^{n-S} \cdot p^{a-1}(1-p)^{b-1}}{\int_0^1 p^{a-1}(1-p)^{b-1}\,dp}, \quad p \in (0,1)$$

← not required as it is a constant

$$\propto p^{S}(1-p)^{n-S}\, p^{a-1}(1-p)^{b-1}, \quad p \in (0,1)$$

$$\propto p^{a+S-1}(1-p)^{b+n-S-1}, \quad p \in (0,1)$$

So, $\pi(p|x) \equiv \text{Beta}(a_1, b_1)$

$$a_1 = a + S = a + x_1 + \cdots + x_n$$

$$b_1 = b + n - S = b + n - (x_1 + \cdots + x_n)$$

4. **Posterior Inference**

$$E[p|\underset{\sim}{x}] = \frac{a_1}{a_1 + b_1}$$

$$V[p|\underset{\sim}{x}] = \frac{a_1 b_1}{(a_1+b_1)^2(a_1+b_1+1)}$$

L = qbeta(.025, a_1, b_1)

U = qbeta(.975, a_1, b_1)