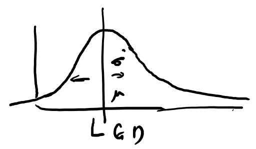
LGD

Parameter: (μ, σ^2)



Prior distribution for (μ, σ^2) or (μ, θ) $\quad \theta = \frac{1}{\sigma^2}$

- $\mu | \theta \sim N\left(\overset{\mu_0}{10}, \underset{\uparrow \frac{\sigma^2}{2}}{\frac{1}{2\theta}}\right)$
- $\theta \sim \text{Gamma}(\nu_0, \lambda_0)$

Data: $X_1 \cdots X_n \sim N(\mu, \sigma^2)$

$$L(\underset{\sim}{X} | \theta) \propto \theta^{\frac{n}{2}} e^{-\frac{\sum (x_i - \mu)^2}{2} \cdot \theta}$$

Posterior for (μ, θ)

$$\pi\left((\mu, \theta) \big| \underset{\sim}{x}\right) \propto L(\underset{\sim}{X}|\theta)\, \pi(\mu, \theta) = L(\underset{\sim}{X}|\theta) \cdot \pi(\mu|\theta)\, \pi(\theta)$$

$$\propto \theta^{\frac{n}{2}} e^{-\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{2} \theta} \cdot e^{-\frac{(\mu - \mu_0)^2}{2} \cdot (2\theta)} \cdot (2\theta)^{1/2} \cdot \theta^{\nu_0 - 1} e^{-\lambda_0 \theta}$$

$\mu \in (-\infty, \infty)$

$\theta > 0$

To Simulate from $(\mu, \theta) | \underset{\sim}{x}$

Approach is as follows: Gibbs Sampling

$\sigma^2 = \frac{1}{r\theta}$

$\tau_0^2 = \frac{\sigma^2}{2}$

0. Start with some value of $(r\mu, r\theta)$
1. Simulate $r\mu$ from $\mu | X, r\theta \longrightarrow N\left(\frac{\frac{n}{\sigma^2}\bar{X} + \frac{1}{\tau_0^2}\mu_0}{\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}}, \frac{1}{\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}}\right) \equiv N\left(\frac{n\bar{X} + 2\mu_0}{n+2}, \frac{\overset{\downarrow}{\sigma^2}}{n+2}\right)$
2. Simulate $r\theta$ from $\theta | X, r\mu \longrightarrow \text{Gamma}\left(\frac{n}{2} + \frac{1}{2} + \nu_0, \frac{\sum (x_i - r\mu)^2}{2} + \frac{(r\mu - \mu_0)^2}{2} + \lambda_0\right)$
3. Collect $(r\mu, r\theta)$
4. repeat 1, 2, 3