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Bayesian Method

Recap: 1) Motivation

2) Steps

e.g. p = % green in the bag (unknown parameter)

1. Prior distribution for p : $\pi(p)$

For the example:

Possible values p	$\cdot 01$	$\cdot 05$	$\cdot 1$
$\pi(p)$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{1}{5}$

$E[p] = 0.044$

$SD[p] = \cdot 033$

2. Data: Likelihood of data (given p)

X_1	X_2	X_3	X_4	X_5
0	0	0	0	0

$L(\underset{\sim}{X} \mid p) = (1-p)^{n-S} p^S$

$= (1-p)^5$ in our data

$S = X_1 + \cdots + X_n$ (# greens in sample)

n = # draws

3. Posterior distribution for p : $\pi(p \mid \text{Data}) = \pi(p \mid X_1=0, \ldots, X_5=0)$

Possible values	$\cdot 01$	$\cdot 05$	$\cdot 1$
$\pi(p \mid X)$	$\cdot 47$	$\cdot 38$	$\cdot 15$

$E[p \mid \underset{\sim}{X} = \underset{\sim}{0}] = 3.8\%$

$SD[p \mid \underset{\sim}{X} = 0] = 3.1\%$

Recall how we computed Posterior

Idea: Bayes theorem

$\pi(p = \cdot 01 \mid X_1 = 0, \ldots, X_5 = 0) \propto L(X_1 = 0 \ldots X_5 = 0 \mid p = \cdot 01)\, \pi(p = \cdot 01)$

$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$

$\pi(p = \cdot 01 \mid X_1 = 0 \ldots X_5 = 0) \propto (1 - \cdot 01)^5 \frac{2}{5} = C \cdot (1 - \cdot 01)^5 \frac{2}{5}$

$\pi(p = \cdot 05 \mid X_1 = 0 \ldots X_5 = 0) \propto (1 - \cdot 05)^5 \cdot \frac{2}{5} = C \cdot (1 - \cdot 05)^5 \frac{2}{5}$

$\pi(p = \cdot 1 \mid X_1 = 0 \ldots X_5 = 0) \propto (1 - \cdot 1)^5 \frac{1}{5} = C (1 - \cdot 1)^5 \frac{1}{5}$

$C = \frac{1}{(1 - \cdot 01)^5 \frac{2}{5} + (1 - \cdot 05)^5 \frac{2}{5} + (1 - \cdot 1)^5 \cdot \frac{1}{5}} = \frac{1}{P(B)}$