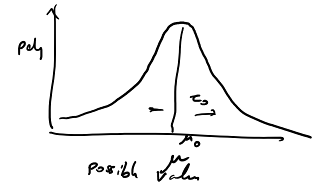
19/3/2022

Bayesian Linear Regression

Parameters: $\underset{\sim}{\beta} = (\beta_1 \dots \beta_k)$, σ^2 or $\theta = \frac{1}{\sigma^2}$



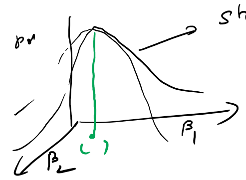
(1) Prior:

Recall for μ we took $\mu \sim N(\mu_0, \tau_0^2)$

recall $\pi(\mu) = \frac{1}{\sqrt{2\pi}\,\tau_0} e^{-\frac{(\mu-\mu_0)^2}{2\tau_0^2}} \longrightarrow \left(\frac{1}{\sqrt{2\pi}}\right)^k \frac{1}{|\Sigma|^{1/2}} e^{-\frac{(\underset{\sim}{\beta}-\underset{\sim}{\beta_0})^T \Sigma^{-1} (\underset{\sim}{\beta}-\underset{\sim}{\beta_0})}{2}}$

$|\Sigma|$ = determinant

$$\Sigma = \begin{bmatrix} V(\beta_1) & Cov(\beta_1,\beta_2) & \dots & Cov(\beta_1,\beta_k) \\ & V(\beta_2) & & \\ & & \ddots & \\ & & & V(\beta_k) \end{bmatrix}$$



stretch depends on $Corr(\beta_1, \beta_2)$, $V(\beta_1)$ $V(\beta_2)$

Prior for $\underset{\sim}{\beta} \sim N_k(\underset{\sim}{\beta_0}, \Sigma)$

Prior for $\frac{1}{\sigma^2} = \theta \sim Gamma(\nu_0, \lambda_0)$

(2) Likelihood:

Recall: $y_i = x_i^T\beta + \epsilon_i$ $\quad \epsilon_i \sim N(0, \sigma^2)$ $\quad (\beta_1 \dots \beta_k) = \underset{\sim}{\beta}^T$

$y_i \sim N(x_i^T\beta, \sigma^2)$

$$L(Y|\beta,\sigma^2) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n e^{-\frac{\sum_{i=1}^n (y_i - x_i^T\beta)^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n e^{-\frac{(Y-X\beta)^T(Y-X\beta)}{2\sigma^2}}$$