16|3|2022

Bayesian Methods

Beta - Binomial model

Problem: Estimation of proportions

Parameter: p $\quad 0 \le p \le 1$

Bayesian approach steps

1) Prior distribution for p $\rightarrow \pi(p) \propto p^{a-1}(1-p)^{b-1} \quad p \in (0,1)$, $a>0$, $b>0$

2) Likelihood of observed data given p:

$$L(\underset{\sim}{x} \mid p) \propto p^{\sum_{i=1}^{n} x_i} (1-p)^{n-\sum_{i=1}^{n} x_i}$$

rhs Involves only terms involving p (the unknown parameter)

3) Posterior distribution for $p \rightarrow$

$$\pi(p \mid \underset{\sim}{x}) \propto L(x \mid p)\,\pi(p) \quad \text{where } p \in [0,1]$$

$$\propto p^{a+\sum_{i=1}^{n} x_i - 1} (1-p)^{b+n-\sum_{i=1}^{n} x_i - 1} \quad p \in (0,1) \equiv Beta(a_1, b_1)$$

$$a_1 = a + \sum_{i=1}^{n} x_i \qquad b_1 = b + n - \sum_{i=1}^{n} x_i$$

Conjugate Property

i.e Posterior distribution belongs to the Same family as the Prior distribution.

we say Beta family or distribution is Conjugate for the Binomial proportion.

4) Posterior Inference

$$E[p \mid \underset{\sim}{x}] = \frac{a_1}{a_1+b_1} \qquad SD(p \mid \underset{\sim}{x}) = \sqrt{\frac{a_1 b_1}{(a_1+b_1)^2(a_1+b_1+1)}}$$

95% Credible Interval : [qbeta(.025, a_1, b_1), qbeta(.975, a_1, b_1)]

5) Predictive distribution.

question from yesterday

$P(Y_1 = 2)$?