Normal Distribution Problems

1. A bank is considering an application for loan by a new upcoming business. In order to determine the appropriate interest rate for the loan, the bank needs to assess the expected loss from the transaction in the event of a future default on the loan. A specific parameter of interest in making this assessment is the "loss given default (LGD)" defined as the percentage of loan amount that goes unrecovered. The bank manager thinks the business has potential to succeed but has limited experience from the past data that can provide an idea on LGD. After a careful look at past deals, the chief risk manager with the help of his team identifies 5 historical deals that had gone into default in the past. The LGDs for these deals were 5%, 8%, 10%, 12% and 5%. This suggests an average LGD of 8%. Given the limited past experience, the chief risk manager also sought opinion from five experienced risk professionals on his team and asked them to suggest a reasonable LGD based on their experience on similar deals in their career. Three of them generally agreed that such deals are riskier than what is apparent from the 5 historical deals that the company has pulled together but the other two were more consistent with the data. The LGDs suggested by the three of them were 5%, 10% , 40%, 50% and 60%.

   a) Assuming a normal model for observed LGDs (i.e. $N(\mu, \sigma^2)$) with unknown mean μ and known sd of $\sigma = 2\%$, and a discrete uniform prior on the LGD values from experts, obtain the posterior estimate for μ on such deals.
   b) Construct a normal distribution prior for μ, using expert information. Obtain the posterior distribution, 95% credible intervals. Compute the posterior mean and variance.
   c) Compare the posterior and prior pdfs. What happens if n (number of past deals) were to increase in support of the 8% average? What happens if the prior variance decreases?
   d) What is the predictive distribution for LGD on the new deal?
   e) What is the probability that LGD on the new deal will be greater than 15% based on the predictive distribution?
   f) Now assume a normal-gamma prior on (μ, σ^2). i.e. $\mu|\sigma^2 \sim N(10, \frac{\sigma^2}{2})$ and Inverse gamma distribution for σ^2 , i.e. we will assume $\frac{1}{\sigma^2}$ follows Gamma distribution with shape=40 and rate=10. Numerically, using simulations, compute the posterior distribution for μ.

Picture of I-Beam, and I-beam supporting roof of a house.
(https://en.wikipedia.org/wiki/I-beam)

2. An engineer takes a sample of 5 steel-I beams from a batch and measures the amount they sag under a standard load. The amounts in mm are 5.19, 4.72, 4.81, 4.87 and 4.88. It is known that the sag is $N(\mu, \sigma^2)$.
   a) Suppose SD is fixed, i.e. $\sigma = 0.5$. Use the prior $\pi(\mu) \sim N(5, .6^2)$ and obtain the posterior distribution for μ.
   b) For a batch of Steel-I beams to be acceptable, the mean sag under the standard load must be less than 5.20. What is the probability that μ is less than 5.20 under the posterior distribution?
   c) Suppose now that the mean is fixed, i.e. $\mu = 5$ but we have a prior on σ^2. More specifically, assume an inverse gamma prior for σ^2, i.e. $\frac{1}{\sigma^2}$ follows Gamma distribution with shape=80 and rate=20. Using the data 5.19, 4.72, 4.81, 4.87 and 4.88, find the posterior distribution for σ^2.
   d) Based on c), find the predictive distribution for the amount of sag experienced by a new item from batch.