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Bayesian Methods

	Proportions	LGD
· Parameter	p	μ σ known
· Prior for parameter $\pi(\theta)$	→ Discrete → Beta (a, b).	→ Discrete → $N(\mu_0, \tau_0^2)$ -
Data Likelihood $L(x\mid\theta)$	Binomial $\propto p^s (1-p)^{n-s}$	Normal $\propto \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n e^{-\frac{\sum (x_i-\mu)^2}{2\sigma^2}}$
· Posterior $\pi(\theta\mid x) \propto L(x\mid\theta) \cdot \pi(\theta)$ $\theta \in$ suitable range	$\pi(p\mid x) \equiv Beta(a+s, b+n-s)$	$\pi(\mu\mid x) \equiv N\left(\underbrace{\frac{\frac{n}{\sigma^2}\bar{x} + \frac{1}{\tau_0^2}\mu_0}{\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}}}_{\mu_1}, \underbrace{\frac{1}{\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}}}_{\tau_1^2}\right)$ ← We could have used truncated Normal by restricting $\mu \in (0,1)$
· Prediction	$P(Y_1 = 2) = E\left[P(Y_1=2 \mid p)\right]$ $p \sim \pi(p\mid y)$	$P(Y_1 \geq .15) = E\left[1 - pnorm(.5, \mu, .02)\right]$ $\mu \sim N(\mu_1, \tau_1^2)$