LGD example

$\pi(\mu) \equiv N(\mu_0, \tau_0^2)$ — Recall $\pi(p) \equiv$ Beta(a, b)

$a = ?$ $b = ?$

Obtain μ_0, τ_0^2 by matching moments of experts

$\mu_0 = .243$ $\tau_0^2 = .33^2$

1.) Prior: $\pi(\mu) \equiv N(\mu_0, \tau_0^2)$

$\pi(\mu) \propto \frac{1}{\sqrt{2\pi}\,\tau_0} e^{-\frac{(\mu-\mu_0)^2}{2\tau_0^2}}$ $\mu \in (-\infty, \infty)$

[If you want say $\mu \in (0,1)$]

2) Data: $L(\underset{\sim}{x}|\mu) \propto \left(\frac{1}{\sqrt{2\pi}}\right)^5 \frac{1}{\sigma^5} \cdot e^{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}}$

3) Posterior distribution for μ:

$\pi(\mu | \underset{\sim}{x}) \propto L(\underset{\sim}{x}|\mu) \cdot \pi(\mu)$ $\mu \in (-\infty, \infty)$

$\propto \cancel{\left(\frac{1}{\sqrt{2\pi}}\right)^5 \frac{1}{\sigma^5}} e^{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}} \cdot \cancel{\frac{1}{\sqrt{2\pi}\,\tau_0}} e^{-\frac{(\mu-\mu_0)^2}{2\tau_0^2}}$ $\mu \in (-\infty, \infty)$

$A\mu^2 - 2\mu B$

$= A\left[\mu^2 - 2\mu\frac{B}{A}\right]$

$= A\left[\mu^2 - 2\mu\frac{B}{A} + \frac{B^2}{A^2}\right] - \frac{B^2}{A}$

$= A\left(\mu - \frac{B}{A}\right)^2 - \frac{B^2}{A}$

$\pi(\mu|\underset{\sim}{x}) \propto e^{-\frac{\sum_{i=1}^{5}(x_i-\mu)^2}{2\sigma^2}} \, e^{-\frac{(\mu-\mu_0)^2}{2\tau_0^2}}$ $\mu \in (-\infty, \infty)$

Qn: Is this a known distribution (pdf)

Can I write

$\pi(\mu|\underset{\sim}{x}) \propto e^{-\frac{(\mu-\mu_1)^2}{2\tau_1^2}}$?

$\pi(\mu|\underset{\sim}{x}) \propto e^{-\frac{A(\mu-\mu_1)^2}{2}}$

Only look at (exponent.)

$\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{\sigma^2} + \frac{(\mu-\mu_0)^2}{\tau_0^2}$

$= \left(\frac{\sum x_i^2}{\sigma^2} - 2\mu\frac{\sum x_i}{\sigma^2} + \frac{n\mu^2}{\sigma^2}\right) + \left(\frac{\mu^2}{\tau_0^2} - \frac{2\mu\mu_0}{\tau_0^2} + \frac{\mu_0^2}{\tau_0^2}\right)$

$= \mu^2\left(\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}\right) - 2\mu\left(\frac{\sum x_i}{\sigma^2} + \frac{\mu_0}{\tau_0^2}\right) + \underbrace{\left(\frac{\sum x_i^2}{\sigma^2} + \frac{\mu_0^2}{\tau_0^2}\right)}$

$= A\mu^2 - 2\mu B + C$

$= A(\mu - \mu_1)^2 + C_1$

$A = \frac{n}{\sigma^2} + \frac{1}{\tau_0^2}$ $\mu_1 = \frac{B}{A}$ $\tau_1^2 = \frac{1}{A}$

$B = \frac{\sum x_i}{\sigma^2} + \frac{\mu_0}{\tau_0^2}$

$V(\bar{x}) = \frac{\sigma^2}{n}$

So $\pi(\mu|\underset{\sim}{x}) \equiv N(\mu_1, \tau_1^2)$.

$n = 5$

$$\mu_1 = \frac{\frac{\sum_{i=1}^{n} x_i}{\sigma^2} + \frac{\mu_0}{\tau_0^2}}{\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}} \qquad \tau_1^2 = \frac{1}{\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}}$$

data mean, Prior Mean

$$\mu_1 = \frac{\frac{n}{\sigma^2}\bar{x} + \frac{1}{\tau_0^2}\mu_0}{\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}}$$

$\frac{n}{\sigma^2}$ = Precision of data estimator

$\frac{1}{\tau_0^2}$ = Precision of the expert/Prior

In our example $\mu_1 =$ $\tau_1 =$ 95% CI =