

$$L(\mathbf{x}|p) \propto p^s (1-p)^{n-s} = p^{\sum_{i=1}^n x_i} (1-p)^{n-\sum_{i=1}^n x_i}$$

$$\pi(p) \equiv \text{Beta}(a, b)$$

$$\pi(p|\mathbf{x}) \equiv \text{Beta}\left(\underbrace{a + \sum_{i=1}^n x_i}_{a_1}, \underbrace{b + n - \sum_{i=1}^n x_i}_{b_1}\right)$$

$$E[p|\mathbf{x}] = \frac{a_1}{a_1 + b_1} = \frac{a + \sum_{i=1}^n x_i}{a + \sum_{i=1}^n x_i + b + n - \sum_{i=1}^n x_i} = \frac{a + \sum_{i=1}^n x_i}{a + b + n} = \left(\frac{a}{a+b}\right) \underbrace{\frac{(a+b)}{(a+b+n)}}_{\omega} + \bar{X} \underbrace{\frac{n}{(a+b+n)}}_{1-\omega}$$

Mean of  
Prior

Data  
Mean

$$V(p) = \frac{n(1-n)}{a+b+1}$$

If  $a+b$  increases expert is more certain  
If  $n$  increases data is more certain

$$V(\bar{X}) = \frac{p(1-p)}{n}$$

"Credible Interval"