$P(Y=2) = ?$

Suppose $\pi(p|y) \sim \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ .4 & .6 \end{pmatrix}$

$P(Y=2|p) = \binom{5}{2} p^2 (1-p)^3$

$$P(Y=2) = \underbrace{P(Y=2|p=\tfrac{1}{3}) \cdot P(p=\tfrac{1}{3})}_{\substack{P(A \cap B) \\ P(A|B)\,P(B)}} + \underbrace{P(Y=2|p=\tfrac{2}{3}) \cdot P(p=\tfrac{2}{3})}_{\substack{P(A \cap B^c) \\ P(A|B^c)\,P(B^c)}} = E\left[P(Y=2|p)\right]$$

$P(Y=2\|p)$	$\pi(p\|x)$
⋮	⋮

So, if $\pi(p|y) \sim$ pdf $f(p)$

$$P(Y_1=2) = \int_0^1 P(Y_1=2|p)\,\pi(p|y)\,dp = E\left[P(Y_1=2|p)\right]$$

new   old

when $E[\ ]$ is w.r.t $\pi(p|y)$

Compute using Simulation

1. Simulate $N = 1000$ values of p from $\pi(p|y) \equiv \text{Beta}(a_1, b_1)$
2. For each value in (1) compute $P(Y_1=2|p) = \binom{5}{2} p^2 (1-p)^3$
3. Take Average of $P(Y_1=2|p)$ computed in (2)