18|3|2022

Bayesian Method (I-beam)

Parameter: Variability of process σ^2

$\theta = \frac{1}{\sigma^2}$

Prior : $\pi(\theta) \propto \lambda_0^{\nu_0} \theta^{\nu_0 - 1} e^{-\lambda_0 \theta}$ $\theta \in (0, \infty)$

$\pi(\theta) = \frac{\lambda_0^{\nu_0} \theta^{\nu_0-1} e^{-\lambda_0\theta}}{\int_0^\infty \lambda_0^{\nu_0} \theta^{\nu_0-1} e^{-\lambda_0\theta} d\theta}$

Gamma(ν_0, λ_0)
shape rate

Data (Likelihood) : Sag measurements
$X_1 \; X_2 \; \dots \; X_n \overset{iid}{\sim} N(\mu, \sigma^2)$
assume $\mu = 5$ known , σ^2 unknown

$L(\underset{\sim}{x} | \theta) = \frac{1}{\sqrt{2\pi}} \theta^{\frac{n}{2}} e^{-\frac{\sum_{i=1}^n (x_i-\mu)^2}{2}\theta}$

Posterior $\pi(\theta | x) \propto L(\underset{\sim}{x} | \theta) \pi(\theta)$, $\theta > 0$

$\propto \theta^{\frac{n}{2}+\nu_0-1} e^{-[\lambda_0 + \frac{\sum(x_i-\mu)^2}{2}]\theta}$, $\theta \in (0, \infty)$

$\int_0^1 p^{a-1}(1-p)^{b-1} dp = B(a, b)$
Beta function

In R , beta & gamma functions are readily available

$\int_0^\infty \lambda_0^{\nu_0} \theta^{\nu_0-1} e^{-\lambda_0\theta} d\theta$ $x = \lambda_0\theta$

$= \int_0^\infty x^{\nu_0-1} e^{-x} dx$

$= \Gamma(\nu_0)$ Gamma function

$E(\theta) = \frac{\nu_1}{\lambda_1}$ $V(\theta) = \frac{\nu_1}{\lambda_1^2}$

95% CI qgamma( )

Gamma(ν_1, λ_1)

$\nu_1 = \frac{n}{2} + \nu_0$

$\lambda_1 = \lambda_0 + \frac{\sum(x_i-\mu)^2}{2}$