

Heuristic Prior $\pi(\beta) \sim N_n(\beta_0, \Sigma)$

OLS Classical regression : estimator $\hat{\beta} = (X^T X)^{-1} X^T y \rightarrow X^T X \beta = X^T y$

$V(\hat{\beta}) = \sigma^2 (X^T X)^{-1}$

Posterior dist of β given $\sigma^2 \sim N_n(\quad)$

Univariat : $\frac{\frac{n}{\sigma^2} \bar{X} + \frac{1}{\tau_0^2} \mu_0}{\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}} \quad \frac{\tau_0^2}{n} = V(\bar{X})$

$$\left((\sigma^2 X^T X)^{-1} + \Sigma^{-1} \right)^{-1} \left((\sigma^2 X^T X)^{-1} \hat{\beta} + \Sigma^{-1} \beta_0 \right)$$

$$= \left[\frac{X^T X}{\sigma^2} + \Sigma^{-1} \right]^{-1} \left[\underbrace{\frac{X^T X}{\sigma^2} \hat{\beta}}_{\frac{X^T y}{\sigma^2}} + \Sigma^{-1} \beta_0 \right]$$

$X^T X \hat{\beta} = X^T y$