I- beam

$\nu \to$ nu

unknow parameter: σ^2, variability in the production process

① Prior: $\frac{1}{\sigma^2} = \theta \sim \text{Gamma}(\underset{\text{Shape}}{\nu_0}, \underset{\text{Scale}}{\lambda_0})$

$$\pi(\theta) \propto \lambda_0^{\nu_0}\, \theta^{\nu_0-1}\, e^{-\lambda_0\theta}, \quad \theta \in (0, \infty);$$

$\nu_0 > 0$, $\lambda_0 > 0$

$\nu_0 = 1 \to$ exponential (λ_0) → waiting time for 1 arrival

$\nu_0 = k \to$ Gamma (k, λ_0) → Waiting time for k arrivals

Recall chi-Squared Test: $\sum_{i=1}^{k} \frac{(O_i - e_i)^2}{e_i} \underset{\text{approx}}{\sim} \chi^2_{k-1}$

$$\chi^2_{k-1} \equiv \text{Gamma}\left(\frac{k-1}{2}, \frac{1}{2}\right)$$

② Data: x_1 x_2 x_3 x_4 x_5

5.19 4.72 4.81 4.87 4.88

$x_1 \ldots x_5 \overset{iid}{\sim} N(\underset{\substack{5\\ \text{known}}}{\mu}, \underset{\text{unknown}}{\sigma^2})$ $\quad \theta = \frac{1}{\sigma^2}$

$$L(\underset{\sim}{x}|\theta) = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^5 e^{-\frac{\sum_{i=1}^{5}(x_i-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi}}\, \theta^{\frac{5}{2}}\, e^{-\frac{\sum_{i=1}^{5}(x_i-\mu)^2}{2}\cdot\theta}$$

③ Posterior for θ

$$\pi(\theta|x) \propto L(x|\theta)\cdot\pi(\theta), \quad \theta > 0$$

$$\pi(\theta|x) \propto \cancel{\frac{1}{\sqrt{2\pi}}}\, \theta^{5/2}\, e^{-\frac{\sum_{i=1}^{5}(x_i-\mu)^2}{2}\cdot\theta} \cdot \cancel{\lambda_0^{\nu_0}}\, \theta^{\nu_0-1} e^{-\lambda_0\theta}, \quad \theta > 0$$

$$\propto \theta^{\frac{5}{2}+\nu_0-1}\, e^{-\left(\lambda_0 + \frac{\sum_{i=1}^{5}(x_i-\mu)^2}{2}\right)\theta}, \quad \theta > 0$$

$$\pi(\theta|x) \equiv \text{Gamma}(\nu_1, \lambda_1)$$

$\nu_1 = \frac{n}{2} + \nu_0$ $(n=5)$

$\lambda_1 = \lambda_0 + \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2}$