Example

(a) $Y \sim Bin(n = 116, p)$ p unknown

(b) $\hat{p} = \frac{y}{n} = \frac{17}{116} = .146$ 95% CI $= \left[\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right] = [.0821, .2109]$

$sd(\hat{p}) = se = .032$

(c) $\pi(p) = Beta(a, b) \propto p^{a-1}(1-p)^{b-1}$

$L(Y|p) = \binom{n}{y} p^{y}(1-p)^{n-y}$

$\pi(p|y) \propto L(Y|p)\pi(p)$ $p \in (0,1)$

$\propto \cancel{\binom{n}{y}} p^{y}(1-p)^{n-y} p^{a-1}(1-p)^{b-1}$ $p \in (0,1)$

$\propto p^{a+y-1}(1-p)^{b+n-y-1}$

$\pi(p|y) = Beta(a_1, b_1)$

$a_1 = a+y$, $b_1 = b+n-y$

For $a = .5$ $b = .5$ (non-informative prior)

Posterior mean $= \frac{a_1}{a_1+b_1} = .1495$

95% Credible Interval: $[.09, .219]$

(d) $a = 1$ $b = 4$

Posterior mean $= .148$

95% Credible Interval $= [.09, .217]$

If $a = 100$ $b = 400$ 95% CI $= [.159, .221]$