

17/3/2022

Bayesian Methods

Proportions
 p

Parameter

Prior for parameter $\pi(\theta)$ \rightarrow Discrete \rightarrow Beta(a, b)

Data Likelihood $L(x|\theta)$

Binomial $\propto p^S (1-p)^{n-S}$

Posterior $\pi(\theta|x) \propto L(x|\theta) \cdot \pi(\theta)$

$\pi(p|x) \equiv \text{Beta}(a+S, b+n-S)$

$\theta \in \text{suitable range}$

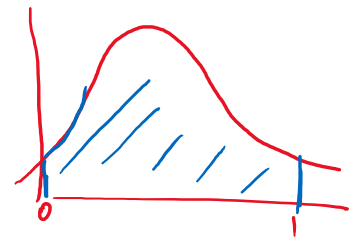
$P(Y_i=2) = E\left[P(Y_i=2|p)\right]$
 $p \sim \pi(p|Y)$

Prediction

LGD
 μ

σ known

\rightarrow Discrete $\rightarrow N(\mu_0, \tau_0^2)$



Normal $\propto \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$\pi(\mu|x) \equiv N\left(\frac{\frac{n}{\sigma^2}\bar{x} + \frac{1}{\tau_0^2}\mu_0}{\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}}, \frac{1}{\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}}\right)$

We could have used truncated Norm by restricting $\mu \in (0,1)$

$P(Y_i \geq 1.5) = E\left[1 - \Phi\left(\frac{1.5 - \mu}{\sigma}\right)\right]$
 $\mu \sim N(\mu_1, \tau_1^2)$