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Exercise 1

$$x_t = \begin{bmatrix} x_t \\ y_t \\ \psi_t \\ m^1_x \\ m^1_y \\ m^2_x \\ \vdots \\ m^n_x \\ m^n_y \end{bmatrix}, \quad y_t = \underbrace{\begin{bmatrix} \|m^1 - p_t\| \\ \vdots \\ \|m^n - p_t\| \\ \operatorname{atan2}(m^1_y - y_t, m^1_x - x_t) - \psi_t \\ \vdots \\ \operatorname{atan2}(m^n_y - y_t, m^n_x - x_t) - \psi_t \end{bmatrix}}_{h(x_t)} + \begin{bmatrix} v^1_t, \text{distance} \\ \vdots \\ v^n_t, \text{distance} \\ v^1_t, \text{bearing} \\ \vdots \\ v^n_t, \text{bearing} \end{bmatrix}$$

also, $x_{t+1} = x_t + \delta t\, \dot{x}_t + \omega^x_t$; $y_{t+1} = y_t + \delta t\, \dot{y}_t + \omega^y_t$; $\psi_{t+1} = \psi_t + \delta t\, \dot{\psi}_t + \omega^\psi_t$

where $\dot{x}_t = v_t \cos\psi_t - \dot{y}_t \sin\psi_t$; $\dot{y}_t = v_t \sin\psi_t + \dot{y}_t \cos\psi_t$

$$F_t = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

$$\Rightarrow F_t = \begin{bmatrix} 1 & 0 & \delta t(-v_t \sin\psi_t - \dot{y}_t \cos\psi_t) & 0 & \cdots & 0 \\ 0 & 1 & \delta t(v_t \cos\psi_t - \dot{y}_t \sin\psi_t) & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$h(x_t) = \begin{bmatrix} \|m^1 - p_t\| \\ \vdots \\ \|m^n - p_t\| \\ \operatorname{atan2}(m^1_y - y_t, m^1_x - x_t) - \psi_t \\ \vdots \\ \operatorname{atan2}(m^n_y - y_t, m^n_x - x_t) - \psi_t \end{bmatrix} = \begin{bmatrix} \left[(m^1_x - x_t)^2 + (m^1_y - y_t)^2\right]^{1/2} \\ \vdots \\ \left[(m^n_x - x_t)^2 + (m^n_y - y_t)^2\right]^{1/2} \\ \operatorname{atan2}(m^1_y - y_t, m^1_x - x_t) - \psi_t \\ \vdots \\ \operatorname{atan2}(m^n_y - y_t, m^n_x - x_t) - \psi_t \end{bmatrix}$$

$$H_t = \frac{\partial h}{\partial x}$$

$$\Rightarrow H_t = \left[\begin{array}{cc|c|cc|c} \frac{-(m^1_x - x_t)}{\left[(m^1_x - x_t)^2 + (m^1_y - y_t)^2\right]^{1/2}} & \frac{-(m^1_y - y_t)}{\left[(m^1_x - x_t)^2 + (m^1_y - y_t)^2\right]^{1/2}} & 0 & \frac{(m^1_x - x_t)}{\left[(m^1_x - x_t)^2 + (m^1_y - y_t)^2\right]^{1/2}} & \frac{(m^1_y - y_t)}{\left[(m^1_x - x_t)^2 + (m^1_y - y_t)^2\right]^{1/2}} \cdots 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{-(m^n_x - x_t)}{\left[(m^n_x - x_t)^2 + (m^n_y - y_t)^2\right]^{1/2}} & \frac{-(m^n_y - y_t)}{\left[(m^n_x - x_t)^2 + (m^n_y - y_t)^2\right]^{1/2}} & 0 & 0 & 0 \cdots \frac{(m^n_x - x_t)}{\left[(m^n_x - x_t)^2 + (m^n_y - y_t)^2\right]^{1/2}} & \frac{(m^n_y - y_t)}{\left[(m^n_x - x_t)^2 + (m^n_y - y_t)^2\right]^{1/2}} \\ \frac{(m^1_y - y_t)}{(m^1_x - x_t)^2 + (m^1_y - y_t)^2} & \frac{-(m^1_x - x_t)}{(m^1_x - x_t)^2 + (m^1_y - y_t)^2} & -1 & \frac{-(m^1_y - y_t)}{(m^1_x - x_t)^2 + (m^1_y - y_t)^2} & \frac{(m^1_x - x_t)}{(m^1_x - x_t)^2 + (m^1_y - y_t)^2} \cdots 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{(m^n_y - y_t)}{(m^n_x - x_t)^2 + (m^n_y - y_t)^2} & \frac{-(m^n_x - x_t)}{(m^n_x - x_t)^2 + (m^n_y - y_t)^2} & -1 & 0 & 0 \cdots \frac{-(m^n_y - y_t)}{(m^n_x - x_t)^2 + (m^n_y - y_t)^2} & \frac{(m^n_x - x_t)}{(m^n_x - x_t)^2 + (m^n_y - y_t)^2} \end{array}\right]$$