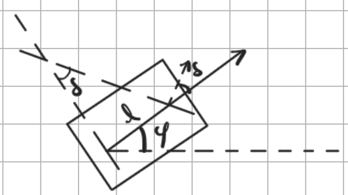
$$\begin{bmatrix} x \\ y \\ \varphi \\ \dot x \\ \dot y \\ \dot\varphi \end{bmatrix} \longrightarrow \text{states}$$

Note: Exercise 1 solved on page 2



$$m a_y = F_{yf}\cos\delta_f + F_{yr} = m(\ddot y + \dot\varphi \dot x)$$

$$\ddot y = -\dot\varphi\dot x + \frac{1}{m}(F_{yf}\cos\delta + F_{yr})$$

$$\ddot\varphi I_z = l_f F_{yf} - l_r F_{yr}$$

$$F_{yf} = 2C_\alpha\left(\delta - \frac{\dot y + l_f\dot\varphi}{\dot x}\right)$$

$$F_{yr} = 2C_\alpha\left(-\left(\frac{\dot y - l_r\dot\varphi}{\dot x}\right)\right)$$

$$\ddot x = \dot\varphi\dot y + a_x$$

and $m a_x = F - F_f = F - fmg$

dynamics in local coordinates

In global coordinates:

$$\dot X = \dot x\cos\varphi - \dot y\sin\varphi$$
$$\dot Y = \dot x\sin\varphi + \dot y\cos\varphi$$

$$\boxed{\begin{aligned} \ddot y &= -\dot\varphi\dot x + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot y + l_f\dot\varphi}{\dot x}\right) - \frac{\dot y - l_r\dot\varphi}{\dot x}\right) \\ \ddot x &= \dot\varphi\dot y + \frac{1}{m}(F - fmg) \\ \ddot\varphi &= \frac{2l_f C_\alpha}{I_z}\left(\delta - \frac{\dot y + l_f\dot\varphi}{\dot x}\right) - \frac{2l_r C_\alpha}{I_z}\left(-\left(\frac{\dot y - l_r\dot\varphi}{\dot x}\right)\right) \end{aligned}} \quad \text{—— } (1)$$

$$\left.\begin{aligned} \dot X &= \dot x\cos\varphi - \dot y\sin\varphi \\ \dot Y &= \dot x\sin\varphi + \dot y\cos\varphi \end{aligned}\right\} \begin{aligned} \dot y &= \dot Y\cos\varphi - \dot X\sin\varphi \\ \dot x &= \dot X\cos\varphi + \dot Y\sin\varphi \end{aligned}$$

$$y = [\dot x \quad \dot y \quad \dot\varphi \quad X \quad Y \quad \varphi]$$

$|\delta| \le \pi/6$ rad/s

$F \ge 0$ & $F \le 15763$ N

$\dot x \ge 10^{-5}$ m/s

$s_1 = [y \quad \dot y \quad \varphi \quad \dot\varphi]^T$, $s_2 = [x \quad \dot x]$, $u = [\delta \quad F]$

Exercise 1

$$\dot{s}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ (\ ) \\ \dot{\psi} \\ (\ ) \end{bmatrix}$$ → dynamics equations in ①

$$\frac{\partial f_1}{\partial s_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{2C_\alpha}{m}\left(\cos\delta\left(\frac{-1}{\dot{x}}\right) - \frac{1}{\dot{x}}\right) & 0 & \frac{2C_\alpha}{m}\left(\cos\alpha\left(-\frac{l_f}{\dot{x}}\right) + \frac{l_r}{\dot{x}}\right) \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2l_f C_\alpha}{I_z}\left(\frac{-1}{\dot{x}}\right) - \frac{2l_r C_\alpha}{I_z}\left(-\frac{1}{\dot{x}}\right) & 0 & \frac{2l_f C_\alpha}{I_z}\left(\frac{-l_f}{\dot{x}}\right) - \frac{2l_r C_\alpha}{I_z}\left(\frac{l_r}{\dot{x}}\right) \end{bmatrix}$$

$$\frac{\partial f_1}{\partial u} = \begin{bmatrix} 0 & 0 \\ -\frac{2C_\alpha}{m}\sin\delta\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) + \frac{2C_\alpha\cos\delta}{m} & 0 \\ 0 & 0 \\ \frac{2l_f C_\alpha}{I_z} & 0 \end{bmatrix}$$

$$\dot{s}_2 = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ (\ ) \end{bmatrix}$$ → from ①

$$\frac{\partial f_2}{\partial s_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\frac{\partial f_2}{\partial u} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix}$$

So, the linearized dynamics can be written as:

$$\dot{s}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_\alpha}{m}(\cos\delta + 1)\frac{1}{\dot{x}} & 0 & \frac{2C_\alpha}{m\dot{x}}(l_r - \cos\delta\, l_f) \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha}{I_z\dot{x}}(l_r - l_f) & 0 & -\frac{2C_\alpha}{I_z\dot{x}}(l_f^2 + l_r^2) \end{bmatrix} s_1 + \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m}\left(\cos\delta - \sin\delta\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right)\right) & 0 \\ 0 & 0 \\ \frac{2l_f C_\alpha}{I_z} & 0 \end{bmatrix} u$$

$$\dot{s}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} s_2 + \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix} u$$

here, $s_1 = [y \quad \dot{y} \quad \psi \quad \dot{\psi}]^T$, $s_2 = [x \quad \dot{x}]^T$, $u = [\delta \quad F]$