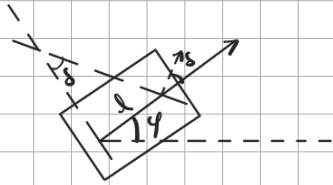


Note: Exercise 1 solved on page 2

$$\begin{bmatrix} x \\ y \\ \varphi \\ \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix}$$

→ states



$$m a_y = F_{y_f} \cos \delta_f + F_{y_n} = m(\ddot{y} + \varphi \ddot{x})$$

$$\ddot{y} = -\varphi \ddot{x} + \frac{1}{m}(F_{y_f} \cos \delta + F_{y_n})$$

$$\ddot{\varphi} I_z = l_f F_{y_f} - l_n F_{y_n}$$

$$F_{y_f} = 2C_x \left( \delta - \frac{\dot{y} + l_f \dot{\varphi}}{\dot{x}} \right)$$

$$F_{y_n} = 2C_n \left( -\frac{(\dot{y} - l_n \dot{\varphi})}{\dot{x}} \right)$$

$$\ddot{x} = \varphi \ddot{y} + a_n$$

$$\text{and } m a_n = F - F_f = F - f m g$$

dynamics in local coordinates

In global coordinates:

$$\dot{x} = \dot{x} \cos \varphi - \dot{y} \sin \varphi$$

$$\dot{y} = \dot{x} \sin \varphi + \dot{y} \cos \varphi$$

$$\ddot{y} = -\varphi \ddot{x} + \frac{2C_x}{m} \left( \cos \delta \left( \delta - \frac{\dot{y} + l_f \dot{\varphi}}{\dot{x}} \right) - \frac{(\dot{y} - l_n \dot{\varphi})}{\dot{x}} \right)$$

$$\ddot{x} = \varphi \ddot{y} + \frac{1}{m}(F - f m g)$$

$$\ddot{\varphi} = \frac{2l_f C_x}{I_z} \left( \delta - \frac{\dot{y} + l_f \dot{\varphi}}{\dot{x}} \right) - \frac{2l_n C_n}{I_z} \left( -\frac{(\dot{y} - l_n \dot{\varphi})}{\dot{x}} \right)$$

①

$$\begin{cases} \dot{x} = \dot{x} \cos \varphi - \dot{y} \sin \varphi \\ \dot{y} = \dot{x} \sin \varphi + \dot{y} \cos \varphi \end{cases} \quad \begin{cases} \dot{y} = \dot{y} \cos \varphi - \dot{x} \sin \varphi \\ \dot{x} = \dot{x} \cos \varphi + \dot{y} \sin \varphi \end{cases}$$

$$y = [x \quad y \quad \varphi \quad \dot{x} \quad \dot{y} \quad \dot{\varphi}]^T$$

$$|\delta| \leq \pi/6 \text{ rad/s}$$

$$F \geq 0 \text{ \& } F \leq 15763 \text{ N}$$

$$\ddot{x} \geq 10^5 \text{ m/s}$$

$$s_1 = [y \quad \dot{y} \quad \varphi \quad \dot{\varphi}]^T, \quad s_2 = [x \quad \dot{x}], \quad u = [\delta \quad F]$$

# Exercise 1

$$\dot{s}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\varphi} \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\varphi} \\ \ddot{\varphi} \end{bmatrix} \rightarrow \text{dynamics equations in (1)}$$

$$\frac{\partial f_1}{\partial s_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{2C_d}{m} \left( \cos \delta \left( \frac{-1}{\dot{x}} \right) \frac{-1}{\dot{n}} \right) & 0 & \frac{2C_d}{m} \left( \cos \delta \left( \frac{l_f}{\dot{n}} \right) + \frac{l_n}{\dot{n}} \right) \\ 0 & 0 & 0 & 0 \\ 0 & \frac{2l_f C_d}{I_z} \left( \frac{-1}{\dot{n}} \right) - \frac{2l_n C_d}{I_z} \left( \frac{-1}{\dot{x}} \right) & 0 & \frac{2l_f C_d}{I_z} \left( \frac{-l_f}{\dot{n}} \right) - \frac{2l_n C_d}{I_z} \left( \frac{l_n}{\dot{n}} \right) \end{bmatrix}$$

$$\frac{\partial f_1}{\partial u} = \begin{bmatrix} 0 & 0 \\ -\frac{2C_d \sin \delta}{m} \left( \delta - \frac{\dot{y} + l_f \dot{\varphi}}{\dot{x}} \right) + \frac{2C_d \cos \delta}{m} & 0 \\ 0 & 0 \\ \frac{2l_f C_d}{I_z} & 0 \end{bmatrix}$$

$$\dot{s}_2 = \begin{bmatrix} \dot{n} \\ \ddot{n} \end{bmatrix} = \begin{bmatrix} \dot{n} \\ \ddot{n} \end{bmatrix} \rightarrow \text{from (1)}$$

$$\frac{\partial f_2}{\partial s_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\frac{\partial f_2}{\partial u} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix}$$

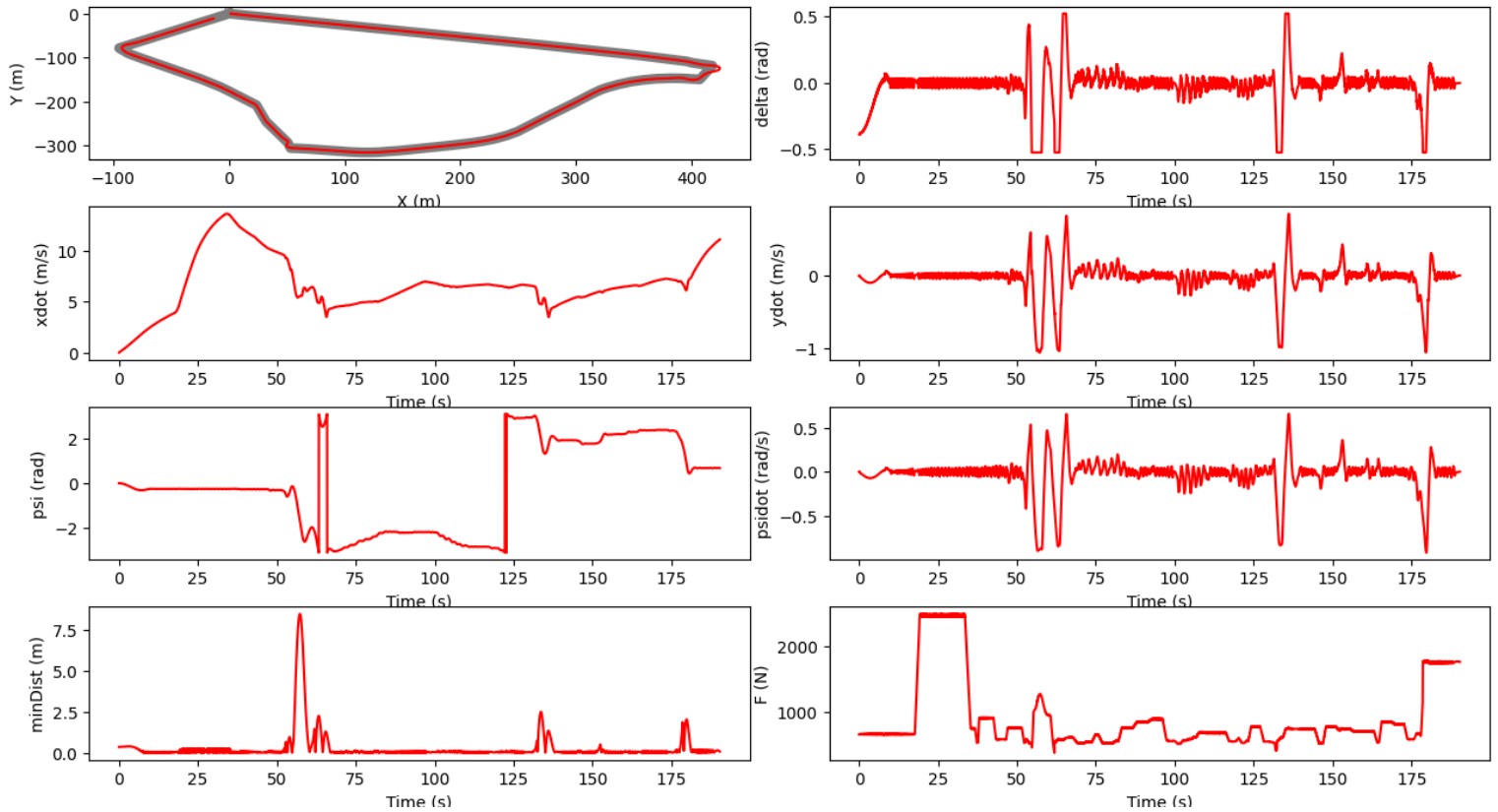
So, the linearized dynamics can be written as:

$$\dot{s}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_d}{m} \left( \cos \delta + 1 \right) \frac{1}{\dot{n}} & 0 & \frac{2C_d}{m \dot{n}} (l_n - \cos \delta l_f) \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_d}{I_z \dot{n}} (l_n - l_f) & 0 & -\frac{2C_d}{I_z \dot{n}} (l_f^2 + l_n^2) \end{bmatrix} s_1 + \begin{bmatrix} 0 & 0 \\ \frac{2C_d}{m} \left( \cos \delta - \sin \delta \left( \delta - \frac{\dot{y} + l_f \dot{\varphi}}{\dot{x}} \right) \right) & 0 \\ 0 & 0 \\ \frac{2l_f C_d}{I_z} & 0 \end{bmatrix} u$$

$$\dot{s}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} s_2 + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix} u$$

$$\text{here, } s_1 = [y \ \ddot{y} \ \varphi \ \ddot{\varphi}]^T, s_2 = [n \ \ddot{n}]^T, u = [\delta \ F]$$

### Performance Plot



### Evaluation Results

Evaluating...

Score for completing the loop: 30.0/30.0

Score for average distance: 30.0/30.0

Score for maximum distance: 30.0/30.0

Your time is 190.304

Your total score is : 100.0/100.0

total steps: 190304

maxMinDist: 8.462192300832887

avgMinDist: 0.28349594319153687