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Problem 1) Jacobians

1.4) In the given configuration, the rank of the spatial manipulator Jacobian is 5.

So in the given configuration, there is a kinematic singularity and J^s is not invertible.

Hence, we cannot find a set of joint velocities $\dot{\theta}$, that can move the end-effector in the way mentioned in the question.

$$V^s = J^s \dot{\theta}$$

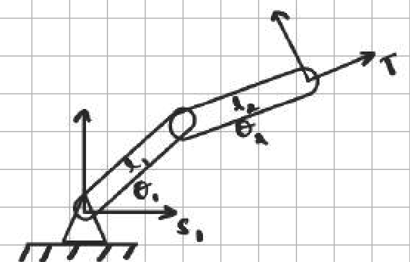
$$\Rightarrow \dot{\theta} = (J^s)^{-1} V^s$$

Since $\rho(J^s) = 5 < 6$, J^s is not invertible $\Rightarrow$ we cannot find a solution for $\dot{\theta}$ using inverse velocity kinematics.

In HW3, we had to write an optimization function and find a "numerical" solution to the problem as opposed to an analytical solution.

This is consistent with the conclusion from above, stating that a singularity does exist in the home configuration.

Problem 2) Statics



2.1) $$g_{sl_1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{l_1 l_2} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_1 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{l_2 t} = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

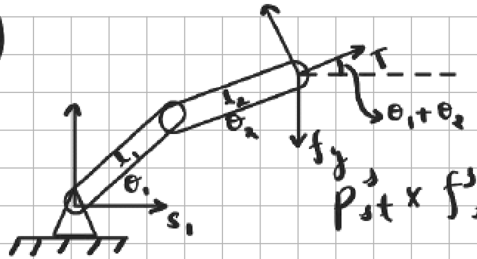
$$g_{st} = g_{sl_1}\, g_{l_1 l_2}\, g_{l_2 t} = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & l_2\cos(\theta_1+\theta_2)+l_1\cos\theta_1 \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & l_2\sin(\theta_1+\theta_2)+l_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.2) When $\theta_2 = 0/n\pi$, the body manipulator Jacobian becomes:

$$J^b = \begin{bmatrix} 0 & 0 \\ l_2 & l_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$ and this matrix has rank 1 due to its columns being linearly dependant.

So, when $\theta_2 = 0, \pi, 2\pi$ etc., there is a kinematic singularity, wherein we will not be able to compute joint velocities analytically for a given V^b or V^s of the end-effector.

2.3)



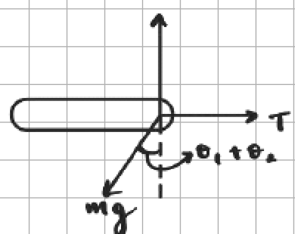
$f^s_{st} = [0 \ -mg \ 0]^T$

$p^s_{st} = [l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2),\ l_1 \sin\theta + l_1 \sin(\theta_1+\theta_2),\ 0]$

$p^s_{st} \times f^s_{st} = [0,\ 0,\ -mg(l_2 \cos(\theta_1+\theta_2) + l_1 \cos\theta_1)]$

$F_t = Ad(g_{st})^T F_s$

or simply computing in tool frame:

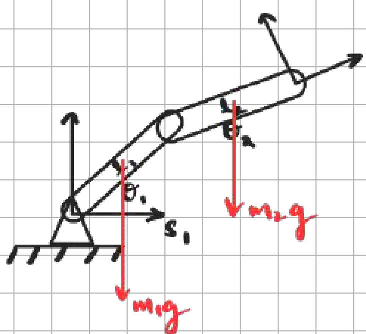


$f = [-mg \sin(\theta_1+\theta_2) \quad -mg \cos(\theta_1+\theta_2) \quad 0]^T$

$\tau = [0 \quad 0 \quad 0]$

$\Rightarrow F_t = [-mg\, s(\theta_1+\theta_2) \quad -mg \cos(\theta_1+\theta_2) \quad 0 \quad 0 \quad 0 \quad 0]^T$

2.5)



$f^s_2 = [\ 0\ ,\ -m_2 g\ ,\ 0]^T$

$p^s_2 = [\ l_1 \cos\theta_1/2,\ l_1 \sin\theta_1/2,\ 0]^T$

$\Rightarrow \tau^s_2 = p^s_2 \times f^s_2 = [0,\ 0,\ -m_1 g\, l_1 \cos\theta_1/2]^T$

$f^s_1 = [0,\ -m_1 g,\ 0]^T$

$p^s_1 = [l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2)/2,\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2)/2,\ 0]^T$

$\Rightarrow \tau^s_1 = p^s_1 \times f^s_1 = [0,\ 0,\ -m_2 g((l_2 \cos(\theta_1+\theta_2))/2 + l_1 \cos\theta_1)]^T$

$F_{2s} = [f_2^T \quad \tau_2^T]^T$; $F_{1s} = [f_1^T \quad \tau_2^T]^T$

$F_s = F_{1s} + F_{2s}$

F_t can be computed using the relation:

$F_t = Ag(g_{st})^{-1} F_s$