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## Problem 2) ABB IRB 620

### General forward kinematics

$$g_{s,l_1} = \begin{bmatrix} R_{z,q_1} & p_{l_1} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

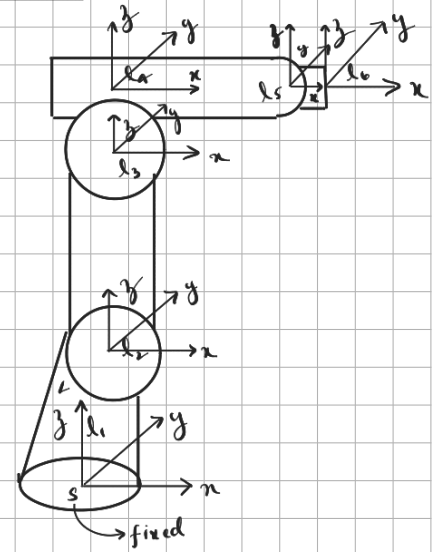
$$g_{l_1,l_2} = \begin{bmatrix} R_{y,q_2} & p_{l_1,l_2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & 0 & \sin q_2 & l_{x_2} \\ 0 & 1 & 0 & 0 \\ -\sin q_2 & 0 & \cos q_2 & l_{z_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{l_2,l_3} = \begin{bmatrix} R_{z,q_3} & p_{l_2,l_3} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_3 & 0 & \sin q_3 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin q_3 & 0 & \cos q_3 & l_{z_3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{l_3,l_4} = \begin{bmatrix} R_{x,q_4} & p_{l_3,l_4} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos q_4 & -\sin q_4 & 0 \\ 0 & \sin q_4 & \cos q_4 & l_{z_4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

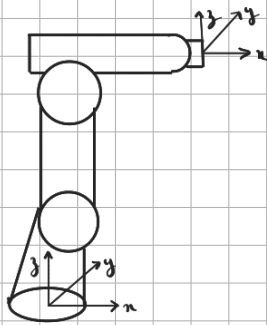
$$g_{l_4,l_5} = \begin{bmatrix} R_{y,q_5} & p_{l_4,l_5} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_5 & 0 & \sin q_5 & l_{x_5} \\ 0 & 1 & 0 & 0 \\ -\sin q_5 & 0 & \cos q_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{l_5,l_6} = g_{l_5,t} = \begin{bmatrix} R_{x,q_6} & p_{l_5,l_6} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & l_{x_6} \\ 0 & \cos q_6 & -\sin q_6 & 0 \\ 0 & \sin q_6 & \cos q_6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Now,  $g_{st} = g_{s,l_1} g_{l_1,l_2} g_{l_2,l_3} g_{l_3,l_4} g_{l_4,l_5} g_{l_5,t}$

# Power of exponents method



$$\text{For } g_{st}(0) \Rightarrow R_{st}(0) = R_y(0) = I_{3 \times 3}$$

$$p_{st}(0) = [(320 + 1837 + 200) \quad 0 \quad (680 + 975 + 200)]^T$$

$$= [1407 \quad 0 \quad 1855]^T$$

$$\Rightarrow g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 1407 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1855 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xi_i = [-\omega_i \times p_i \quad \omega_i]^T$$

In reference configuration:

$$\xi_1: \omega_1 = [0 \quad 0 \quad 1]^T; \quad p_1 = [0 \quad 0 \quad 0]^T$$

$$\Rightarrow \xi_1 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]^T$$

$$\xi_2: \omega_2 = [0 \quad 1 \quad 0]^T; \quad p_2 = [320 \quad 0 \quad 680]^T$$

$$\Rightarrow \xi_2 = [-680 \quad 0 \quad 320 \quad 0 \quad 1 \quad 0 \quad 0]^T$$

$$\xi_3: \omega_3 = [0 \quad 1 \quad 0]^T; \quad p_3 = [320 \quad 0 \quad 1655]^T$$

$$\xi_3 = [-1655 \quad 0 \quad 320 \quad 0 \quad 1 \quad 0 \quad 0]^T$$

$$\xi_4: \omega_4 = [1 \quad 0 \quad 0]^T; \quad p_4 = [320 \quad 0 \quad 1855]^T$$

$$\xi_4 = [0 \quad 1855 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]^T$$

$$\xi_5: \omega_5 = [0 \quad 1 \quad 0]^T; \quad p_5 = [1207 \quad 0 \quad 1855]^T$$

$$\xi_5 = [-1855 \quad 0 \quad 1207 \quad 0 \quad 1 \quad 0 \quad 0]^T$$

$$\xi_6: \omega_6 = [1 \quad 0 \quad 0]^T; \quad p_6 = [1407 \quad 0 \quad 1855]^T$$

$$\xi_6 = [0 \quad 1855 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]^T$$

$$\text{Now, } g_{st} = \xi_1 \xi_2 \xi_3 \xi_4 \xi_5 \xi_6 g_{st}(0)$$