Homework 10: Hybrid Systems

24-760 Robot Dynamics & Analysis
Fall 2024

Name: _____

Please turn in a PDF with the answers to the following questions.

Problem 1) Falling Block

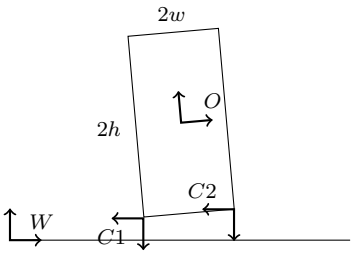

Figure 1: A block.

Consider a planar, rectangular block as shown above and considered in the last homework. The block has mass m, width $2w$, and height $2h$. Assume a tall block, where $h > 2w$. The state of the block in local coordinates is $q = [x, y, \theta]^T$ where each coordinate is expressed relative to the W frame. The gravity vector points in the $-y$ direction in the W frame, and there are no other applied wrenches or friction. There are two contact points on the bottom corners of the block, C_1 and C_2 that can make frictionless contact. Their position constraints are,

$$a_1(q) = y - h\,\cos(\theta) - w\,\sin(\theta)$$
$$a_2(q) = y - h\,\cos(\theta) + w\,\sin(\theta)$$

1.1) What are the possible contact modes, $\mathcal{J}$? Assume the block doesn't tip over (i.e. the only two possible contacts are at C_1 and C_2).

1.2) What is the domain, D_I, of each contact mode?

1.3) Write down the flow for the system, $\mathcal{F}$, i.e. the dynamics of the system in all possible contact modes. Please specify the matrices M, C, N, A, and Υ for the unconstrained contact mode $\{\}$ (i.e. neither C_1 or C_2 touching the ground), then for all other contact modes write down the updated version of any matrices that change. Feel free to use your solutions from HW9 as a start.

1.4) What are the feasible transitions between contact modes, $\tilde{\Gamma}$, based on the dynamics?

1.5) For each transition above, what are the corresponding guard conditions?

1.6) For each transition above, what is the corresponding reset map?

1.7) Now, let's drop the assumption that $h > 2w$, and for this problem only assume that $h > w$. Is the transition from $\{1\}$ to $\{1, 2\}$ achievable? Is the transition from $\{2\}$ to $\{1, 2\}$ achievable? That is, will it ever reach a state where it comes to rest?