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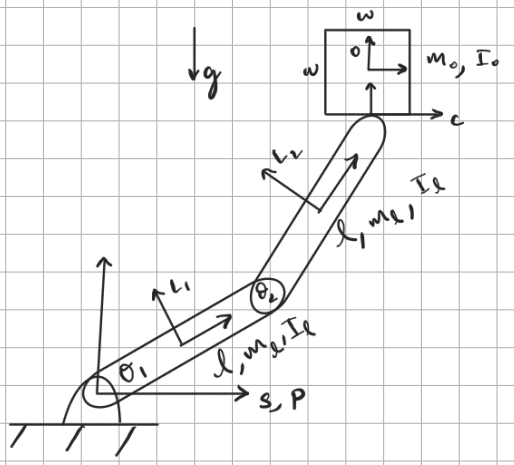

$q = [\theta_1, \theta_2, x_o, y_o, \phi_o]^T$

frictional contact between the link and the object.

1.1) To compute J_1^b,

$\xi_1 = [0, l/2, 0, 0, 0, 1]^T$

$\xi_2 = [0, 0, 0, 0, 0, 0]^T$

$$\Rightarrow J_1^b = [\xi_1 \ \xi_2] = \begin{bmatrix} 0 & 0 \\ l/2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$J_2^b \rightarrow \xi_1 = [-l\sin\theta_2, l/2 + l\cos\theta_2, 0, 0, 0, 1]^T$

$\xi_2 = [0, l/2, 0, 0, 0, 1]^T$

$$\Rightarrow J_2^b = [\xi_1 \ \xi_2] = \begin{bmatrix} -l\sin\theta_2 & 0 \\ l/2 + l\cos\theta_2 & l/2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

The individual mass matrices for the links are: $M_l = \begin{bmatrix} m_l & 0 & 0 & 0 & 0 & 0 \\ 0 & m_l & 0 & 0 & 0 & 0 \\ 0 & 0 & m_l & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_l \end{bmatrix}$

For the object: $M_o = \begin{bmatrix} m_o & 0 & 0 & 0 & 0 & 0 \\ 0 & m_o & 0 & 0 & 0 & 0 \\ 0 & 0 & m_o & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_o \end{bmatrix}$

The inertia matrix for the system hence becomes:

$$M = \frac{1}{2} J_1^{b^T} M_l J_1^b + \frac{1}{2} J_2^{b^T} M_l J_2^b + M_0$$

In planar coordinates:

$$J_1^b = \begin{bmatrix} 0 & 0 \\ l/2 & 0 \\ 1 & 0 \end{bmatrix}, \quad J_2^b = \begin{bmatrix} -l\sin\theta_2 & 0 \\ l/2 + l\cos\theta_2 & l/2 \\ 1 & 1 \end{bmatrix}$$

$$M_l = \begin{bmatrix} m_l & 0 & 0 \\ 0 & m_l & 0 \\ 0 & 0 & I_l \end{bmatrix}$$

$$M_0 = \begin{bmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & I_0 \end{bmatrix}$$

1.6) For the massless and frictionless case, we are unable to compute the instantaneous accelerations, as the rank of block matrix is insufficient and thus it becomes non-invertible.

Since the contact is frictionless, we have only one constraint. This is not able to make up for the loss off 2 masses. If like the previous case there was friction, it would have been possible to compute the accelerations.

In the case where only one of the links are massless, and the contact is frictionless, we are still able to compute the accelerations, as the one constraint is able to make up for the loss of rank.