Homework 4: Kinematics & Statics

24-760 Robot Dynamics & Analysis
Fall 2024

Name: ______________________________

Note: For homework submission, please submit a zipped folder of your Matlab files to "HW4: Kinematics & Statics_Programming" in Gradescope. Submit a Matlab script following the given template `HW4_sudent.m` and include all used functions in .zip format. Fill in all the `TODO` sections and clearly label sections based on which part they are for. Please use the precise variable names that we define in the template and do not overwrite them in later sections.

Problem 1) Jacobians

Consider the ABB IRB6620 that was used in HW3. As always, you may use the functions you defined in past homeworks as well as any of the Robotics System Toolbox functions.

1.1) Compute the spatial Jacobian, J^s_{st}, by calculating $\left(\frac{\partial g_{st}}{\partial q_i} g_{st}^{-1}\right)^\vee$ for each joint using the `diff` function. Submit in the template a script that produces a symbolic expression called `Js`.

1.2) Compute the spatial Jacobian, J^s_{st}, again by using exponential twists and compare the result to the last part. Submit in the template a script that produces a symbolic expression called `Js_exp` and performs the comparison.

1.3) When the robot is in the initial configuration, what body and spatial velocities correspond to moving the end effector at unit velocity in the tool's $+y$ direction (without rotation)? Submit in the template a script that produces numerical expressions called `Vb` and `Vs`.

1.4) Can we move the joints in some direction $\dot{q}$ to achieve this velocity? In HW3 Q2.4 we found a configuration that was $100mm$ offset in the $+y$ direction – are the answers to that problem and this problem consistent? Submit in the template a script that performs the singularity check and add a comment discussing the relationship between this answer and the answer from HW3.

Problem 2) Statics

Consider a planar robot with two revolute joints, with angles θ_1 and θ_2, and two links of length l_1 and l_2 (Figure 1).

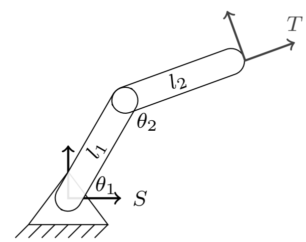

Figure 1: Two link robot.

2.1) Compute the forward kinematics of the robot `g_st`.

2.2) What kinematic singularities are there for this robot? Start by computing the spatial Jacobian `J_s` and body Jacobian `J_b` (you may use your lecture notes). Write a Matlab script with both the spatial and body Jacobian's (use your lecture notes to compare and confirm their correctness).

2.3) Assume that our arm is holding a point mass m (located at the origin of the tool frame) against gravity. What body wrench does this point mass apply? Compute the body wrench `F_t` as a function of the configuration variables $[\theta_1, \theta_2]^T$ of the arm.

2.4) What joint torques `tau_pm` need to be applied at joint 1 and joint 2 to counteract this body wrench (i.e. hold the arm in place)? Assume massless links.

2.5) For the rest of the problem ignore the point mass from previous sections and assume that our links have masses m_1 and m_2 respectively, with all of the mass located at the geometric center of each link (no inertia). What joint torques `tau_lm` need to be applied to counteract just the weight of the links themselves?

2.6) (Optional, 0 points – this section is **ungraded**, but interesting) Assume both links have unit length and unit mass. What configuration θ minimizes the actuator effort required to maintain? You can use $\tau^T\tau$ as a model for actuator effort where τ is the column vector of torques applied to joint 1 and joint 2 respectively.