Homework 9: Impact

24-760 Robot Dynamics & Analysis
Fall 2024

Name: _____

Compose a **single MATLAB script** (excluding helper functions), including all reasoning (either typed or as a picture of handwritten results), and all calculations with required output, using the exact variable names provided. Complete the `TODO` sections and clearly label sections according to the corresponding parts. Ensure variable names remain consistent without being overwritten in later sections. Place any helper functions alongside the main script in a **MATLAB Drive folder**, using an account associated with your Andrew ID. Name the folder as ***andrewID_24760_HW9***, share the folder link in the writeup, and submit this writeup to Gradescope..

Problem 1) Rocking Block

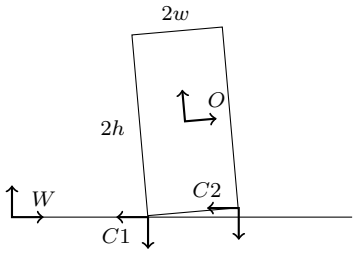

Figure 1: A block rotates about its left corner until impact occurs at the right corner.

Consider a planar, rectangular block as shown in Figure 1. The block has mass m, width $2w$, and height $2h$. The state of the block in local coordinates is $q = [x, y, \theta]^T$ where each coordinate is expressed relative to the W frame. The gravity vector points in the $-y$ direction in the W frame, and there are no other applied wrenches or friction.

The block is initially rotating clockwise with the bottom left corner (frame C_1) in contact with the ground. As the block rotates, the bottom right corner (frame C_2) will eventually come into contact with the ground, and a plastic impact event will occur ($e = 0$). We want to calculate the post-impact motion of this block as a function of the pre-impact states. In other words, given some q and $\dot{q}^-$, we wish to calculate $\dot{q}^+$ as well as the constraint impulse $\hat{P}$.

We can begin by writing the constraint functions $a(q) = [a_1(q), a_2(q)]^T$ associated with the C_1 and C_2 frames. Since there is no friction, there are only two constraints (one constraint for the contact normal of each frame). These functions are given by

$$a(q) = \begin{bmatrix} a_1(q) \\ a_2(q) \end{bmatrix} = \begin{bmatrix} y - h\cos(\theta) - w\sin(\theta) \\ y - h\cos(\theta) + w\sin(\theta) \end{bmatrix}$$

1.1) What are the A and M matrices for this system? Explain (in words) why we don't need to consider C, N, or Υ for analyzing impact.

Please compute and save them in the symbolic variables `A` and `M`.

1.2) Consider a wide block of mass $m = 1$ and size $w = 2$, $h = 1$ with pre-impact states $q = [2, 1, 0]^T$ and $\dot{q}^- = [1, -2, -1]^T$. Calculate $\dot{q}^+$ and $\hat{P}$ for a plastic impact into mode $\{C_1, C_2\}$, and verify that both the post-impact constraint velocities and impulses are valid (show that $A\dot{q}^+ \geq 0$ and $\hat{P} \leq 0$).

Please compute and save them in the numerical variables `dq_plus_wide`, `P_hat_wide`, and `A_dq_plus_wide`.

1.3) Now consider a narrow block of mass $m = 1$ and size $w = 1$, $h = 2$ with pre-impact states $q = [1, 2, 0]^T$ and $\dot{q}^- = [2, -1, -1]^T$. Calculate $\dot{q}^+$ and $\hat{P}$ for a plastic impact into mode $\{C_1, C_2\}$, and show that either the post-impact constraint velocities or impulses are invalid.

Please compute and save them in the numerical variables `dq_plus_narrow`, `P_hat_narrow`, and `A_dq_plus_narrow`.

1.4) For the same block as in Problem 1.3, find the correct contact mode and recalculate $\dot{q}^+$ and $\hat{P}$. Verify that both the post-impact constraint velocities and impulses are valid.

Please compute and save them in the numerical variables `dq_plus_correct`, `P_hat_correct`, and `A_dq_plus_correct`.