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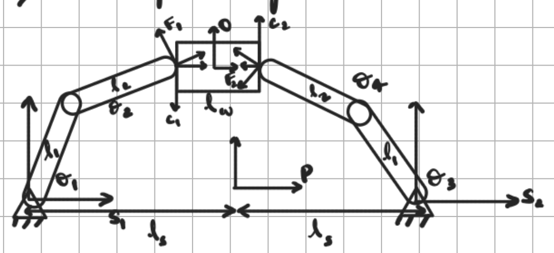
Problem 1) Grasp Properties



$P_{op} = [x_o, y_o, \phi_o]^T$

$\theta = [\theta_1, \theta_2, \theta_3, \theta_4]$

friction contact with friction coefficient μ.

1.1) Grasp map G is given by:

$$G = \left[Ad^T_{g^{-1}_{oc_1}} B_{c_1} \quad Ad^T_{g^{-1}_{oc_2}} B_{c_2} \right]$$

For finger 1 (left side on figure):

$$B_{c_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} ; \quad f_{c_1} = \begin{bmatrix} f_{cx_1} \\ f_{cy_1} \end{bmatrix} \begin{matrix} \rightarrow \text{friction} \\ \rightarrow \text{normal} \end{matrix}$$

$g_{oc_1} \Rightarrow \quad p_{oc_1} = \left[-\frac{l_w}{2} \quad 0 \quad 0 \right]^T$

$$R_{oc_1} = R_z(270^\circ) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow g_{oc_1} = \begin{bmatrix} 0 & 1 & 0 & -l_w/2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using MATLAB to compute $Ad^T_{g^{-1}_{oc_1}}$, we get

$$Ad^T_{g^{-1}_{oc_1}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{l_w}{2} & -1 & 0 & 0 \\ \frac{l_w}{2} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Ad^T_{g^{-1}_{oc_1}} B_{c_1} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{l_w}{2} & 0 \end{bmatrix}$$

Similarly for finger 2:

$$B_{c_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$g_{oc2} \Rightarrow p_{oc2} = \left[\frac{l_w}{2}, 0, 0\right]^T$$

$$R_{oc2} = R_z(90^\circ) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow g_{oc2} = \begin{bmatrix} 0 & -1 & 0 & l_w/2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

using MATLAB, $Ad^T_{g_{oc2}^{-1}} \Rightarrow$

$$Ad^T_{g_{oc2}^{-1}} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -\frac{l_w}{2} & 1 & 0 & 0 \\ \frac{l_w}{2} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Ad^T_{g_{oc2}^{-1}} B_{c_2} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{l_w}{2} & 0 \end{bmatrix}$$

$$\Rightarrow G = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{l_w}{2} & 0 & \frac{l_w}{2} & 0 \end{bmatrix}$$

The contact forces at each contact point are subject to the friction cone constraint.

Friction cone at finger i is given by:

$$V_i(f_{c_i}) = \begin{bmatrix} f_{cy_i} \\ \mu f_{cy_i} - f_{cx_i} \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The combined friction cone then becomes:

$$\begin{bmatrix} f_{cy_1} \\ f_{cy_2} \\ \mu f_{cy_1} - f_{cn_1} \\ \mu f_{cy_2} - f_{cn_2} \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

1.2) For force-closure grasp, we need G to be onto.

For G to be onto, G has to be full rank. i.e., $\rho(G) = 4$

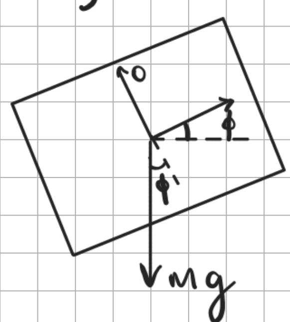
(also $null(G) = \{0\}$)

$$G = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{l_w}{2} & 0 & \frac{l_w}{2} & 0 \end{bmatrix}$$

From observation, $c_2 = -c_4 \Rightarrow$ these columns are linearly dependant.

So $\rho(G) = 3 \Rightarrow$ This is a force-closure for the planar system.

1.3)



In frame O,

$$f_o = [-mg\sin\phi, -mg\cos\phi, 0]^T$$

$$p = [0, 0, 0]$$

$$\Rightarrow \tau = [0, 0, 0]$$

$$\Rightarrow F_o = \begin{bmatrix} -mg\sin\phi \\ -mg\cos\phi \\ 0 \\ 0 \\ 0 \end{bmatrix} = F_e$$

Using the grasp map,

$$F_o = G f_c$$

$$\rightarrow \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{l_w}{2} & 0 & \frac{l_w}{2} & 0 \end{bmatrix} f_c = \begin{bmatrix} -mg\sin\phi \\ -mg\cos\phi \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

dropping the non-planar terms, we get:

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix} f_c = \begin{bmatrix} -mg\sin\phi \\ -mg\cos\phi \\ 0 \end{bmatrix}$$

where $f_c = \begin{bmatrix} f_{cx_1} \\ f_{cy_1} \\ f_{cx_2} \\ f_{cy_2} \end{bmatrix}$

$f_{cy_1} - f_{cy_2} = -mg\sin\phi$, $-f_{cx_1} + f_{cx_2} = -mg\cos\phi$; $f_{cx_1} + f_{cx_2} = 0$

$\Rightarrow f_{cx_1} = -f_{cx_2} = \frac{mg\cos\phi}{2}$; $f_{cy_1} - f_{cy_2} = -mg\sin\phi$

$$\rightarrow f_c = \begin{bmatrix} mg\cos\phi/2 \\ f_{cy_1} \\ -mg\cos\phi/2 \\ f_{cy_1} + mg\sin\phi \end{bmatrix}$$

subject to: $\begin{bmatrix} f_{cy_1} \\ f_{cy_1} + mg\sin\phi \\ \mu f_{cy_1} - mg\cos\phi/2 \\ \mu f_{cy_1} + \mu mg\sin\phi - mg\cos\phi/2 \end{bmatrix} \geq 0$

So, feasible contact forces to resist F_e:

$$f_c = \begin{bmatrix} mg\cos\phi/2 \\ f_{cy_1} \\ -mg\cos\phi/2 \\ f_{cy_1} + mg\sin\phi \end{bmatrix} \quad \text{s.t.} \quad \begin{bmatrix} f_{cy_1} \\ f_{cy_1} + mg\sin\phi \\ \mu f_{cy_1} - mg\cos\phi/2 \\ \mu f_{cy_1} + \mu mg\sin\phi - mg\cos\phi/2 \end{bmatrix} \geq 0$$

where $F_e = \begin{bmatrix} -mg\sin\phi \\ -mg\cos\phi \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

1.4) Internal forces have no effect on the body, i.e., they produce 0 body wrench. So, internal forces $\in$ null(G)

$$Gf = 0 \Rightarrow \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix} \begin{bmatrix} f_{cx_1} \\ f_{cy_1} \\ f_{cx_2} \\ f_{cy_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow f_{cy_1} = f_{cy_2}$, $f_{cx_1} = f_{cx_2}$, $f_{cx_1} + f_{cx_2} = 0$

Thus, $f_{cx_i} = 0$. which indicates 0 friction force

So for $f_{cx_1} = f_{cx_2} = 0$, $f_{cy_1} = f_{cy_2} = f_{cy}$,

$$f = \begin{bmatrix} f_{cx_1} \\ f_{cy_1} \\ f_{cx_2} \\ f_{cy_2} \end{bmatrix} = \begin{bmatrix} 0 \\ f_{cy} \\ 0 \\ f_{cy} \end{bmatrix}$$

can be an internal force, given friction cone constraints are met.

$$\begin{bmatrix} f_{cy_2} \\ f_{cy_1} \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

1.5) Since $f_{cy_1} = f_{cy_2}$ are internal forces, and from 1.3, to hold the object up, $f_c = \begin{bmatrix} mg\cos\phi/2 \\ f_{cy_1} \\ -mg\cos\phi/2 \\ f_{cy_1} + mg\sin\phi \end{bmatrix}$, s.t. $\begin{bmatrix} f_{cy_1} \\ \mu f_{cy_1} - mg\cos\phi/2 \end{bmatrix} \geq 0$

So for minimum value, $f_{cy_1} \geq mg\cos\phi / 2\mu$

1.6) The hand Jacobian J_h is given by:

$$J_h = \begin{bmatrix} B_{c_1}^T Ad_{g_{s_1c_1}}^{-1} J^s_{s_1f_1} & 0 \\ 0 & B_{c_2}^T Ad_{g_{s_2c_2}}^{-1} J^S_{s_2f_2} \end{bmatrix}$$

$$B_{c_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{c_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$g_{s_1c_1}$: $R_{s_1c_1} = R_z(270^\circ + \phi) = \begin{bmatrix} \sin\phi & \cos\phi & 0 \\ -\cos\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$p_{s_1c_1} = [$ ... $\sin(\theta_1+\theta_2), 0]^T$

$$\Rightarrow g_{s_1c_1} = \begin{bmatrix} s\phi & c\phi & 0 & l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) \\ -c\phi & s\phi & 0 & l_1 s\theta_1 + l_2 s(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \left\{ \begin{array}{l} c\phi = \cos\phi \\ s\phi = \sin\phi \end{array} \right\}$$

Similarly for $g_{s_2c_2}$:

$$R_{s_2c_2} = R_z(90^\circ + \phi) = \begin{bmatrix} -\sin\phi & -\cos\phi & 0 \\ \cos\phi & -\sin\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{s_2c_2} = [l_1 c\theta_3 + l_2 c(\theta_1+\theta_2),\ l_1 s\theta_3 + l_2 s(\theta_3+\theta_4),\ 0]^T$$

$$\Rightarrow g_{s_2c_2} = \begin{bmatrix} -s\phi & -c\phi & 0 & l_1 c\theta_3 + l_1 c(\theta_3+\theta_4) \\ c\phi & -s\phi & 0 & l_1 s\theta_3 + l_2 s(\theta_3+\theta_4) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To compute $J^s_{s_1f_1}$:

$$J^s_{s_1f_1} = \left[\left(\left(\frac{\partial g_{s_1f_1}}{\partial\theta_1} \right) g_{s_1f_1}^{-1} \right)^\vee \quad \left(\left(\frac{\partial g_{s_1f_1}}{\partial\theta_2} \right) g_{s_1f_1}^{-1} \right)^\vee \right]$$

or $\left[\xi_1' \quad \xi_2' \right]$

$$\xi_1' = \begin{bmatrix} -\omega_1 \times p_1 \\ \omega_1 \end{bmatrix}$$

$\omega_1 = [0, 0, 1]^T$; $p_1 = [0, 0, 0]^T$

$\Rightarrow \xi_1' = [0, 0, 0, 0, 0, 1]^T$

For ξ_2',

$\omega_2 = [0, 0, 1]^T$; $p_2 = [l_1 \cos\theta_1, l_2 \sin\theta_1, 0]^T$

$\omega_2 \times p_2 = [-l_1 \sin\theta_1, l_1 \cos\theta_1, 0]^T$

$\Rightarrow \xi_2' = [l_1 \sin\theta_1, -l_1 \cos\theta_1, 0, 0, 0, 1]^T$

$$\Rightarrow J^s_{s_1f_1} = \begin{bmatrix} 0 & l_1 \sin\theta_1 \\ 0 & -l_1\cos\theta_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

Similarly for $J^s_{s_2f_2}$, we get:

$$J^s_{s_2f_2} = \begin{bmatrix} 0 & l_1 \sin\theta_3 \\ 0 & -l_1 \cos\theta_3 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

And computing the adjoints of $g_{s_1c_1}^{-1}$ and $g_{s_2c_2}^{-1}$ using MATLAB, we get:

$$Ad_{g_{s_1c_1}^{-1}} = \begin{bmatrix} s\phi & -c\phi & 0 & 0 & 0 & -l_2 c(\theta_1 - \phi + \theta_2) - l_1 c(\phi - \theta_1) \\ c\phi & s\phi & 0 & 0 & 0 & l_1 s(\phi - \theta_1) - l_2 s(\theta_1 - \phi + \theta_2) \\ 0 & 0 & 1 & l_2 s(\theta_1+\theta_2) + l_1 s\theta_1 & -l_2 c(\theta_1+\theta_2) - l_1 c\theta_1 & 0 \\ 0 & 0 & 0 & \sin\phi & -c\phi & 0 \\ 0 & 0 & 0 & \cos\phi & \sin\phi & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Ad_{g_{s_2c_2}^{-1}} = \begin{bmatrix} -s\phi & c\phi & 0 & 0 & 0 & l_2 c(\theta_3 - \phi + \theta_4) + l_1 c(\phi - \theta_3) \\ -c\phi & -s\phi & 0 & 0 & 0 & l_2 s(\theta_3 - \phi - \theta_4) - l_1 s(\phi - \theta_3) \\ 0 & 0 & 1 & l_2 s(\theta_3+\theta_4) + l_1 s\theta_3 & -l_2 c(\theta_3+\theta_4) - l_1 c\theta_3 & 0 \\ 0 & 0 & 0 & -s\phi & \cos\phi & 0 \\ 0 & 0 & 0 & -c\phi & -\sin\phi & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Computing $B_{c_i}^T Ad_{g_{s_ic_i}^{-1}} J_{s_if_i}$ using MATLAB, we get:

$$B_{c_1}^T Ad_{g_{s_1c_1}^{-1}} J_{s_1f_1} = \begin{bmatrix} -l_2 c(\theta_1+\theta_2-\phi) - l_1 \cos(\phi - \theta_1) & -l_2 c(\theta_1+\theta_2-\phi) \\ l_1 s(\phi-\theta_1) - l_2 s(\theta_1+\theta_2-\phi) & -l_2 s(\theta_1+\theta_2-\phi) \end{bmatrix}$$

$$B_{c_2}^T Ad_{g_{s_2c_2}^{-1}} J_{s_2f_2} = \begin{bmatrix} l_2 c(\theta_3+\theta_4-\phi) - l_1 c(\phi-\theta_3) & -l_2 c(\theta_3+\theta_4-\phi) \\ l_2 s(\theta_3+\theta_4-\phi) - l_1 s(\phi-\theta_3) & l_2 s(\theta_3+\theta_4-\phi) \end{bmatrix}$$

$$J_h = \left[\begin{array}{c:c:c:c} -\sigma_4 - l_1 \cos(\phi - \theta_1) & -\sigma_4 & \sigma_3 + l_1 \cos(\phi - \theta_3) & \sigma_3 \\ \hdashline l_1 \sin(\phi - \theta_3) - \sigma_2 & -\sigma_2 & \sigma_1 - l_1 \sin(\phi - \theta_3) & \sigma_1 \end{array}\right]$$

$\sigma_1 = l_2 \sin(\theta_3 + \theta_4 - \phi)$, $\sigma_2 = l_2 \sin(\theta_1 + \theta_2 - \phi)$,
$\sigma_3 = l_2 \cos(\theta_3 + \theta_4 - \phi)$, $\sigma_4 = l_2 \cos(\theta_1 + \theta_2 - \phi)$

1.7) For internal motions, we have to look at the null space of J_h, because these are motions that have no effect on the contact/body velocity.

$\dot{\theta}_{int}$ gives internal motions if $J_h \dot{\theta}_{int} = 0$. i.e., $\dot{\theta}_{int} \in null(J_h)$

Using MATLAB to compute,

$$\dot{\theta}_{int} = \begin{bmatrix} \dfrac{l_2 \psi_1 + l_1 \psi_3}{l_1 \sin\theta_2} & \dfrac{l_2 \psi_1}{l_1 \sin\theta_2} \\ \dfrac{-l_1^2 \sin(\theta_1 - \theta_3) + l_2^2 \psi_1 + l_1 l_2 \psi_3 - l_1 l_2 \psi_2}{l_1 l_2 \sin\theta_2} & \dfrac{l_1 \psi_2 - l_2 \psi_1}{l_1 \sin\theta_2} \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\psi_1 = \sin(\theta_1 + \theta_2 - \theta_3 - \theta_4)$

$\psi_2 = \sin(\theta_3 + \theta_4 - \theta_1)$

$\psi_3 = \sin(\theta_1 + \theta_2 - \theta_3)$

1.8) The robot independently has 4 DOF.

The object by itself has 3 DOF.

When the robot and object are in contact, the robot has some motions that have no effect on the object's velocity or wrench. This is the extra DOF that is missing or lost. This comes at the cost of keeping contact with the object.

To summarise, the extra DOF is the grasping mechanism or the internal forces of the robot after grasping the object.