Homework 7: Lagrangian Dynamics

24-760 Robot Dynamics & Analysis
Fall 2024

Name: ________________________________

Submission Details:

1. Compose everything in a single Matlab script (except helper functions).
2. Include all reasoning in the main script (either typed or a picture of handwritten results).
3. Include calculations and required output, maintaining the exact variable names given in the problem statements.
4. Fill in all the `TODO` sections.
5. Clearly label sections according to which part they correspond to.
6. Use precise variable names defined in the template without overwriting them in later sections.
7. If helper functions are used, place them together with the main script in a **Matlab Drive folder**. (create a account/login on Matlab Drive only though your andrewID)
8. Name the folder as ***andrewID_24760_HW7***, where andrewID is your Andrew ID.
9. **Share the link** of the above Matlab drive folder in the writeup and submit this writeup to *Gradescope*.

Please make sure to use the predefined symbolic variables in the code template, especially the differential state, for example, we defined `q1` and its first and second derivative `dq1` and `ddq1` there. You should be able to complete the homework without defining any new symbolic variables.

Hint: Look at the Matlab functions `diff`, `gradient`, and `jacobian`.

Problem 1) Unconstrained Lagrangian

Consider the dynamics of the two link robot shown in Figure 2, with one rotational joint and one prismatic joint in the plane. Each link is a rod of length l and mass m with uniform mass distribution and center of mass (COM) at frame L_i. The second joint, q_2, extends the second link from length 0 (fully retracted) to length l (fully extended).

For this question we will consider the generalized coordinates, $q_g = [q_1, q_2]^T$, with no constraints. In the next question we will consider maximal coordinates with constraints. We use subscript g and m to distinguish them (i.e. L_g and L_m). *(HINT: It may be beneficial to create reusable functions that perform Lagrange's equation to differentiate L to get the EOM, or that calculates C from M, etc).*

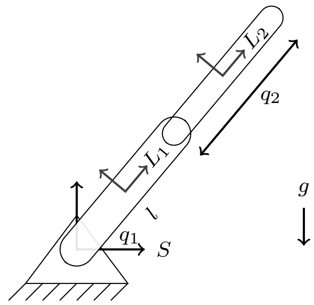

Figure 1: Two link robot. For each frame, the label indicates the x-axis.

1.1) What is the kinematic energy $T_g(q_1, q_2, \dot{q}_1, \dot{q}_2)$, potential energy $V_g(q_1, q_2)$, and Lagrangian in generalized coordinates, $L_g(q_1, q_2, \dot{q}_1, \dot{q}_2)$?

Please compute and save them in the symbolic variables `T_g`, `V_g`, and `L_g` respectively in the script.

1.2) Assuming that joints 1 and 2 have motor torque and force τ and F, compute the applied forces to each generalized coordinate Υ_g and the dynamic equations of motion by using the Lagrange equations to differentiate the Lagrangian in generalized coordinates.

Please compute and save them in the symbolic variables `Y_g` and `EOM_g1` in the script. `EOM_g1` should be a 2 by 1 symbolic matrix that is equal to [0; 0]. It is obtained by subtracting Υ_g on both sides of the equations.

1.3) Re-compute the dynamic equations of motion by directly computing the M_g, C_g, N_g, and Υ_g matrices in the manipulator equation,

$$M_g(q_g)\ddot{q}_g + C_g(q_g, \dot{q}_g)\dot{q}_g + N_g(q_g, \dot{q}_g) = \Upsilon_g$$

Check that you get the same answer as Problem 1.2.

Please compute and save them in the symbolic variables `M_g`, `C_g`, `N_g`, and `EOM_g2` in the script. `EOM_g2` should be a 2 by 1 symbolic matrix that is equal to [0; 0] and should be the same as `EOM_g1` from Problem 1.2.

Problem 2) Constrained Lagrangian

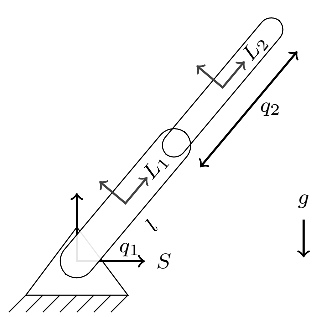

Figure 2: Two link robot. For each frame, the label indicates the x-axis.

In the last question we considered the generalized coordinates, $q_g = [q_1, q_2]^T$, with no constraints. In this question we consider maximal coordinates with constraints.

Maximal coordinates represent the full position and orientation of each link. For each link, use local coordinates for the link frame at the COM, namely L_1 is at (x_1, y_1, ϕ_1) and L_2 is at (x_2, y_2, ϕ_2) with ϕ_i measured counter-clockwise from the S frame. The new combined state is $q_m = [x_1, y_1, \phi_1, x_2, y_2, \phi_2]^T$.

2.1) What position and velocity constraints are there on the system, $a_m(q_m)$ and $A_m \dot{q}_m$?

Please compute and save them in the symbolic variables `a_m`, `A_m` respectively in the script.

2.2) What is the kinematic energy $T_m(q_m, \dot{q}_m)$, potential energy $V_m(q_m)$, and Lagrangian in maximal coordinates, $L_m(q_m, \dot{q}_m)$?

Please compute and save them in the symbolic variables `T_m`, `V_m`, and `L_m` respectively in the script.

2.3) The actuator effort is harder to represent in maximal coordinates. For each link, consider the force or torque applied to a frame at the end of the link (with equal and opposite signs for the joint effort between the links). Assuming that joints 1 and 2 have motor torque and force τ and F, calculate Υ_m, the resulting applied force on the two links in maximal coordinates.

Please compute and save it in the symbolic variable `Y_m` in the script.

2.4) Compute the dynamic equations of motion by using the constrained Lagrange equations to differentiate the Lagrangian in maximal coordinates.

Please compute and save it in the symbolic variable `EOM_m1` in the script. `EOM_m1` should be a 6 by 1 symbolic matrix that is equal to [0; 0; 0; 0; 0; 0]. It is obtained by subtracting Υ_m on both sides of the equations.

2.5) Re-compute the dynamic equations of motion by directly computing the M_m, C_m, N_m, and Υ_m matrices in the constrained manipulator equation,

$$M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + N_m(q_m, \dot{q}_m) + A^T(q_m)\lambda = \Upsilon_m$$

Check that you get the same answer as Problem 2.4.

Please compute and save them in the symbolic variables `M_m`, `C_m`, `N_m`, and `EOM_m2` in the script. `EOM_m2` should be a 6 by 1 symbolic matrix that is equal to [0; 0; 0; 0; 0; 0] and should be the same as `EOM_m1` from Problem 2.4.

2.6) What are the constraint forces?

Please compute and save it in the symbolic variable `lambdaVec` in the script.

2.7) (Optional) Finally, show that the dynamic equations in maximal coordinates (Problem 2.4 or 2.5) are equivalent to the dynamic equations in generalized coordinates (Problem 1.2 or 1.3) by using the constraint equations and the change of basis between q_g and q_m so that $q_m = h(q_g)$ and $\dot{q}_m = H\dot{q}_g$. The converted EOM in generalized coordinates from maximal coordinates should yield the same results as Problem 1.2 or 1.3.

Please compute and save them in the symbolic variables `h`, `H`, and `EOM_g3` respectively in the script.