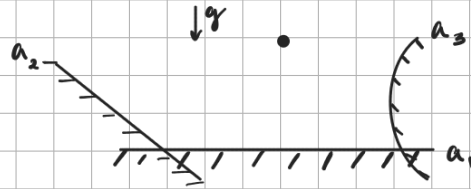


Problem 1



$$m=1, g=9.8$$

$$a_1 = y; a_2 = x+y+1; a_3 = (x-2)^2 + (y-1)^2 - 2$$

$$1.1) \mathcal{I} : \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}\}$$

$$\Gamma : \{(\emptyset, 1), (\emptyset, 2), (\emptyset, 3), (1, 2), (1, \{1,2\}), (1, \{1,3\}), (2, 1), (3, \emptyset), (3, \{1,3\}), (\{1,2\}, 1), (\{1,3\}, 1)\}$$

D:

$$\emptyset = \{(x, y) : a_1(x, y) > 0, a_2(x, y) > 0, a_3(x, y) > 0\}$$

$$1 = \{(x, y) : a_1(x, y) = 0, a_2(x, y) > 0, a_3(x, y) > 0\}$$

$$2 = \{(x, y) : a_2(x, y) = 0, a_1(x, y) > 0, a_3(x, y) > 0\}$$

$$3 = \{(x, y) : a_3(x, y) = 0, a_1(x, y) > 0, a_2(x, y) > 0\}$$

$$1,2 = \{(x, y) : a_1(x, y) = a_2(x, y) = 0, a_3(x, y) > 0\}$$

$$1,3 = \{(x, y) : a_1(x, y) = a_3(x, y) = 0, a_2(x, y) > 0\}$$

F:

$$M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}; N = \begin{bmatrix} 0 \\ mg \end{bmatrix}; C = [0]_{2 \times 2}; Y = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a_1 = y \Rightarrow A_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$a_2 = x+y+1 \Rightarrow A_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$a_3 = (x-2)^2 + (y-1)^2 - 2 \Rightarrow A_3 = \begin{bmatrix} 2(x-2) & 2(y-1) \end{bmatrix}$$

$$A_{1,2} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, A_{1,3} = \begin{bmatrix} 0 & 1 \\ 2(x-2) & 2(y-1) \end{bmatrix}$$

$$\text{Taking } q = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$F_{\{ \}} : M \ddot{q} + N = 0$$

$$F_1 : M \ddot{q} + N + A_1^T \lambda_1 = 0, A_1 \ddot{q} + \dot{A}_1 \dot{q} = 0$$

$$F_2 : M \ddot{q} + N + A_2^T \lambda_2 = 0, A_2 \ddot{q} + \dot{A}_2 \dot{q} = 0$$

$$F_3 : M \ddot{q} + N + A_3^T \lambda_3 = 0, A_3 \ddot{q} + \dot{A}_3 \dot{q} = 0$$

$$F_{1,2} : M \ddot{q} + N + A_{1,2}^T \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = 0, A_{1,2} \ddot{q} + \dot{A}_{1,2} \dot{q} = 0$$

$$F_{1,3} : M \ddot{q} + N + A_{1,3}^T \begin{bmatrix} \lambda_1 \\ \lambda_3 \end{bmatrix} = 0, A_{1,3} \ddot{q} + \dot{A}_{1,3} \dot{q} = 0$$

G: $G_{\emptyset,1} = \{(q, \dot{q}) \in \mathcal{D}_{\emptyset} : a_1(q) < 0\}$ → trading condition

$G_{\emptyset,2} = \{(q, \dot{q}) \in \mathcal{D}_{\emptyset} : a_2(q) < 0\}$

$G_{\emptyset,3} = \{(q, \dot{q}) \in \mathcal{D}_{\emptyset} : a_3(q) < 0\}$

$G_{1,2} = \{(q, \dot{q}) \in \mathcal{D}_1 : a_2(q) < 0\}$

$G_{1,\{1,2\}} = \{(q, \dot{q}) \in \mathcal{D}_1 : a_2(q) = 0\}$

$G_{1,\{1,3\}} = \{(q, \dot{q}) \in \mathcal{D}_1 : a_3(q) = 0\}$

$G_{2,1} = \{(q, \dot{q}) \in \mathcal{D}_2 : a_1(q) < 0\}$

$G_{3,\emptyset} = \{(q, \dot{q}) \in \mathcal{D}_3 : U_3(-\lambda) = 0, a_3(q) > 0\}$ lift off

$G_{3,\{1,3\}} = \{(q, \dot{q}) \in \mathcal{D}_3 : a_1 = 0\}$

$G_{\{1,2\},1} = \{(q, \dot{q}) \in \mathcal{D}_{12} : U_2(-\lambda) = 0, a_2(q) > 0\}$

$G_{\{1,3\},1} = \{(q, \dot{q}) \in \mathcal{D}_{13} : U_3(-\lambda) = 0, a_3(q) > 0\}$

R:

$R_{\emptyset,1} : (q, \dot{q}^-) \in \mathcal{D}_{\emptyset} \mapsto (q, \dot{q}^+) \in \mathcal{D}_1$

$R_{\emptyset,2} : (q, \dot{q}^-) \in \mathcal{D}_{\emptyset} \mapsto (q, \dot{q}^+) \in \mathcal{D}_2$

$R_{\emptyset,3} : (q, \dot{q}^-) \in \mathcal{D}_{\emptyset} \mapsto (q, \dot{q}^+) \in \mathcal{D}_3$

$R_{1,2} : (q, \dot{q}^-) \in \mathcal{D}_1 \mapsto (q, \dot{q}^+) \in \mathcal{D}_2$

$R_{1,\{1,2\}} : (q, \dot{q}^-) \in \mathcal{D}_1 \mapsto (q, \dot{q}^+) \in \mathcal{D}_{12}$

$R_{1,\{1,3\}} : (q, \dot{q}^-) \in \mathcal{D}_1 \mapsto (q, \dot{q}^+) \in \mathcal{D}_{13}$

$R_{2,1} : (q, \dot{q}^-) \in \mathcal{D}_2 \mapsto (q, \dot{q}^+) \in \mathcal{D}_1$

$R_{3,\emptyset} : (q, \dot{q}^-) \in \mathcal{D}_3 \mapsto (q, \dot{q}^+) \in \mathcal{D}_{\emptyset}$

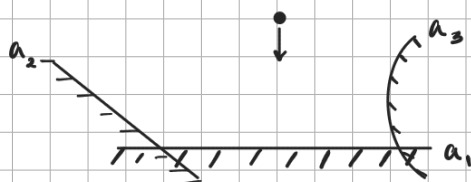
$R_{3,\{1,3\}} : (q, \dot{q}^-) \in \mathcal{D}_3 \mapsto (q, \dot{q}^+) \in \mathcal{D}_{13}$

$R_{\{1,2\},1} : (q, \dot{q}^-) \in \mathcal{D}_{12} \mapsto (q, \dot{q}^+) \in \mathcal{D}_1$

$R_{\{1,3\},1} : (q, \dot{q}^-) \in \mathcal{D}_{13} \mapsto (q, \dot{q}^+) \in \mathcal{D}_1$

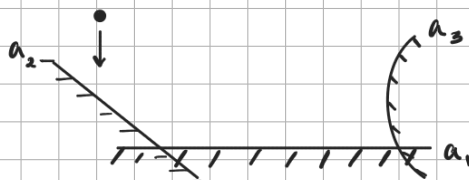
1.3) Results from simulation:

i) $q_0 = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$



Transitions: $\emptyset \rightarrow 1$ at $t = 1.0102s$

ii) $q_0 = \begin{bmatrix} -1.5 \\ 5 \end{bmatrix}$



Transitions: $\emptyset \rightarrow 2$ at $t = 0.9583s$
 $2 \rightarrow 1$ at $t = 1.0595s$
 $1 \rightarrow \{1,3\}$ at $t = 1.4447s$

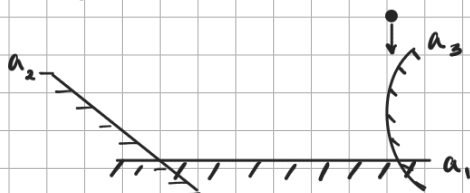
iii) $q_0 = \begin{bmatrix} 1.5 \\ 5 \end{bmatrix}$



Transitions: $\emptyset \rightarrow 3$ at $t = 0.7392s$
 $3 \rightarrow \emptyset$ at $t = 0.8920s$
 $\emptyset \rightarrow 1$ at $t = 1.3706s$
 $1 \rightarrow 2$ at $t = 1.7155s$
 $2 \rightarrow 1$ at $t = 2.2402s$
 $1 \rightarrow \{1,3\}$ at $t = 3.7959s$

(goes through $1 \rightarrow \{1,2\} \rightarrow 2$ in an instant)

iv) $q_0 = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$



Transitions: $\emptyset \rightarrow 3$ at $t = 0.7825s$
 $3 \rightarrow \{1,3\}$ at $t = 1.1073s$