Homework 2: Motion

24-760 Robot Dynamics & Analysis
Fall 2024

Name: _____

Note: For homework submission, please submit the PDF of the written portion to "Homework 2" and a zipped folder of your Matlab code to "Homework 2 Programming" in Gradescope.

Problem 1) Lunar Motion

Consider a simplified model of the motion of the earth and the moon. Attach a stationary coordinate frame to the center of the earth (s), a frame with the same origin that rotates with the earth (e), and a frame to the moon (m). The axes of rotation are all aligned with each other and pointing in the $+z$ direction of each frame. Assume the moon's orbit around the earth is circular with radius l_m. The earth's radius is r_e and the moon's radius is r_m. The moon rotates about the earth at a rate of 1 revolution per 28 days, and about it's own axis at a rate of 1 revolution per 28 days. The earth rotates about it's own axis at a rate of 1 revolution per day. *Hint: Draw a figure to keep track of the different frames.*

1.1) Just consider the earth's rotation to start. At time t, assume the earth is rotated so that the earth's $+x$ axis is aligned with the stationary $-y$. What is R_{se}? What is g_{se}? Use this configuration for the other parts of this question.

1.2) For a point q on the surface of the earth, $q_e = [0, r_e, 0]^T$, calculate the location of this point in the stationary frame using a rigid body transformation.

1.3) What is the body velocity of the earth's rotation, V^b_{se}? What is the spatial velocity V^s_{se}?

1.4) Using that body and spatial velocity, what is the instantaneous velocity of the point q_e in the earth's frame, v_{qe}? What is the velocity in the stationary frame, v_{qs}?

1.5) Now consider the position of the moon relative to the earth. Assume at time t that the moon is located at $[l_m, 0, 0]^T$ in the stationary frame, with the moon's x axis pointing to the earth. Calculate g_{sm}, then calculate g_{em} based on g_{sm} and g_{se}.

1.6) What is the body velocity of the moon's motion, V^b_{sm}?

1.7) Calculate the spatial velocity, V^s_{sm}, using an adjoint operation. What is special about v^s_{sm}?

1.8) Based on the rotation of the earth and the orbit of the moon, how long is a lunar day on earth? That is, from the earth's perspective, how long does the moon take to come back over the same spot on the surface of the earth?

Problem 2) Matlab

The Matlab Robotics System Toolbox provides many of the functions needed to calculate rigid body motions. In particular, there are useful tools for converting between different representations of angles and transformations:
https://www.mathworks.com/help/robotics/coordinate-system-transformations.html
https://www.mathworks.com/help/robotics/ug/coordinate-systems-in-robotics.html
However, it does not have all of the functions we want. Create functions for each of the following operations. You may then call and reuse these functions for other sections of this homework or in future homeworks.

2.1) In folder `studentDefinedFunctions`, complete the functions `angvel2skew(w)` and `skew2angvel(w)`.

`angvel2skew(w)` is a function that maps the 3-vector w to the 3x3 skew-symmetric matrix $\hat{w}$, and `skew2angvel(w)` is the inverse mapping. Write a unit test script that generates random w and checks that the composition of these functions is identity.

2.2) In folder `studentDefinedFunctions`, complete the functions `twist2rbvel(x)` and `rbvel2twist(x)`.

`twist2rbvel(x)` is the function that maps the 6-vector twist `V` to the 4x4 rigid body velocity matrix in homogeneous coordinates, $\hat{\mathtt{V}}$, and `rbvel2twist(x)` is the inverse mapping. Write a unit test script that generates random `V` and checks that the composition of these functions is identity.

2.3) In folder `studentDefinedFunctions`, complete the function `tform2adjoint(g)`.

`tform2adjoint(g)` is the function that maps the rigid body transformation g, in homogeneous coordinates, to the transformation adjoint matrix, Ad_g.

2.4) In folder `studentDefinedTests`, complete the function `compare_twist()`.

Generate a random transformation `g`. Compute the spatial velocity `V_s_hat` and the body velocity `V_b_hat` base on definition. Convert the spatial and body velocities in homogeneous coordinates into twists `V_s` and `V_b` with your function `rbvel2twist`.

2.5) In folder `studentDefinedTests`, complete the function `compare_twist()`.

Compute the conversion from twist `V_b` to twist `V_s_Ad_g` using `tform2adjoint`. Then compare `V_s` with `V_s_Ad_g` and make sure they are identical.

The function should return `g, V_s_Ad_g, V_s` and `V_b`.