```
syms l1 l2 t1 t2 t3 t4 p real
w1 = [0, 0, 1]'
```

```
w1 = 3×1
     0
     0
     1
```

```
p1 = [0, 0, 0]'
```

```
p1 = 3×1
     0
     0
     0
```

```
e1 = [- cross(w1, p1); w1]
```

```
e1 = 6×1
     0
     0
     0
     0
     0
     1
```

```


w2 = [0, 0, 1]';
p2 = [l1*cos(t1), l1*sin(t1), 0]'
```

p2 =

$$\begin{pmatrix} l_1 \cos(t_1) \\ l_1 \sin(t_1) \\ 0 \end{pmatrix}$$

```
e2 = [- cross(w2, p2); w2]
```

e2 =

$$\begin{pmatrix} l_1 \sin(t_1) \\ -l_1 \cos(t_1) \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

```


w3 = [0, 0, 1]'
```

```
w3 = 3×1
     0
     0
     1
```

```
p3 = [0, 0, 0]'
```

```
p3 = 3×1
     0
     0
     0
```

```
e3 = [- cross(w3, p3); w3]
```

```
e3 = 6×1
     0
     0
     0
     0
     0
     1
```

```
w4 = [0, 0, 1]'
```

```
w4 = 3×1
     0
     0
     1
```

```
p4 = [l1*cos(t3), l1*sin(t3), 0]'
```

p4 =

$$\begin{pmatrix} l_1 \cos(t_3) \\ l_1 \sin(t_3) \\ 0 \end{pmatrix}$$

```
e4 = [- cross(w4, p4); w4]
```

e4 =

$$\begin{pmatrix} l_1 \sin(t_3) \\ -l_1 \cos(t_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

```
Js1f1 = [e1, e2]
```

Js1f1 =

$$\begin{pmatrix} 0 & l_1 \sin(t_1) \\ 0 & -l_1 \cos(t_1) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}$$

```
Js2f2 = [e3, e4]
```

```
Js2f2 =
```

$$\begin{pmatrix} 0 & l_1 \sin(t_3) \\ 0 & -l_1 \cos(t_3) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}$$

```
gs1c1 = [sin(p), cos(p), 0, l1*cos(t1) + l2*cos(t1 + t2);...
    -cos(p), sin(p), 0, l1*sin(t1) + l2*sin(t1 + t2); 0, 0, 1, 0; 0, 0, 0, 1]
```

```
gs1c1 =
```

$$\begin{pmatrix} \sin(p) & \cos(p) & 0 & l_2 \cos(t_1 + t_2) + l_1 \cos(t_1) \\ -\cos(p) & \sin(p) & 0 & l_2 \sin(t_1 + t_2) + l_1 \sin(t_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
gs2c2 = [-sin(p), -cos(p), 0, l1*cos(t3) + l2*cos(t3 + t4);...
    cos(p), -sin(p), 0, l1*sin(t3) + l2*sin(t3 + t4); 0, 0, 1, 0; 0, 0, 0, 1]
```

```
gs2c2 =
```

$$\begin{pmatrix} -\sin(p) & -\cos(p) & 0 & l_2 \cos(t_3 + t_4) + l_1 \cos(t_3) \\ \cos(p) & -\sin(p) & 0 & l_2 \sin(t_3 + t_4) + l_1 \sin(t_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Adgs1c1 = simplify(tform2adjoint(inv(gs1c1)))
```

```
Adgs1c1 =
```

$$\begin{pmatrix} \sin(p) & -\cos(p) & 0 & 0 & 0 & -l_2 \cos(t_1 - p + t_2) - l_1 \cos(p \\ \cos(p) & \sin(p) & 0 & 0 & 0 & l_1 \sin(p - t_1) - l_2 \sin(t_1 - p \cdot \\ 0 & 0 & 1 & l_2 \sin(t_1 + t_2) + l_1 \sin(t_1) & -l_2 \cos(t_1 + t_2) - l_1 \cos(t_1) & 0 \\ 0 & 0 & 0 & \sin(p) & -\cos(p) & 0 \\ 0 & 0 & 0 & \cos(p) & \sin(p) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Adgs2c2 = simplify(tform2adjoint(inv(gs2c2)))
```

```
Adgs2c2 =
```

$$\begin{pmatrix} -\sin(p) & \cos(p) & 0 & 0 & 0 & l_2\cos(t_3-p+t_4)+l_1\cos(p \\ -\cos(p) & -\sin(p) & 0 & 0 & 0 & l_2\sin(t_3-p+t_4)-l_1\sin(p \\ 0 & 0 & 1 & l_2\sin(t_3+t_4)+l_1\sin(t_3) & -l_2\cos(t_3+t_4)-l_1\cos(t_3) & 0 \\ 0 & 0 & 0 & -\sin(p) & \cos(p) & 0 \\ 0 & 0 & 0 & -\cos(p) & -\sin(p) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Bc1 = [1 0; 0 1; 0 0; 0 0; 0 0; 0 0]
```

```
Bc1 = 6×2
     1     0
     0     1
     0     0
     0     0
     0     0
     0     0
```

```
Bc2 = [1 0; 0 1; 0 0; 0 0; 0 0; 0 0]
```

```
Bc2 = 6×2
     1     0
     0     1
     0     0
     0     0
     0     0
     0     0
```

```
Jh = [simplify(Bc1' * Adgs1c1 * Js1f1), simplify(Bc2' * Adgs2c2 * Js2f2)]
```

Jh =

$$\begin{pmatrix} -\sigma_4 - l_1\cos(p-t_1) & -\sigma_4 & \sigma_3 + l_1\cos(p-t_3) & \sigma_3 \\ l_1\sin(p-t_1) - \sigma_2 & -\sigma_2 & \sigma_1 - l_1\sin(p-t_3) & \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = l_2\sin(t_3 - p + t_4)$$

$$\sigma_2 = l_2\sin(t_1 - p + t_2)$$

$$\sigma_3 = l_2\cos(t_3 - p + t_4)$$

$$\sigma_4 = l_2\cos(t_1 - p + t_2)$$

```
t_int = simplify(null(Jh))
```

t_int =

$$\begin{pmatrix} \frac{l_2\,\sigma_1 + l_1\,\sigma_3}{l_1\,\sin(t_2)} & \frac{l_2\,\sigma_1}{l_1\,\sin(t_2)} \\ -\frac{{l_1}^2\,\sin(t_1 - t_3) + {l_2}^2\,\sigma_1 + l_1\,l_2\,\sigma_3 - l_1\,l_2\,\sigma_2}{l_1\,l_2\,\sin(t_2)} & -\frac{l_2\,\sigma_1 - l_1\,\sigma_2}{l_1\,\sin(t_2)} \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \sin(t_1 + t_2 - t_3 - t_4)$$

$$\sigma_2 = \sin(t_3 - t_1 + t_4)$$

$$\sigma_3 = \sin(t_1 + t_2 - t_3)$$