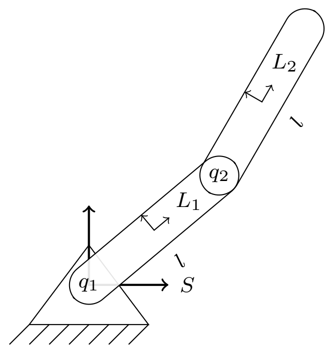
Homework 13: Trajectory Optimization

24-760 Robot Dynamics & Analysis
Fall 2024

Name: ________________________________

Submission: Submit the Matlab Drive folder link on Gradescope in Writeup

Problem 1) Two-link Robot



Consider a planar two link robot, as shown above. Each link has length $l = 1$, mass $m_l = 1$, and inertia $I_l = 1/12$. Each joint i of the robot is actuated with some torque τ_i. The gravity vector points in the $-y$ direction in the world frame. You may use code from prior homeworks to help with the calculations.

1.1) Write out in equations a direct collocation trajectory optimization problem for this system that finds a trajectory from an initial state $q(t_0) = [-\frac{\pi}{2}, 0]^T$ and $\dot{q}(t_0) = [0, 0]^T$ to final state $q(t_f) = [\frac{\pi}{2}, 0]^T$ and $\dot{q}(t_f) = [0, 0]^T$. Use fixed timesteps of 20 ms, a total duration of 1.5 s, and an objective function to minimize the thermal cost of actuation, given by $\int \tau^T \tau dt$. Use linear interpolation for control and acceleration dynamics, and quadratic interpolation for velocity and state trajectories. How many decision variables and how many constraints are there?

1.2) Now implement this problem in Matlab using `fmincon`. Write separate functions to handle the objective and constraints. Attached is a helper function that returns the M, C, N, and Υ matrices for a given q, $\dot{q}$, and τ, as well as a function that will animate the results. Note that `fmincon` may take several minutes to complete. We recommend using the following options (though you may need to change `MaxFunctionEvaluations`):

```
options = optimoptions('fmincon','Display', 'iter', 'MaxFunctionEvaluations', 1e5);
```

These tutorials may also help:

https://www.mathworks.com/help/optim/ug/fmincon.html

https://www.mathworks.com/help/optim/ug/nonlinear-equality-and-inequality-constraints.html

https://www.mathworks.com/help/optim/ug/example-nonlinear-constrained-minimization.html

1.3) Run the optimization for both the original problem and again with the added constraint that $q_i \in (-\frac{3\pi}{4}, \frac{3\pi}{4})$. How do the results change, in terms of trajectory and cost? Submit a figure of each trajectory using the animation script.