

Problem 1)

1.2) Transition information:

$\{ \} \rightarrow 1$ at $0.42857s$
 $1 \rightarrow 2$ at $0.5342s$
 $2 \rightarrow 1$ at $1.1759s$
 $1 \rightarrow 2$ at $1.5608s$
 $2 \rightarrow 1$ at $1.7917s$
 \vdots
 $2 \rightarrow 1$ at $2.138s$

The ball goes back and forth between modes 1 & 2 for a large number of times in a short period of time. The time between each transition reduces, and the ball finally comes to a stop at around $2.138s$. We can observe the zero effect in action here.

1.4)

$h = 0.04s$

Initial mode: $\{ \}$

Transitions:

$\{0,0\} \rightarrow \{1,0\}$ at $0.44s$
 $\{1,0\} \rightarrow \{1,1\}$ at $0.52s$
 $\{1,1\} \rightarrow \{0,1\}$ at $0.56s$
 $\{0,1\} \rightarrow \{1,0\}$ at $1.04s$
 $\{1,0\} \rightarrow \{1,1\}$ at $1.2s$
 Termination in mode $\{1,1\}$ at $t=3s$

$h = 0.02s$

Initial mode: $\{ \}$

Transitions:

$\{0,0\} \rightarrow \{1,0\}$ at $0.42s$
 $\{1,0\} \rightarrow \{0,1\}$ at $0.54s$
 $\{0,1\} \rightarrow \{1,1\}$ at $1.12s$
 $\{1,1\} \rightarrow \{1,0\}$ at $1.14s$
 $\{1,0\} \rightarrow \{0,1\}$ at $1.44s$
 $\{0,1\} \rightarrow \{1,1\}$ at $1.56s$
 Termination in mode $\{1,1\}$ at $t=3s$

$h = 0.01s$

Initial mode: $\{ \}$

Transitions:

$\{0,0\} \rightarrow \{1,0\}$ at $0.43s$
 $\{1,0\} \rightarrow \{1,1\}$ at $0.53s$
 $\{1,1\} \rightarrow \{0,1\}$ at $0.54s$
 $\{0,1\} \rightarrow \{1,0\}$ at $1.13s$
 $\{1,0\} \rightarrow \{0,1\}$ at $1.45s$
 $\{0,1\} \rightarrow \{1,0\}$ at $1.61s$
 $\{1,0\} \rightarrow \{1,1\}$ at $1.68s$
 $\{1,1\} \rightarrow \{0,1\}$ at $1.69s$
 $\{0,1\} \rightarrow \{1,1\}$ at $1.7s$
 Termination in mode $\{1,1\}$ at $t=3s$

In event-based simulation, every change in contact mode is accounted for, making it very accurate. However, as observed above, too many changes in the contact in a short amount of time can cause unwanted effects like the Zeno effect.

Time-stepping on the other hand ignores what happens between two time steps, and directly looks at the scenario at the end of each step.

So in cases where contact modes or changes in contact modes are fewer, event-based simulation might be the better choice owing to its accuracy. However when there are frequent changes in the contact modes, time-stepping might be the better choice as it is computationally cheaper.