Problem 1

Model 2

Problem 2

Option 4: $G = -y^*[x_1,x_2,1], h = -1$

Problem 3

Model 1 due to the presence of lesser support vectors

Problem 4

The RBF kernel classifies the data the best

Problem 5

One vs One (since number of classifiers would be the same, smaller datasets in one vs one classification would lead to lower training times)

Problem 6

4 points that lie outside the margin

M4-L1 Problem 1

In this problem, you will perform support vector classification on a linearly separable dataset. You will do so without using an SVM package

That is, you will be solving the large margin linear classifier optimization problem:

$$\min_{\mathbf{w},b} \frac{1}{2} ||\mathbf{w}||^2$$
subject to: $y_i(\mathbf{w}^T \mathbf{x}_i + b) \ge 1$

As described in lecture, you will convert the problem into a form compatible with the quadratic programming solver in the cvxopt package in Python:

$$\min \quad \frac{1}{2}x^T P x + q^T x$$

subject to: $Gx \le h; Ax = b$

Your job in this notebook is to define P, q, G, and h from above.

Please install the cvxopt package. (You can do that in the notebook directly with !pip install cvxopt) Then run the next cell to make the necessary imports.

```
In [17]: # Import modules
         import numpy as np
         import matplotlib.pyplot as plt
         from matplotlib.colors import ListedColormap
         from cvxopt import matrix, solvers
         solvers.options['show_progress'] = False
         def plot_boundary(x, y, w1, w2, b, e=0.1):
              x1min, x1max = min(x[:,0]), max(x[:,0])
              x2min, x2max = min(x[:,1]), max(x[:,1])
             xb = np.linspace(x1min,x1max)
             y_0 = 1/w2*(-b-w1*xb)
              y_1 = 1/w2*(1-b-w1*xb)
             y_m1 = 1/w2*(-1-b-w1*xb)
              cmap = ListedColormap(["purple", "orange"])
              plt.scatter(x[:,0],x[:,1],c=y,cmap=cmap)
             plt.plot(xb,y_0,'-',c='blue')
plt.plot(xb,y_1,'--',c='green')
              plt.plot(xb,y_m1,'--',c='green')
              plt.xlabel('$x_1$')
              plt.ylabel('$x 2$')
              plt.axis((x1min-e,x1max+e,x2min-e,x2max+e))
```

Load the data

Quadratic Programming

Create the P, q, G, and h matrices as described in the lecture:

- P (3x3): Identity matrix, but with 0 instead of 1 for the bias (third) row/column
- q (3x1): Vector of zeros
- G (Nx3): Negative y multiplied element-wise by [x1, x2, 1]
- h (Nx1): Vector of -1

Make sure the sizes of your matrices match the above. Use numpy arrays. These will be converted into cvxopt matrices later.

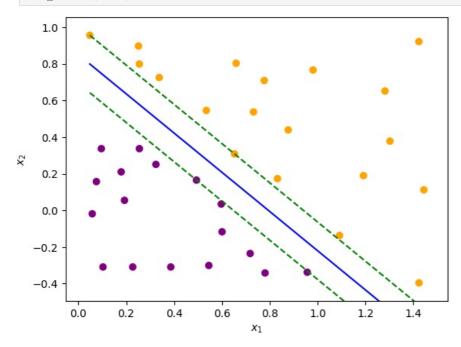
```
In [32]: # YOUR CODE GOES HERE
          # Define P, q, G, h
          P = np.array([[1.0, 0.0, 0.0], [0.0, 1.0, 0.0], [0.0, 0.0, 0.0]])
          q = np.array([0.0, 0.0, 0.0]).reshape(-1, 1)
          c1 = np.multiply(y, x1).reshape(-1, 1)
          c2 = np.multiply(y, x2).reshape(-1, 1)
          c3 = np.multiply(y, 1).reshape(-1, 1)
          G = np.hstack([c1, c2, c3])
          h = - np.ones([len(y), 1]).reshape(-1, 1) * 1.0
          print("P: ",P.shape)
print("q: ",q.shape)
print("G: ",G.shape)
          print("h: ",h.shape)
             (3, 3)
         q:
             (3, 1)
             (36, 3)
         G:
            (36, 1)
```

Using cvxopt for QP

Now we convert these arrays into cvxopt matrices and solve the quadratic programming problem. Then we get the weights w1, w2, and b and plot the decision boundary.

```
In [33]: z = solvers.qp(matrix(P),matrix(q),matrix(G),matrix(h))
w1 = z['x'][0]
w2 = z['x'][1]
b = z['x'][2]

plot_boundary(X, y, w1, w2, b)
```

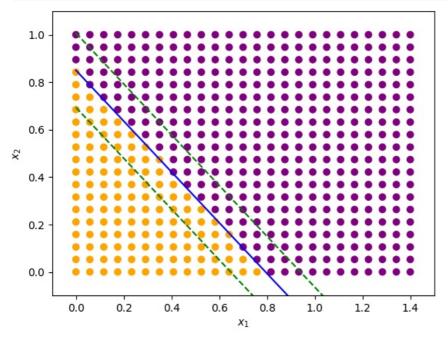


Using the SVM

Finally, we will generate a grid of (x1,x2) points and evaluate our support vector classifier on each of these points. Given the array X grid, determine y grid, the class of each point in X grid according to the support vector machine you trained.

```
In [37]: x1vals = np.linspace(0,1.4,25)
    x2vals = np.linspace(0,1,20)
    x1s, x2s = np.meshgrid(x1vals, x2vals)
    X_grid = np.vstack([x1s.flatten()],x2s.flatten()]).T
```

```
# YOUR CODE GOES HERE
# Get y_grid
y_grid = X_grid @ np.array([w1, w2]).reshape(2, 1) + b
y_grid = np.where(y_grid > 0, 1, -1)
plot_boundary(X_grid, y_grid, w1, w2, b)
```



Processing math: 100%

M4-L1 Problem 2

The UCI Machine Learning Repository (https://archive.ics.uci.edu/ml/index.php) contains hundreds of public datasets donated by researchers to test machine learning/statistical methods. Here we will look at a curated version of one of these datasets and try to perform classification using SVM.

Tsanas and Xifara, cited below, performed simulations of buildings using a program called Ecotect. They modified 8 building features, and measured energy efficiency with 2 metrics: heating load requirement and cooling load requirement. For the purpose of demonstration, we have truncated the dataset to only look at a subset of the data points and building attributes.

You will be training an SVM (with sklearn) to use "relative compactness" and "wall area" to classify whether "heating load" is high (>20) or low (<=20).

```
Dataset source:
A. Tsanas, A. Xifara: 'Accurate quantitative estimation of energy performance of residential buildings using statistical machine learning tools', Energy and Buildings, Vol. 49, pp. 560-567, 2012
```

Run the following cell to perform the necessary imports and load the data:

```
In [1]: import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        from sklearn.svm import SVC
        from matplotlib.colors import ListedColormap
        def plot data(x,y,e=0.1):
            x1min, x1max = min(x[:,0]), max(x[:,0])
            x2min, x2max = min(x[:,1]), max(x[:,1])
            xb = np.linspace(x1min,x1max)
            cmap = ListedColormap(["blue","red"])
            plt.scatter(x[:,0],x[:,1],c=y,cmap=cmap)
            plt.colorbar()
            plt.xlabel('$x_1$')
            plt.ylabel('$x 2$')
            plt.axis((x1min-e,x1max+e,x2min-e,x2max+e))
        def plot SV decision boundary(svm, extend=True):
            ax = plt.gca()
            xlim = ax.get xlim()
            ylim = ax.get_ylim()
            xrange = xlim[1] - xlim[0]
            yrange = ylim[1] - ylim[0]
            x = np.linspace(xlim[0] - extend*xrange, xlim[1] + extend*xrange, 100)
            y = np.linspace(ylim[0] - extend*yrange, ylim[1] + extend*yrange, 100)
            X,Y = np.meshgrid(x,y)
            xy = np.vstack([X.ravel(), Y.ravel()]).T
            P = svm.decision_function(xy)
            P = P.reshape(X.shape)
            ax.contour(X, Y, P, colors='k',levels=[0],linestyles=['-'])
            ax.contour(X, Y, P, colors='k',levels=[-1, 1], alpha=0.6,linestyles=['--'])
            plt.xlim(xlim)
            plt.ylim(ylim)
        relative compactness = np.array([0.98, 0.9, 0.86, 0.82, 0.79, 0.76, 0.74, 0.71, 0.69, 0.66, 0.64,
               0.621)
        wall_area = np.array([294. , 318.5, 294. , 318.5, 343. , 416.5, 245. , 269.5, 294. ,
               318.5, 343., 367.5])
        heating_load = np.array([24.58, 29.03, 26.28, 23.53, 35.56, 32.96, 10.36, 10.71, 11.11,
               11.68, 15.41, 12.96])
```

Train an SVM in sklearn

- Combine relative_compactness and wall_area into one 2-column input feature array
- Transform heating load into an array of classes with -1 where heating load entries are less than 20, and +1 otherwise.
- Create a Support Vector Classification model in sklearn. Make sure to use a "linear" kernel! Also set the argument "C" to a large number, like 1e5.
- · Fit the SVC to your data

```
In [24]: # YOUR CODE GOES HERE

X = np.vstack([relative_compactness, wall_area]).T
y = np.where(heating_load > 20, 1, -1)
X_scaled = np.vstack([relative_compactness * 20, wall_area / 20]).T
```

Plotting results

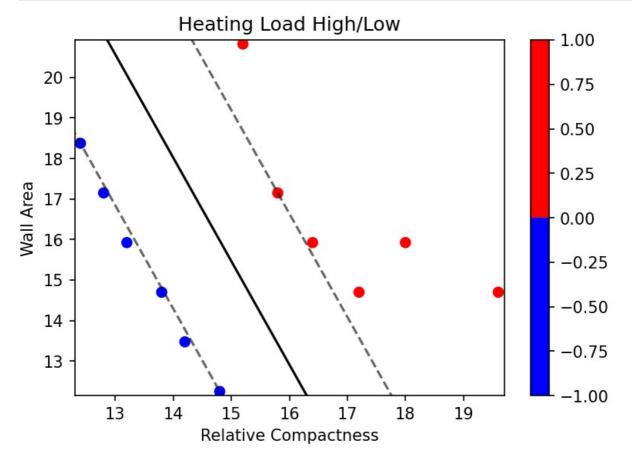
You can make predictions on any X data using the .predict(X) method of your SVC model. The .decision_function() method will return a continuous class evaluation, 0 at the boundary and 1 or -1 at the margin edges.

Now use the provided function <code>plot_SV_decision_boundary()</code> , which takes an sklearn model as its input, to plot the decision boundary.

```
In [27]: plt.figure(figsize=(6,4),dpi=150)
plot_data(X_scaled,y)

# YOUR CODE GOES HERE
model = SVC(kernel = 'linear')
model.fit(X_scaled, y)
# print(model.decision_function(X))
# print(model.predict(X))
plot_SV_decision_boundary(model)

plt.xlabel("Relative Compactness")
plt.ylabel("Wall Area")
plt.title("Heating Load High/Low")
plt.show()
```



M4-L1 Problem 3

In this problem you will use sklearn's support vector classification to study the effect of changing the parameter C, which represents inverse regularization strength.

Run the following cell to import libraries, define functions, and load data:

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from sklearn.svm import SVC
        from matplotlib.colors import ListedColormap
        # Plotting functions:
        def plot data(X,c,s=30):
            lims = [0,1]
            markers = [dict(marker="o", color="royalblue"), dict(marker="s", color="crimson"), dict(marker="^", color="
            x,y = X[:,0], X[:,1]
            iter = 0
            for i in np.unique(c):
                marker = markers[iter]
                iter += 1
                plt.scatter(x[c==i], y[c==i], s=s, **(marker), edgecolor="black", linewidths=0.4, label="y = " + str(i)
        def plot SVs(svm, s=120):
            sv = svm.support vectors
            x, y = sv[:,0], sv[:,1]
            plt.scatter(x, y, s=s, edgecolor="black", facecolor="none", linewidths=1.5)
        def plot SV decision boundary(svm, margin=True, extend=True, shade margins=False, shade decision=False):
            ax = plt.gca()
            xlim = ax.get_xlim()
           ylim = ax.get_ylim()
           xrange = xlim[1] - xlim[0]
           yrange = ylim[1] - ylim[0]
            x = np.linspace(xlim[0] - extend*xrange, xlim[1] + extend*xrange, 200)
            y = np.linspace(ylim[0] - extend*yrange, ylim[1] + extend*yrange, 200)
           X,Y = np.meshgrid(x,y)
            xy = np.vstack([X.ravel(), Y.ravel()]).T
            P = svm.decision function(xy)
            P = P.reshape(X.shape)
            ax.contour(X, Y, P, colors='k',levels=[0],linestyles=['-'])
            if margin:
                ax.contour(X, Y, P, colors='k',levels=[-1, 1], alpha=0.6,linestyles=['--'])
            if shade_margins:
                cmap = ListedColormap(["white","lightgreen"])
                plt.pcolormesh(X,Y,np.abs(P)<1,shading="nearest",cmap=cmap,zorder=-999999)</pre>
            if shade decision:
                cmap = ListedColormap(["lightblue","lightcoral"])
                pred = (svm.predict(xy).reshape(X.shape) == 1).astype(int)
                plt.pcolormesh(X,Y,pred,shading="nearest",cmap=cmap,zorder=-1000)
            plt.xlim(xlim)
            plt.ylim(ylim)
        def make_plot(title,svm_model,Xdata,ydata):
            plt.figure(figsize=(5,5))
            plot_data(Xdata,ydata)
            plot SVs(svm model)
            plot_SV_decision_boundary(svm_model,margin=True,shade_decision=True)
            plt.legend()
            plt.xlabel("$x_1$")
            plt.ylabel("$x_2$")
            plt.title(title)
            plt.show()
        # Dataset 1:
        x1 = np.array([0.48949729, 0.93403431, 0.77318605, 0.99708798, 0.7453347,
                                                                                        0.62782192, 0.88728846, 0.71619
               0.74459769, 0.75305792, 0.79103743, 0.63603483, 0.7035605 ,
                                                                                0.84037653, 0.47648924, 0.82480262, 0.0
               0.69268775,\ 0.74637666,\ 0.62823845,\ 0.92394124,\ 0.52824645,
                                                                                0.66571952, 0.5772065 , 0.8942154 , 0.8
               0.63920244, 0.75007448, 0.69128972, 0.9
                                                                                0.72136467, 0.70606728, 0.65459534, 0.0
```

```
0.660455 , 0.54130881, 0.99176949, 0.41660508, 0.61517452,
                                                                                                         0.76214 , 0.92212188, 0.90712313, 0.0
          0.26571114,\ 0.51712792,\ 0.17642698,\ 0.38630807,\ 0.27326383,
                                                                                                         0.4757757 , 0.43221499, 0.29701567, 0.2
          0.41828429, 0.55323218, 0.30897445, 0.51987077, 0.25015929,
                                                                                                         0.29285768, 0.06361631, 0.32100622, 0.4
          0.43747171, 0.41560485, 0.40850384, 0.53710681, 0.2458796 ,
                                                                                                         0.36389757, 0.34206599, 0.44241723, 0.4
          0.53785843, 0.56305326, 0.18442455, 0.4783044 , 0.341153 ,
                                                                                                        0.59226031, 0.34403529, 0.64020965, 0.5
          0.54259663, 0.36260852, 0.28089588, 0.28126787, 0.5046967,
                                                                                                        0.32032048, 0.25728685, 0.30410956, 0.1
          0.37573027, 0.43281125, 0.10385945, 0.45855828, 0.12496919,
                                                                                                        0.43889099, 0.30972969, 0.32992047, 0.4
x2 = np.array([0.82692832, 0.64782992, 0.51168806, 0.66255369, 0.80959079,
                                                                                                                    0.74825032, 0.62810149, 0.7752
          0.74030383, 0.76234673, 0.57673835, 0.76739864, 0.70551825,
                                                                                                        0.76417749, 0.68736246, 0.68255718, 0.0
          0.72477217,\ 0.81890284,\ 0.75486623,\ 0.57160741,\ 0.71961768,
                                                                                                         0.69643131, 0.78733278, 0.68253707, 0.
          0.6174821 \ , \ 0.69385581, \ 0.72352607, \ 0.57192729, \ 0.69906178,
                                                                                                         0.85159439, 0.65319918, 0.77788724, 0.
          0.81828425, 0.61449583, 0.54882155, 0.61557563, 0.76571808,
                                                                                                         0.63905784, 0.82482057, 0.71437531, 0.
          0.79516325, 0.71840235, 0.67254172, 0.58651416, 0.5778736 ,
                                                                                                        0.8128274 , 0.77131005, 0.83007228, 0.
                                                                                                        0.37373449, 0.33908254, 0.47230067, 0.4
          0.3216439 \ , \ 0.43068008, \ 0.48166151, \ 0.29743746, \ 0.45100559,
           \hbox{0.3776663 , 0.39820282, 0.43011064, 0.32873478, 0.35169937, } 
                                                                                                         0.25739568, 0.34931656, 0.2860302, 0.4
          0.26646292,\ 0.44178363,\ 0.28835415,\ 0.45468991,\ 0.19393014,
                                                                                                        0.42472115, 0.21083439, 0.3441914 , 0.1
          0.38262922, 0.36293124, 0.4006077, 0.34750469, 0.35023348,
                                                                                                        0.3905313 , 0.17185166, 0.44013747, 0.3
           0.40579986 \,, \; 0.23702401 \,, \; 0.38844385 \,, \; 0.29752652 \,, \; 0.18619147 \,, \\
                                                                                                        0.46662002, 0.33503445, 0.43295842, 0.4
          0.32186971, 0.37281822, 0.36488808, 0.37194919, 0.30829606,
                                                                                                        0.39365028, 0.48855396, 0.40258577, 0.4
-1, -1, -1, -1, -1, -1
          -1, -1, -1, -1, -1, -1, -1
           1, 1, 1, 1, 1, 1, 1
                                                                                                                     11)
X1 = np.vstack([x1,x2]).T
# Dataset 2:
z1 = np.array([0.4623709 , 0.68787981, 0.22665386, 0.42140211, 0.30510439,
                                                                                                                    0.53488987, 0.2040148, 0.39919
          0.58131992,\ 0.21989461,\ 0.41031163,\ 0.2825145\ ,\ 0.71079507, \\ 0.4301869\ ,\ 0.29867119,\ 0.35561876,\ 0.5967119,\ 0.35561876,\ 0.5967119,\ 0.35561876,\ 0.5967119,\ 0.35561876,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.5967119,\ 0.
          0.25007082, 0.40050165, 0.45727726, 0.45009186, 0.3127013,
                                                                                                        0.24118917, 0.37026561, 0.29343492, 0.3
                                                                                                         0.51600837, 0.26834863, 0.52915085, 0.4
           0.62142011, \ 0.24273132, \ 0.63236235, \ 0.39114511, \ 0.48803606, 
          0.51381933, 0.34398293, 0.61695795, 0.
                                                                                                        0.66135962, 0.68646158, 0.53325602, 0.4
          0.82291395,\ 0.6414185 , 0.54730993,\ 0.67858451,\ 0.53265047,
                                                                                                         0.49505561, 0.64200182, 0.36407551, 0.
          0.64641634, 0.41411608, 0.64992294, 0.60316402, 0.88008764,
                                                                                                        0.75418984, 0.4862578 , 0.66244808, 0.
z2 = np.array([0.83290004, 0.66234451, 0.65801115, 0.84029466, 0.70126933,
                                                                                                                    0.82112621, 0.83142114, 0.8078
          0.81111503, 0.69181695, 0.81230644, 0.68982279, 0.70037483,
                                                                                                        0.79716711, 0.85375938, 0.63633106, 0.0
          0.87396874, 0.63583241, 0.62337179, 0.71575062, 0.59439517,
                                                                                                         0.59527384, 0.57959709, 0.56120683, 0.
          0.81318113,\ 0.74471739,\ 0.76689873,\ 0.74142189,\ 0.58628648,
                                                                                                         0.58050036, 0.83946113, 0.51560503, 0.
          0.49047076, 0.61580307, 0.46660621, 0.41485462, 0.50601875,
                                                                                                         0.55752863, 0.53187983, 0.53825942, 0.5
          0.37757746\,,\; 0.47083258\,,\; 0.59490871\,,\; 0.4743862\,\;,\; 0.41337164\,,
                                                                                                        0.30688374, 0.48155856, 0.42810555, 0.0
          0.41406711, 0.58475545, 0.43525632, 0.61888062, 0.47842385,
                                                                                                        0.40661197, 0.71625865, 0.61275964, 0.4
           0.64427582 \,, \; 0.37797242 \,, \; 0.59767007 \,, \; 0.2815758 \;\;, \; 0.5679225 \;\;, \\
                                                                                                        0.35863786, 0.50579416, 0.3072999 , 0.0
-1, -1, -1, -1, -1, -1
                                                                                                                   1, 1, 1, 1, 1, 1, 1
X2 = np.vstack([z1,z2]).T
```

Linearly Separable Dataset

X1 and y1 are the features and classes for a linearly separable dataset. Train 4 SVC models on the data. Set kernel="linear", but use four different regularization values:

- C = 0.1
- C = 1
- C = 10
- C = 1000

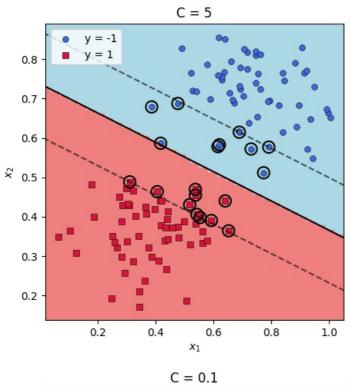
For each of these models, create a plot that shows the data, decision boundary, and support vectors, complete with a title that states the

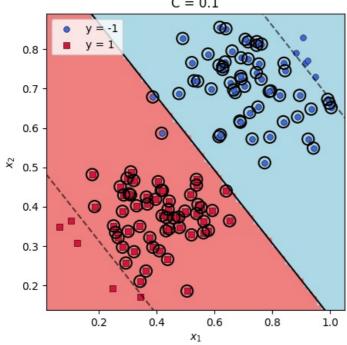
Use the provided function make plot(title,svm model,Xdata,ydata)

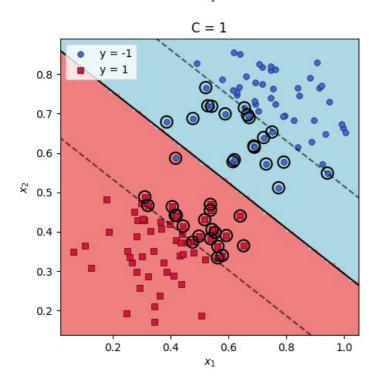
One example has been provided. Please repeat for all of the requested C values:

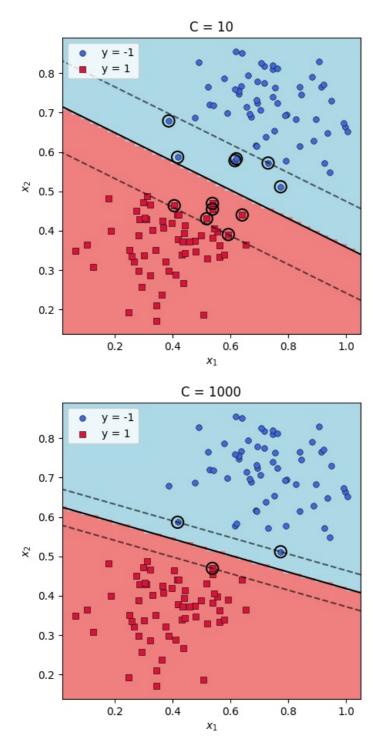
```
In [3]: C = 5
    svm = SVC(C=C,kernel="linear")
    svm.fit(X1,y1)
    make_plot(f"C = {C}",svm,X1,y1)

for C in [0.1,1,10,1000]:
    # YOUR CODE GOES HERE
    svm = SVC(C = C, kernel="linear")
    svm.fit(X1, y1)
    make_plot(f"C = {C}",svm,X1,y1)
```





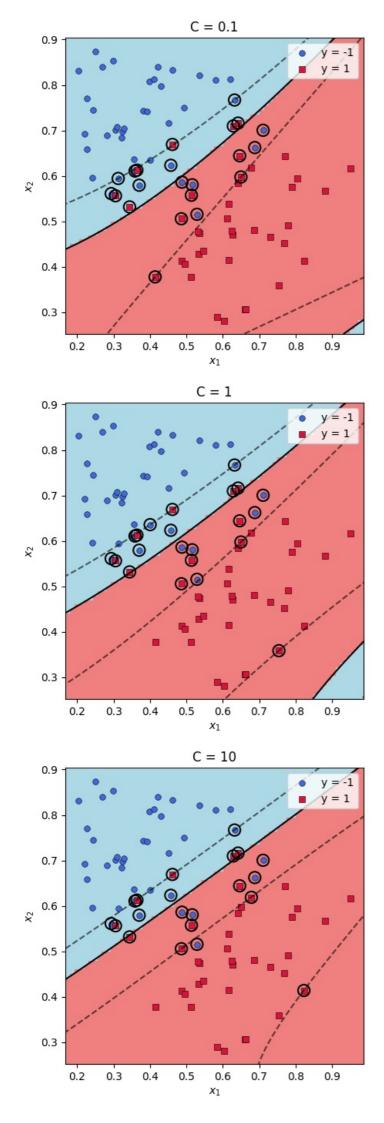


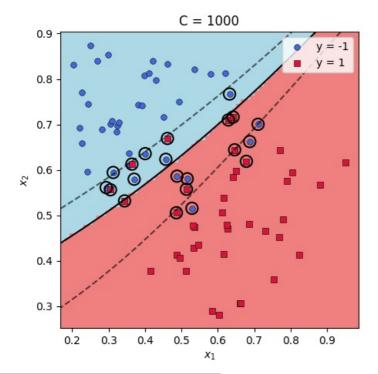


Linearly Non-Separable Dataset

Repeat the above for the linearly non-separable dataset (X2 and y2).

```
In [8]: # YOUR CODE GOES HERE
for C in [0.1,1,10,1000]:
    # YOUR CODE GOES HERE
    svm = SVC(C = C, kernel = "poly")
    svm.fit(X2, y2)
    make_plot(f"C = {C}",svm,X2,y2)
```





Loading [MathJax]/jax/output/CommonHTML/fonts/TeX/fontdata.js

M4-I 2 Problem 1

Now you will try support vector classification on data with nonlinear decision boundaries. You will use the sklearn SVC tool on four datasets. Your job is to find an appropriate choice of kernel and regularization strength that does a qualitatively good job separating the data

Run this cell first:

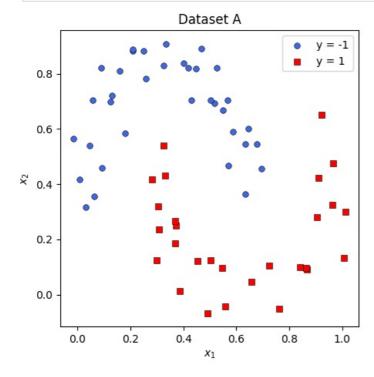
```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from sklearn.svm import SVC
        from matplotlib.colors import ListedColormap
        # Plotting functions:
        def plot_data(X,c,s=30):
            lims = [0,1]
            markers = [dict(marker="o", color="royalblue"), dict(marker="s", color="red"), dict(marker="^", color="limegents"),
            x,y = X[:,0], X[:,1]
            iter = 0
            for i in np.unique(c):
                marker = markers[iter]
                iter += 1
                plt.scatter(x[c==i], y[c==i], s=s, **(marker), edgecolor="black", linewidths=0.4, label="y = " + str(i)
        def plot_SVs(svm, s=120):
            sv = svm.support vectors
            x, y = sv[:,0], sv[:,1]
            plt.scatter(x, y, s=s, edgecolor="black", facecolor="none", linewidths=1.5)
        def plot SV decision boundary(svm, margin=True, extend=True, shade margins=False, shade decision=False):
            ax = plt.gca()
            xlim = ax.get_xlim()
            ylim = ax.get_ylim()
            xrange = xlim[1] - xlim[0]
            yrange = ylim[1] - ylim[0]
            x = np.linspace(xlim[0] - extend*xrange, xlim[1] + extend*xrange, 200)
            y = np.linspace(ylim[0] - extend*yrange, ylim[1] + extend*yrange, 200)
            X,Y = np.meshgrid(x,y)
            xy = np.vstack([X.ravel(), Y.ravel()]).T
            P = svm.decision_function(xy)
            P = P.reshape(X.shape)
            ax.contour(X, Y, P, colors='k',levels=[0],linestyles=['-'])
            if margin:
                ax.contour(X, Y, P, colors='k',levels=[-1, 1], alpha=0.6,linestyles=['--'])
            if shade margins:
                cmap = ListedColormap(["white","lightgreen"])
                plt.pcolormesh(X,Y,np.abs(P)<1,shading="nearest",cmap=cmap,zorder=-999999)
            if shade decision:
                cmap = ListedColormap(["lightblue","lightcoral"])
                pred = (svm.predict(xy).reshape(X.shape) == 1).astype(int)
                plt.pcolormesh(X,Y,pred,shading="nearest",cmap=cmap,zorder=-1000)
            plt.xlim(xlim)
            plt.ylim(ylim)
        def plot(Xdata, ydata, svm_model=None, title=""):
            plt.figure(figsize=(5,5))
            plot data(Xdata,ydata)
            if svm_model is not None:
                plot SVs(svm model)
                plot_SV_decision_boundary(svm_model,margin=True,shade_decision=True)
            plt.legend()
            plt.xlabel("$x_1$")
            plt.ylabel("$x_2$")
            plt.title(title)
            plt.show()
```

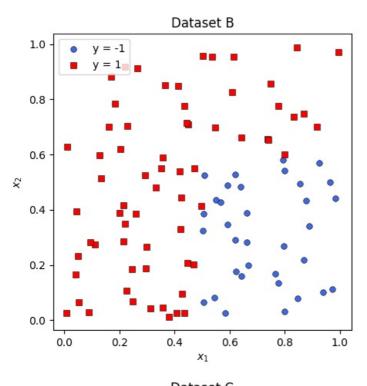
There are four datasets, all 2D and with X and y names as follows:

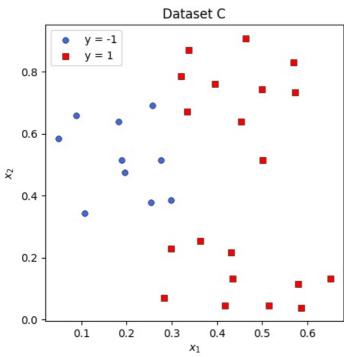
- Xa, ya
- Xb, yb
- Xc, yc
- Xd, yd

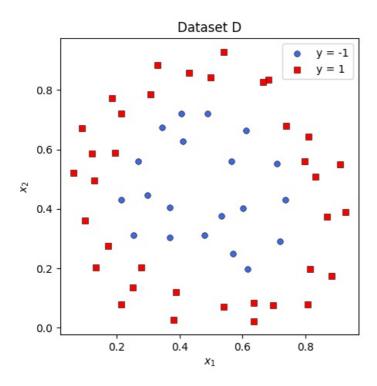
Run this cell to load and plot the data:

```
xa0 = np.array([0.00806452, 0.0467742, -0.0145161, 0.0564516, 0.130645, 0.0887097, 0.208065, 0.25, 0.324194, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130645, 0.130
 ya0 = np.array([0.418367, 0.540816, 0.565306, 0.704082, 0.720408, 0.822449, 0.883673, 0.883673, 0.830612, 0.7810
 xa1 = np.array([0.324194, 0.304839, 0.372581, 0.369355, 0.453226, 0.546774, 0.724194, 0.866129, 1.00806, 1.01129
 ya1 = np.array([0.540816, 0.320408, 0.25102, 0.185714, 0.120408, 0.0959184, 0.104082, 0.0918367, 0.132653, 0.3,
 Xa = np.concatenate([np.vstack([xa0,ya0]).T,np.vstack([xa1,ya1]).T],0)
 plot(Xa,ya,title="Dataset A")
 xb1 = np.array([0.43599, 0.54966, 0.42037, 0.20465, 0.29965, 0.62113, 0.13458, 0.18444, 0.85398, 0.84656, 0.50525, 0.42812]
 xb2 = np.array([0.025926, 0.43532, 0.33033, 0.61927, 0.26683, 0.52914, 0.51358, 0.78534, 0.49424, 0.079645, 0.065287, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.090888, 0.0908
 Xb = np.vstack([xb1,xb2]).T
 plot(Xb,yb,title="Dataset B")
 xc1 = np.array([0.05, 0.08871, 0.18226, 0.18871, 0.27581, 0.25323, 0.10806, 0.19516, 0.25645, 0.29839, 0.3371, 0.4629, 0.4983, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.2083, 0.208
 xc2 = np.array([0.58571, 0.65918, 0.63878, 0.51633, 0.51633, 0.37755, 0.3449, 0.47551, 0.69184, 0.38571, 0.87143, 0.90816, 0.47551, 0.69184, 0.38571, 0.87143, 0.90816, 0.47551, 0.47551, 0.69184, 0.38571, 0.87143, 0.90816, 0.47551, 0.47551, 0.47551, 0.69184, 0.38571, 0.87143, 0.90816, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47551, 0.47
 Xc = np.vstack([xc1,xc2]).T
 plot(Xc,yc,title="Dataset C")
 xd1 = np.array([0.062903, 0.08871, 0.18548, 0.33065, 0.54032, 0.68226, 0.81129, 0.91129, 0.92742, 0.88548, 0.80807, 0.6371, 0.88548, 0.88548, 0.88887, 0.88887, 0.88887, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.888888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.888888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.888888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.888888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.888888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.888888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.88888, 0.8888
 xd2 = np.array([0.52041, 0.67143, 0.77347, 0.88367, 0.92857, 0.83469, 0.64286, 0.54898, 0.3898, 0.17347, 0.079592, 0.022449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.02449, 0.079592, 0.00440, 0.079592, 0.00440, 0.079592, 0.00440, 0.079592, 0.00440, 0.079592, 0.00440, 0.079592, 0.00440, 0.079592, 0.00440, 0.079592, 0.00440, 0.079592, 0.00440, 0.079592, 0.00440, 0.079592, 0.00440, 0.079592, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.004400, 0.004400, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440, 0.00440,
 Xd = np.vstack([xd1,xd2]).T
 plot(Xd,yd,title="Dataset D")
```









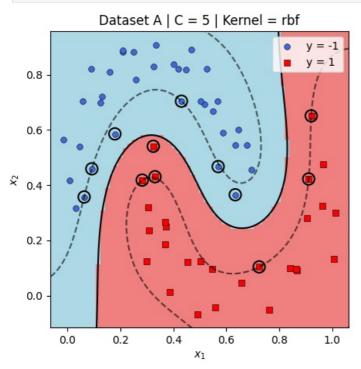
Using the Kernel Trick

Now, train four SVC models, one for each dataset. Try out different combinations of 'kernel' and 'C', until you find a satisfactory classifier in each case.

Please generate a plot for each dataset showing the results of a trained support vector classifier, using the provided function:

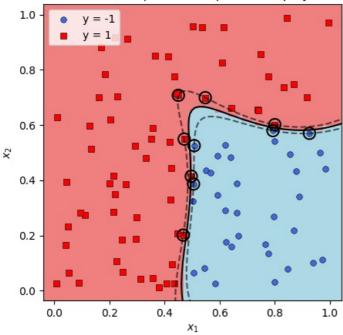
```
plot(Xdata, ydata, svm_model, title)
```

```
In [18]: # YOUR CODE GOES HERE
# (Dataset A)
svm_a = SVC(C = 5, kernel = 'rbf')
svm_a.fit(Xa, ya)
plot(Xa, ya, svm_a, "Dataset A | C = 5 | Kernel = rbf")
```



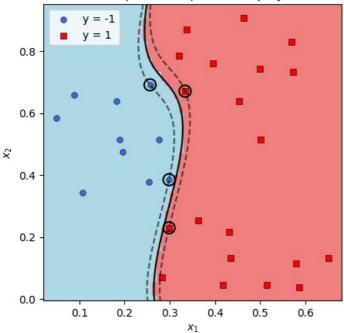
```
In [64]: # YOUR CODE GOES HERE
# (Dataset B)
svm_b = SVC(C = 1000, kernel = 'rbf')
svm_b.fit(Xb, yb)
plot(Xb, yb, svm_b, "Dataset B | C = 1000 | Kernel = poly")
```

Dataset B | C = 1000 | Kernel = poly

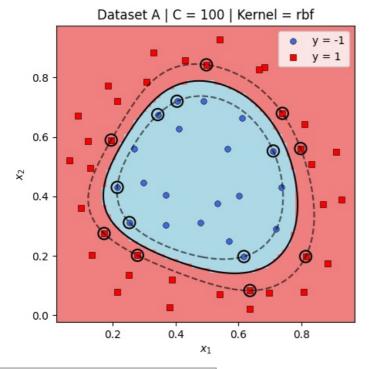


```
In [70]: # YOUR CODE GOES HERE
# (Dataset C)
svm_c = SVC(C = 100, kernel = 'poly', degree = 4)
svm_c.fit(Xc, yc)
plot(Xc, yc, svm_c, "Dataset C | C = 100 | Kernel = polynomial")
```

Dataset C | C = 100 | Kernel = polynomial



```
In [58]: # YOUR CODE GOES HERE
# (Dataset D)
svm_d = SVC(C = 100, kernel = 'rbf')
svm_d.fit(Xd, yd)
plot(Xd, yd, svm_d, "Dataset A | C = 100 | Kernel = rbf")
```



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M4-L2 Problem 2

Here we will revisit the phase diagram problem from the logistic regression module. Your task will be to code a one-vs-rest support vector classifier.

Work through this notebook, filling in code as requested, to implement the OvR classifier.

```
In [1]: import numpy as np
       import matplotlib.pyplot as plt
       from matplotlib.colors import ListedColormap
       from sklearn.svm import SVC
       x2 = np.array([0.11120957227224215, 0.1116933996874757, 0.14437480785146242, 0.11818202991034835, 0.0859507900573786]
       X = np.vstack([x1,x2]).T
       def plot_data(X, y, title="Phase of simulated material", newfig=True):
          xlim = [0,52.5]
          ylim = [0,1.05]
          markers = [dict(marker="o", color="royalblue"), dict(marker="s", color="crimson"), dict(marker="^", color="
          labels = ["Solid", "Liquid", "Vapor"]
          if newfig:
              plt.figure(dpi=150)
          for i in range(1+max(v)):
              plt.scatter(X[y==i,0], X[y==i,1], s=60, **(markers[i]), edgecolor="black", linewidths=0.4,label=labels[i]
          plt.title(title)
          plt.legend(loc="upper right")
          plt.xlim(xlim)
          plt.ylim(ylim)
          plt.xlabel("Temperature, K")
          plt.ylabel("Pressure, atm")
          plt.box(True)
       def plot_ovr_colors(classifiers, res=40):
          xlim = [0,52.5]
          ylim = [0,1.05]
          xvals = np.linspace(*xlim,res)
          yvals = np.linspace(*ylim,res)
          x,y = np.meshgrid(xvals,yvals)
          XY = np.concatenate((x.reshape(-1,1),y.reshape(-1,1)),axis=1)
          if type(classifiers) == list:
              color = classify_ovr(classifiers,XY).reshape(res,res)
          else:
              color = classifiers(XY).reshape(res,res)
          cmap = ListedColormap(["lightblue","lightcoral","palegreen"])
          plt.pcolor(x, y, color, shading="nearest", zorder=-1, cmap=cmap,vmin=0,vmax=2)
          return
```

Binomial classification function

You are given a function that performs binomial classification by using sklearn's SVC tool: prob = get ovr decision function(X, y, A, kernel, C)

To use it, input:

- X , an array in which each row contains (x,y) coordinates of data points
- \bullet $\;$ y , an array that specifies the class each point in $\;$ X $\;$ belongs to
- A , the class of the group (0, 1, or 2 in this problem) -- classifies into A or "rest"
- kernel, the kernel to use for the SVM
- C , the inverse regularization strength to use for the SVM

The function outputs a decision function (decision () in this case), which can be used to evaluate each X, giving positive values for class A, and negative values for [not A].

```
In [2]: def get_ovr_decision_function(X, y, A, kernel="linear",C=1000):
    y_new = -1 + 2*(y == A).astype(int)

model = SVC(kernel=kernel,C=C)
    model.fit(X, y_new)
```

```
def decision(X):
    pred = model.decision_function(X)
    return pred.flatten()

return decision
```

Coding an OvR classifier

Now you will create a one-vs-rest classifier to do multinomial classification. This will generate a binomial classifier for each class in the dataset, when compared against the rest of the classes. Then to predict the class of a new point, classify it using each of the binomial classifiers, and select the class whose binomial classifier decision function returns the highest value.

Complete the two functions we have started:

- generate_ovr_decision_functions(X, y) which returns a list of binary classifier probability functions for all possible classes (0, 1, and 2 in this problem)
- classify_ovr(decisions, X) which loops through a list of ovr classifiers and gets the decision function evaluation for each point in X. Then taking the highest decision function value for each, return the overall class predictions for each point.

```
In [4]: def generate_ovr_decision_functions(X, y, kernel="linear", C=1000):
    # YOUR CODE GOES HERE
    decision_0 = get_ovr_decision_function(X, y, 0, kernel, C)
    decision_1 = get_ovr_decision_function(X, y, 1, kernel, C)
    decision_2 = get_ovr_decision_function(X, y, 2, kernel, C)

    decisions = [decision_0, decision_1, decision_2]
    return decisions

def classify_ovr(decisions, X):
    # YOUR CODE GOES HERE

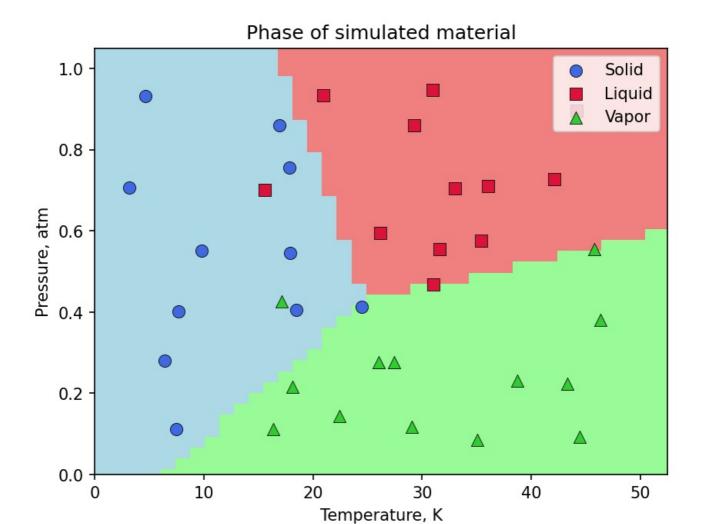
    preds0 = decisions[0](X)
    preds1 = decisions[1](X)
    preds2 = decisions[2](X)

    predictions = np.vstack((preds0, preds1, preds2))
    pred = np.argmax(predictions, axis=0)
```

Testing the classifier

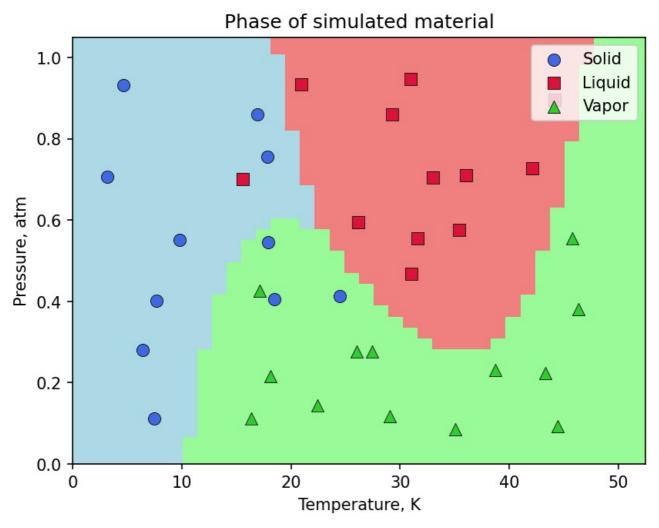
Plotting results

```
In [6]: plot_data(X,y)
    plot_ovr_colors(decisions)
    plt.show()
```



Modifying the SVC

Now go back and change the kernel and C value; observe how the results change.



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M4-L2 Problem 3

In this problem, we will investigate kernel selection and regularization strength in support vector regression for a 1-D problem.

Run each cell below, then try out the interactive plot to answer the questions.

```
In [1]: import numpy as np
                                   import matplotlib.pyplot as plt
                                   from sklearn.svm import SVR
                                   xs = np.array([0.094195, 0.10475, 0.12329, 0.12767, 0.1343, 0.11321, 0.16134, 0.16622, 0.15704, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.19564, 0.16892, 0.1707, 0.18892, 0.1707, 0.18892, 0.1707, 0.18892, 0.1707, 0.18892, 0.1707, 0.18892, 0.1707, 0.18892, 0.1707, 0.18892, 0.1707, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892, 0.18892,
                                   ys = np.array([0.51123, 0.50881, 0.50546, 0.50756, 0.51653, 0.50797, 0.49658, 0.50899, 0.50218, 0.50242, 0.50906, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50466, 0.50
                                   y = p_0.4756(0.46193, 0.47566, 0.48699, 0.49609, 0.50315, 0.50836, 0.51189, 0.51393, 0.51467, 0.51428, 0.51294, 0.510819, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51294, 0.51244, 0.51244, 0.51244, 0.512
In [2]: %matplotlib inline
                                   from ipywidgets import interact, interactive, fixed, interact manual, Layout, FloatSlider, Dropdown
                                   def plotting_function(kernel, log_C, log_epsilon):
                                                   C = np.power(10.,log_C)
                                                   epsilon = np.power(10.,log_epsilon)
                                                   model = SVR(kernel=kernel,C=C,epsilon=epsilon)
                                                   model.fit(xs.reshape(-1,1),ys)
                                                   xfit = np.linspace(0,1,200)
                                                  yfit = model.predict(xfit.reshape(-1,1))
                                                   plt.figure(figsize=(12,7))
                                                   plt.scatter(xs,ys,s=10,c="k",label="Data")
                                                   plt.plot(xfit,yfit,linewidth=3, label="SVR")
                                                   plt.plot(x_gt,y_gt,"--",label="Ground Truth")
                                                   title = f"Kernel: {kernel}, C = {C:.1e}, eps = {epsilon:.1e}"
                                                   plt.legend(loc="lower left")
                                                   plt.xlabel("$x 1$")
                                                   plt.ylabel("$y$")
                                                   plt.title(title)
                                                   plt.show()
                                   slider1 = FloatSlider(
                                                   value=0.
                                                   min=-5,
                                                   max=5,
                                                   step=.5,
                                                   description='C',
                                                  disabled=False,
                                                   continuous_update=True,
                                                   orientation='horizontal',
                                                   readout=False,
                                                   layout = Layout(width='550px')
                                   slider2 = FloatSlider(
                                                   value=-1,
                                                   min=-7.
                                                   max=-1,
                                                   step=.5.
                                                   description='epsilon',
                                                  disabled=False.
                                                   continuous update=True,
                                                   orientation='horizontal',
                                                   readout=False,
                                                   layout = Layout(width='550px')
                                   dropdown = Dropdown(
                                                   options=['linear', 'rbf', 'sigmoid'],
                                                   value='linear',
                                                   description='kernel',
                                                   disabled=False,
                                   interactive_plot = interactive(
                                                   plotting_function,
                                                   kernel = dropdown,
                                                   log_C = slider1,
```

```
log_epsilon = slider2
)
output = interactive_plot.children[-1]
output.layout.height = '500px'
interactive_plot
```

Out[2]: interactive(children=(Dropdown(description='kernel', options=('linear', 'rbf', 'sigmoid'), value='linear'), Fl...

Questions

- 1. Which kernel produced the best fit overall? (Assume this kernel for subsequent questions.)
- 2. As 'C' increases, does model performance on in-sample data generally improve or worsen?
- 3. As 'C' increases, does model performance on out-of-sample data (on the intervals [0.0, 0.1] and [0.9, 1.0]) generally improve or worsen?
- 4. What 'C' value would you recommend for this kernel?
- 5. What 'epsilon' value would you recommend?
- 1. rbf
- 2. As 'C' increases, model is fit very well to in-sample data, increasing its performance at those points
- 3. When 'C' increases beyond a certain level, due to overfitting to in-sample data, model performance on out-of-sample data tends to worsen
- 4. The model performs considerably well across a big range of the data, especially for out-of-sample data for C = 3.2. If performance in ranges close to in-sample data is more important, C = 10 might be a better choide.
- 5. eps = 3.2e-2

Loading [MathJax]/jax/output/CommonHTML/fonts/TeX/fontdata.js

Problem 1

Problem Description

As a lecture activity, you performed support vector classification on a linearly separable dataset by solving the quadratic programming optimization problem to create a large margin classifier.

Now, you will use a similar approach to create a soft margin classifier on a dataset that is not cleanly separable.

Fill out the notebook as instructed, making the requested plots and printing necessary values.

You are welcome to use any of the code provided in the lecture activities.

Summary of deliverables:

Functions (described later):

soft_margin_svm(X,y,C)

Results:

Print the values of w1, w2, and b for the C=0.05 case

Plots:

- Plot the data with the optimized margin and decision boundary for the case C=0.05
- · Make 4 such plots for the requested C values

Discussion:

• Respond to the prompt asked at the end of the notebook

Imports and Utility Functions:

```
In [1]: import numpy as np
         {\color{red} \textbf{import}} \ \texttt{matplotlib.pyplot} \ {\color{red} \textbf{as}} \ \texttt{plt}
         from matplotlib.colors import ListedColormap
         from cvxopt import matrix, solvers
         solvers.options['show_progress'] = False
         def plot_boundary(x, y, w1, w2, b, e=0.1):
              x1min, x1max = min(x[:,0]), max(x[:,0])
              x2min, x2max = min(x[:,1]), max(x[:,1])
              xb = np.linspace(x1min,x1max)
              y_0 = 1/w2*(-b-w1*xb)
              y_1 = 1/w2*(1-b-w1*xb)
              y m1 = 1/w2*(-1-b-w1*xb)
              cmap = ListedColormap(["purple","orange"])
              plt.scatter(x[:,0],x[:,1],c=y,cmap=cmap)
              plt.plot(xb,y_0,'-',c='blue')
plt.plot(xb,y_1,'--',c='green')
              plt.plot(xb,y_m1,'--',c='green')
              plt.xlabel('$x_1$')
              plt.ylabel('$x 2$')
              plt.axis((x1min-e,x1max+e,x2min-e,x2max+e))
```

Load data

Data is loaded as follows:

- X: input features, Nx2 array
- · y: output class, length N array

```
In [2]: data = np.load("data/w4-hw1-data.npy")
X = data[:, 0:2]
y = data[:, 2]
```

For soft-margin SVM, we introduce N slack variables ξ_i (one for each point), and reformulate the optimization problem as:

$$\min_{\mathbf{w},b} \frac{1}{2} ||\mathbf{w}||^2 + C \sum_i \xi_i$$

subject to: $y_i(\mathbf{w}^T \mathbf{x}_i + b) \ge 1 - \xi_i; \quad \xi_i \ge 0$

To put this into a form compatible with cvxopt, we will need to assemble large matrices as described in the next section.

Soft Margin SVM function

Define a function soft margin svm(X, y, C) with inputs:

- X: (Nx2) array of input features
- y : Length N array of output classes, -1 or 1
- C : Regularization parameter

In this function, do the following steps:

- 1. Create the P, q, G, and h arrays for this problem (each comprised of multiple sub-matrices you need to combine into one)
- P: (3+N) x (3+N)
 - Upper left: Identity matrix, but with 0 instead of 1 for the bias (third) row/column
 - Upper right (3xN): Zeros
 - Lower left (Nx3): Zeros
 - Lower right: (NxN): Zeros
- q: (3+N) x (1)
 - Top (3x1): Vector of zeros
 - Bottom (Nx1): Vector filled with 'C'
- G: (N+N) x (N+3):
 - Upper left (Nx3): Negative y multiplied element-wise by [x1 , x2 , 1]
 - Upper right (NxN): Negative identity matrix
 - Lower left (Nx3): Zeros
 - Lower right (NxN): Negative identity matrix
- h: (N+N) x (1)
 - Top: Vector of -1
 - Bottom: Vector of zeros

You can use np.block() to combine multiple submatrices into one.

- 2. Convert each of these into cvxopt matrices (Provided)
- 3. Solve the problem using cvxopt.solvers.qp (Provided)
- 4. Extract the w1, w2, and b values from the solution, and return them (Provided)

```
In [39]: def soft_margin_svm(X, y, C):
             N = np.shape(X)[0]
             # YOUR CODE GOES HERE
             # Define P, q, G, h
             P = np.zeros([3 + N, 3 + N])
             q = np.ones([3 + N, 1])
             G = np.zeros([N + N, 3 + N])
             h = np.zeros([N + N, 1])
             P[0:3, 0:3] = np.identity(3) * 1.0
             P[2, 2] = 0.0
             q[0:3] = np.zeros([3, 1])
             q[3:N + 3] = C
             G[0:N, 0] = - np.multiply(y, X[:, 0]) * 1.0
             G[0:N, 1] = - np.multiply(y, X[:, 1]) * 1.0
             G[0:N, 2] = - np.multiply(y, np.ones([N, 1]).T) * 1.0
             G[0:N, 3:N + 3] = - np.identity(N) * 1.0
             G[N:N + N, 0:3] = np.zeros([N, 3]) * 1.0
             G[N: N + N, 3:N + 3] = - np.identity(N) * 1.0
             h[0:N] = - np.ones([N, 1]) * 1.0
             z = solvers.qp(matrix(P), matrix(q), matrix(G), matrix(h))
             w1 = z['x'][0]
             w2 = z['x'][1]
```

```
b = z['x'][2]
return w1, w2, b
```

Demo: C = 0.05

Run the cell below to create the plot for the N = 0.05 case

```
In [40]: C = 0.05
          w1, w2, b = soft_margin_svm(X, y, C)
          print(f"\nSolution\n-----\nw1: \{w1:8.4f\}\nw2: \{w2:8.4f\}\n b: \{b:8.4f\}")
          plt.figure()
          plot_boundary(X,y,w1,w2,b,e=1)
         plt.title(f"C = {C}")
         plt.show()
        Solution
        w1: -0.2685
        w2: 0.1857
             1.1785
                                              C = 0.05
              7.5
              5.0
              2.5
              0.0
         XZ
            -2.5
            -5.0
            -7.5
           -10.0
                        -7.5
                                        -2.5
                                -5.0
                                                 0.0
                                                         2.5
                                                                  5.0
                                                                          7.5
                                                                                 10.0
                                                  x_1
```

Varying C

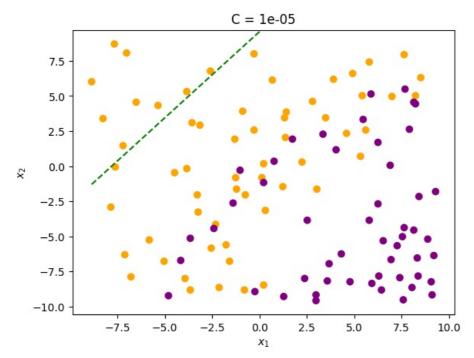
Now loop over the C values [1e-5, 1e-3, 1e-2, 1] and generate soft margin decision boundary plots like the one above for each case.

```
In [43]: # YOUR CODE GOES HERE
C = [1e-5, 1e-3, 1e-2, 1]

for i in C:
    w1, w2, b = soft_margin_svm(X, y, i)
    print(f"\nSolution\n----\nw1: {w1:8.4f}\nw2: {w2:8.4f}\n b: {b:8.4f}")

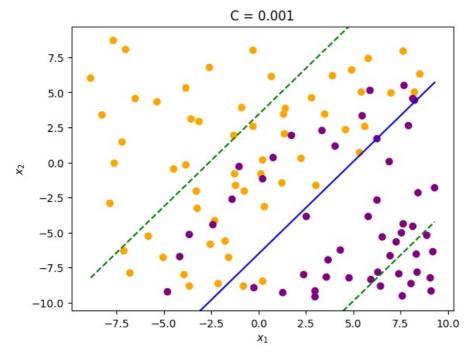
    plt.figure()
    plot_boundary(X,y,w1,w2,b,e=1)
    plt.title(f"C = {i}")
    plt.show()
Solution
```

w1: -0.0025 w2: 0.0020 b: 0.9807



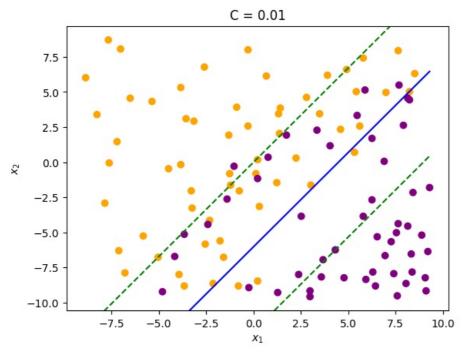
${\tt Solution}$

w1: -0.1323 w2: 0.1006 b: 0.6562



${\tt Solution}$

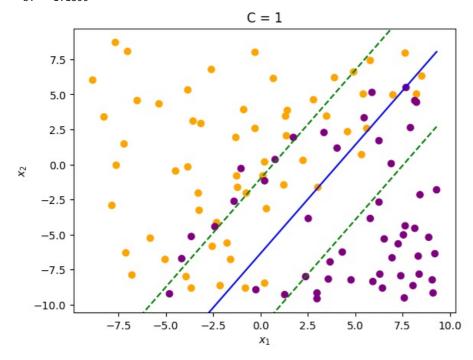
w1: -0.2231 w2: 0.1661 b: 1.0017



Solution

..... A

w1: -0.2899 w2: 0.1873 b: 1.1899



Discussion

Please write a sentence or two discussing what happens to the decision boundary and margin as you vary C, and try to provide some rationale for why.

As the value of C - the inverse regularization strength - increases, the strength of regularization applied on the model decreases, increasing the complexity of the model. As seen in the plots, for a very low value of C, the model complexity is sacrificed too much, causing the model to be fit very poorly to the data and the decision boundary being quite poor at describing the data. As C increases, the model complexity increases, meaning it is fit more well to the in-sample data and the decision boundary does a good job at splitting the classes. However, when C is increased too much, the model is overfit to do the class that has more data points due to high model

complexity.

Processing math: 100%

Problem 2

Problem Description

In this problem you will use sklearn.svm.SVC to classify thermal imaging data of a CPU die. We are interested in classifying points on the die as critical or non-critical, to inform where thermal paste should be applied to the die. The thermal imaging data is noisy, so your boss has asked you to develop a model that can produce a smoother profile of where the die is expected to be at, or above critical temperature.

The thermal imaging data is contained in cputemp.npy, where the first two columns correspond to the x and y position on the die, and the third column corresponds to the temperature at that point in degrees Celsius.

Fill out the notebook as instructed, making the requested plots and printing necessary values.

You are welcome to use any of the code provided in the lecture activities.

Summary of deliverables:

Functions:

accuracy(model, X, y)

Results:

• Print the accuracy of the two models requested on classifying the training set points as critical or non-critical temperature

Plots:

• Plot the decision boundary of each trained model with the provided plotting functions

Discussion:

. Compare the plots and accuracy of the two models, and reason which model is the better of the two

Imports and Utility Functions:

ax.set_ylabel('y')

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from matplotlib.colors import ListedColormap
        from sklearn.svm import SVC
        def plot svc decision function(model, ax=None):
             """Plot the decision function for a 2D SVC"""
            if ax is None:
                ax = plt.gca()
            xlim = ax.get_xlim()
            ylim = ax.get_ylim()
            # create grid to evaluate model
            x = np.linspace(xlim[0], xlim[1], 50)
            y = np.linspace(ylim[0], ylim[1], 50)
            Y, X = np.meshgrid(y, x)
            xy = np.vstack([X.ravel(), Y.ravel()]).T
            P = model.decision function(xy).reshape(X.shape)
            # plot decision boundary and margins
            ax.contour(X, Y, P, colors='k',
                        levels=[-1, 0, 1],
linestyles=['--', '-', '--'],
                        linewidths = [2,4,2])
            ax.set xlim(xlim)
            ax.set_ylim(ylim)
            plt.show()
        def plot_temp_profile(X, T, ax = None):
            if ax == None:
                ax = plt.gca()
            # Plot points colored by temperature
            sc = ax.scatter(X[:,0],X[:,1],c = T)
            # Add colorbar to plot
            cbar = plt.colorbar(sc)
            # Add labels
            cbar.set label('Temperature ($\degree C$)')
            ax.set xlabel('x')
```

```
plt.show()
 def plot_temp_critical(X, y, ax = None):
     if ax is None:
        ax = plt.gca()
         showflag = True
     else:
         showflag = False
     ax.scatter(X[:,0],X[:,1], c = y, cmap = ListedColormap(['blue', 'red']))
     ax.set_xlabel('x')
     ax.set_ylabel('y')
     ax.set_aspect(0.8)
     if showflag:
         plt.show()
     else:
         return ax
 def plot model(model, X, y):
     # Wrapper function to generate plot and decision boundary
     ax = plt.gca()
     ax = plot temp critical(X,y,ax)
     plot_svc_decision_function(model, ax)
<>:38: SyntaxWarning: invalid escape sequence '\d'
<>:38: SyntaxWarning: invalid escape sequence '\d'
C:\Users\barat\AppData\Local\Temp\ipykernel 17076\3633420468.py:38: SyntaxWarning: invalid escape sequence '\d'
 cbar.set_label('Temperature ($\degree C$)')
```

Load and visualize the data

Data is contained in cputemp.npy and can be loaded with np.load(). The first two columns of the file correspond to the x and y position on the die, and the third columns corresponds to the temperature at that position in degrees Celsius.

Store the data as:

- X (Nx2) array of position data
- T (Nx1) array of temperature data

Then visualize the data with plot_temp_profile(X,T)

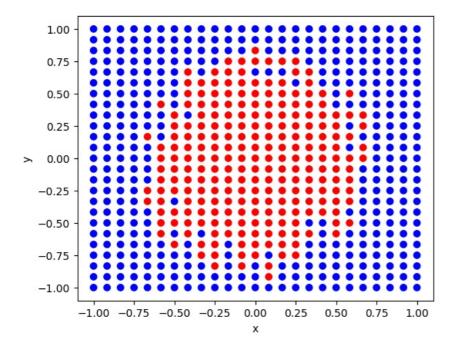
```
In [13]: # YOUR CODE GOES HERE
  data = np.load("data/cputemp.npy")
X = data[:, 0:2]
temp = data[:, 2]
```

Assign labels to data

Now we need to assign labels to the data for the support vector machine to be able to classify points as critical or non-critical. Generate a boolean vector y that is True for points at or above $180 \times C$, and False otherwise. Then use $plot_temp_critical(X,y)$ to plot the points on the die that are critical and non-critical.

```
In [18]: # YOUR CODE GOES HERE
y = np.where(temp > 180, 1, -1)

plot_temp_critical(X, y)
```



Train Support Vector Classifiers

Now you can train a SVC to classify the region on the die that you expect to be at or above the critical temperature. Using sklearn.svm.SVC train the following two models:

- RBF Kernel with C = 100
- 8th order polynomial Kernel with C = 100

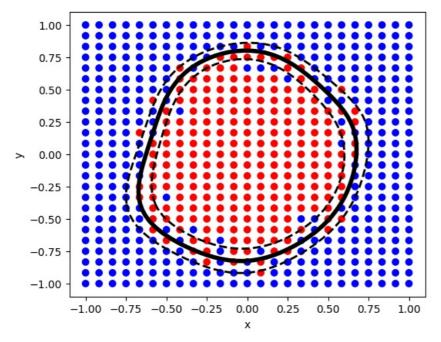
acc_poly = accuracy(model_poly, X, y)

print("Model accuracy with polynomial kernel, C = 100: ", acc poly)

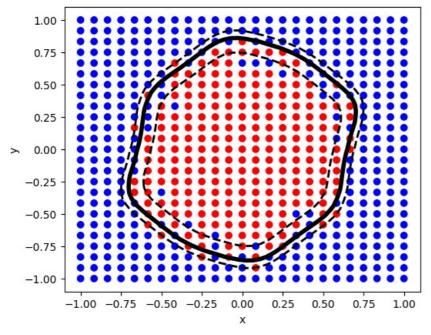
Write a function accuracy (model, X, y) that takes in the model, evaluates the points in X, and computes an accuracy between the predictions and ground truth labels in y. Accuracy is defined as the number of correctly classified points, divided by the total number of points. For a more in depth discussion of accuracy please see: Accuracy - Wikipedia. We will cover this topic more later in the course.

For each model, report the accuracy on the training data and use plot_model(model, X, y) to visualize the decision boundary.

```
In [21]: # YOUR CODE GOES HERE
         # Define accuracy function
         def accuracy(model, X, y):
             preds = model.predict(X)
             acc = (np.sum(preds == y) / len(y)) * 100
             return acc
In [26]: # YOUR CODE GOES HERE
         # Train and plot SVC models
         model rbf = SVC(C = 100, kernel = 'rbf')
         model_rbf.fit(X, y)
         plot_model(model_rbf, X, y)
         acc rbf = accuracy(model rbf, X, y)
         print("Model accuracy with RBF kernel, C = 100: ", acc_rbf)
         model_poly = SVC(C = 100, kernel = 'poly', degree = 8)
         model_poly.fit(X, y)
         plot model(model_poly, X, y)
```



Model accuracy with RBF kernel, C = 100: 93.28



Model accuracy with polynomial kernel, C = 100: 92.3200000000001

Discussion

Briefly discuss the performance of the two models, both with regard to their accuracy and the appearance of the decision boundary. Which model would you submit to your boss?

Both models have very similar accuracy levels for the given dataset (RBF = 93.28 and 8th order polynomial = 92.32). The RBF kernel, although quite smooth and radially symmetric, leaves quite a few failure points in or outside the margin. This could result in failures without the model being able to predict it. From the 2 models trained above, I would submit the polynomial function model to my boss. However, due to the nature of the problem and the kernels, an RBF kernel with a higher C value (less regularization strength) might do a better job at predicting failures.

Processing math: 100%

Problem 3

Problem Description

In this problem you will use sklearn.svm.SVR to train a support vector machine for a regression problem. Your model will predict G forces experienced by a sports car as it travels through a chicane in the Nurburgring.

Fill out the notebook as instructed, making the requested plots and printing necessary values.

You are welcome to use any of the code provided in the lecture activities.

Summary of deliverables:

Results:

- Plot the fitted SVR function for three different epsilon values
- Compute the R2 score for each of the fitted functions

Discussion:

· Discuss the performance of the models and the effect of epsilon

Imports and Utility Functions:

```
In [1]:
        import numpy as np
        import matplotlib.pyplot as plt
        from sklearn.svm import SVR
        def plot data(X, y, ax = None):
            if ax is None:
                ax = plt.gca()
                showflag = True
               showflag = False
            ax.scatter(X,y, c = 'blue')
            ax.set_xlabel('Normalized Position')
            ax.set_ylabel('G Force')
            if showflag:
                plt.show()
            else:
                return ax
        def plot_svr(model, X, y):
            ax = plt.gca()
            ax = plot_data(X, y, ax)
            xs = np.linspace(min(X), max(X), 1000).reshape(-1,1)
            ys = model.predict(xs)
            ax.plot(xs,ys,'r-')
            plt.legend(['Data', 'Fitted Function'])
            plt.show()
```

Load and visualize the data

The data is contained in nurburgring.npy and can be loaded with np.load(). The first column corresponds to the normalized position of the car in the chicane, and the second column corresponds to the measured G force experienced at that point in the chicane.

Store the data as:

- X (Nx1) array of position data
- y N-dimensional vector of G force data

Then visualize the data with plot data(X, y)

Note: use X.reshape(-1,1) to make the X array two dimensional as required by 'SVR.fit(X,y)'

```
In [6]: # YOUR CODE GOES HERE
  data = np.load("data/nurburgring.npy")
X = data[:, 0].reshape(-1, 1)
y = data[:, 1]
```

Train Support Vector Regressors

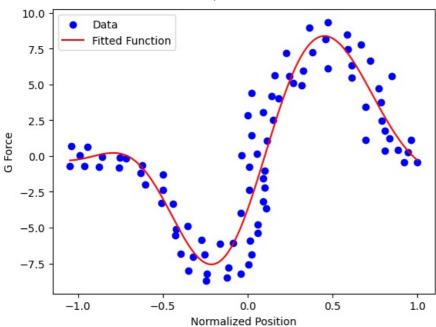
Train three different support vector regressors using the RBF Kernel, C = 100, and epsilon = [1, 5, 10]. For each model, report the coefficient of determination (R^2) for the fitted model using the builtin sklearn function} model.score(X,y), and plot the fitted function against the data using plot svr(model, X, y)

```
In [7]: # YOUR CODE GOES HERE
eps = [1, 5, 10]

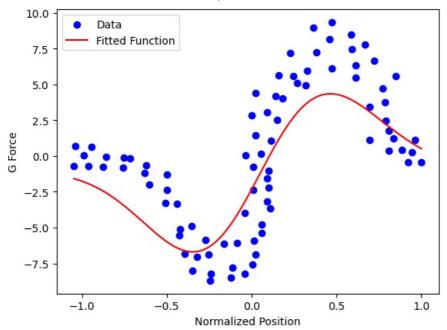
for i in eps:
    model = SVR(C = 100, kernel = 'rbf', epsilon = i)
    model.fit(X, y)
    coeff_d = model.score(X, y)

    print(" Coefficient of determination for epsilon = ", i, ": ", coeff_d)
    plot_svr(model, X, y)
```

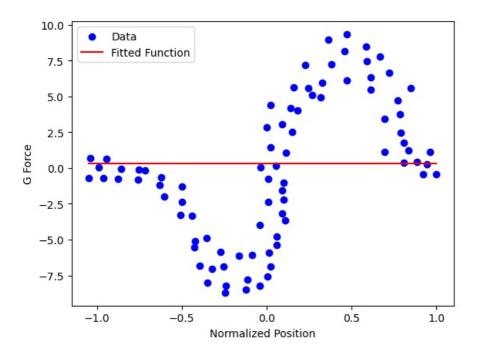
Coefficient of determination for epsilon = 1: 0.803896252951426



Coefficient of determination for epsilon = 5 : 0.6412302705136446



Coefficient of determination for epsilon = 10 : -0.005585141758953638



Discussion

Briefly discuss the performance of the three models, and explain how the value of epsilon influences the fitted model within the context of epsilon insenstive loss introduced in lecture.

 ϵ = 1 shows a good fit to the data, ϵ = 5 shows a moderately good fit and ϵ = 10 gives a poor fit to the sample data. Smaller epsilon values results in a tighter fit, so predictions are penalized more severely for deviations from the actual values. However, larger epsilon values allow for more deviations to go unpenalized, which can lead to underfitting. A small epsilon value causes the model to be more complex, that can fit the data more accurately but may also increase the risk of overfitting. A larger epsilon simplifies the model, but it may miss trends in the data.

Processing math: 100%

```
In [1]: import numpy as np
        import pandas as pd
        from sklearn.svm import SVC
        from sklearn.linear model import LogisticRegression
        from sklearn.pipeline import make pipeline
        from sklearn.preprocessing import StandardScaler
        X_data_train = pd.read_csv("bonus/train_input_data.txt", sep = ' ')
        y data train = pd.read csv("bonus/train output data.txt")
        X train = X data train.to numpy()
        y_train = y_data_train.to_numpy().flatten()
        y_train_svm = np.where(y_train > 0.5, 1, -1)
        X_data_test = pd.read_csv("bonus/test_input_data.txt", sep = ' ')
        y_data_test = pd.read_csv("bonus/test_output_data.txt")
        X_test = X_data_test.to_numpy()
        y_test = y_data_test.to_numpy().flatten()
        y_{test_svm} = np.where(y_{test} > 0.5, 1, -1)
In [2]: LR = make_pipeline(StandardScaler(), LogisticRegression(C = 100))
        LR.fit(X_train, y_train)
        LR_train_accuracy = LR.score(X_train, y_train) * 100
        print("\nAccuracy of Logistic Regression model for training data: ", LR_train_accur
        LR_test_accuracy = LR.score(X_test, y_test) * 100
        print("Accuracy of Logistic Regression model for test data: ", LR_test_accuracy)
       Accuracy of Logistic Regression model for training data: 87.98107897324607
       Accuracy of Logistic Regression model for test data: 88.09523809523809
In [ ]: | svm = make_pipeline(StandardScaler(), SVC(kernel = 'linear', gamma = 'scale'))
        svm.fit(X_train, y_train_svm)
        svm_train_accuracy = svm.score(X_train, y_train_svm) * 100
        print("\nAccuracy of SVM with linear kernel for training data: ", svm_train_accurac
        svm_test_accuracy = svm.score(X_test, y_test_svm) * 100
        print("Accuracy of SVM with linear kernel for test data: ", svm_test_accuracy)
        svm = make_pipeline(StandardScaler(), SVC(kernel = 'sigmoid', gamma = 'scale'))
        svm.fit(X_train, y_train_svm)
        svm train accuracy = svm.score(X train, y train svm) * 100
        print("\nAccuracy of SVM with sigmoid kernel for training data: ", svm_train_accura
        svm test accuracy = svm.score(X test, y test svm) * 100
        print("Accuracy of SVM with sigmoid kernel for test data: ", svm_test_accuracy)
        svm = make pipeline(StandardScaler(), SVC(kernel = 'poly', gamma = 'scale'))
        svm.fit(X_train, y_train_svm)
```

```
svm_train_accuracy = svm.score(X_train, y_train_svm) * 100
print("\nAccuracy of SVM with polynomial kernel for training data: ", svm_train_acc
svm_test_accuracy = svm.score(X_test, y_test_svm) * 100
print("Accuracy of SVM with polynomial kernel for test data: ", svm_test_accuracy)
svm = make_pipeline(StandardScaler(), SVC(kernel = 'rbf', gamma = 'scale'))
svm.fit(X_train, y_train_svm)
svm_train_accuracy = svm.score(X_train, y_train_svm) * 100
print("\nAccuracy of SVM with rbf kernel for training data: ", svm_train_accuracy)
svm_test_accuracy = svm.score(X_test, y_test_svm) * 100
print("Accuracy of SVM with rbf kernel for test data: ", svm_test_accuracy)
```

Accuracy of SVM with linear kernel for training data: 88.07432776101184 Accuracy of SVM with linear kernel for test data: 88.26312576312576