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1.0 – Filter Specifications

In short, these following specifications have to be determined:

1. Passband and stopband frequency range
2. Stopband attenuation and passband ripple
3. Transition band & filter order
4. Cutoff frequency

1.1 – Passband and Stopband Frequency Range

The difference between the original and noisy audio can be determined by plotting the time and frequency domain graph. The time-domain graph can be plotted by plotting the absolute values of the audio data (imported in MATLAB), whereas the frequency-domain graph can be attained by converting the finite time-domain sequence into a finite frequency-domain sequence using the discrete-time Fourier Transform function (DTFT). This is achieved using the fast Fourier transform (FFT) function in MATLAB, *fft(x)*. A time and frequency domain plot of both the noisy and original audio are shown:

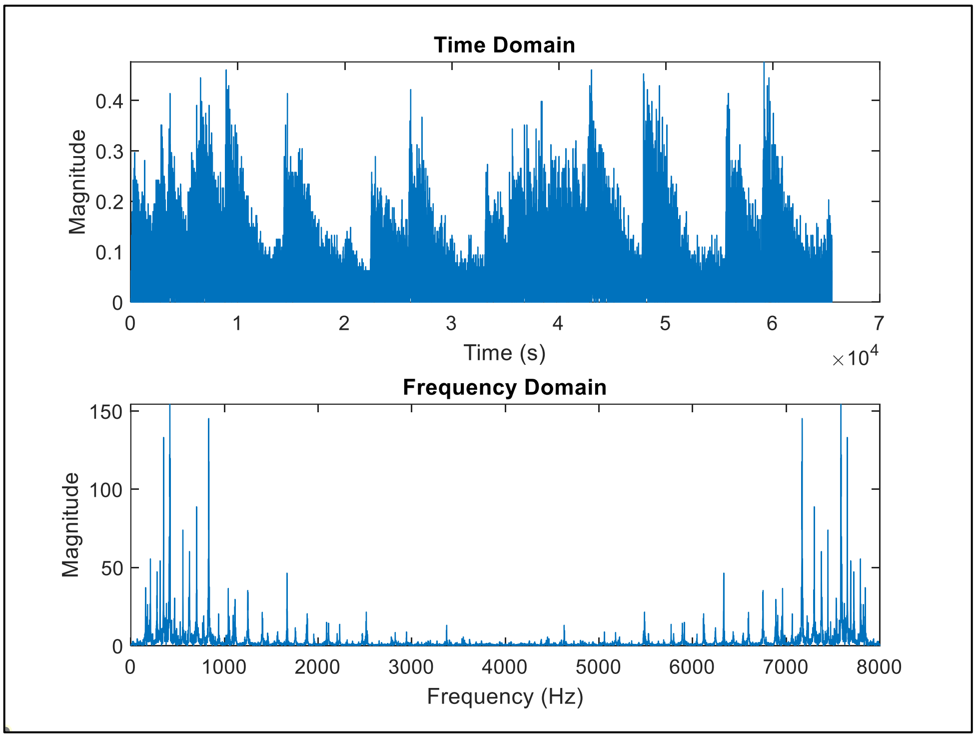

Figure 1.1.1 – Original Audio Time/Frequency Domain Plot

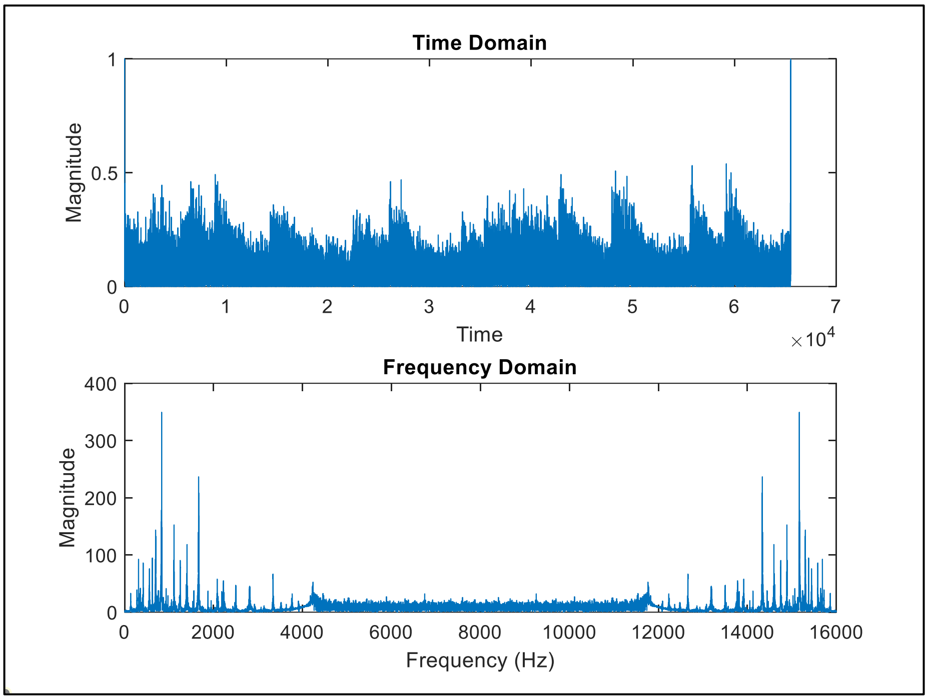

Figure 1.1.2 – Noisy Audio Time/Frequency Domain Plot

In the above plots, it is noticed that the original audio's time-domain plot magnitude is greater at average and peaks at a higher value many times than the noisy audio's time-domain plot magnitude and peak. This can be further verified when listening to both audios; the original audio is louder, clearer and has a good mix of low/high frequencies, whereas the noisy audio has static/hiss, was less louder compared to the original audio, and sounds even overall.

In terms of the frequency-domain plot, there is a noticeable "bump" in the noisy audio (between 4200Hz and 11800Hz) compared to the original audio. Since there is no such "bump" in the original audio, it is assumed that this frequency range contains the static/hiss in the noisy audio. Based on this assumption, the passband and stopband frequency ranges can be identified.

To get a clearer plot of the “bump,” a new plot can be graphed by differentiating the frequency-domain plot of the noisy audio with the original audio. An example is shown below:

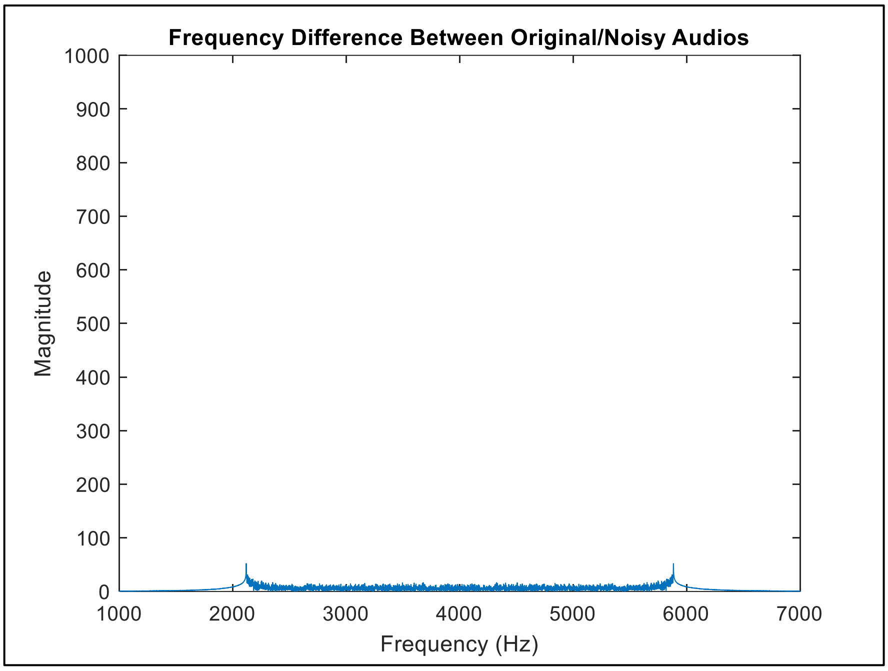

Figure 1.1.3 – Frequency Difference Between Original/Noisy Audio

From there, the stopband and passband (edge) frequencies can be obtained clearly:

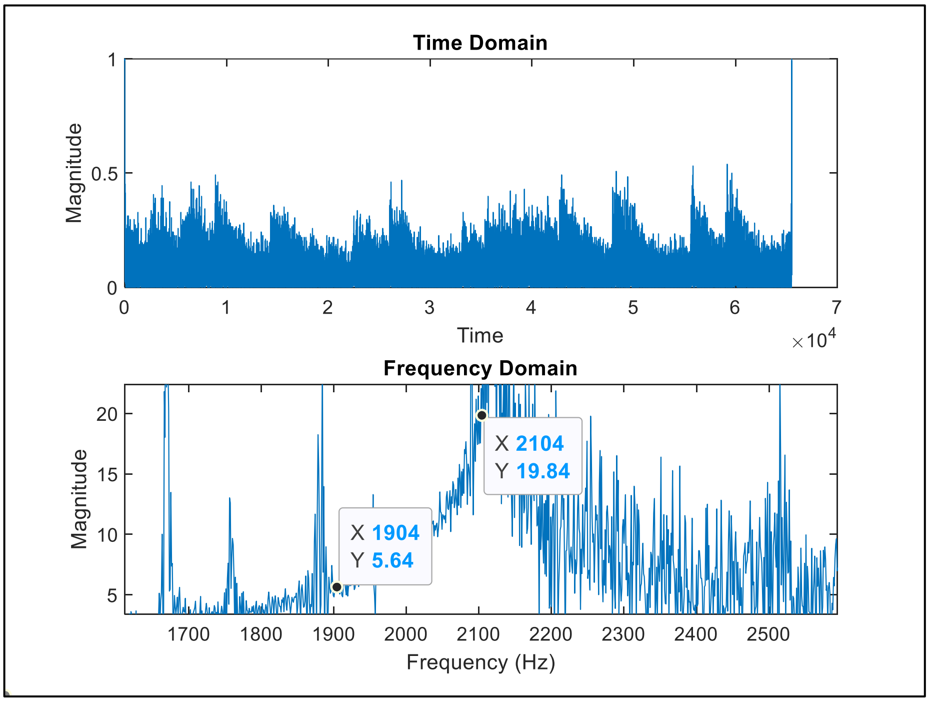

Figure 1.1.4 – Obtaining passband/stopband edge frequency

The passband frequency range is considered as the point where the filter will not attenuate the signal, whereas the stopband frequency range is considered as the point of signal attenuation by the filter. Based on the above figure, the frequency ranges were recorded as:

1. **Passband frequency range: 0 Hz< x < 1904 Hz (edge frequency)**
2. **Stopband frequency range: 2104 Hz (edge frequency) < x < 5918 Hz**

1.2 – Stopband Attenuation and Passband Ripple

The passband ripple was assumed to be **3dB** in order to reduce the complexities involved in identifying the filter specifications, and to allow large but fixed ripple amplitudes in the passband region instead of having a monotonic or unequal ripple filter. This can help keep the FIR filter length short, which in turn reduces computational time. In addition, the IIR filters require a whole number for the passband ripple input before performing the computations.

The stopband attenuation will be taken from any one of the four filters (Rectangular Window/Hanning/Hamming/Blackman) that starts attenuating closest to the cutoff frequency. In this way, the best filter can be chosen without any restrictions (such as selecting the next highest stopband attenuation value that correlates to a certain type of [inferior] filter, provided a value has already been assumed).

1.3 – Transition Band and Filter Order

The transition band calculation is shown below:

$$\Delta f = \frac{|f_{stop} - f_{pass}|}{f_s}$$

$$\Delta f = \frac{2104 - 1904}{8000} = 0.025$$

The filter orders for all four FIR filters windows are calculated below:

1.3.1 – Rectangular Window (FIR)

$$N = \frac{0.9}{\Delta f} = \frac{0.9}{0.025} = 36 \cong 37$$

The smallest odd number that is larger than 36 is 37, therefore **N = 37.**

1.3.2 – Hanning Window (FIR)

$$N = \frac{3.1}{\Delta f} = \frac{3.1}{0.025} = 124 \cong 125$$

The smallest odd number that is larger than 124 is 125, therefore **N = 125.**

1.3.3 – Hamming Window (FIR)

$$N = \frac{3.3}{\Delta f} = \frac{3.3}{0.025} = 132 \cong 133$$

The smallest odd number that is larger than 132 is 133, therefore **N = 133.**

1.3.4 – Blackman Window (FIR)

$$N = \frac{5.5}{\Delta f} = \frac{5.5}{0.025} = 220 \cong 221$$

The smallest odd number that is larger than 220 is 221, therefore **N = 221.**

The filter order for the IIR filter is shown below:

1.3.5 – Chebyshev Type I Filter (IIR)

Passband ripple A_p: 3db
Stopband attenuation A_s: 45db
Stopband frequency f_s: 2104
Passband frequency f_p: 1904

Number of orders in Chebyshev type I filter:

$$k = \frac{f_s}{f_p} = \frac{2104}{1904} = 1.105$$

$$A_1 = \sqrt{\frac{10^{0.1A_s} - 1}{10^{0.1A_p} - 1}} = \sqrt{\frac{10^{(0.1)(45)} - 1}{10^{(0.1)(3)} - 1}} = 178.248$$

$$n \geq \frac{\cosh^{-1} A_1}{\cosh^{-1} k}$$

$$n \geq \frac{\cosh^{-1}(178.248)}{\cosh^{-1}(1.105)} = 12.93$$

Thus, the order of selected filter is 13.

1.3.6 – Butterworth Filter (IIR)

MATLAB can calculate the order number directly using the *buttord* function. It needs the passband/stopband edge frequencies, along with the passband ripple and stopband attenuation value. Due to that, no manual calculation will be shown for this type of filter.

1.4 – Cutoff Frequency

The cutoff frequency calculation is done below:

$$f_c = \frac{f_{pass} + f_{stop}}{2}$$

$$f_c = \frac{1904 + 2104}{2} = 2004\ HZ$$

The cutoff frequency is **2004 Hz**.

2.0 – FIR Filter Design

Since a stopband attenuation value was not assumed during the filter specifications, all the filters will be used to filter the noisy audio in the initial stages. The filter that attenuates closest to the cutoff frequency will be selected as the best performing filter.

A frequency spectrum of all the four filters in action is shown below:

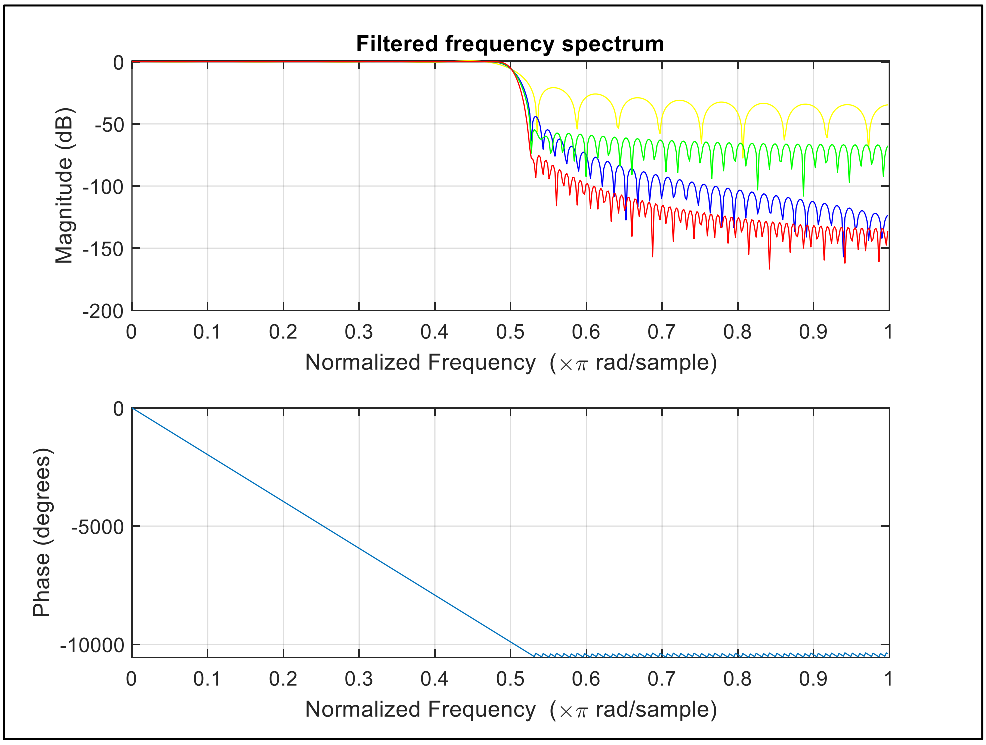

*Figure 2.1 – Frequency spectrum of filtered data (**Yellow**: Rectangular window, **Blue**: Hanning, **Green**: Hamming, **Red**: Blackman)*

When the normalized cutoff frequency is 0.501, a closer look at the transition region of all the four filters are shown below:

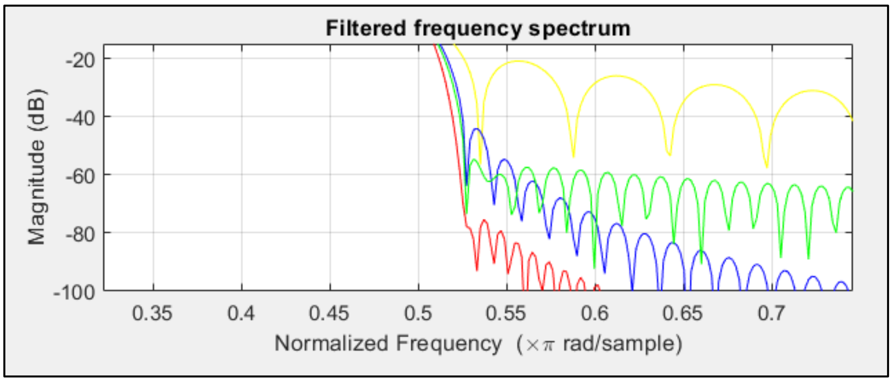

Figure 2.2 – Zoomed-in frequency spectrum of all four filters

Both the Blackman & Rectangular windows attenuate the furthest from the cutoff frequency. Zooming in even more towards the Hanning and Hamming filters show:

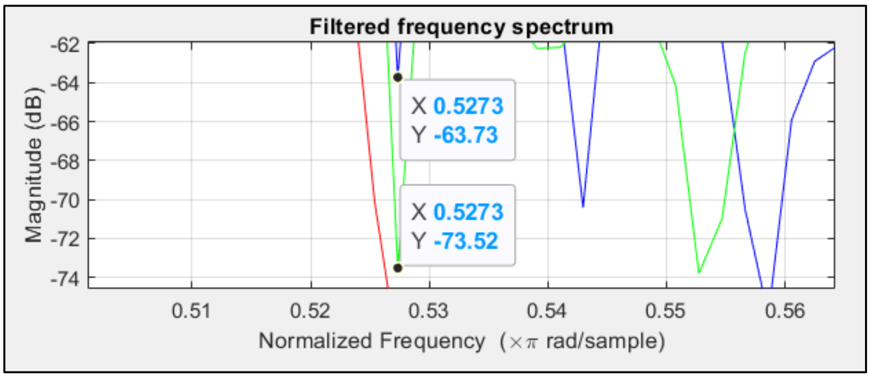

Figure 2.3 – Zoomed-in frequency spectrum of Hanning & Hamming filters

It is noticed that both the Hanning and Hamming filters are equally closer to the cutoff frequency. For the sake of this assignment, the **Hanning window** will be selected as the FIR filter of choice, even though the only difference between both filters is in how the main/nearest/other side lobes are attenuated (Hamming performs better than Hanning in cancelling the nearest side lobe, although it performs poorly in cancelling the rest).

From the Hanning filter, the stopband attenuation was assumed to be **-39.89dB (approximated to 40dB).**

3.0 – IIR Filter Design

For the IIR filter, both the Butterworth and Chebyshev filter will be tested. Based on the design in MATLAB, the frequency responses of both filters are:

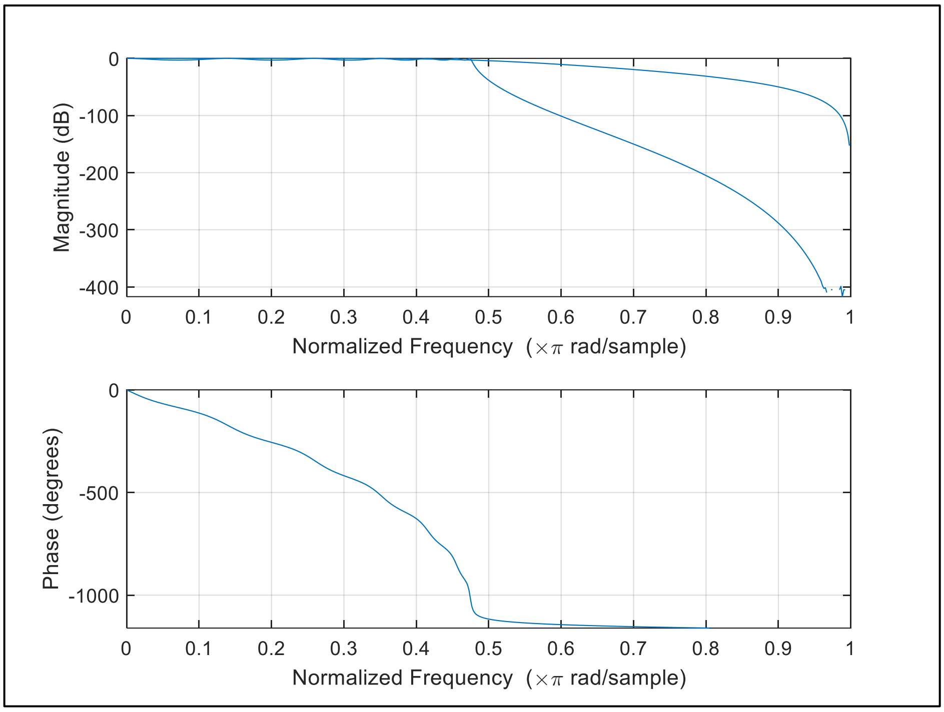

Figure 3.1 – Frequency response of Butterworth/Chebyshev filters

The properties of the responses match the characteristics of a typical Butterworth/Chebyshev filter. For a Butterworth filter, the passband region is maximally flat with an extremely smooth roll-off towards 1 at the expense of not attenuating as fast as possible. For the Chebyshev filter, the signal is attenuated immediately and reaches 1 quickly at the expense of having passband ripples.

When the Butterworth filtered audio is played, the static/hiss noise can still be heard. This is due to the filter prioritizing a smoother roll-off curve towards the stopband region instead of immediately attenuating towards the stopband region. This is further justified in the frequency-domain graph:

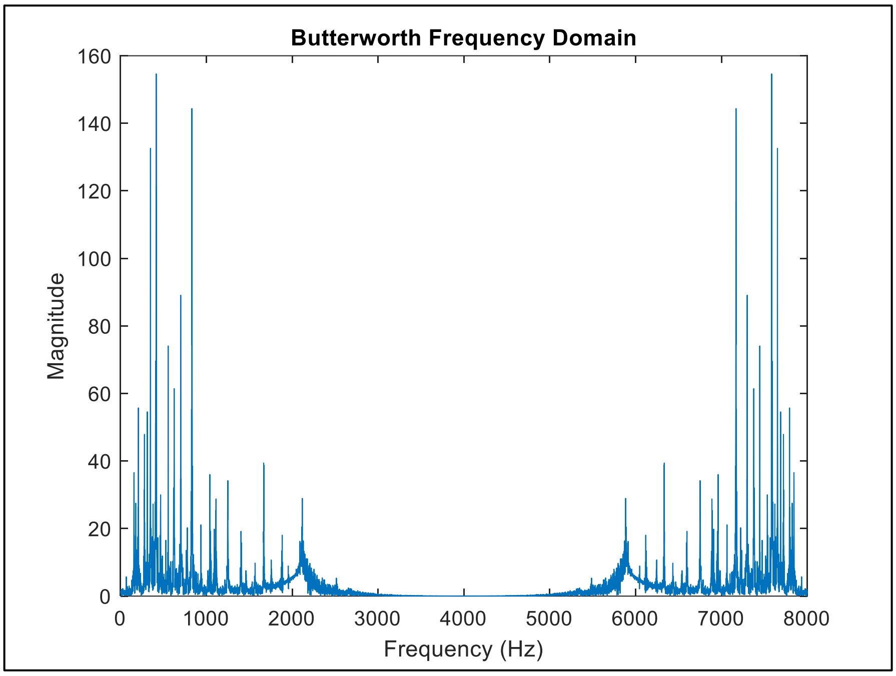

Figure 3.2 – Butterworth filtered audio's frequency-domain plot

As seen in the above plot, the signal attenuation happens gradually instead of immediately near the starting and ending point of the stopband frequency range (2104 Hz – 5918 Hz). Since the entire stopband frequency range is not being attenuated, this leads to some of the static/hiss being present in the filtered audio. In the end, this also means that the Butterworth filter is not the right type of filter to be used.

Because of the above, the selected IIR filter will be the **Chebyshev filter.** When the filtered audio is played, the noise is mostly eliminated due to the immediate attenuation towards the stopband region.

4.0 – Filtered Signal Analysis

4.1 – FIR Filter (Hanning Window)

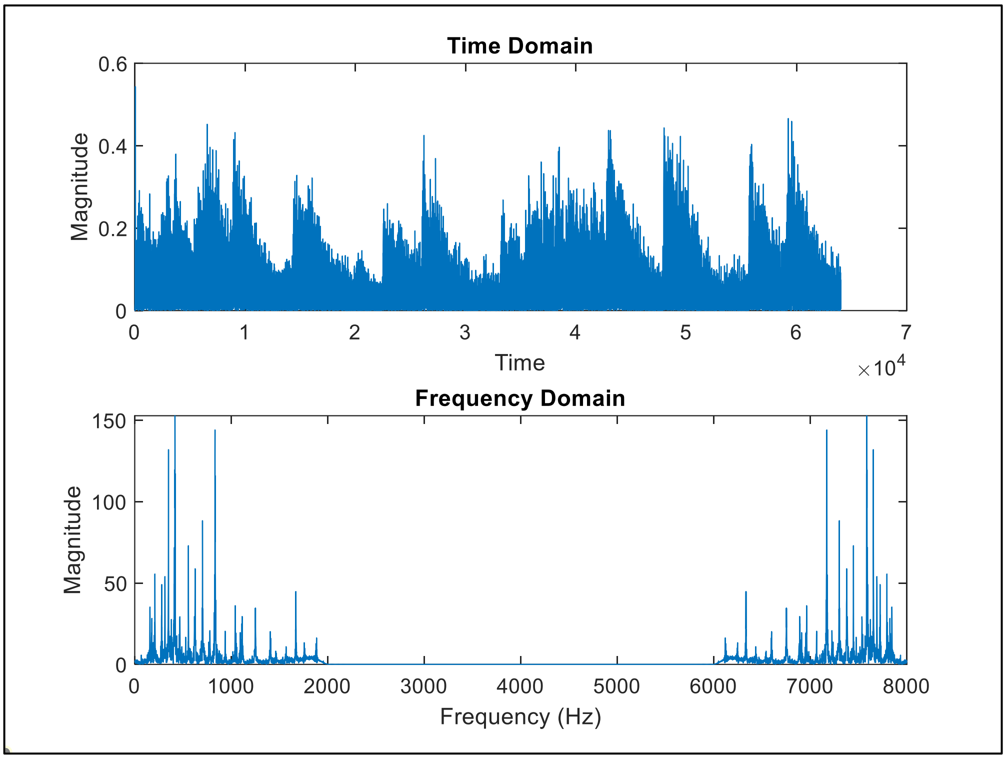

Figure 4.1.1 – Time & frequency domain plot

Although the "bump" is attenuated properly, the quality of the audio has been noticeably decreased due to the filtering (as seen in the time-domain plot).

4.2 – IIR Filter (Chebyshev)

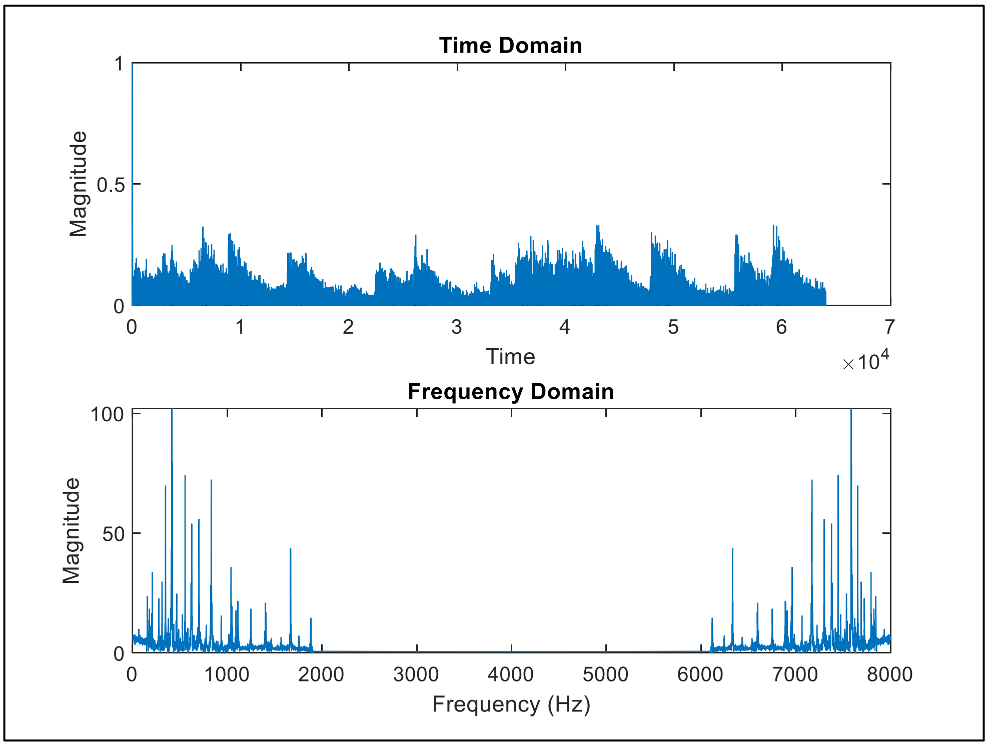

Figure 4.2.1 – Time & frequency domain plot

Although the "bump" is attenuated properly, the quality of the audio has been decreased even further (when compared to the FIR filter) due to the filtering (as seen in the time-domain plot).