Assignment

Barath surya M — EE22BTECH11014

Question Let X be a random variable having poisson distribution with mean $\lambda > 0$. Then $\mathrm{E}\left(\frac{1}{1+X}\middle|X > 0\right)$ equals

1) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda\left(1-e^{-\lambda}\right)}$
2) $\frac{1-e^{-\lambda}}{\lambda}$
3) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda}$
4) $\frac{1-e^{-\lambda}}{\lambda+1}$

Solution:

(A) Theory

$$X \sim Pois\left(\lambda\right) \tag{1}$$

$$\Pr\left(X = k\right) = e^{-\lambda}\frac{\lambda^k}{k!}; k \geq 0 \tag{2}$$

we know that

$$E\left(A|B\right) = \frac{E\left(A, B\right)}{\Pr\left(B\right)} \tag{3}$$

$$\implies E\left(\frac{1}{1+X}\middle|X > 0\right) = \frac{\sum_{k=1}^{\infty}\frac{1}{k+1}\Pr\left(X = k\right)}{\Pr\left(X > 0\right)} \tag{4}$$

$$= \frac{\sum_{k=1}^{\infty}\frac{1}{k+1}e^{-\lambda}\frac{\lambda^k}{k!}}{1 - \Pr\left(X \leq 0\right)} \tag{5}$$

$$= \frac{e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k+1)!}}{1 - \Pr\left(X = 0\right)} \tag{6}$$

from equation (2)

$$= \frac{e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k+1)!}}{1 - e^{\lambda}} \tag{7}$$

Now simplifying Just the Summation

$$\implies \sum_{k=1}^{\infty}\frac{\lambda^k}{(k+1)!} \tag{8}$$

$$= \frac{1}{\lambda}\sum_{k=1}^{\infty}\frac{\lambda^{k+1}}{k+1} \tag{9}$$

Letting $k + 1 = m$,

$$\implies \frac{1}{\lambda}\sum_{m=2}^{\infty}\frac{\lambda^m}{m!} \tag{10}$$

We Know from Taylor series

$$e^x = \sum_{k=0}^{\infty}\frac{x^k}{k!} \tag{11}$$

$$\implies \frac{1}{\lambda}\sum_{m=2}^{\infty}\frac{\lambda^m}{m!} = \frac{1}{\lambda}\left(e^{\lambda} - 1 - \lambda\right) \tag{12}$$

Substituting back we get,

$$= \frac{e^{-\lambda}}{\left(1 - e^{\lambda}\right)}\left(\frac{1}{\lambda}\left(e^{\lambda} - 1 - \lambda\right)\right) \tag{13}$$

$$= \frac{e^{-\lambda}}{\lambda\left(1 - e^{\lambda}\right)}\left(e^{\lambda} - 1 - \lambda\right) \tag{14}$$

$$= \frac{1 - e^{-\lambda} - \lambda e^{-\lambda}}{\lambda\left(1 - e^{\lambda}\right)} \tag{15}$$

(B) Simulation

(i) Initialize a Random variable X and equate to 0. Calculate the probability pp for $X = 0$ which is $e^{-\lambda}$ from equation (2).
(ii) Generate a Uniform random variable U between 0 and 1 by dividing (double)rand() by RAND_MAX.
(iii) While U is greater than pp, iterate and Increase X by value of 1.Update pp according to poisson distribution to calculate the cumulative distribution for increasing values of X.
(iv) When U does not meet the condition the value of random variable X is found and is a Poisson random variable.
(v) Then for the second part Check if the generated value of X is greater than 0. If X is greater than 0, calculate the value Y as $\frac{1}{X+1}$ and add it to the sumY.Increment the validCount to keep track of valid X values.
(vi) If validCount is greater than 0, calculate the estimate of the conditional expectation by dividing sumY by validCount.