GATE 2023 ST 17

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Question

Let X be a random variable having Poisson distribution with mean $\lambda > 0$. Then $\mathrm{E}\left(\frac{1}{1+X} \middle| X > 0\right)$ equals

1. $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda\left(1-e^{-\lambda}\right)}$
2. $\frac{1-e^{-\lambda}}{\lambda}$
3. $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda}$
4. $\frac{1-e^{-\lambda}}{\lambda+1}$

Defining Poisson random variable

Let X be a Poisson random variable, then pmf of X is defined as

$$X \sim Pois\left(\lambda\right) \tag{1}$$

$$\Pr\left(X = k\right) = e^{-\lambda}\frac{\lambda^k}{k!}; k \geq 0 \tag{2}$$

where λ is the Poisson parameter and also,

$$E\left(X\right) = \lambda \tag{3}$$

Conditional Expectance

Conditional Expectance $E(X|Y)$ is defined as

$$E(X \mid A) = \sum_x xP(X = x \mid A) \tag{4}$$

$$= \sum_x x \frac{P(X = x, A)}{P(A)} \tag{5}$$

$$\implies E\left(\frac{1}{1+X}\middle| X > 0\right) = \frac{\sum_{k=1}^{\infty} \frac{1}{k+1} \Pr(X = k)}{\Pr(X > 0)} \tag{6}$$

$$= \frac{\sum_{k=1}^{\infty} \frac{1}{k+1} e^{-\lambda} \frac{\lambda^k}{k!}}{1 - \Pr(X \leq 0)} \tag{7}$$

$$= \frac{e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k+1)!}}{1 - \Pr(X = 0)} \tag{8}$$