Assignment

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Question Let X be a random variable having poisson distribution with mean $\lambda > 0$. Then $\mathrm{E}\left(\frac{1}{1+X}\middle|X > 0\right)$ equals

1) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda\left(1-e^{-\lambda}\right)}$
2) $\frac{1-e^{-\lambda}}{\lambda}$
3) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda}$
4) $\frac{1-e^{-\lambda}}{\lambda+1}$

Solution:

$$X \sim Pois\left(\lambda\right) \tag{1}$$

$$\Pr\left(X = k\right) = e^{-\lambda}\frac{\lambda^k}{k!}; k \geq 0 \tag{2}$$

we know that

$$E\left(A|B\right) = \frac{E\left(A, B\right)}{\Pr\left(B\right)} \tag{3}$$

$$\implies E\left(\frac{1}{1+X}\middle|X > 0\right) = \frac{\sum_{k=1}^{\infty}\frac{1}{k+1}\Pr\left(X = k\right)}{\Pr\left(X > 0\right)} \tag{4}$$

$$= \frac{\sum_{k=1}^{\infty}\frac{1}{k+1}e^{-\lambda}\frac{\lambda^k}{k!}}{1 - \Pr\left(X \leq 0\right)} \tag{5}$$

$$= \frac{e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k+1)!}}{1 - \Pr\left(X = 0\right)} \tag{6}$$

$$= \frac{e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k+1)!}}{1 - e^{\lambda}} \tag{7}$$

let $k + 1 = m$,

$$= \frac{e^{-\lambda}}{\left(1 - e^{\lambda}\right)}\left(\sum_{m=2}^{\infty}\frac{\lambda^{m-1}}{m!}\right) \tag{8}$$

$$= \frac{e^{-\lambda}}{\lambda\left(1 - e^{\lambda}\right)}\left(\sum_{m=0}^{\infty}\frac{\lambda^m}{m!} - 1 - \lambda\right) \tag{9}$$

$$= \frac{e^{-\lambda}}{\lambda\left(1 - e^{\lambda}\right)}\left(e^{\lambda} - 1 - \lambda\right) \tag{10}$$

$$= \frac{1 - e^{-\lambda} - \lambda e^{-\lambda}}{\lambda\left(1 - e^{\lambda}\right)} \tag{11}$$