GATE 2023 ST 17

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Let U be a continuous random variable having a standard uniform distribution. Then, the random variable

$$X = F^{-1}(U) \tag{1}$$

has a probability distribution characterized by the invertible cumulative distribution function $F_X(x)$.
proof: The cumulative distribution function of the transformation $X = F_X^{-1}(U)$ can be derived as

$$\Pr\left(X \le x\right) \tag{2}$$
$$= \Pr\left(F_X^{-1}(U) \le (x)\right) \tag{3}$$
$$= \Pr\left(U \le F_X(x)\right) \tag{4}$$
$$= F_X(x) \tag{5}$$

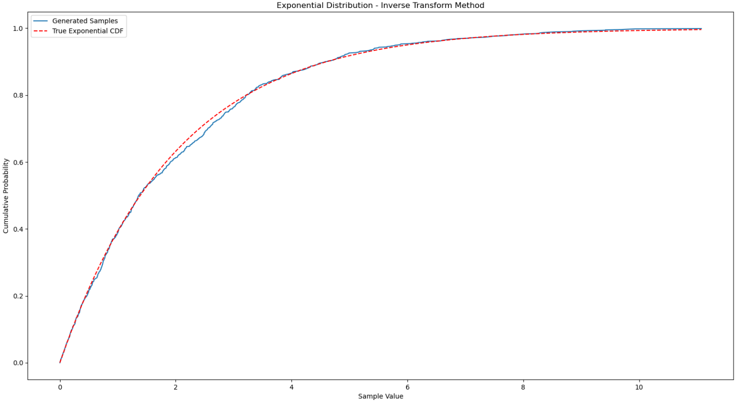
because the cumulative distribution function of the standard uniform distribution U(0,1) is

$$U \sim U(0,1) \implies F_U(u) = \Pr\left(U \le u\right) = u \tag{6}$$

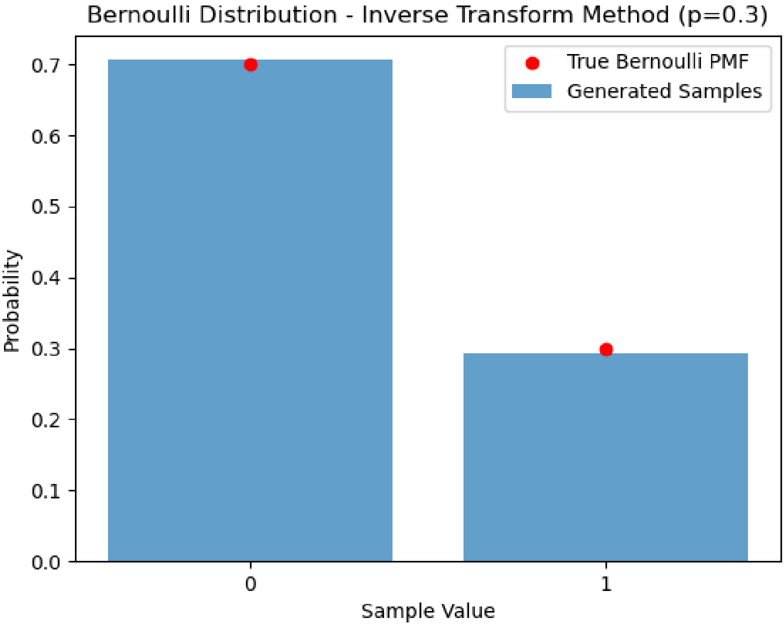
But in case of discreate distributions, taking Inverse Might be difficult so we will check for index k such that

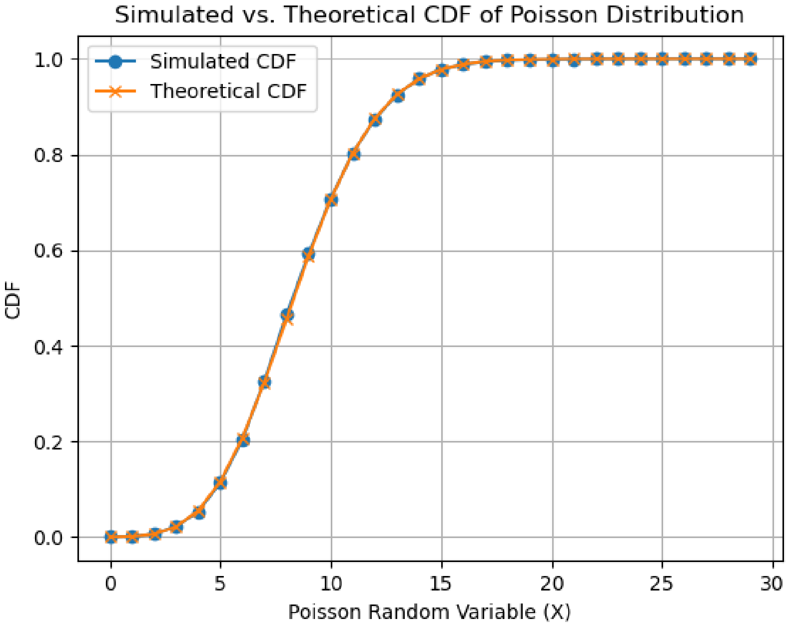
$$\sum_{j=1}^{k-1} p_j \leq U \leq \sum_{j=1}^{k} p_j \tag{7}$$

```
algorithm Poisson generator based upon the inversion by sequential search:
    init:
        Let x ← 0, p ← e^-λ, s ← p.
        Generate uniform random number u in [0,1].
    while u > s do:
        x ← x + 1.
        p ← p × λ / x.
        s ← s + p.
    return x.
```


Exponential Distribution - Inverse Transform Method
Generated Samples
True Exponential CDF
Cumulative Probability
Sample Value
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10

```python
def generate_exponential_samples(lambda_param, size):
    u = np.random.uniform(0, 1, size)
    x = -np.log(1 - u) / lambda_param
    return x
```


Bernoulli Distribution - Inverse Transform Method (p=0.3)
True Bernoulli PMF
Generated Samples
Probability
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
Sample Value


Simulated vs. Theoretical CDF of Poisson Distribution
Simulated CDF
Theoretical CDF
CDF
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
30
Poisson Random Variable (X)