Assignment

Barath surya M — EE22BTECH11014

Question 9.3.3 Five cards are drawn successively with replacement from well shuffled deck of 52 cards, what is the probability that

1) all the five cards are spades?
2) only 3 cards are spades
3) None is a spade

Solution:

Binomial

Parameter	Value	Description
X	{0,1,2,3,4,5}	Number of spade cards drawn
n	5	Number of cards drawn
p	0.25	Drawing a spade card
q	0.75	Drawing any other card
$\mu = np$	1.25	Mean of Binomial distribution
$\sigma^2 = npq$	0.9375	Varience of Binomial distribution

TABLE 1: Random variable and Parameter

PMF of the distribution is,

$$\Pr(X = k) = {}^{n}C_k p^k (1-p)^{n-k} \tag{1}$$

1)

$$k = 5 \tag{2}$$
$$\implies \Pr(X = 5) = {}^{5}C_5 (0.25)^5 (0.75)^0 \tag{3}$$
$$= 0.0009765625 \tag{4}$$

2)

$$k = 3 \tag{5}$$
$$\implies \Pr(X = 3) = {}^{5}C_3 (0.25)^3 (0.75)^2 \tag{6}$$
$$= 0.087890625 \tag{7}$$

3)

$$k = 0 \tag{8}$$
$$\implies \Pr(X = 0) = {}^{5}C_0 (0.25)^0 (0.75)^5 \tag{9}$$
$$= 0.2373046875 \tag{10}$$

Gaussian

Central limit theroem states that ,Z be a random variable

$$Z \approx \frac{X - \mu}{\sigma} \tag{11}$$

Z converges to normal distribution for large value of n

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{12}$$

Q function is defined

$$Q(x) = \int_x^{\infty} f(x)\,dx \tag{13}$$

then CDF of X is:

$$\Pr(X < x) = \int_{-\infty}^{x} f(x)\,dx \tag{14}$$

$$= 1 - \int_x^{\infty} f(x)\,dx \tag{15}$$

$$= 1 - Q(x) \tag{16}$$

and for finding $\Pr\left(Z = \frac{X-\mu}{\sigma}\right)$ Using approximation,

$$\Pr\left(Z = \frac{X-\mu}{\sigma}\right) \approx \Pr\left(\frac{X+0.5-\mu}{\sigma} < Z < \frac{X-0.5-\mu}{\sigma}\right) \tag{17}$$

$$\approx \Pr\left(Z < \frac{X+0.5-\mu}{\sigma}\right) - \Pr\left(Z < \frac{X-0.5-\mu}{\sigma}\right) \tag{18}$$

$$\approx Q\left(\frac{X-0.5-\mu}{\sigma}\right) - Q\left(\frac{X+0.5-\mu}{\sigma}\right) \tag{19}$$

1)

$$X = 5 \tag{20}$$

$$\Pr(Z = 3.872) \approx Q(3.356) - Q(4.389) \tag{21}$$

$$\approx 0.0003888 \tag{22}$$

2)

$$X = 3 \tag{23}$$

$$\Pr(Z = 1.8073) \approx Q(1.2909) - Q(2.3237) \tag{24}$$

$$\approx 0.08828 \tag{25}$$

3)

$$X = 0 \tag{26}$$

$$\Pr(Z = -1.2909) \approx Q(-1.8073) - Q(-0.7745) \tag{27}$$

$$\approx 0.1839 \tag{28}$$

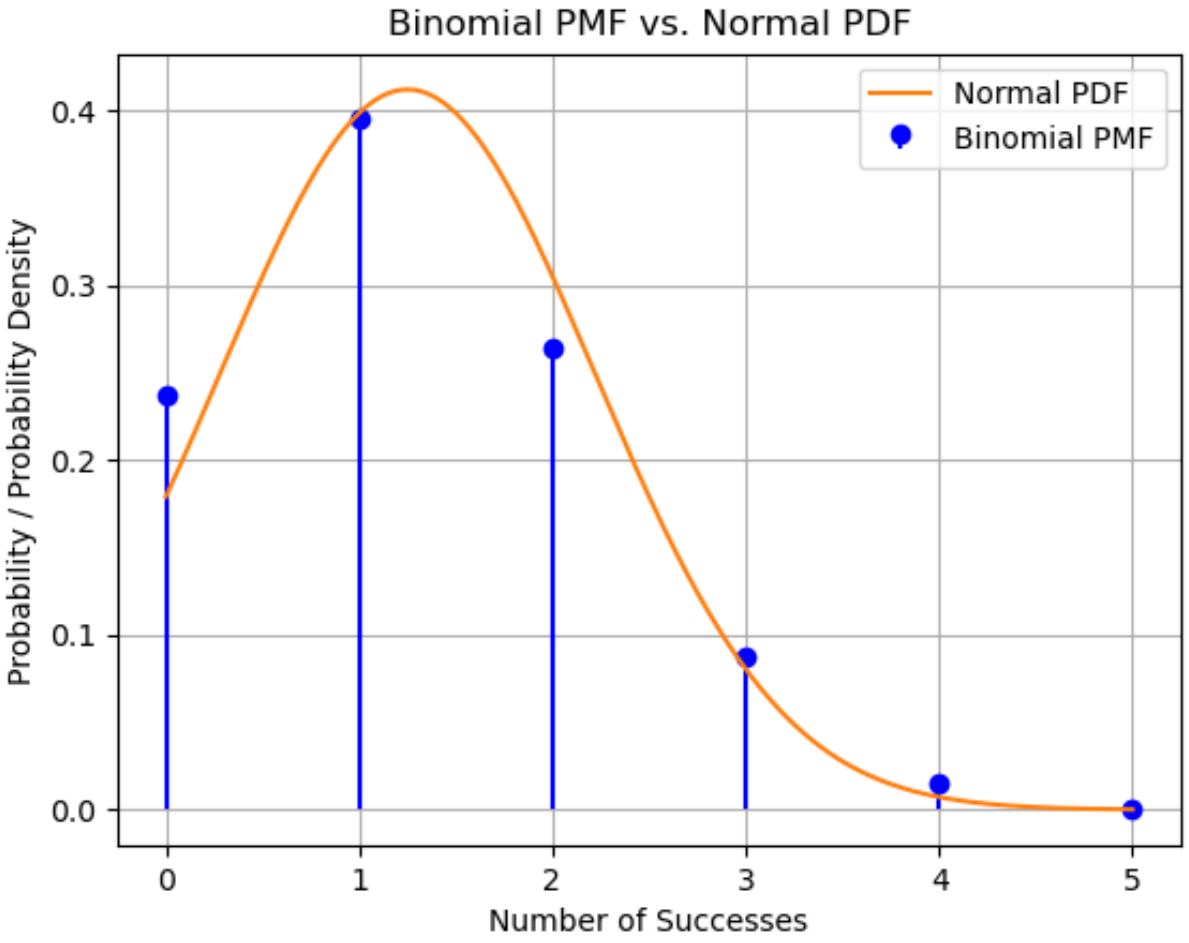

Fig. 1: Binomial and gaussian distribution