

Assignment

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Question 9.3.3 Five cards are drawn successively with replacement from well shuffled deck of 52 cards, what is the probability that

- 1) all the five cards are spades?
- 2) only 3 cards are spades
- 3) None is a spade

Solution:

Binomial

PMF of the distribution is,

Parameter	Value	Description
X	{0,1,2,3,4,5}	Number of spade cards drawn
n	5	Number of cards drawn
p	0.25	Drawing a spade card
q	0.75	Drawing any other card

TABLE 1: Random variable and Parameter

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \quad (1)$$

1)

$$k = 5 \quad (2)$$

$$\Rightarrow \Pr(X = 5) = {}^5C_5 (0.25)^5 (0.75)^0 \quad (3)$$

$$= 0.0009765625 \quad (4)$$

2)

$$k = 3 \quad (5)$$

$$\Rightarrow \Pr(X = 3) = {}^5C_3 (0.25)^3 (0.75)^2 \quad (6)$$

$$= 0.087890625 \quad (7)$$

3)

$$k = 0 \quad (8)$$

$$\Rightarrow \Pr(X = 0) = {}^5C_0 (0.25)^0 (0.75)^5 \quad (9)$$

$$= 0.2373046875 \quad (10)$$

Gaussian

$$X \sim \text{Bin}(n, p) \quad (11)$$

$$\sim \text{Bin}(5, 0.25) \quad (12)$$

Let, Y be a random variable such that,

$$Y = \frac{X - \mu_X}{\sigma_X} \quad (13)$$

Parameter	Value	Description
n	5	Number of cards drawn
p	0.25	Drawing a spade card
q	0.75	Drawing any other card
$\mu=np$	1.25	Mean of the distribution
$\sigma^2 =npq$	0.9375	Variance of the distribution

TABLE 2: Random variable and Parameter

Y converges to normal distribution for large value of n

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad (14)$$

And Finding CDF using Q funtion by

$$\Pr(Y > x) = Q(x) \quad (15)$$

and for finding $\Pr\left(Y = \frac{X-\mu_X}{\sigma_X}\right)$ Using approximation,

$$\Pr\left(Y = \frac{X - \mu_X}{\sigma_X}\right) \approx \Pr\left(\frac{X + 0.5 - \mu_X}{\sigma_X} < Y < \frac{X - 0.5 - \mu_X}{\sigma_X}\right) \quad (16)$$

$$\approx \Pr\left(Y < \frac{X + 0.5 - \mu_X}{\sigma_X}\right) - \Pr\left(Y < \frac{X - 0.5 - \mu_X}{\sigma_X}\right) \quad (17)$$

$$\approx \Pr\left(Y > \frac{X - 0.5 - \mu_X}{\sigma_X}\right) - \Pr\left(Y > \frac{X + 0.5 - \mu_X}{\sigma_X}\right) \quad (18)$$

$$\approx Q\left(\frac{X - 0.5 - \mu_X}{\sigma_X}\right) - Q\left(\frac{X + 0.5 - \mu_X}{\sigma_X}\right) \quad (19)$$

1)

$$X = 5 \quad (20)$$

$$\Pr(Y = 3.872) \approx Q(3.356) - Q(4.389) \quad (21)$$

$$\approx 0.0003888 \quad (22)$$

2)

$$X = 3 \quad (23)$$

$$\Pr(Y = 1.8073) \approx Q(1.2909) - Q(2.3237) \quad (24)$$

$$\approx 0.08828 \quad (25)$$

3)

$$X = 0 \quad (26)$$

$$\Pr(Y = -1.2909) \approx Q(-1.8073) - Q(-0.7745) \quad (27)$$

$$\approx 0.1839 \quad (28)$$

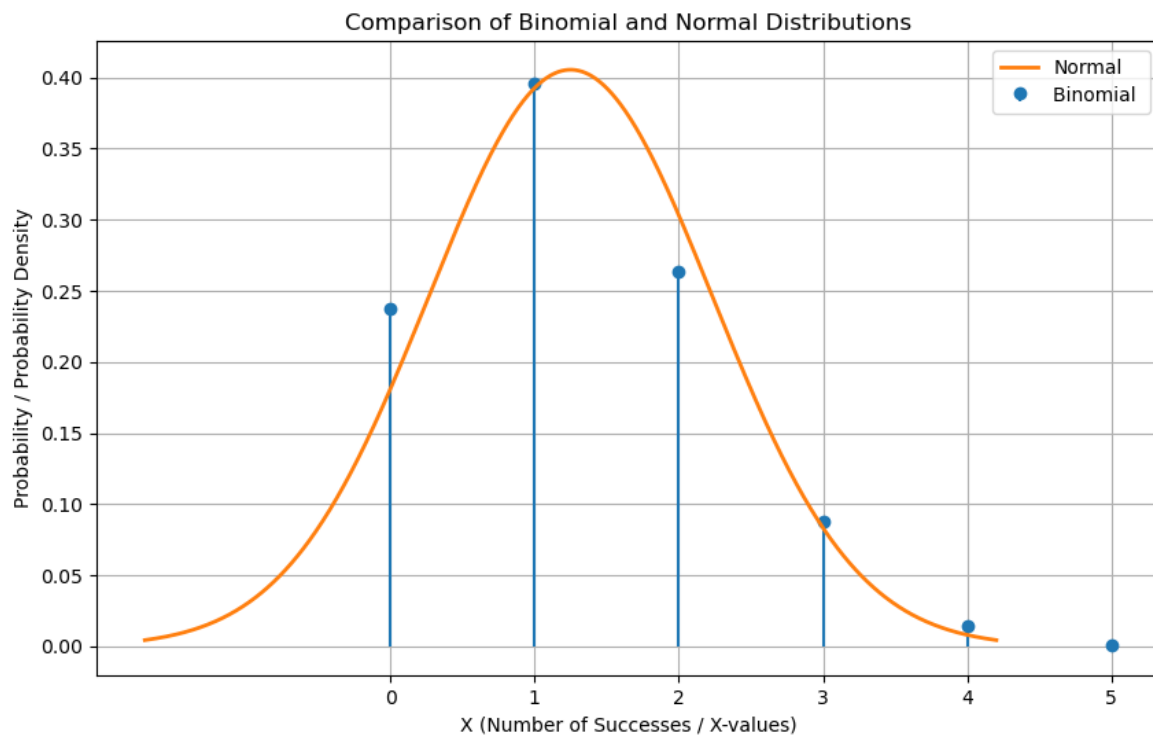


Fig. 1: Binomial and gaussian distribution