

Assignment

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Question 9.3.3 Five cards are drawn successively with replacement from well shuffled deck of 52 cards, what is the probability that

- 1) all the five cards are spades?
- 2) only 3 cards are spades
- 3) None is a spade

Solution:

Binomial

Parameter	Value	Description
X	$\{0,1,2,3,4,5\}$	Number of spade cards drawn
n	5	Number of cards drawn
p	0.25	Drawing a spade card
q	0.75	Drawing any other card
$\mu = np$	1.25	Mean of Binomial distribution
$\sigma^2 = npq$	0.9375	Variance of Binomial distribution

TABLE 1: Random variable and Parameter

PMF of the distribution is,

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \quad (1)$$

1)

$$k = 5 \quad (2)$$

$$\Rightarrow \Pr(X = 5) = {}^5C_5 (0.25)^5 (0.75)^0 \quad (3)$$

$$= 0.0009765625 \quad (4)$$

2)

$$k = 3 \quad (5)$$

$$\Rightarrow \Pr(X = 3) = {}^5C_3 (0.25)^3 (0.75)^2 \quad (6)$$

$$= 0.087890625 \quad (7)$$

3)

$$k = 0 \quad (8)$$

$$\Rightarrow \Pr(X = 0) = {}^5C_0 (0.25)^0 (0.75)^5 \quad (9)$$

$$= 0.2373046875 \quad (10)$$

Gaussian

let Y be a gaussian Random variable

$$Y \sim N(np, npq) \quad (11)$$

$$\sim N(1.25, 0.9375) \quad (12)$$

Due to continuity correction $\Pr(X = x)$ can be approximated using gaussian distribution as

$$\Pr(X = x) \approx \Pr(x - 0.5 < Y < x + 0.5) \quad (13)$$

$$\approx \Pr(Y < x + 0.5) - \Pr(Y < x - 0.5) \quad (14)$$

$$\approx F_Y(x + 0.5) - F_Y(x - 0.5) \quad (15)$$

CDF of Y is defined as:

$$F_Y(x) = \Pr(Y < x) \quad (16)$$

$$= \Pr\left(\frac{Y - \mu}{\sigma} < \frac{x - \mu}{\sigma}\right) \quad (17)$$

$$\Rightarrow \frac{Y - \mu}{\sigma} \sim N(0, 1) \quad (18)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{x - \mu}{\sigma}\right) \quad (19)$$

$$= 1 - Q\left(\frac{x - \mu}{\sigma}\right) \quad (20)$$

Then PDF in terms of Q funtion is

$$\Pr(X = x) \approx Q\left(\frac{(x - 0.5) - \mu}{\sigma}\right) - Q\left(\frac{(x + 0.5) - \mu}{\sigma}\right) \quad (21)$$

1)

$$X = 5 \quad (22)$$

The Gaussian approximation for $\Pr(X = 5)$ is

$$\Pr(X = 5) \approx Q\left(\frac{4.5 - 1.25}{0.9375}\right) - Q\left(\frac{5.5 - 1.25}{0.9375}\right) \quad (23)$$

$$\approx Q(3.356) - Q(4.389) \quad (24)$$

$$\approx 0.0003888 \quad (25)$$

2)

$$X = 3 \quad (26)$$

The Gaussian approximation for $\Pr(X = 3)$ is

$$\Pr(X = 3) \approx Q\left(\frac{2.5 - 1.25}{0.9375}\right) - Q\left(\frac{3.5 - 1.25}{0.9375}\right) \quad (27)$$

$$\approx Q(1.2909) - Q(2.3237) \quad (28)$$

$$\approx 0.08828 \quad (29)$$

3)

$$X = 0 \quad (30)$$

The Gaussian approximation for $\Pr(X = 0)$ is

$$\Pr(X = 0) \approx Q\left(\frac{-0.5 - 1.25}{0.9375}\right) - Q\left(\frac{0.5 - 1.25}{0.9375}\right) \quad (31)$$

$$\approx Q(-1.8073) - Q(-0.7745) \quad (32)$$

$$\approx 0.1839 \quad (33)$$

Comparison			
Number of spade cards	Binomial distribution	Gaussian approximation	Error (%)
5	0.0009765625	0.00038880	60.18688
3	0.087890625	0.088279	0.4430
0	0.2373046875	0.18390	22.5046

TABLE 2: Comparison between the approximation

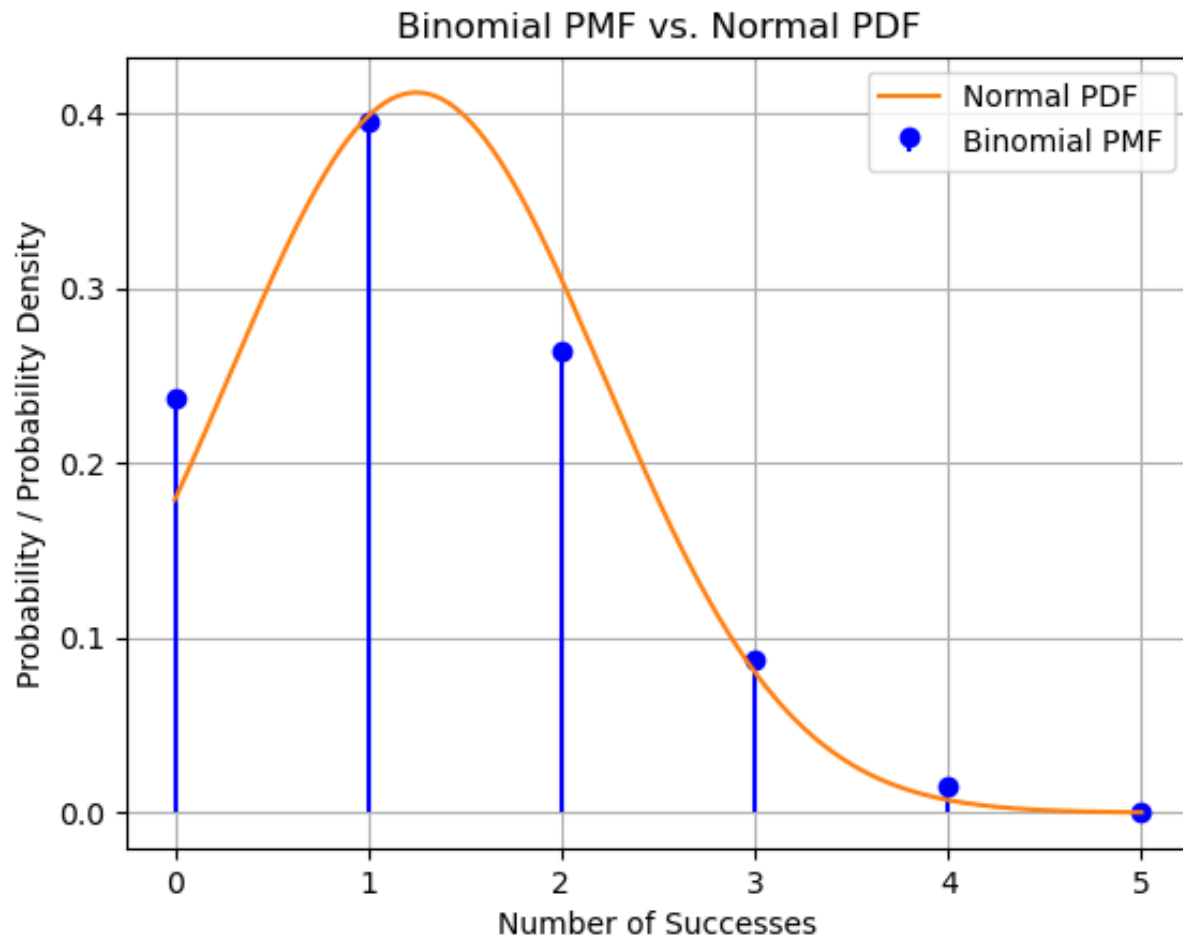


Fig. 1: Binomial and gaussian distribution