Assignment

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Question Let X be a random variable having poisson distribution with mean $\lambda > 0$. Then $\mathrm{E}\left(\frac{1}{1+X}\middle|X > 0\right)$ equals

1) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda\left(1-e^{-\lambda}\right)}$
2) $\frac{1-e^{-\lambda}}{\lambda}$
3) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda}$
4) $\frac{1-e^{-\lambda}}{\lambda+1}$

Solution:

(A) Theory

$$X \sim Pois\left(\lambda\right) \tag{1}$$

$$\Pr\left(X = k\right) = e^{-\lambda}\frac{\lambda^k}{k!}; k \geq 0 \tag{2}$$

we know that

$$E\left(A|B\right) = \frac{E\left(A, B\right)}{\Pr\left(B\right)} \tag{3}$$

$$\implies E\left(\frac{1}{1+X}\middle|X > 0\right) = \frac{\sum_{k=1}^{\infty}\frac{1}{k+1}\Pr\left(X = k\right)}{\Pr\left(X > 0\right)} \tag{4}$$

$$= \frac{\sum_{k=1}^{\infty}\frac{1}{k+1}e^{-\lambda}\frac{\lambda^k}{k!}}{1 - \Pr\left(X \leq 0\right)} \tag{5}$$

$$= \frac{e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k+1)!}}{1 - \Pr\left(X = 0\right)} \tag{6}$$

from equation (2)

$$= \frac{e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k+1)!}}{1 - e^{\lambda}} \tag{7}$$

Now simplifying Just the Summation

$$\implies \sum_{k=1}^{\infty}\frac{\lambda^k}{(k+1)!} \tag{8}$$

$$= \frac{1}{\lambda}\sum_{k=1}^{\infty}\frac{\lambda^{k+1}}{k+1} \tag{9}$$

Letting $k + 1 = m$,

$$\implies \frac{1}{\lambda}\sum_{m=2}^{\infty}\frac{\lambda^m}{m!} \tag{10}$$

We Know from Taylor series

$$e^x = \sum_{k=0}^{\infty}\frac{x^k}{k!} \tag{11}$$

$$\implies \frac{1}{\lambda}\sum_{m=2}^{\infty}\frac{\lambda^m}{m!} = \frac{1}{\lambda}\left(e^{\lambda} - 1 - \lambda\right) \tag{12}$$

Substituting back we get,

$$= \frac{e^{-\lambda}}{\left(1 - e^{\lambda}\right)}\left(\frac{1}{\lambda}\left(e^{\lambda} - 1 - \lambda\right)\right) \tag{13}$$

$$= \frac{e^{-\lambda}}{\lambda\left(1 - e^{\lambda}\right)}\left(e^{\lambda} - 1 - \lambda\right) \tag{14}$$

$$= \frac{1 - e^{-\lambda} - \lambda e^{-\lambda}}{\lambda\left(1 - e^{\lambda}\right)} \tag{15}$$

(B) Simulation

(i) In the code, it simulates the generation of RV X using the CDF of Poisson distribution.

$$F\left(x\right) = \sum_{k=0}^{x}e^{-\lambda}\frac{\lambda^k}{k!} \tag{16}$$

(ii) initialize $X = 0$
(iii) initialize $F = e^{-\lambda}$ which is $F\left(0\right)$ of a poisson distribution
(iv) generate a uniform random variable between 0 and 1
(v) enter the loop that continues as long as $u > F$ Inside the loop,
(vi) Increment X by 1
(vii) Add next term of Poisson pmf ($\Pr\left(X = k + 1\right)$) to $F\left(x\right)$
(viii) the loop continues until u is no longer greater than F. At this point, X represents the generated value of the poisson random variable that follows the desired poisson distribution with mean parameter λ.
(ix) Save all the values of poisson random variable X in pois.dat so to open it in python an plot the cdf graph
(x) Then for the second part Check if the generated value of X is greater than 0. If X is greater than 0, calculate the value Y as $\frac{1}{X+1}$ and add it to the

sumY.Increment the validCount to keep track of valid X values.

(xi) If validCount is greater than 0, calculate the estimate of the conditional expectation by dividing sumY by validCount.

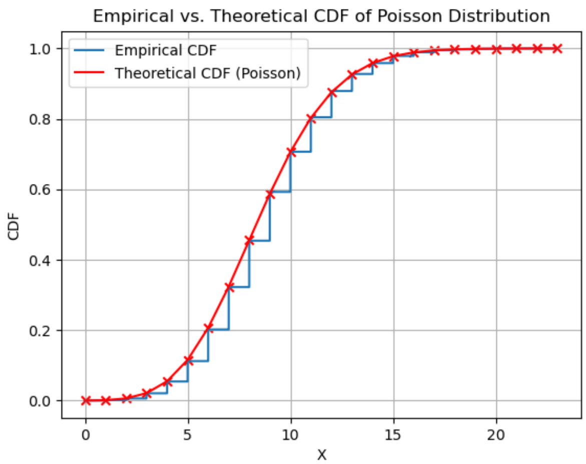

Fig. 1: Simulation Vs theoretical cdf of poisson distribution with $\lambda = 9$