Assignment

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Question 9.3.3 Five cards are drawn successively with replacement from well shuffled deck of 52 cards, what is the probability that

1) all the five cards are spades?
2) only 3 cards are spades
3) None is a spade

Solution:

Binomial

Parameter	Value	Description
X	{0,1,2,3,4,5}	Number of spade cards drawn
n	5	Number of cards drawn
p	0.25	Drawing a spade card
q	0.75	Drawing any other card
$\mu = np$	1.25	Mean of Binomial distribution
$\sigma^2 = npq$	0.9375	Varience of Binomial distribution

TABLE 1: Random variable and Parameter

PMF of the distribution is,

$$\Pr(X = k) = {}^{n}C_k p^k (1-p)^{n-k} \tag{1}$$

1)

$$k = 5 \tag{2}$$
$$\implies \Pr(X = 5) = {}^{5}C_5 (0.25)^5 (0.75)^0 \tag{3}$$
$$= 0.0009765625 \tag{4}$$

2)

$$k = 3 \tag{5}$$
$$\implies \Pr(X = 3) = {}^{5}C_3 (0.25)^3 (0.75)^2 \tag{6}$$
$$= 0.087890625 \tag{7}$$

3)

$$k = 0 \tag{8}$$
$$\implies \Pr(X = 0) = {}^{5}C_0 (0.25)^0 (0.75)^5 \tag{9}$$
$$= 0.2373046875 \tag{10}$$

Gaussian

let Y be a gaussian Random variable

$$Y \sim N(np, npq) \tag{11}$$
$$\sim N(1.25, 0.9375) \tag{12}$$

Due to continuity correction $\Pr(X = x)$ can be approximated using gaussian distribution as

$$\Pr(X = x) \approx \Pr(x - 0.5 < Y < x + 0.5) \tag{13}$$
$$= \Pr(Y < x + 0.5) - \Pr(Y < x - 0.5) \tag{14}$$

Standardising Y to $\mu_Y = 0$ and $\sigma_Y = 1$, the approximation becomes a Normal distribution

$$\Pr(X = x) \approx \Pr\left(Z < \frac{(x + 0.5) - np}{\sqrt{npq}}\right) - \Pr\left(Z < \frac{(x - 0.5) - np}{\sqrt{npq}}\right) \tag{15}$$

Q function is defined as

$$Q(x) = \int_x^\infty f(x)\, dx \tag{16}$$

then CDF of Normal distribution is defined as:

$$\Pr(Z < x) = \int_{-\infty}^x f(x)\, dx \tag{17}$$
$$= 1 - \int_x^\infty f(x)\, dx \tag{18}$$
$$= 1 - Q(x) \tag{19}$$
$$\implies \Pr(X = x) \approx \left(1 - Q\left(\frac{(x + 0.5) - np}{\sqrt{npq}}\right)\right) - \left(1 - Q\left(\frac{(x - 0.5) - np}{\sqrt{npq}}\right)\right) \tag{20}$$
$$\approx Q\left(\frac{(x - 0.5) - np}{\sqrt{npq}}\right) - Q\left(\frac{(x + 0.5) - np}{\sqrt{npq}}\right) \tag{21}$$

1)

$$X = 5 \tag{22}$$

The Gaussian approximation for $\Pr(X = 5)$ is

$$\Pr(X = 5) \approx Q\left(\frac{4.5 - 1.25}{0.9375}\right) - Q\left(\frac{5.5 - 1.25}{0.9375}\right) \tag{23}$$
$$\approx Q(3.356) - Q(4.389) \tag{24}$$
$$\approx 0.0003888 \tag{25}$$

2)

$$X = 3 \tag{26}$$

The Gaussian approximation for $\Pr(X = 3)$ is

$$\Pr(X = 3) \approx Q\left(\frac{2.5 - 1.25}{0.9375}\right) - Q\left(\frac{3.5 - 1.25}{0.9375}\right) \tag{27}$$
$$\approx Q(1.2909) - Q(2.3237) \tag{28}$$
$$\approx 0.08828 \tag{29}$$

3)

$$X = 0 \tag{30}$$

The Gaussian approximation for $\Pr(X = 0)$ is

$$\Pr(X = 0) \approx Q\left(\frac{-0.5 - 1.25}{0.9375}\right) - Q\left(\frac{0.5 - 1.25}{0.9375}\right) \tag{31}$$
$$\approx Q(-1.8073) - Q(-0.7745) \tag{32}$$
$$\approx 0.1839 \tag{33}$$

Comparison			
Number of spade cards	Binomial distribution	Gaussian approximation	Error (%)
5	0.0009765625	0.00038880	60.18688
3	0.087890625	0.088279	0.4430
0	0.2373046875	0.18390	22.5046

TABLE 2: Comparison between the approximation

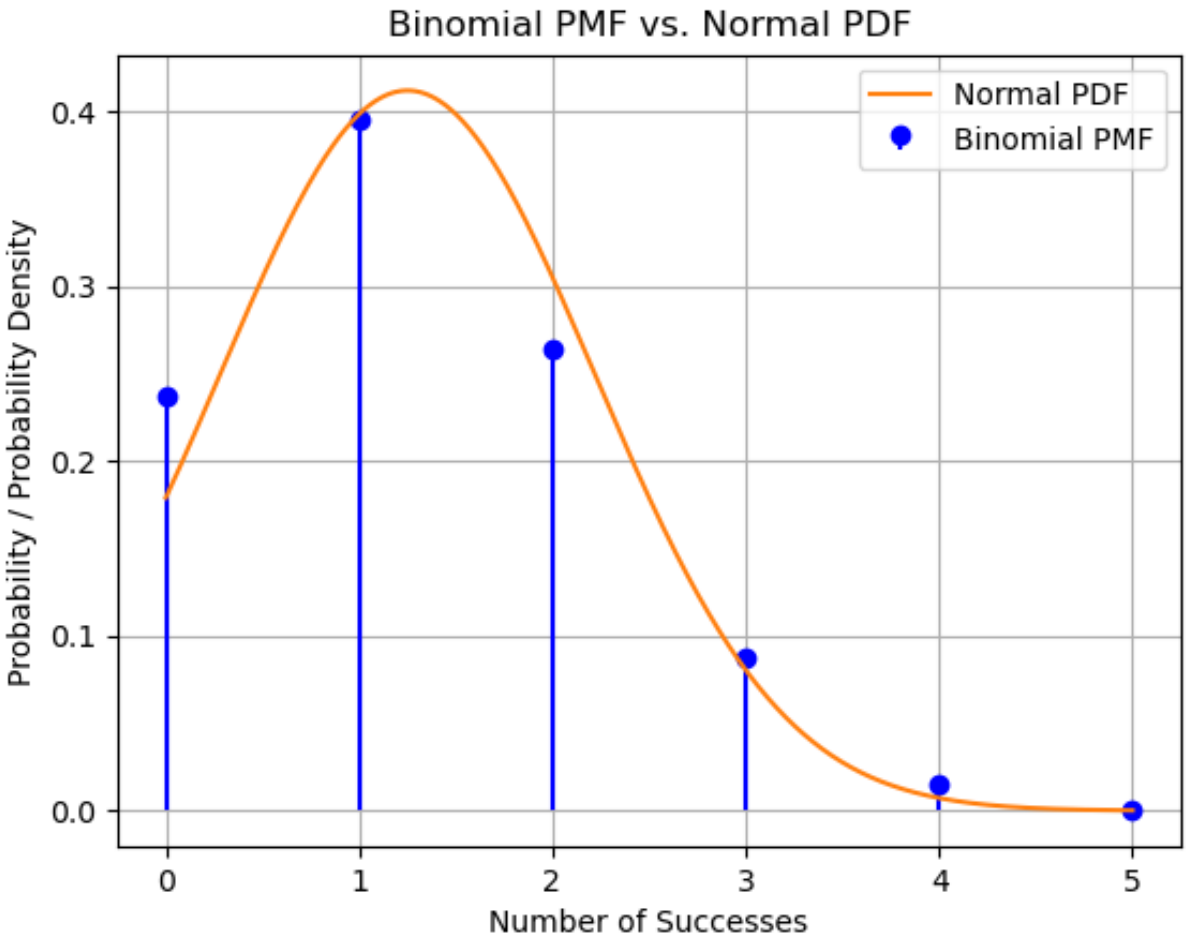

Fig. 1: Binomial and gaussian distribution