Assignment

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Question 9.3.3 Five cards are drawn successively with replacement from well shuffled deck of 52 cards, what is the probability that

1) all the five cards are spades?
2) only 3 cards are spades
3) None is a spade

Solution:

Binomial

PMF of the distribution is,

Parameter	Value	Description
X	{0,1,2,3,4,5}	Number of spade cards drawn
n	5	Number of cards drawn
p	0.25	Drawing a spade card
q	0.75	Drawing any other card

TABLE 1: Random variable and Parameter

$$\Pr(X = k) = {}^{n}C_k p^k (1-p)^{n-k} \tag{1}$$

1)

$$k = 5 \tag{2}$$
$$\implies \Pr(X = 5) = {}^{5}C_5 (0.25)^5 (0.75)^0 \tag{3}$$
$$= 0.0009765625 \tag{4}$$

2)

$$k = 3 \tag{5}$$
$$\implies \Pr(X = 3) = {}^{5}C_3 (0.25)^3 (0.75)^2 \tag{6}$$
$$= 0.087890625 \tag{7}$$

3)

$$k = 0 \tag{8}$$
$$\implies \Pr(X = 0) = {}^{5}C_0 (0.25)^0 (0.75)^5 \tag{9}$$
$$= 0.2373046875 \tag{10}$$

Gaussian

$$X \sim Bin(n, p) \tag{11}$$
$$\sim Bin(5, 0.25) \tag{12}$$

Let,Z be a random variable with mean $\mu_Z = 0$ and $\sigma_Z = 1$, such that,

$$Z = \frac{X - \mu_X}{\sigma_X} \tag{13}$$

Parameter	Value	Description
n	5	Number of cards drawn
p	0.25	Drawing a spade card
q	0.75	Drawing any other card
μ=np	1.25	Mean of the distribution
σ^2 =npq	0.9375	Varience of the distribution

TABLE 2: Random variable and Parameter

Z converges to normal distribution for large value of n

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{14}$$

And the Q funtion is

$$\Pr(Z > x) = Q(x) \tag{15}$$

and for finding $\Pr\left(Z = \frac{X-\mu_X}{\sigma_X}\right)$

$$\Pr\left(Z = \frac{X-\mu_X}{\sigma_X}\right) \approx \Pr\left(\frac{X+0.5-\mu_X}{\sigma_X} < Z < \frac{X-0.5-\mu_X}{\sigma_X}\right) \tag{16}$$

$$\approx \Pr\left(Z < \frac{X+0.5-\mu_X}{\sigma_X}\right) - \Pr\left(Z < \frac{X-0.5-\mu_X}{\sigma_X}\right) \tag{17}$$

$$\approx \Pr\left(Z > \frac{X-0.5-\mu_X}{\sigma_X}\right) - \Pr\left(Z > \frac{X+0.5-\mu_X}{\sigma_X}\right) \tag{18}$$

$$\approx Q\left(\frac{X-0.5-\mu_X}{\sigma_X}\right) - Q\left(\frac{X+0.5-\mu_X}{\sigma_X}\right) \tag{19}$$

1)

$$X = 5 \tag{20}$$

$$\Pr(Z = 3.872) \approx Q(3.356) - Q(4.389) \tag{21}$$

$$\approx 0.0003888 \tag{22}$$

2)

$$X = 3 \tag{23}$$

$$\Pr(Z = 1.8073) \approx Q(1.2909) - Q(2.3237) \tag{24}$$

$$\approx 0.08828 \tag{25}$$

3)

$$X = 0 \tag{26}$$

$$\Pr(Z = -1.2909) \approx Q(-1.8073) - Q(-0.7745) \tag{27}$$

$$\approx 0.1839 \tag{28}$$

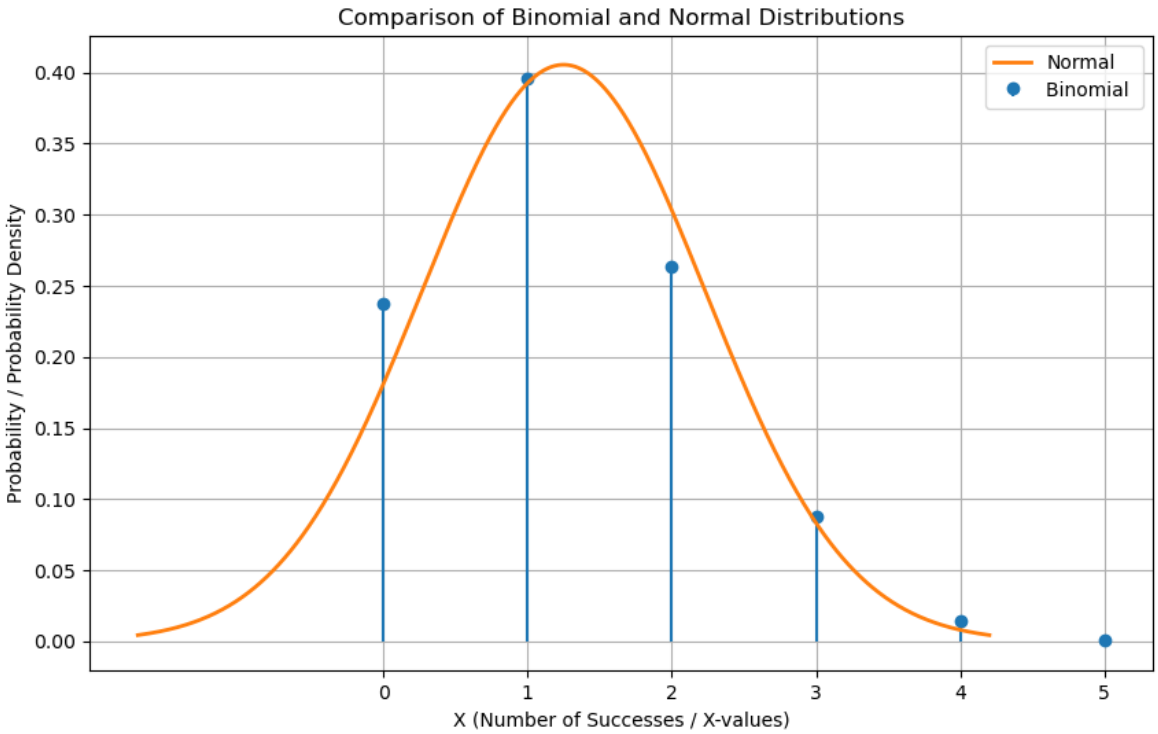

Fig. 1: Binomial and gaussian distribution