Assignment

Barath surya M — EE22BTECH11014

Question Let X be a random variable having poisson distribution with mean $\lambda > 0$. Then $E\left(\frac{1}{1+X}\big|X > 0\right)$ equals

1) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda(1-e^{-\lambda})}$
2) $\frac{1-e^{-\lambda}}{\lambda}$
3) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda}$
4) $\frac{1-e^{-\lambda}}{\lambda+1}$

Solution:

(A) Theory

$$X \sim Pois\,(\lambda) \tag{1}$$

$$\Pr\,(X = k) = e^{-\lambda}\frac{\lambda^k}{k!}; k \geq 0 \tag{2}$$

we know that

$$E\,(A|B) = \frac{E\,(A, B)}{\Pr\,(B)} \tag{3}$$

$$\implies E\left(\frac{1}{1 + X}\bigg|X > 0\right) = \frac{\sum_{k=1}^{\infty} \frac{1}{k+1} \Pr\,(X = k)}{\Pr\,(X > 0)} \tag{4}$$

$$= \frac{\sum_{k=1}^{\infty} \frac{1}{k+1} e^{-\lambda}\frac{\lambda^k}{k!}}{1 - \Pr\,(X \leq 0)} \tag{5}$$

$$= \frac{e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k+1)!}}{1 - \Pr\,(X = 0)} \tag{6}$$

from equation (2)

$$= \frac{e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k+1)!}}{1 - e^{\lambda}} \tag{7}$$

Now simplifying Just the Summation

$$\implies \sum_{k=1}^{\infty} \frac{\lambda^k}{(k + 1)!} \tag{8}$$

$$= \frac{1}{\lambda} \sum_{k=1}^{\infty} \frac{\lambda^{k+1}}{k + 1} \tag{9}$$

Letting $k + 1 = m$,

$$\implies \frac{1}{\lambda} \sum_{m=2}^{\infty} \frac{\lambda^m}{m!} \tag{10}$$

We Know from Taylor series

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \tag{11}$$

$$\implies \frac{1}{\lambda} \sum_{m=2}^{\infty} \frac{\lambda^m}{m!} = \frac{1}{\lambda}\left(e^\lambda - 1 - \lambda\right) \tag{12}$$

Substituting back we get,

$$= \frac{e^{-\lambda}}{(1 - e^\lambda)}\left(\frac{1}{\lambda}\left(e^\lambda - 1 - \lambda\right)\right) \tag{13}$$

$$= \frac{e^{-\lambda}}{\lambda\,(1 - e^\lambda)}\left(e^\lambda - 1 - \lambda\right) \tag{14}$$

$$= \frac{1 - e^{-\lambda} - \lambda e^{-\lambda}}{\lambda\,(1 - e^\lambda)} \tag{15}$$

(B) Simulation

(i) In the code, it simulates the generation of RV X using the CDF of Poisson distribution.

$$F\,(x) = \sum_{k=0}^{x} e^{-\lambda}\frac{\lambda^k}{k!} \tag{16}$$

(ii) initialize $X = 0$

(iii) initialize $F = e^{-\lambda}$ which is $F\,(0)$ of a poisson distribution

(iv) generate a uniform random variable between 0 and 1

(v) enter the loop that continues as long as $u > F$ Inside the loop,

(vi) Increment X by 1

(vii) Add next term of Poisson pmf ($\Pr\,(X = k + 1)$) to $F\,(x)$

(viii) the loop continues until u is no longer greater than F. At this point, X represents the generated value of the poisson random variable that follows the desired poisson distribution with mean parameter λ.

(ix) Save all the values of poisson random variable X in pois.dat so to open it in python an plot the cdf graph

(x) Then for the second part Check if the generated value of X is greater than 0. If X is greater than 0, calculate the value Y as $\frac{1}{X+1}$ and add it to the

sumY.Increment the validCount to keep track of valid *X* values.

(xi) If validCount is greater than 0, calculate the estimate of the conditional expectation by dividing sumY by validCount.
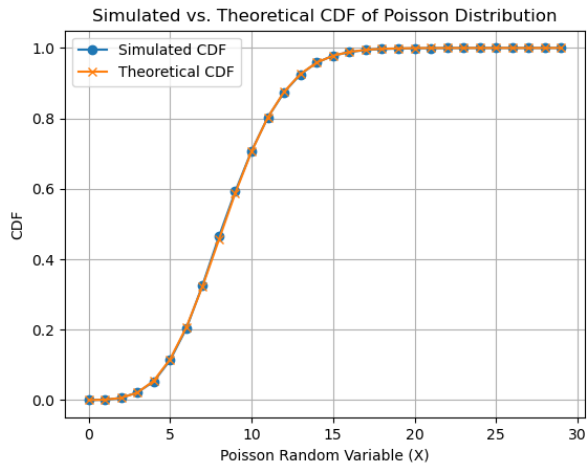

Fig. 1: Simulation Vs theoretical cdf of poisson distribution with $\lambda = 9$