

Assignment

Barath surya M — EE22BTECH11014

Question Let X be a random variable having poisson distribution with mean $\lambda > 0$. Then $E\left(\frac{1}{1+X} \mid X > 0\right)$ equals

- 1) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda(1-e^{-\lambda})}$
- 2) $\frac{1-e^{-\lambda}}{\lambda}$
- 3) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{1-e^{-\lambda}}$
- 4) $\frac{1-e^{-\lambda}}{\lambda+1}$

Solution:

(A) Theory

$$X \sim \text{Pois}(\lambda) \quad (1)$$

$$\Pr(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}; k \geq 0 \quad (2)$$

we know that

$$E(A|B) = \frac{E(A, B)}{\Pr(B)} \quad (3)$$

$$\Rightarrow E\left(\frac{1}{1+X} \mid X > 0\right) = \frac{\sum_{k=1}^{\infty} \frac{1}{k+1} \Pr(X = k)}{\Pr(X > 0)} \quad (4)$$

$$= \frac{\sum_{k=1}^{\infty} \frac{1}{k+1} e^{-\lambda} \frac{\lambda^k}{k!}}{1 - \Pr(X \leq 0)} \quad (5)$$

$$= \frac{e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k+1)!}}{1 - \Pr(X = 0)} \quad (6)$$

from equation (2)

$$= \frac{e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k+1)!}}{1 - e^{-\lambda}} \quad (7)$$

Now simplifying Just the Summation

$$\Rightarrow \sum_{k=1}^{\infty} \frac{\lambda^k}{(k+1)!} \quad (8)$$

$$= \frac{1}{\lambda} \sum_{k=1}^{\infty} \frac{\lambda^{k+1}}{k+1} \quad (9)$$

Letting $k+1 = m$,

$$\Rightarrow \frac{1}{\lambda} \sum_{m=2}^{\infty} \frac{\lambda^m}{m!} \quad (10)$$

We Know from Taylor series

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \quad (11)$$

$$\Rightarrow \frac{1}{\lambda} \sum_{m=2}^{\infty} \frac{\lambda^m}{m!} = \frac{1}{\lambda} (e^{\lambda} - 1 - \lambda) \quad (12)$$

Substituting back we get,

$$= \frac{e^{-\lambda}}{(1 - e^{-\lambda})} \left(\frac{1}{\lambda} (e^{\lambda} - 1 - \lambda) \right) \quad (13)$$

$$= \frac{e^{-\lambda}}{\lambda(1 - e^{-\lambda})} (e^{\lambda} - 1 - \lambda) \quad (14)$$

$$= \frac{1 - e^{-\lambda} - \lambda e^{-\lambda}}{\lambda(1 - e^{-\lambda})} \quad (15)$$

(B) Simulation

(i) In the code, it simulates the generation of RV X using the CDF of Poisson distribution.

$$F(x) = \sum_{k=0}^x e^{-\lambda} \frac{\lambda^k}{k!} \quad (16)$$

(ii) initialize $X = 0$

(iii) initialize $F = e^{-\lambda}$ which is $F(0)$ of a poisson distribution

(iv) generate a uniform random variable between 0 and 1

(v) enter the loop that continues as long as $u > F$
Inside the loop,

(vi) Increment X by 1

(vii) Add next term of Poisson pmf ($\Pr(X = k+1)$) to $F(x)$

(viii) the loop continues until u is no longer greater than F . At this point, X represents the generated value of the poisson random variable that follows the desired poisson distribution with mean parameter λ .

(ix) Then for the second part Check if the generated value of X is greater than 0. If X is greater than 0, calculate the value Y as $\frac{1}{X+1}$ and add it to the sum Y . Increment the validCount to keep track of valid X values.

(x) If validCount is greater than 0, calculate the es-

estimate of the conditional expectation by dividing
sumY by validCount.