

EE1205

EE22BTECH11014 - Barath Surya M

General form of GP

$$x(n) = x(0) r^n \quad (1)$$

Sum of first $n + 1$ terms of a geometric progression is defined as

$$y(n + 1) = \sum_{k=0}^n x(k) \quad (2)$$

$$y(n + 1) = \sum_{k=-\infty}^{\infty} x(k) u(n - k) \quad (3)$$

$$y(n + 1) = x(n) * u(n) \quad (4)$$

On Taking Z-Transform

$$Y(z) = X(z) U(z) \quad (5)$$

From Z-Transform of a term of GP

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad |rz^{-1}| < 1 \quad (6)$$

and Z-Transform of unit step is

$$U(z) = \frac{1}{1 - z^{-1}} \quad |z^{-1}| < 1 \quad (7)$$

$$\Rightarrow Y(z) = \left(\frac{x(0)}{1 - rz^{-1}} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad |rz^{-1}| < 1 \quad \text{and} \quad |z^{-1}| < 1 \quad (8)$$

By using Partial Fractions

$$Y(z) = x(0) \left(\frac{A}{1 - rz^{-1}} + \frac{B}{1 - z^{-1}} \right) \quad (9)$$

$$A = \frac{r}{r - 1} \quad (10)$$

$$B = \frac{-1}{r - 1} \quad (11)$$

Taking inverse Z-transform

$$y(n + 1) = \frac{x(0)}{r - 1} (r(r^n) - 1) u(n) \quad (12)$$

$$= x(0) \left(\frac{r^{n+1} - 1}{r - 1} \right) u(n) \quad (13)$$

for Sum of n terms $y(n)$ substitute $n + 1$ to n

$$y(n) = x(0) \left(\frac{r^n - 1}{r - 1} \right) u(n - 1) \quad (14)$$

$$= x(0) \left(\frac{r^n - 1}{r - 1} \right) \quad n > 0 \quad (15)$$