EE1205

EE22BTECH11014 - Barath Surya M

General form of GP

$$x(n) = x(0)\, r^n u(n) \tag{1}$$

Let

$$y(n) = x(n) * u(n) \tag{2}$$
$$= \sum_{k=-\infty}^{\infty} x(k)\, u(n-k) \tag{3}$$

on substituting (1) in The convolution sum

$$y(n) = \sum_{k=-\infty}^{\infty} x(0)\, r^n u(n)\, u(n-k) \tag{4}$$
$$= \sum_{k=0}^{n} x(0)\, r^n \tag{5}$$

Which gives us sum of GP, So On Taking Z-Transform on (2)

$$Y(z) = X(z)\, U(z) \tag{6}$$

From Z-Transform of a term in GP

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad |z| > |r| \tag{7}$$

and Z-Transform of unit step is

$$U(z) = \frac{1}{1 - z^{-1}} \quad \left|z^{-1}\right| < 1 \tag{8}$$
$$\implies Y(z) = \left(\frac{x(0)}{1 - rz^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right) \quad |z| > |r| \quad \text{and} \quad |z| > |1| \tag{9}$$

By using Partial Fractions

$$Y(z) = x(0)\left(\frac{A}{1 - rz^{-1}} + \frac{B}{1 - z^{-1}}\right) \tag{10}$$
$$A = \frac{r}{r-1} \tag{11}$$
$$B = \frac{-1}{r-1} \tag{12}$$

Using (7) and Taking inverse Z-transform

$$y(n) = \frac{x(0)}{r-1}\left(r\left(r^n\right) - 1\right) u(n) \tag{13}$$
$$= x(0)\left(\frac{r^{n+1} - 1}{r-1}\right) u(n) \tag{14}$$