EE1205

EE22BTECH11014 - Barath Surya M

General form of GP

$$x(n) = x(0)\, r^n \tag{1}$$

Sum of first $n+1$ terms of a geometric progression is defined as

$$y(n+1) = \sum_{k=0}^{n} x(k) \tag{2}$$

$$y(n+1) = \sum_{k=-\infty}^{\infty} x(k)\, u(n-k) \tag{3}$$

$$y(n+1) = x(n) * u(n) \tag{4}$$

On Taking Z-Transform

$$Y(z) = X(z)\, U(z) \tag{5}$$

From Z-Transform of a term of GP

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad \left|rz^{-1}\right| < 1 \tag{6}$$

and Z-Transform of unit step is

$$U(z) = \frac{1}{1 - z^{-1}} \quad \left|z^{-1}\right| < 1 \tag{7}$$

$$\implies Y(z) = \left(\frac{x(0)}{1 - rz^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right) \quad \left|rz^{-1}\right| < 1 \quad \text{and} \quad \left|z^{-1}\right| < 1 \tag{8}$$

By using Partial Fractions

$$Y(z) = x(0)\left(\frac{A}{1 - rz^{-1}} + \frac{B}{1 - z^{-1}}\right) \tag{9}$$

$$A = \frac{r}{r-1} \tag{10}$$

$$B = \frac{-1}{r-1} \tag{11}$$

Taking inverse Z-transform

$$y(n+1) = \frac{x(0)}{r-1}\left(r\left(r^n\right) - 1\right) u(n+1) \tag{12}$$

$$= x(0)\left(\frac{r^{n+1} - 1}{r-1}\right) u(n+1) \tag{13}$$

for Sum of n terms $y(n)$ substitute $n+1$ to n

$$y(n) = x(0)\left(\frac{r^n - 1}{r-1}\right) \tag{14}$$