EE1205

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let $x(n)$ have a z transform of $X(z)$, then

$$x(n) \xrightarrow{Z} X(z) \tag{1}$$

$$\implies X(z) = \sum_{k=-\infty}^{\infty} x(k) z^{-k} \tag{2}$$

Multiplying both side with z^{k-1} and integrating on a contour integral enclosing the region of convergence. Where C is a counter-clockwise closed contour in region of convergence.

$$\frac{1}{2\pi j} \oint_C X(z) z^{k-1}.dz = \frac{1}{2\pi j} \oint_C \sum_{k=-\infty}^{\infty} x(k) z^{-n+k-1}.dz \tag{3}$$

$$= \sum_{k=-\infty}^{\infty} x(k) \frac{1}{2\pi j} \oint_C z^{-n+k-1}.dz \tag{4}$$

From cauchy's integral theorem

$$\frac{1}{2\pi j} \oint_C z^{-k}.dz = \begin{cases} 1, & k = 1 \\ 0, & k \neq 1 \end{cases} \tag{5}$$

$$= \delta(1-k) \tag{6}$$

So eq (4) becomes

$$\frac{1}{2\pi j} \oint_C X(z) z^{k-1}.dz = \sum_{k=-\infty}^{\infty} x(k) \delta(k-n) \tag{7}$$

$$\implies x(n) = \frac{1}{2\pi j} \oint_C X(z) z^{n-1}.dz \tag{8}$$

Contour integrals like (8) can be evaluated using Cauchy's residue theorem.

$$x(n) = \frac{1}{2\pi j} \oint_C X(z) z^{n-1}.dz \tag{9}$$

$$= \sum \left[\text{Residue of } X(z) z^{n-1} \text{ at poles inside } C \right] \tag{10}$$

if $X(z) z^{n-1}$ is a Rational function of z then it can be expressed as

$$X(z) z^{n-1} = \frac{X'(z)}{(z - z_o)^s} \tag{11}$$

where $X(z)$ has s poles at $z = z_o$ and $X'(z)$ has no poles at $z = z_o$, Then

$$Res\left[X(z) z^{n-1} \text{ at } z = z_0 \right] = \frac{1}{(m-1)!} \left(\frac{d^{m-1}(X'(z))}{dz^{m-1}} \right) \tag{12}$$

for simple first order pole

$$Res\left[X(z) z^{n-1} \text{ at } z = z_0 \right] = X'(z_0) \tag{13}$$