

EE1205

EE22BTECH11014 - Barath Surya M

SECTION I

- 1) If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x .
- 2) Use elementary column operation $C_2 \rightarrow C_2 + 2C_1$ in the following matrix equation:

$$\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix}$$

- 3) Write the number of all possible matrices of order 2×2 with each entry 1,2 or 3.
- 4) Write the position vector of the point which divides the join of the points s with position vectors $3\vec{d} - 2\vec{b}$ and $2\vec{d} + 3\vec{b}$ in the ratio 2:1.
- 5) Write the number of vectors of unit length perpendicular to both the vector $\vec{d} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$.
- 6) Find the vector equation of the plane with intercepts 3,-4 and 2 on x, y and z -axis respectively.
- 7) Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the XZ plane. Also find the angle which this line makes with the XZ plane.
- 8) The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.
- 9) In a game, a man wins ₹5 for getting a number greater than 4 and loses ₹1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.
- 10) A bag contains 4 balls. Two balls are drawn at random (*without replacement*) and are found to be white. What is the probability that all balls in the bag are white ?
- 11) differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x .
- 12) if $y = \cos(\log x) + 2 \sin(\log x)$, prove that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$.
- 13) If $x = a \sin 2t(1 + \cos 2t)$ and $y = b \cos 2t(1 - \cos 2t)$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.
- 14) The equation of tangent at $(2, 3)$ on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b
- 15) Find : $\int \frac{x^2}{x^4 + x^2 - 2} dx$.
- 16) Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$.
- 17) Evaluate : $\int_0^{\frac{3}{2}} |x \cos \pi x| dx$
- 18) Find: $\int (3x + 1) \sqrt{4 - 3x - 2x^2} dx$.
- 19) Solve the differential equation : $y + x \frac{dy}{dx} = x - y \frac{dy}{dx}$.
- 20) Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.
- 21) Solve the equation for x : $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$.
- 22) if $\cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha$, prove that $\frac{x^2}{a^2} - 2 \frac{xy}{ab} \cos \alpha + \frac{y^2}{b^2} = \sin^2 \alpha$.
- 23) A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹2,800 as interest. However, if trust had interchanged money in bonds, they would have got ₹100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this question ?
- 24) There are two types of fertilisers "A" and "B". "A" consists of 12% nitrogen and 5% phosphoric acid whereas "B" consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions,

farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If "A" costs ₹10 per kg and "B" cost ₹8 per kg, then graphically determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost.

- 25) Five bad oranges are accidentally mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence find the mean and variance of the distribution.
- 26) Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i} + 3\hat{j} + \hat{k}$ to the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$. Also find image of P in the plane.
- 27) Show that the binary operation $*$ on $A = R - \{-1\}$ defined as $a * b = a + b + ab$ for all $a, b \in A$ is commutative and associative on A . Also find the identity element of $*$ in A and prove that every element of A is invertible.
- 28) Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}r$.
- 29) If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.
- 30) Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of square bounded by $x = 0, y = 4$ and $y = 0$ into three equal parts.
- 31) Using Properties of determinants, show that $\triangle ABC$ is isosceles if :
- $$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$
- 32) A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of ₹21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for ₹60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for ₹70. Using matrix method, find cost of each variety of pen.
- 33) Write the value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.
- 34) if $A = \begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}$ and $BA = (b_{ij})$ find $b_{21} + b_{32}$.
- 35) Write the number of all possible matrices of order 2×3 with each entry 1 or 2.
- 36) Write the coordinates for the point which is the reflection of the point (α, β, γ) in the XZ -plane.
- 37) find the position vector of the point which divides the join of points with position vectors $\vec{a} + 3\vec{b}$ and $\vec{a} - \vec{b}$ internally in the ratio 1:3.
- 38) if $|\vec{a}| = 4, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 6\sqrt{3}$, then find the value of $|\vec{a} \times \vec{b}|$.
- 39) solve for x : $\tan^{-1}\left(\frac{2-x}{2+x}\right) = \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right), x > 0$.
- 40) prove that $2 \sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$
- 41) On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹10 more. However, if there were 16 children more, every one would have got ₹10 less. Using matrix method, find the number of children and the amount distributed by Seema. What values are reflected by Seema's decision ?
- 42) if $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.
- 43) Verify Mean Value theorem for the function $f(x) = 2 \sin x + \sin 2x$ on $[0, \pi]$.
- 44) Show that the function f given by :

$$f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$$

is discontinuous at $x = 0$.

- 45) Find the equation of the tangent line to the curve $y = \sqrt{5x-3}-5$, which is parallel to line $4x-2y+5=0$.
- 46) Evaluate : $\int_1^5 \{|x-1| + |x-2| + |x-3|\} dx$.
- 47) Evaluate : $\int_0^{\pi} \frac{x \sin x}{1+3 \cos^2 x} dx$.
- 48) Find : $\int \frac{2x+1}{(x^2+1)(x^2+4)} dx$.
- 49) Find : $\int (3x+5) \sqrt{5+4x-2x^2} dx$.
- 50) Solve the differential equation $x \frac{dy}{dx} + y - x + xy \cot x = 0$; $x \neq 0$.
- 51) Solve the differential equation: $(x^2 + 3xy + y^2) dx - x^2 dy = 0$ given that $y = 0$, when $x = 1$.
- 52) Find the angle between the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ if $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$, and hence find a vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.
- 53) Show that the lines $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$ intersect. Find their point of intersection.
- 54) A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.
- 55) A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6
Pr(X)	C	2C	2C	3C	C ²	2C ²	7C ² + C

Find the value of C and also calculate mean of the distribution.

- 56) Solve for x: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$, using properties of determinants.
- 57) Using elementary row operations find the inverse of matrix $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ and hence solve the following system of equations $3x - 3y + 4z = 21$, $2x - 3y + 4z = 20$, $-y + z = 5$.
- 58) Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi-vertical angle α is one-third that of the cone and the greatest volume of the cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.
- 59) Find the intervals in which the function $f(x) = \frac{4 \sin x}{2 + \cos x} - x$; $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.
- 60) Using integration, find the area of the triangle formed by negative x-axis and tangent and normal to the circle $x^2 + y^2 = 9$ at $(-1, 2\sqrt{2})$.
- 61) Find the coordinates of the foot of perpendicular and perpendicular distance from the point $P(4, 3, 2)$ to the plane $x + 2y + 3z = 2$. Also find the image of P in the plane.
- 62) A, B and C throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning, if A starts first.
- 63) A company manufactures two types of cardigans : type A and type B. It costs ₹360 to make a type A cardigan and ₹120 to make a type B cardigan. The company can make at most 300 cardigans and spend at most ₹72,000 a day. The number of cardigans of type B cannot exceed the number of cardigans of type A by more than 200. The company makes a profit of ₹100 for each cardigan of type A and ₹50 for every cardigan of type B. Formulate this problem as a linear programming problem to maximise the profit to the company. Solve it graphically and find maximum profit.