EE1205

EE22BTECH11014 - Barath Surya M

$$x(n) \xrightarrow{\text{Z}} X(z) \tag{1}$$

$$\implies X(z) = \sum_{k=-\infty}^{\infty} x(k) z^{-k} \tag{2}$$

Multiplying both side with z^{k-1} and integrating on a contour integral enclosing the region of convergence. Where C is a counter-clockwise closed contour in region of convergence.

$$\frac{1}{2\pi j} \oint_C X(z) z^{k-1} dz = \frac{1}{2\pi j} \oint_C \sum_{k=-\infty}^{\infty} x(k) z^{-n+k-1} dz \tag{3}$$

$$= \sum_{k=-\infty}^{\infty} x(k) \frac{1}{2\pi j} \oint_C z^{-n+k-1} .dz \tag{4}$$

From cauchy's integral theorem

$$\frac{1}{2\pi j} \oint_C z^{-k} dz = \begin{cases} 1, & k = 1 \\ 0, & k \neq 1 \end{cases} \tag{5}$$

$$= \delta(1-k) \tag{6}$$

So eq (4) becomes

$$\frac{1}{2\pi j} \oint_C X(z) z^{k-1} dz = \sum_{k=-\infty}^{\infty} x(k) \delta(k-n) \tag{7}$$

$$\implies x(n) = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz \tag{8}$$

Contour integrals like (8) can be evaluated using Cauchy's residue theorem.

$$x(n) = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz \tag{9}$$

$$= \sum \left[\text{Residue of } X(z) z^{n-1} \text{ at poles inside } C \right] \tag{10}$$