Complex value derivative identities

$$\frac{\partial \mathbf{a}^{\mathrm{H}}\mathbf{x}}{\partial \mathbf{x}} = \mathbf{a}^{*} \tag{1}$$

$$\frac{\partial \mathbf{x}^{\mathrm{H}}\mathbf{a}}{\partial \mathbf{x}} = \mathbf{0} \tag{2}$$

$$\frac{\partial \mathbf{x}^{\mathrm{H}}\mathbf{A}\mathbf{x}}{\partial \mathbf{x}} = \mathbf{A}^{\mathrm{T}}\mathbf{x}^{*} \tag{3}$$

$$\frac{\partial \mathbf{y}^{\mathrm{H}}\mathbf{X}\mathbf{a}}{\partial \mathbf{X}} = \mathbf{y}^{*}\mathbf{a}^{\mathrm{T}} \tag{4}$$

Minimum Mean Square Error (MMSE)

The system model:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{5}$$

The MMSE estimator of $\mathbf{x}$ is:

$$\arg\min_{\mathbf{W}} \mathbb{E}\left\{\|\mathbf{W}\mathbf{y} - \mathbf{x}\|^{2}\right\} \tag{6}$$

where the expectation is with respect to $\mathbf{x}$. To find it:

$$\begin{gathered} \frac{\partial \mathbb{E}\left\{\|\mathbf{W}^{\mathrm{H}}\mathbf{y} - \mathbf{x}\|^{2}\right\}}{\partial \mathbf{W}} = \\ \mathbb{E}\left\{\frac{\partial\left(\mathbf{W}^{\mathrm{H}}\mathbf{y}\mathbf{y}^{\mathrm{H}}\mathbf{W} - \mathbf{y}^{\mathrm{H}}\mathbf{W}\mathbf{x} - \mathbf{x}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{y} + \|\mathbf{x}\|^{2}\right)}{\partial \mathbf{W}}\right\} = 0 \end{gathered} \tag{7}$$

$$\left(\mathbf{y}\mathbf{y}^{\mathrm{H}}\right)^{\mathrm{T}}\mathbf{W}^{*} - \mathbf{y}^{*}\mathbf{x}^{\mathrm{T}} = 0 \tag{8}$$

which results in:

$$\mathbf{W} = \mathbb{E}\left\{(\mathbf{y}\mathbf{y}^{\mathrm{H}})^{-1}\mathbf{y}\mathbf{x}^{\mathrm{H}}\right\} \tag{9}$$

Substitute $\mathbf{y}$ with $\mathbf{H}\mathbf{x} + \mathbf{n}$ and we get:

$$\mathbf{W} = \left(\mathbf{H}\mathbf{R}_{\mathbf{xx}}\mathbf{H}^{\mathrm{H}} + \mathbf{R}_{\mathbf{nn}}\right)^{-1}\mathbf{H}\mathbf{R}_{\mathbf{xx}} \tag{10}$$

Therefore, to estimate $\mathbf{x}$:

$$\hat{\mathbf{x}} = \mathbf{R}_{\mathbf{xx}}\mathbf{H}^{\mathrm{H}}\left(\mathbf{H}\mathbf{R}_{\mathbf{xx}}\mathbf{H}^{\mathrm{H}} + \mathbf{R}_{\mathbf{nn}}\right)^{-1}\mathbf{y} \tag{11}$$