1 Triangulation

Suppose $(\cos(\varphi_1), \sin(\varphi_1)$ and $(\cos(\varphi_2), \sin(\varphi_2)$ are the direction vector from access points 1 and 2 toward the user. Note that φ is the angle of arrival at the AP. Moreover, (x_1, y_1) and (x_2, y_2) represent the location of these APs. The line equation passing through the AP and user are as follow:

$$(x_{\mathrm{u}}, y_{\mathrm{u}}) = (x_1, y_1) + t_1 \cdot ((\cos(\varphi_1), \sin(\varphi_1)) \tag{1}$$

$$(x_{\mathrm{u}}, y_{\mathrm{u}}) = (x_2, y_2) + t_2 \cdot ((\cos(\varphi_2), \sin(\varphi_2)) \tag{2}$$

To find the intersection between two lines, we equate two parametric form:

$$(x_1, y_1) + t_1 \cdot ((\cos(\varphi_1), \sin(\varphi_1)) = (x_2, y_2) + t_2 \cdot ((\cos(\varphi_2), \sin(\varphi_2)) \tag{3}$$

We can seperate the above equation into two seperate equations:

$$x_1 + t_1 \cdot \cos(\varphi_1) = x_2 + t_2 \cdot \cos(\varphi_2) \tag{4}$$

$$y_1 + t_1 \cdot \sin(\varphi_1) = y_2 + t_2 \cdot \sin(\varphi_2) \tag{5}$$

Writing these equation into matrix form:

$$\underbrace{\begin{bmatrix} \cos(\varphi_1) & -\cos(\varphi_2) \\ \sin(\varphi_1) & -\sin(\varphi_2) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} t_1 \\ t_2 \end{bmatrix}}_{\mathbf{t}} = \underbrace{\begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \end{bmatrix}}_{\mathbf{b}} \tag{6}$$

To find user location we should solve the sytem of linear equations for $\mathbf{t}$.

2 Trilateration

Suppose the distance from the user to three access points is denoted by d_i for $i \in \{1, 2, 3\}$. Moreover, (x_i, y_i) denote the location of i-th AP. We have

$$d_1^2 = (x_u - x_1)^2 + (y_u - y_1)^2 \tag{7}$$

$$d_2^2 = (x_u - x_2)^2 + (y_u - y_2)^2 \tag{8}$$

$$d_3^2 = (x_u - x_3)^2 + (y_u - y_3)^2 \tag{9}$$

Expanding the equations:

$$d_1^2 = x_u^2 + x_1^2 - 2x_u x_1 + y_u^2 + y_1^2 - 2y_u y_1 \tag{10}$$

$$d_2^2 = x_u^2 + x_2^2 - 2x_u x_2 + y_u^2 + y_2^2 - 2y_u y_2 \tag{11}$$

$$d_3^2 = x_u^2 + x_3^2 - 2x_u x_3 + y_u^2 + y_3^2 - 2y_u y_3 \tag{12}$$

Subtracting these equation and rearranging:

$$2x(x_1 - x_2) + 2y(y_1 - y_2) = x_1^2 + y_1^2 - x_2^2 - y_2^2 + d_2^2 - d_1^2 \tag{13}$$

$$2x(x_1 - x_3) + 2y(y_1 - y_3) = x_1^2 + y_1^2 - x_3^2 - y_3^2 + d_3^2 - d_1^2 \tag{14}$$

writing it in matrix form

$$\underbrace{\begin{bmatrix} 2(x_1 - x_2) & 2(y_1 - y_2) \\ 2(x_1 - x_3) & 2(y_1 - y_3) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\mathbf{p}} = \underbrace{\begin{bmatrix} x_1^2 + y_1^2 - x_2^2 - y_2^2 + d_2^2 - d_1^2 \\ x_1^2 + y_1^2 - x_3^2 - y_3^2 + d_3^2 - d_1^2 \end{bmatrix}}_{\mathbf{B}} \tag{15}$$

and solving for $\mathbf{p}$, the user location is estimated.