Network Simulation
Model & Analysis

The model presented in this paper aims to simulate the behavior of a small social network. In this model, we assume a dynamic network, thus implementing an adaptive network model. In adaptive networks, the state and the topology of the network interact with each other, and the changes are often observed given the same time scales (Sayama, 2015). This is an extension of the social dynamic model implemented by Prof. Scheffler and presented in class.

In implementation, the assumptions considered are:

- Each person (node) has an opinion in two topics: 1) religion and 2) politics. A value of 0 in politics is assumed to be a liberal person, whereas a value of 1 represents a conservative person. In religion, a value of 0 means that a person is not religious, and a value of 1 means that the person is (no matter which religion);

- The nodes' opinions change with every interaction, given that in a real life scenario, our opinions are constantly questioned;

- The strength of the relationships are based on how close their opinions are;

- In every interaction between two nodes, the interaction is based on only one topic randomly chosen, which in turn affects their overall opinion attribute. The assumption is that religion and politics are two complex topics, and therefore people will only have the energy to talk about one of them during one interaction. However, their friendships are based on all of their opinions, since friends talk about all of them at some point, and thus the strength of their relationship is based on their overall opinions;

- Every node has a pre-defined value for charisma and stubbornness. Their charisma value will make them more likely to influence the other person's opinions whereas their stubbornness value will make them less likely to be influenced;

- Nodes with connections between 0.8 and 0.9 (on a scale of 0 to 1) will have their connections strengthened by 10%. Nodes with connections between 0.2 and 0.06 will have their connections weakened by 20%. This is because we assume that people with really strong relationships will be likely to rely on those more, thus positively reinforcing these ties. On the other hand, people with very weak relationships will be less likely to reach out to each other, and thus their relationship will deteriorate faster than the others. We also assume that the deterioration process is faster than the strengthening because in order to reach out to less-close friends, more energy is required, as the interaction might not come out so naturally as it does with a close friend;

- New relationships are made based on proximity. First, a new relationship will be made with a friend of a friend. If a person is already connected with all their friend's friends, then a random connection will be made. If a person has initially no friends, then a random connection will also be made. The reasoning behind it is that it is more likely that you will be introduced to new people through common connections.

The update process for the opinion value and the strength of the relationship is given by the following functions:

$\Delta o_i = \alpha * w_{ij} * (o_j - o_i)$

$\Delta w_{ij} = \beta * w_{ij} * (1 - w_{ij}) * (1 - \gamma * |o_j - o_i|)$

Where α is the rate at which opinions converge and β indicates the rate at which relationships will change. Both are bounded between 0 and 1. The difference in opinion and the weights has to be between 0 and 1:- 0 means a perfect agreement in opinions while 1 means a perfect disagreement; while for the weights, 0 means that there is no connection while 1 means that there is a perfect connection. Thus, γ is bounded by 0 and $1 + 1/\beta$ so such constraints can be satisfied.

To account for the charisma and stubbornness of each person, a new variable ε is introduced. The update function for the delta opinion is:

$\Delta o_i = (\varepsilon + \alpha) * w_{ij} * (o_j - o_i)$

Thus, $(\varepsilon + \alpha)$ now represents the rate at which opinions converge, which as mentioned before is bounded between 0 and 1. ε is calculated by taking the difference of the other node's charisma and the chosen node's stubbornness. In the simulation, the maximum rate at which the opinions converge is taken as a parameter max_alpha. Then, the alpha value is taken as a random value between max_alpha/2 and max_alpha; ε is then bounded between 0 and (max_alpha - alpha); this way we ensure that $(\varepsilon + \alpha)$ is always within the max_alpha limit.

With this models assumptions, the network behavior can be seen through its vector field plot:

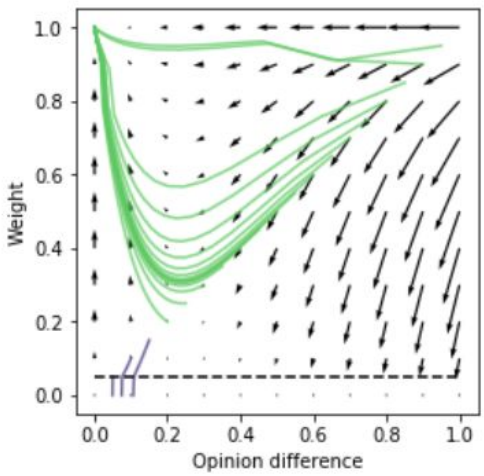

Figure 1-Vector field plot of the social network model. The x axis represents the difference in opinion between two nodes and the y axis represents the connection weight between them. The parameters for this model are: Max alpha=0.25; Beta=0.3; Gamma=4.The dashed line is set at weight = 0.05, which is when the relationship should break.

We can see then that, overall, the nodes move towards converging their opinions and strengthening their values. However, if the existing weight connection is less than 0.2, the tendency is that the relationship between two nodes will be broken. This is coherent with the model description that the relationship should decrease at a 20% faster rate than the usual when the weight is less than 0.2. On the other hand, when the weight is more than 0.3, with a difference in opinion around 0.2 or less, we can observe that there is only an upward tendency, and thus we should expect clusters to form within the network, as the relationships get stronger and the difference in opinions is reduced even more. However, it is important to notice that the given analysis is applicable for the given parameters of alpha, beta, and gamma.

For instance, if the maximum rate of convergence (max alpha) is set to 0.05, we will see a very different trend.

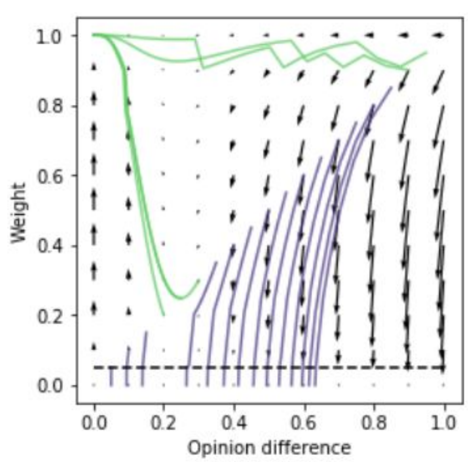

Figure 2-Vector field plot of the social network model. The x axis represents the difference in opinion between two nodes and the y axis represents the connection weight between them. The parameters for this model are: Max

alpha=0.05; Beta=0.3; Gamma=4.The dashed line is set at weight = 0.05, which is when the relationship should break.

In this case, we can see that there is a stronger tendency towards breaking relationships than making strong clusters. This makes sense, as the rate of converge is extremely low. We can note, however, that even if nodes have a strong difference in opinions, if their relationship is around 0.9, the tendency is that the relationship gets stronger regardless of the high opinion difference. We can also see the behavior represented by this vector field, as well as the previous one, in the simulation results ran with the specified parameters below.

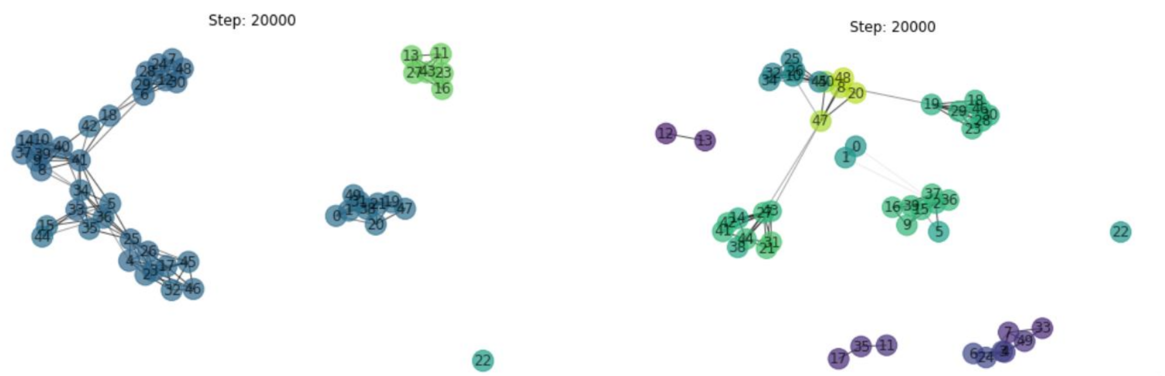

Figure 3- Simulation results at step 2000 for different parameter values. The network on the left represents the first vector field in which the max alpha value is set to 0.25. The network on the right represents the second vector field, in which max alpha is set to 0.05

Although it is possible to see clusters being formed in the network graph on the right, with max alpha of 0.05, small clusters are more common compared to the network graph on the left. Nodes will be less likely to converge to a common opinion (given the low max alpha value), and thus will be less likely to branch out or include more nodes to their own cluster.

Simulation

In this simulation, I chose to implement a Watts-Strogatz graph. This graph initializes the nodes in a ring topology connected with its k nearest neighbors (in this case k is set to 2). In small communities, I assume that a relationship based on physical location is more representative of reality than prioritize connections given nodes' popularity (as it happens in the Barabasi-Albert graph). Take the class of 2020 at Minerva, for instance, as a small community, we know that eventually we will all be connected, so there is no rush in ensuring that we are with the most initially well connected nodes right away. Instead, we initially connect with the people physically close to us, like our roommates, and then eventually branch out.

For this project, I ran different versions of this model to observe how the network would behave given the different modifications added to the network. My focus was to compare the convergence behavior of difference approaches, so I set max alpha to 0.05 so any discrepancies caused by the different modifications in the code could be easily spotted. Commenting out the portion of the code that ensures that new connections are made only with friends of friends, we see that it is less likely that clusters are formed:

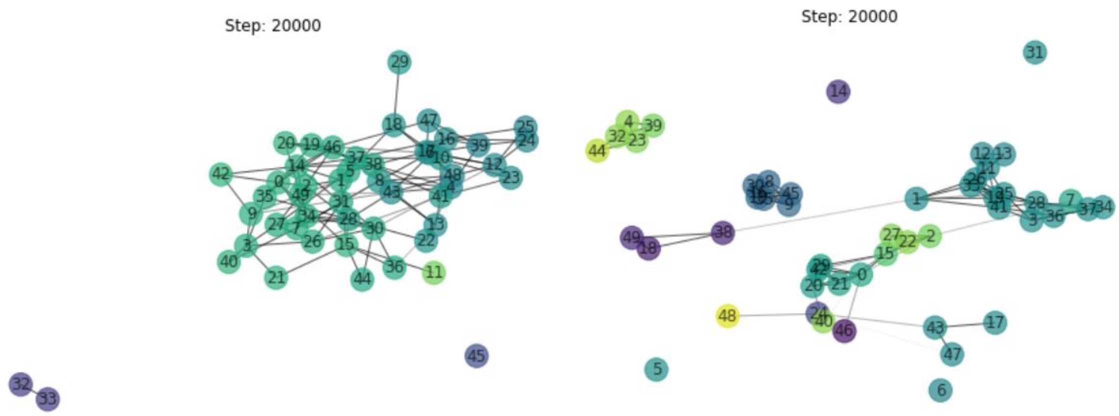

Figure 4 - The graph on the left does not include the modification that only connects friends of friends. The graph on the right does include this modification

We can see that when new connections are not restrained by friends of friends, we get fewer clusters; the network rather closer converges to a consensus. It is interesting to note that new connections only have a 1% chance to be formed, however, even with such a small chance, the difference in the network is remarkable, as we can see in the graph on the right (Figure 4).

Additionally, when removing the charisma and stubbornness factors for convergence of opinions, we can also see that convergence happens faster. In this test I took out the charisma and stubbornness factors and made alpha take on the max alpha value of 0.25.

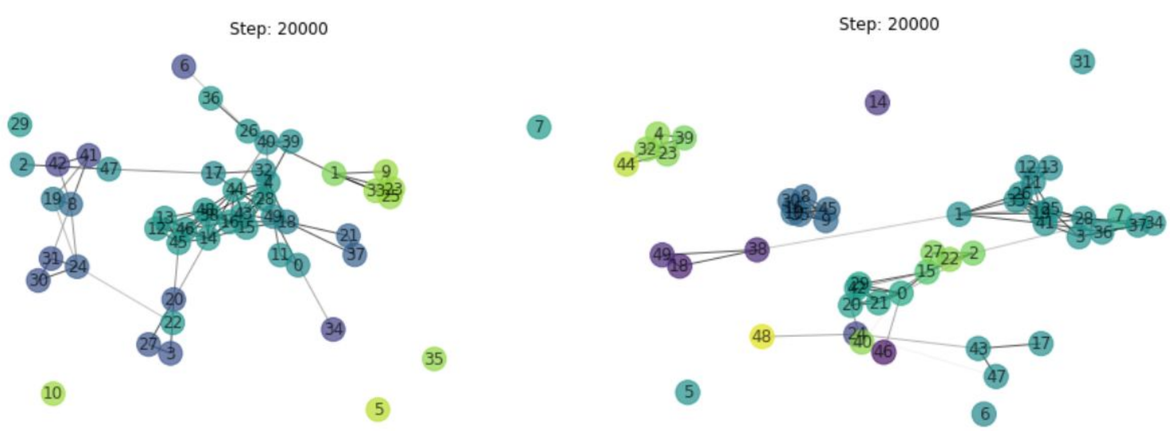

Figure 5 - The graph on the left does not include the modification that takes into consideration the charisma and stubbornness of nodes. The graph on the right does include this modification.

Although the network on the left does not fully converge, it does so better than the one in the right. When we take into consideration the charisma and the stubbornness of nodes, we have a variable total rate of convergence $(\varepsilon + \alpha)$ that can take at most the max alpha value, and thus will converge slower than when considering the full max alpha value for all node interactions, like we do with the graph on the left.

Finally, if we don't take into consideration the strengthening (for weights > 0.8) and weakening of connections (for weights < 0.2), we also observe a different behavior.

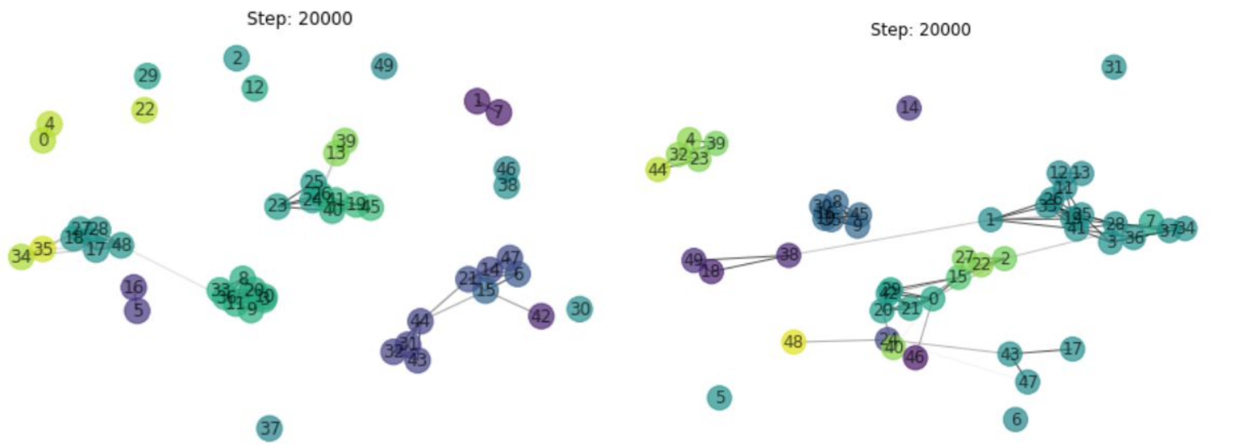

Figure 6 - The graph on the left does not include the modification in which nodes strengthen their connection if the weight is more than 0.8 and weaken the connection if the weight is less than 0.2. The graph on the right does include this modification.

When not taking into consideration the strengthening of weakening effect, we end up with more isolates nodes, as we can see in the graph on the left. That might be most influenced by the strengthening factor. In this case, nodes that reach a high weight will not have it deteriorated even if their opinions change given their interactions with other nodes, and thus makes it less likely that nodes break off. On the other hand, nodes that reach a weight of 0.2 would be most likely to have their relationship deteriorated anyway and eventually have the connection broken off at weight equals to 0.05, so the accelerated break off is not affecting the network that much, as we can see in the graph on the right.

Final Considerations

The proposed model takes more factors inherent to human interactions into account, and thus might provide a more accurate representation of a real small social network than the original base model provided in class. However, it is important to notice that it still holds limitations. For instance, by assuming that the strength of nodes connections is based on the average opinions in politics and religion, it oversimplifies the effect of each in an interaction. Highly politicized people, for instance, might not want to hold a relationship with someone that holds different political views, even if their opinion on religion is well aligned. Additionally, it does not take into consideration the fact that in a real social network, people might leave it and new people can enter it at some point (if we consider that the network represented is a school class, for instance).

References

Sayama, H. (2015). *Introduction to the modeling and analysis of complex systems*. Geneseo, NY: Published by Open SUNY Textbooks, Milne Library, State University of New York at Geneseo.