

DAY 8

Q. D. gm 9/12/2022

ESTIMATORS

TO FAKE OR NOT

• A coin is flipped 6 times and the output is

1 = HEADS

0 = TAILS

What is the probability of it coming out as heads?

$$100101 \rightarrow P(\text{HEADS}) = \frac{3}{6} = 0.5$$

$$11011 \rightarrow P(\text{HEADS}) = \frac{4}{5} = 0.8$$

$$0000000 \rightarrow P(\text{HEADS}) = \frac{0}{7} = 0$$

DATA

x_1, x_2, \dots, x_n

$$0 \leq x_i$$

$$x_i \in \{0, 1\}$$

$$0 \leq \prod_i x_i$$

$$0 \leq \frac{1}{n} \sum_i x_i$$

$$\frac{1}{N} \sum_{i=1}^N x_i$$

between 0, 1

→ Maximum likelihood estimator
(MLE)

DICE



$N=10$

1 6 6 3 2 6 5 4 6 2

$$p(1) = 1/10$$

$$p(2) = 2/10$$

$$p(3) = 1/10$$

$$p(4) = 1/10$$

$$p(5) = 1/10$$

$$p(6) = 4/10$$

Total

$$\frac{10}{10} = 1$$

MAXIMUM LIKELIHOOD ESTIMATOR

ESTIMATION PROBLEM is: DATA $\xrightarrow{\text{Given}}$ $P \xrightarrow{\text{find th}}$ $P \xrightarrow{\text{probability}}$ $P(DATA)$

$P \xrightarrow{\text{find th}}$ $P(DATA)$

DATA 101

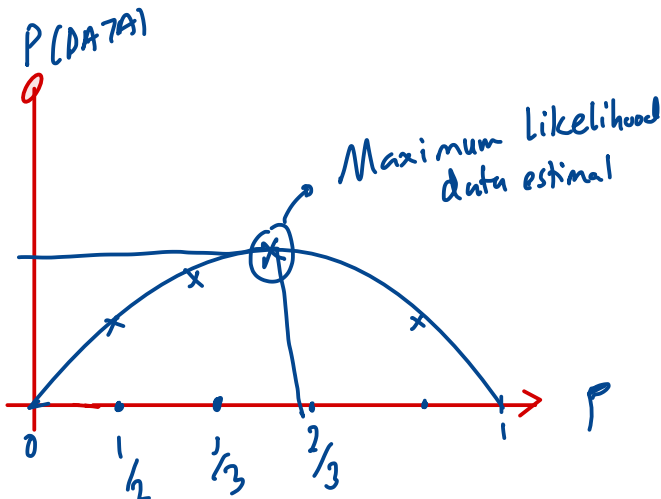
P probability

$P = 1/2$ $P(DATA) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = 0.125$

$P = 1/3$ $P(DATA) = \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{2}{27}$

$P = \frac{2}{3}$ $P(DATA) = \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{27}$

$P = 1$ $P(DATA) = 1 \times 0 \times 1 = 0$



WEAKNES

BE WICKED:

1

0

$$p(\text{HEADS}) = \frac{1}{1} = 1$$

$$p(\text{HEADS}) = \frac{0}{1} = 0$$

From a single coin flip, the ML^E will
always assume a loaded coin?

Soln

If the coin is fair $p = 1/2$

1

0

$$p(\text{HEAD}) = \frac{1}{2} \times 1 = 1/2$$

$$p(\text{HEAD}) = \frac{1}{2} \times 0 = 0$$

⊗ Yes

o No

SOLUTION: Add fake data!

Data \rightarrow DATA \times HEADS + TAILS

1
pHeads

001

1

1

1/3

2/3

101

2/5

✓
TO FAKE OR NOT TO FAKE?

↳ when data is scarce. Gives better and smoother results.