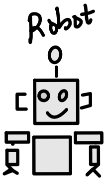
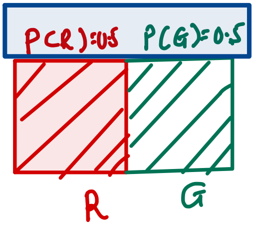
Day 5

6th Dec. 2022





Two conditions exist

- Location : @ R or @ G
- Colour it sees : sees R or sees G

Question : sees R

Posterior probabilities

$P(\text{at } R \mid \text{see } R) =$

$P(\text{at } G \mid \text{see } R) =$

Conditional Probabilities

$P(\text{see } R \mid \text{at } R) = 0.8$

$P(\text{see } G \mid \text{at } G) = 0.5$

↑ colour　　↑ location

Information

- Unknown
- Known

Solution : $P(\text{at } R \mid \text{see } R)$

Bayes R

$$P(\text{at } R \mid \text{see } R) = \frac{P(\text{see } R \mid \text{at } R) \cdot P(R)}{P(\text{see } R)}$$

Conditional Probabilities

$P(\text{see } R \mid \text{at } R) = 0.8$

$P(\text{see } G \mid \text{at } G) = 0.5$

$P(\text{see } G \mid \text{at } R) = 0.2$

$P(\text{see } R \mid \text{at } G) = 0.5$

Joint: sees R

$P(\text{see } R, \text{at } R) = 0.8 \cdot 0.5 = 0.4$

$P(\text{see } R, \text{at } G) = 0.5 \cdot 0.5 = 0.25$

Normalizer $P(\text{see } R) = 0.4 + 0.25 = 0.65$

$$P(\text{at } R \mid \text{see } R) = \frac{0.4}{0.65} = 0.6154$$

$$P(\text{at } G \mid \text{see } R) = \frac{0.25}{0.65} = 0.3846$$

$$\Sigma = 0.6154 + 0.3846 = 1$$

See G

$$P(\text{at } G \mid \text{see } G) = \frac{P(\text{see } G \mid \text{at } G) \cdot P(G)}{P(\text{see } G)}$$

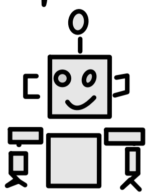
$$= \frac{0.5 \cdot 0.5}{[0.5 \cdot 0.5] + [0.2 \cdot 0.5]} = \frac{0.25}{0.25 + 0.1} =$$

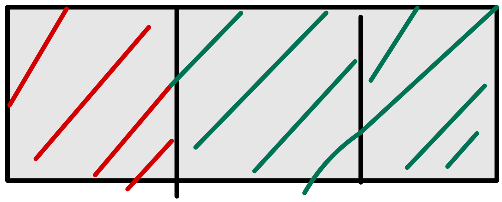
$$= 0.714$$

Question

$$P(A) = P(B) = P(C) = 1/3$$

Robo





A B C

2 colours
3 locations

Conditional probabilities

$P(R \mid A) = 0.9$; $P(G \mid A) = 0.1$

$P(G \mid B) = 0.9$; $P(R \mid B) = 0.1$

$P(G \mid C) = 0.9$; $P(R \mid C) = 0.1$

Robot sees R

Joint : $P(A, R) = 1/3 \cdot 0.9 = 0.3$

$P(B, R) = 1/3 \cdot 0.1 = 0.0333$

$P(C, R) = 1/3 \cdot 0.1 = 0.0333$

$$P(R) = 0.3666$$

$$P(A \mid R) = \frac{\frac{1}{3}\cdot 0.9}{\underbrace{\left[\frac{1}{3}\cdot 0.9\right]}_{\text{At } A\text{, sees } R} + \underbrace{\left[\frac{1}{3}\cdot 0.1\right]}_{\text{At } B\text{, sees } R} + \underbrace{\left[\frac{1}{3}\cdot 0.1\right]}_{\text{At } C\text{, sees } R}} = \frac{0.3}{0.3 + 0.033 + 0.033} = \frac{0.3}{0.3666} = 0.8183$$

$$P(B \mid R) = \frac{0.033}{0.3666} = 0.0908$$

$$P(C \mid R) = \frac{0.033}{0.3666} = 0.0908$$

SEBASTIAN: $P(gone) = 0.6$

$P(home) = 0.4$

$P(rain \mid home) = 0.01$

$P(rain \mid gone) = 0.3$

P gone and it rains

$$P(home \mid rain) = \frac{0.4 \cdot 0.01}{0.4 \cdot 0.01 + 0.6 \cdot 0.3}$$

P at home and rains

$$= \frac{0.004}{0.004 + 0.18}$$

$$= 0.0217$$

Programming assignment

Truth table

Flip 1	Flip 2	Flip 3
H	H	H
T	T	T
H	T	H
T	H	T
H	T	T
T	H	H
H	H	T
T	T	H

One head in 3 flips

Flip 1	Flip 2	Flip 3
H	H	H
H	T	H
T	H	H
T	T	H
H	H	T
H	T	T
T	H	T
T	T	T

$$p(T)\cdot p(T)\, p(H) + p(H)\cdot p(T)\cdot p(T) + p(T)\cdot p(H)\cdot p(T)$$

$$p(T) = 1 - p(H)$$

1i

Flip 1	Flip 2	Flip 3		
H	T	T	0.5 · 0.5 · 0.5	0.125
T	H	T		0.125
T	T	H		0.125

→ p(h) = p

$p \cdot (1-p) \cdot (1-p)$

Coin 1	Coin 2	probability
H	H	$p_1 \cdot p_2$
H	T	
T	H	
T	T	

Pick one coin

C_1 $p(H \mid C_1) = p_1$

$p(H \mid C_2) = p_2$

$p(C_1) = p_0$

$p(C_2) = 1 - p_0$

$p(H) = ?$

$p_0 = 0.3$

$p_1 = 0.5$

$p_2 = 0.9$

$p(H, C_1) = 0.3 \cdot 0.5 = 0.15$

$p(H, C_2) = 0.7 \cdot 0.9 = 0.63$

$p(H) = p(H, C_1) + p(H, C_2) = 0.78$

CANCER EXAMPLE

$P(C) = P_0$ 0.1

$P(Pos \mid C) : P_1$ 0.9

$P(Neg \mid \neg C) : P_2$ 0.8

$P(\neg C) = 0.9$

$P(Neg \mid C) = 0.1$

$P(Pos \mid \neg C) = 0.2$

$$P(Pos) = P(C) \cdot P(Pos \mid C) + P(\neg C) \cdot P(Pos \mid \neg C)$$

$$= 0.1 \cdot 0.9 + 0.9 \cdot 0.2$$

$$= 0.09 + 0.18$$

$$= 0.27$$

$$\begin{array}{r} 0.09 \\ 0.18 \\ \hline 27 \end{array}$$

$$P(C \mid Pos) = \frac{P(C) \cdot P(Pos \mid C)}{P(C) \cdot P(Pos \mid C) + P(\neg C) + P(Pos \mid \neg C)}$$

$$= \frac{0.1 \cdot 0.9}{0.27} = \frac{0.09}{0.27} = 0.333$$

Bayles Rule

$p(\neg B | \neg A) = p_2$

$$\frac{p_0 \cdot p_1}{}$$

$p(A) = p_0$ $\quad p(B) = 1 - p_0$

$p(B|A) = p_1$ $\quad p(\neg B | A) = 1 - p_1$

$p(\neg B | \neg A) = p_2$ $\quad p(B | \neg A) = 1 - p_2$

$$\boxed{p(A|B) = \frac{p(A) \cdot p(B/A)}{p(B)}}$$

$$\boxed{p(B) = p(A) \cdot p(B/A) + p(B) \cdot p(B | \neg A)}$$

$$p(A|B) = \frac{p_0 \cdot p_1}{p_0 \cdot p_1 + (1 - p_0) \cdot p_2}$$

$p(B) =$

given (known)

$$P(A \mid B)$$

unknown (to find)

Prior probability

$p(A) = p0$

$p(B) = 1 - p0$

Conditional probability

$p(B \mid A) = p1$

$p(\neg B \mid \neg A) = p2$

$P(\neg B \mid A) = 1 - p1$

$p(B \mid \neg A) = 1 - p2$

Bayes Rule:

$$P(A \mid B) = \frac{p(A) \cdot p(B \mid A)}{p(A) \cdot p(B \mid A) + p(\neg A) \cdot p(B \mid \neg A)}$$

$$\boxed{p(A \mid B) = \frac{p0 \cdot p1}{p0 \cdot p1 + (1 - p0) \cdot (1 - p2)}}$$

To Code