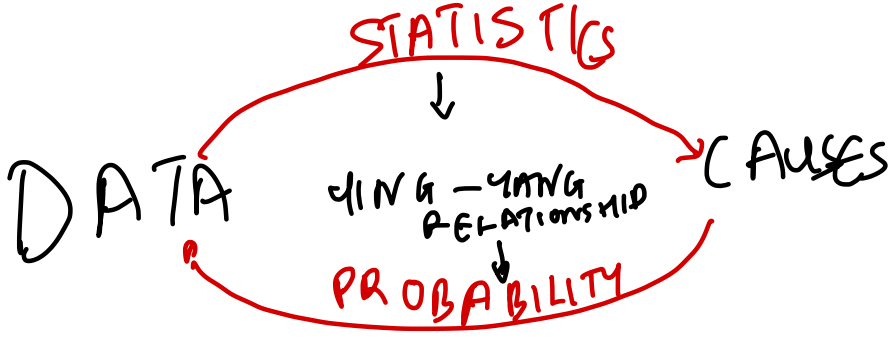
3 of x

8:39 am

4th Dec

Probability



Probability is the means of anticipating the outcomes of an event

Example

FAIR COIN

$P(\text{HEADS}) = 0.5$

↑ probability of obtaining a head when you flip a fair coin

LOADED COIN

Has more chance of obtaining one side than the other

Question

What is the probability for a coin that always comes up heads

$P(\text{Head}) = 1.0$

$P(\text{Tail}) = 0.0$

P(HEADS) : 0.75

P(TAILS) = ? 0.25

P(HEADS) + P(TAILS) = 1

LAW of PROBABILITY

$P(A) = 1 - P(\neg A)$

Probability of A

Probability of not A

Question : unbiased coin

{heads, heads} P(H) = 0.5

P{H, H} = 0.5 ? X → Not sure. I just guessed

0.25

TRUTH TABLE : Possible outcome.

FLIP-1	FLIP 2	P
H	H	0.25
H	T	0.25
T	H	0.25
T	T	0.25
Total		1

or

$P(H, H) = P(H) \cdot P(H) = 0.5 \cdot 0.5 = 0.25$

LOADED COIN

$P(H) = 0.6$

what is the probability of tails

$P(T) = 0.4$

TRUTH TABLE

FILL IT

FLIP 1	FLIP 2	P
H	H	0.36
H	T	0.24
T	H	0.24
T	T	0.16
		1

$= 0.6 \cdot 0.6$

$= 0.4 \cdot 0.6$

$= 0.4 \cdot 0.6$

$0.4 \cdot 0.4$

{ HEADS, HEADS }

P(H) = 1
P(T) = 0

P(H, H) = 1 · 1 = 1

TRUTH TABLE

FLIP 1	FLIP 2	P
H	H	1
H	T	0
T	H	0
T	T	0

1/4

P(H) = 0.5

P(Exactly one H) = 0.5

P(H and, Tail) + P(Tail, Head)

0.5 · 0.5 + 0.5 · 0.5 = 0.5

Truth Table

Flip 1	flip 2	P
H	H	0.25
H	T	0.25
T	H	0.25
T	T	0.25

0.5

P(Exactly one H in 3 Flips) = 0.375

P(H) = 0.5

P(H, T, T) + P(T, H, T) + P(T, T, H)

flip1 flip2 flip3

0.5 · 0.5 · 0.5 + 0.5 · 0.5 · 0.5 + 0.5 · 0.5 · 0.5

3 · (0.125) = 0.375

Truth Table

Flip 1	Flip 2	Flip 3	P
H	H	H	1/8
H	H	T	
H	T	H	3/8
H	T	T	
T	H	H	
T	H	T	
T	T	H	
T	T	T	1/8

P(Exactly one H) in 3 Flips) = 0.288

P(H) = 0.6
P(T) = 0.4

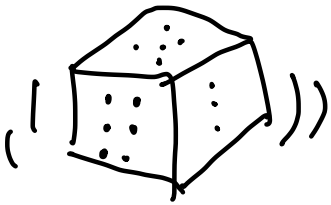
=

P(H, T, T) + P(T, H, T) + P(T, T, H)

0.6 x 0.4 x 0.4 + 0.4 x 0.6 x 0.4 + 0.4 x 0.4 x 0.6

= 3(0.096) = 0.288

FAIR DIE



P(each no shown) = 1/6

= 1/3 x 1/6 = 1/2

Exercise)

P(Die is even)

No of even = {2, 4, 6} = 3

↳ P(2,4,6) = 1/6 + 1/6 + 1/6

= 3/6 = 1/2

Truth table

1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

THROWS A FAIR DIE TWICE !! P(one no. shows) = 1/6

P(DOUBLE) = 1/6

↑ i.e same number shows

P(1,1) + P(2,2) + P(3,3) + P(4,4) + P(5,5) + P(6,6)

$\frac{1}{6}\cdot\frac{1}{6}$ + $\frac{1}{6}\cdot\frac{1}{6}$ + $\frac{1}{6}\cdot\frac{1}{6}$ + $\frac{1}{6}\cdot\frac{1}{6}$ + $\frac{1}{6}\cdot\frac{1}{6}$ + $\frac{1}{6}\cdot\frac{1}{6}$

$6\left(\frac{1}{36}\right) = \frac{1}{6}$

TRUTH TABLE

THROW 1	THROW 2	
1	1	→ 1/6 · 1/6 = 1/36
2	2	1/36
3	3	1/36
4	4	1/36
5	5	1/36
6	6	1/36
	Total	1/6

SUMMARY

- Probability of an event : p
  - Probability of opposite event : $1-p$
  - Probability of composite event : $p \cdot p \cdot p$ (under independence)
    - → coin flip
    - → fair dice

10:00 pm

CONDITIONAL PROBABILITY

DEPENDENT THINGS :

→ Events in life are mostly dependent on other things. Outcome of the first event, influences outcome of the second event.

EXAMPLE : MEDICAL EXAMPLE

An individual is sick @ é hospital. What is the probability of a positive result irrespective of cancer or not.

$P(\text{CANCER}) = 0.1$ $\quad P(\overset{\text{not}}{\neg}\text{cancer}) = 0.9$

probability of having a + result for a cancer patient

$P(\text{POSITIVE} \mid \text{CANCER}) = 0.9$ } 1.0
$P(\text{NEGATIVE} \mid \text{CANCER}) = 0.1$ }
$P(\text{POSITIVE} \mid \neg\text{CANCER}) = 0.2$ } 1.0
$P(\text{NEGATIVE} \mid \neg\text{CANCER}) = 0.8$ }

QUESTION : CREATE THE TRUTH TABLE WITH THEIR LIKELY OUTCOME CP

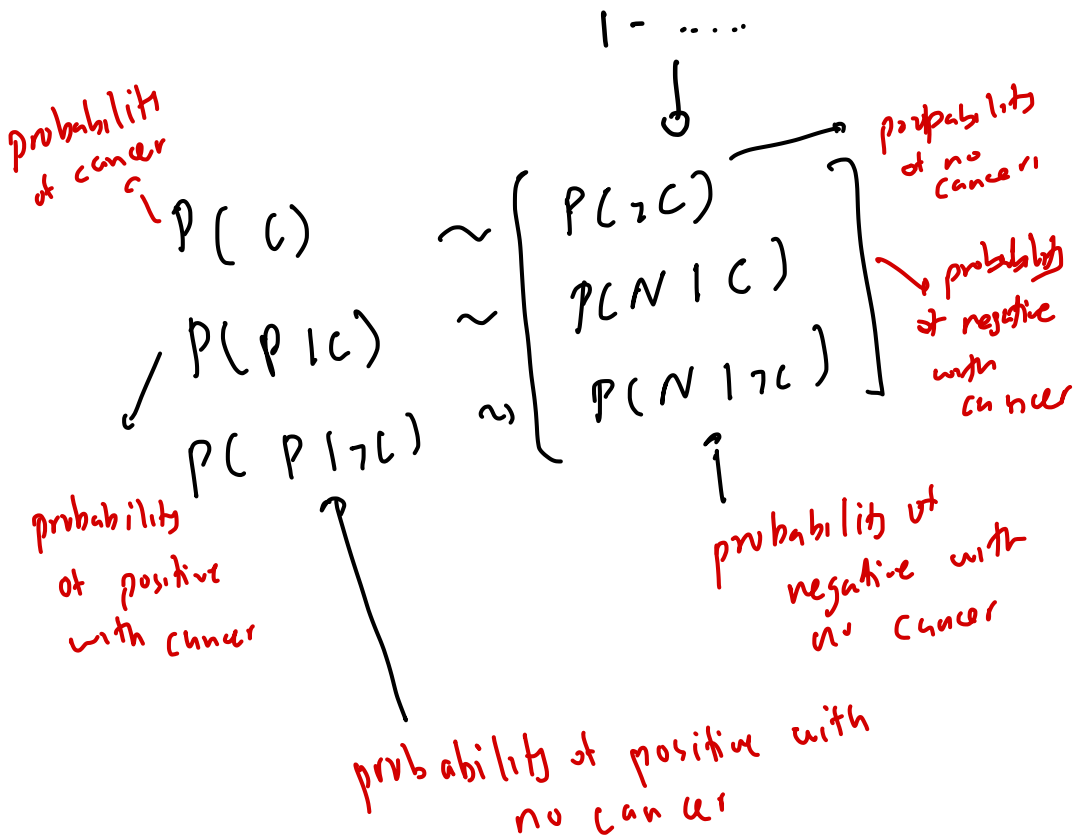
CANCER	TEST	Probab.	
Y	(P)	0.09	$= 0.1 \cdot 0.9$
Y	N	0.01	$= 0.1 \cdot 0.1$
N	(P)	0.18	$= 0.9 \cdot 0.2$
N	N	0.72	$= 0.9 \cdot 0.8$
		1	probability of all outcomes is 1

Question: What is the probability of a positive result irrespective of cancer or not?)

Check truth table

$\therefore 0.04 + 0.18 = 0.27$

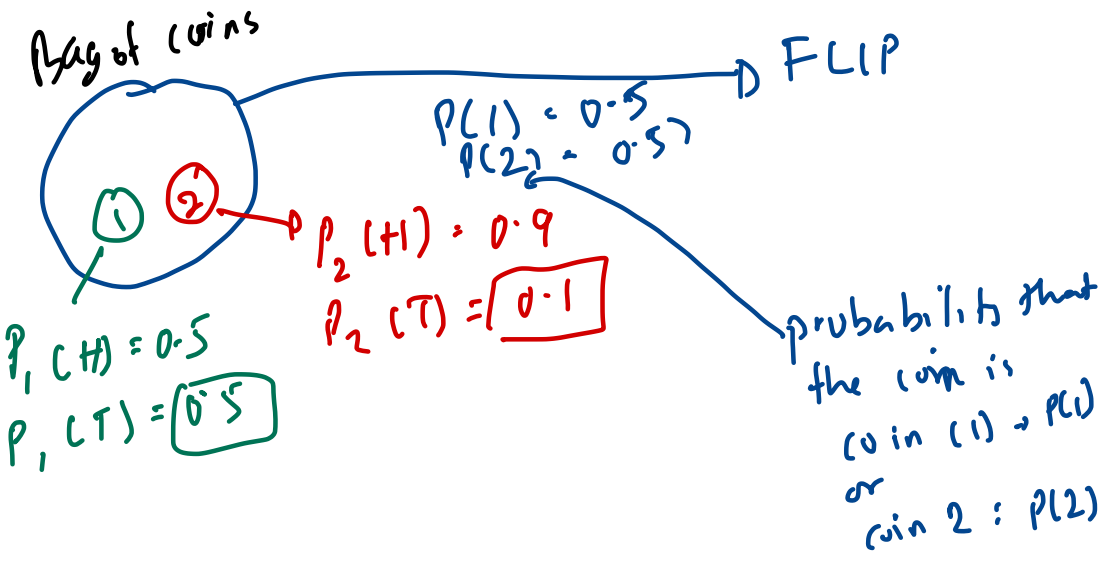
MATHEMATICAL NOTATIONS



$$P(P) = P(P|C) \cdot P(P) + P(P|\neg C) \cdot P(P)$$

TOTAL PROBABILITY

probability of positive



QUESTION

FLIP

Total probability ← $P(H) = P(C_1|H) \cdot P(H) + P(C_2|H) \cdot P(H)$

$P(H) = 0.25 + 0.45 = 0.7$

TRUTH TABLE

COIN	HEAD	Probability
Y	Y	$0.25 = 0.5 \cdot 0.5$
Y	N	$0.25 = 0.5 \cdot 0.5$
N	Y	$0.45 = 0.9 \cdot 0.5$
N	N	$0.05 = 0.1 \cdot 0.5$
		1

* Truth table makes the whole concept of probability interesting and more understable.

Question

You are picking one coin from the bag, then flipping that one coin twice, what is the probability that the first flip is a head and the second a tail?

$P_1(T) = 0.1$ and $P_2(T) = 0.1$

Soln

$P_1(H) = 0.9$ and $P_2(H) = 0.9$

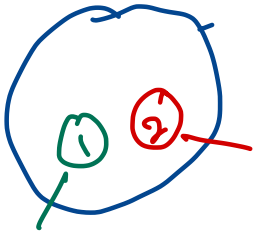
Truth table

PICK	FLIP 1	FLIP 2	Probability
1	H	H	
→ 1	H	T	$0.5 \cdot 0.5 \cdot 0.5 = 0.125$
1	T	H	
1	T	T	+
1	H	H	
2	H	T	$0.5 \cdot 0.9 \cdot 0.1 = 0.045$
→ 2	T	H	
2	T	T	

$P(H \mid T) = 0.125 + 0.045$

$= 0.17$ //

Example / Exercise 2



$P_1(1) = 0.5$

$P(H|1) = 1$

$P(H|2) = 0.6$

Question

$P(T, T)$

?

You choose a coin and flip twice, what is the probability of seeing tails twice?

Truth table

Coin	Flip 1	Flip 2	Probability
1	H	H	
1	T	H	
1	H	T	
1	T	T	0
2	H	H	
2	T	H	
2	H	T	
2	T	T	$0.5 \cdot 0.4 \cdot 0.4 = 0.08$

TRUTH TABLE

TEST	DISEASE
→ P	Y
→ P	N
N	Y
N	N

↳ P(TEST) =