11 am 05/12/2022

# Bayes Rule

cancer

Example: $P(C) = 0.01$

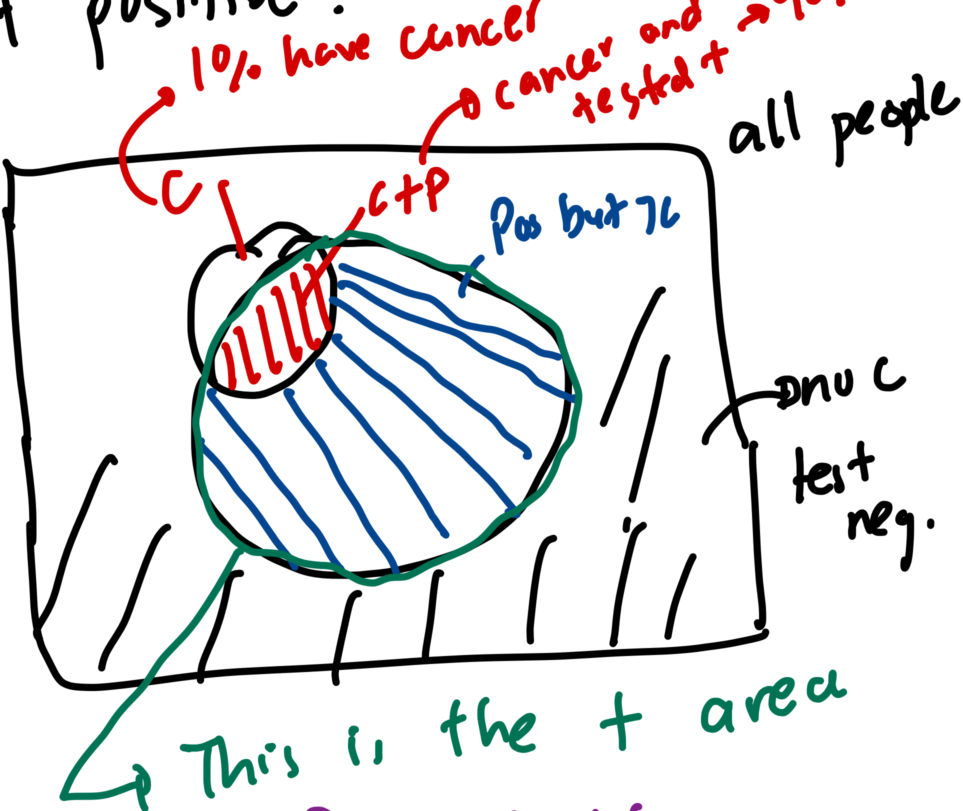
TEST: 90% it is +, if you have c
90% it is −, if you don't have c

?? sensitivity

specificity ??

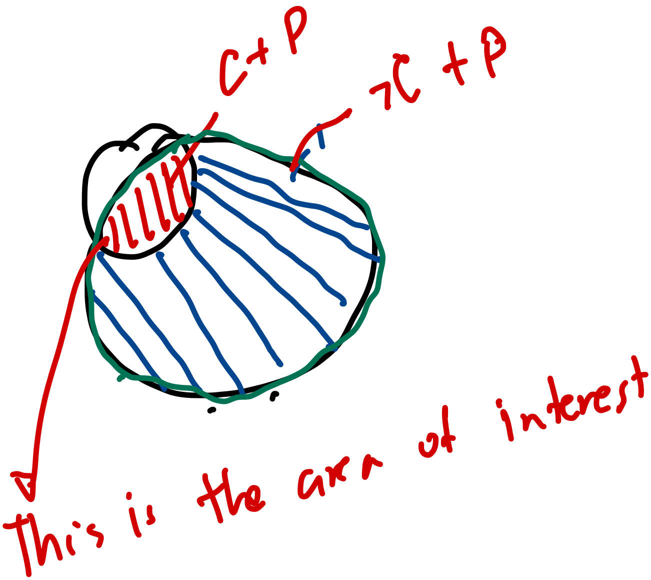
QUESTION:
1% of the population have cancer. Given that there's a 90% chance that you will test positive if you have cancer and that there's a 90% chance you will test negative if you don't have

cancer, what is the probability that you have cancer if you test positive?




TEST = POSITIVE

What is the probability of having cancer?




# BAYES RULE

prior probability · test evidence → posterior probability

$p \cdot T = P$

prior : $P(C) = 0.01 = 1\%$ $P(\neg C) = 0.99$

$P(Pos \mid C) = 0.9 = 90\%$ $P(Pos \mid \neg C) = 0.1\%$

$P(Neg \mid C) = 0.1\%$

$P(Neg \mid \neg C) = 0.9$

prior test sensitivity

joint : $P(C, Pos) = \underbrace{P(C)}_{\text{prior}} \cdot \underbrace{P(Pos \mid C)}_{\text{test sensitivity}}$

$P(\neg C, Pos) = P(\neg C) \cdot P(Pos \mid \neg C)$

probability of cancer given a positive results

what I missed is that there's still a probability of having cancer but the test results will be negative

TEST = POSITIVE

What is the probability of having cancer?

what I missed is that there's still a probability of having cancer but the test results will be negative

Soln $P(Pos) = P(C, Pos) + P(\neg C, Pos)$

$P(C, pos) = P(C) \cdot P(pos | C)$

$= 0.01 \cdot 0.9$

$= 0.009$

$P(\neg C, pos) = P(\neg C) \cdot P(pos | \neg C)$

$= 0.99 \cdot 0.1$

$= 0.099$

$\Sigma = 0.009 + 0.099$

$= 0.108$

Doesn't add to 100

so normal

Normalization to obtain a probability of 1

Posterior:

$$P(C \mid P) = \frac{0.009}{0.108} = 0.0833$$

$$P(\neg C \mid P) = \frac{0.099}{0.108} = 0.9167$$

$$\Sigma = 1$$

$\therefore$ probability of having cancer is 0.0823 //

# Bayes Rule Diagram

$P(C)$ prior

$P(Pos \mid C)$ Sensitivity

$P(Neg \mid \neg C)$ Specitivity

1

Cancer

No cancer

positive : $P(C)$

multiply

$P(Pos \mid C)$

multiply

$P(Pos \mid \neg C)$

don't add to 1 ?

$P(Pos, C)$ $\xrightarrow[\text{add}]{+}$ $P(Pos, \neg C) \Rightarrow P(Pos)$

Normalise divide by $P(pos)$

divide by $P(Pos)$

Posterior probability : $\dfrac{P(Pos, C)}{P(Pos)}$ $\xrightarrow[\text{add}]{+}$ $\dfrac{P(Pos, \neg C)}{P(Pos)}$ = 1 ? adds to 1

$$P(Pos, C) = P(Pos \mid C) \cdot P(C)$$

$$P(Pos, \neg C) = P(Pos \mid \neg C) \cdot P(C)$$

Neg :

Cancer

$P(C) \xrightarrow{\text{multiply}} P(\quad | C)$

$P(\text{Neg}, C) \xrightarrow[\text{add}]{+} P(\text{Neg}, \neg C) \Rightarrow P(\text{Neg})$

No cancer

$P(C) \xrightarrow{\text{multiply}} P(\text{Neg} | \neg C)$

don't add to 1 ?

Normalise: divide by $P(\text{Neg})$

divide by $P(\text{Neg})$

Posterior probability : $\dfrac{P(\text{Neg}, C)}{P(\text{Neg})} \xrightarrow[\text{add}]{+} \dfrac{P(\text{Neg}, \neg C)}{P(\text{Neg})} = 1$ ?

adds to 1

Question

Prior Probability

$P(C) = 0.01$

$P(Pos \mid C) = 0.9$

$P(Neg \mid \neg C) = 0.9$

$P(\neg C) = \boxed{0.99}$ — $1 - 0.01$

$P(Neg \mid C) = \boxed{0.1}$

$P(Pos \mid \neg C) = \boxed{0.1}$

Test: Neg $P(C, Neg) = P(C) \cdot P(Neg \mid C) = 0.01 \times 0.1 = 0.001$

$+$

$P(\neg C, Neg) = P(\neg C) \cdot P(Neg \mid \neg C) = 0.99 \times 0.9 = 0.891$

$\Sigma = 0.892$

Normalizer $P(Neg) = P(C, Neg) + P(\neg C, Neg)$

Posterior probability $= \alpha \dfrac{P(C, Neg)}{P(Neg)}$ $\alpha \dfrac{P(\neg C, Neg)}{P(Neg)}$

$||$ $||$

$\boxed{P(C \mid Neg) = 0.0011}$ $\boxed{P(\neg C \mid Neg) = 0.999}$

Example 2

$P(C) = 0.1$

$P(Pos \mid C) = 0.9$ sensitivity

$P(Neg \mid \neg C) = 0.5 \rightarrow$ Specificity

$P(\neg C) = \boxed{0.9}$

$P(Neg \mid C) = \boxed{0.1}$

$P(Pos \mid \neg C) = \boxed{0.5}$

Test = Neg

Joint $\begin{cases} P(C, Neg) = \boxed{0.01} = P(C) \cdot P(Neg \mid C) \\ P(\neg C, Neg) = \boxed{0.45} = P(\neg C) \cdot P(Neg \mid \neg C) \end{cases}$

Normalize $\rightarrow$ $P(Neg) = \boxed{0.46} = P(C, Neg) + P(\neg C, Neg)$

Posterior probability $\begin{cases} P(C \mid Neg) = \boxed{0.0217} = \dfrac{P(C, Neg)}{P(Neg)} \\ P(\neg C \mid Neg) = \boxed{0.9783} = \dfrac{P(\neg C, Neg)}{P(Neg)} \end{cases}$

$P(C) = 0.1$

$P(Pos \mid C) = 0.9 \rightarrow$ sensitivity

$P(Neg \mid \neg C) = 0.5 \rightarrow$ Specificity

$P(\neg C) = \boxed{0.9}$

$P(Neg \mid C) = \boxed{0.1}$

$P(Pos \mid \neg C) = \boxed{0.5}$

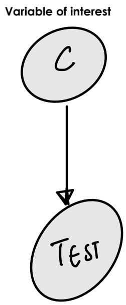
Test = Pos

Joint $\begin{cases} P(C, Pos) = \boxed{0.09} = 0.1 \times 0.9 \\ P(\neg C, Pos) = \boxed{0.45} = 0.9 \times 0.5 \end{cases}$

Normalize $\rightarrow$ $P(Pos) = \boxed{0.54} = 0.09 + 0.45$

Posterior probability $\begin{cases} P(C \mid Pos) = \boxed{0.1667} = \frac{0.09}{0.54} \\ P(\neg C \mid Pos) = \boxed{0.8333} = \frac{0.45}{0.54} \end{cases}$

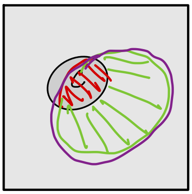
# Bayles Rules Summary




Prior $P(C)$

$P(Pos \mid C)$

$P(Neg \mid \neg C)$




Considering positive case of test

$P(C)$

$* \, P(Pos \mid C)$ $* \, P(Pos \mid \neg C)$

$P(C, Pos)$ $P(\neg C, Pos)$

$+$

$\div \, ($ $P(Pos)$ $)$ $\div$

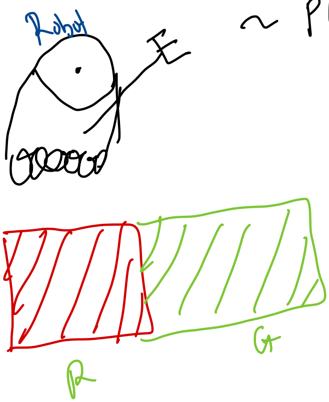
Normalizer

$P(C \mid Pos)$ $P(\neg C \mid Pos)$

# Bayles Rule

$$P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}$$

# Example




$\sim P(R) = P(G) = 0.5$

$P(\text{see } R \mid \text{at } R) = 0.8$

$P(\text{see } G \mid \text{at } G) = 0.8$

$P(\text{see } G \mid \text{at } R) = 0.2$

$P(\text{see } R \mid \text{at } G) = 0.2$

Sees Red

posterior probabilities

$P(\text{at } R \mid \text{see } R) =$

$P(\text{at } G \mid \text{see } R) =$

Joint: $P(\text{at } R, \text{see } R) = P(R) \cdot P(\text{see } R \mid \text{at } R) = 0.5 \cdot 0.8 = 0.4$

$P(\text{at } G, \text{see } R) = P(G) \cdot P(\text{see } R \mid \text{at } G) = 0.5 \cdot 0.2 = 0.1$

Normalizer $= 0.4 + 0.1 = 0.5$

$P(\text{at } R \mid \text{see } R) = 0.4 / 0.5 = 0.8$

$P(\text{at } G \mid \text{see } R) = 0.1 / 0.5 = 0.2$