

Section with Barbara

Week 2

A11



About
That



Bayes !



FREQUENTIST

BAYESIAN

What is our belief from
our observations?

How should we change our
belief from the evidence?

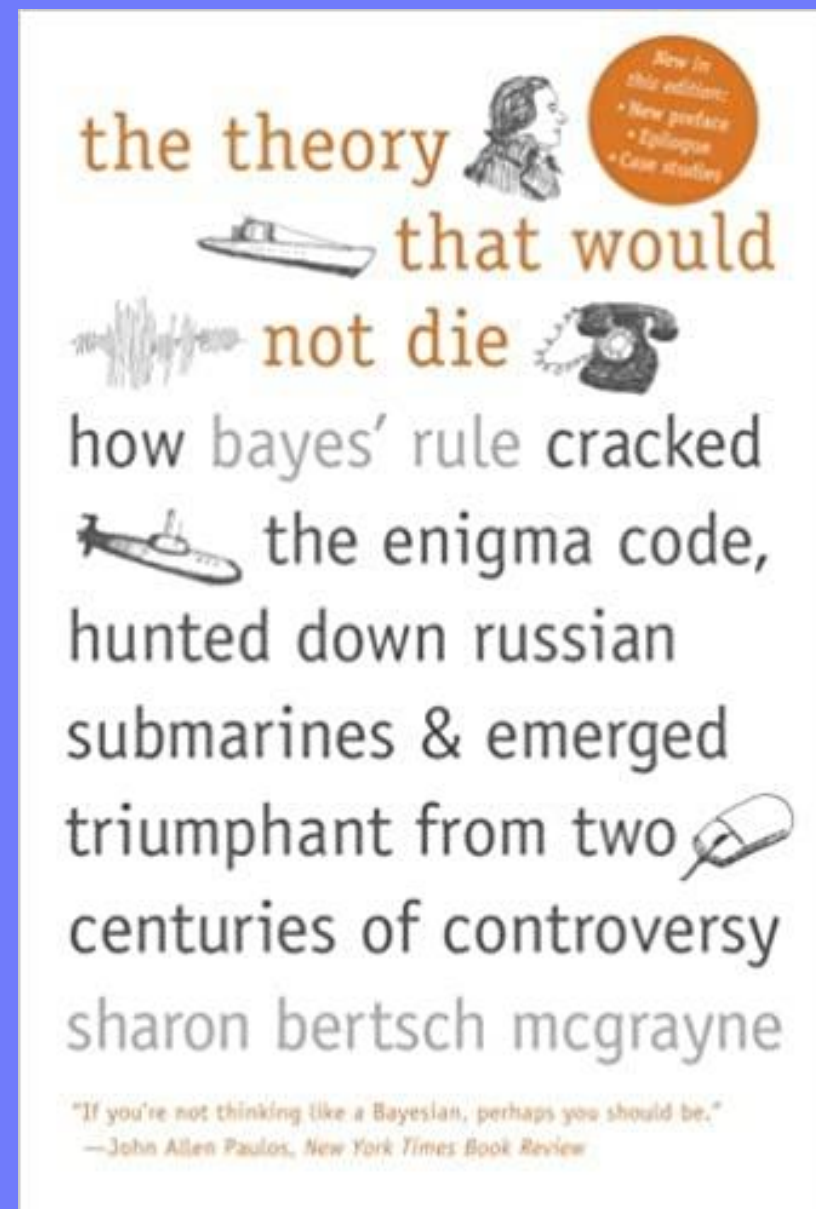
“

The frame of the world
must be the effect of the
wisdom and power of an
intelligent cause

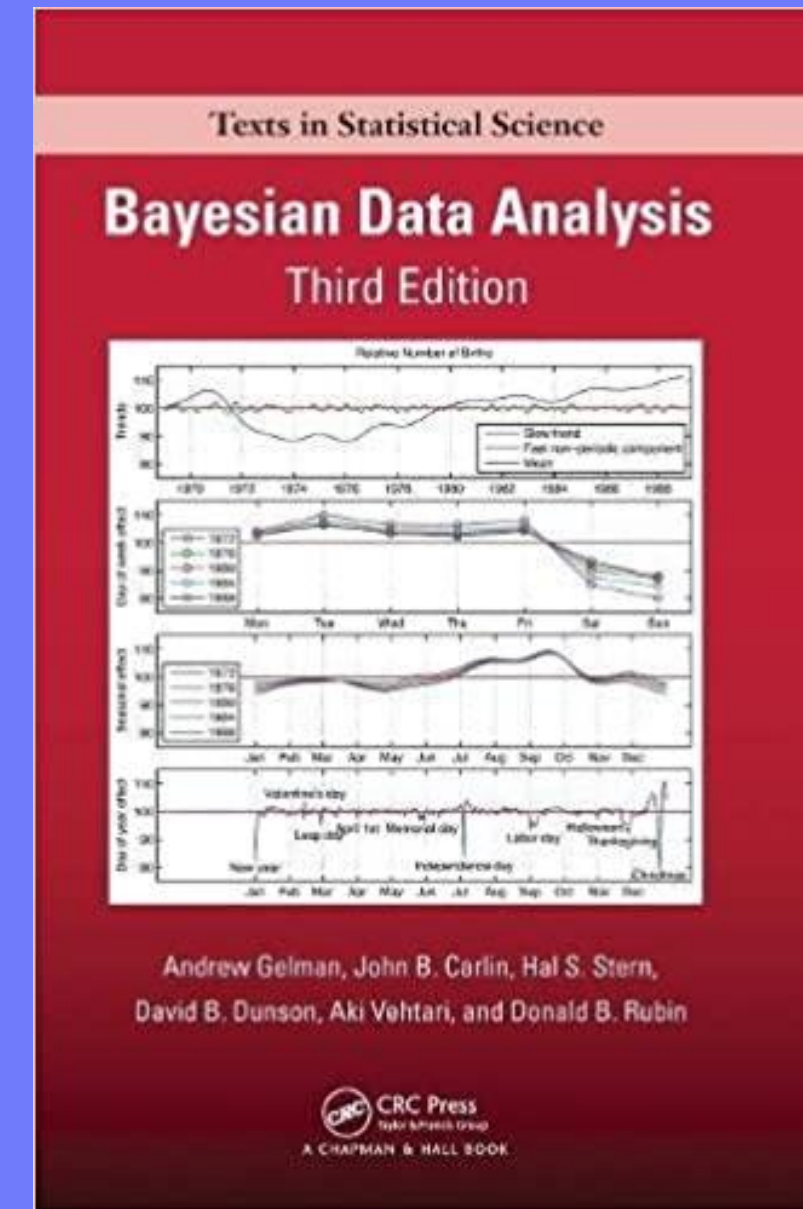
An Essay towards solving a Problem in the
Doctrine of Chances. Richard Price (1763)

”

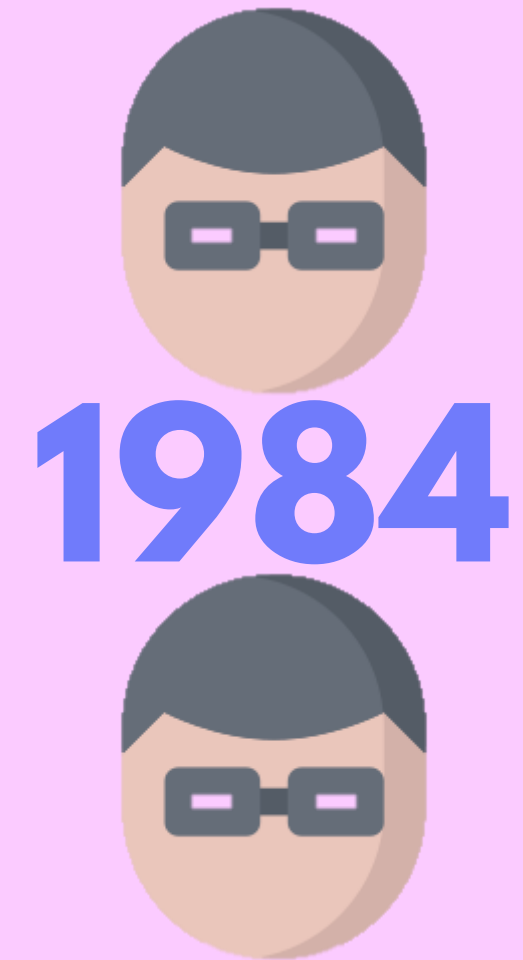
Fun Book on Bayes



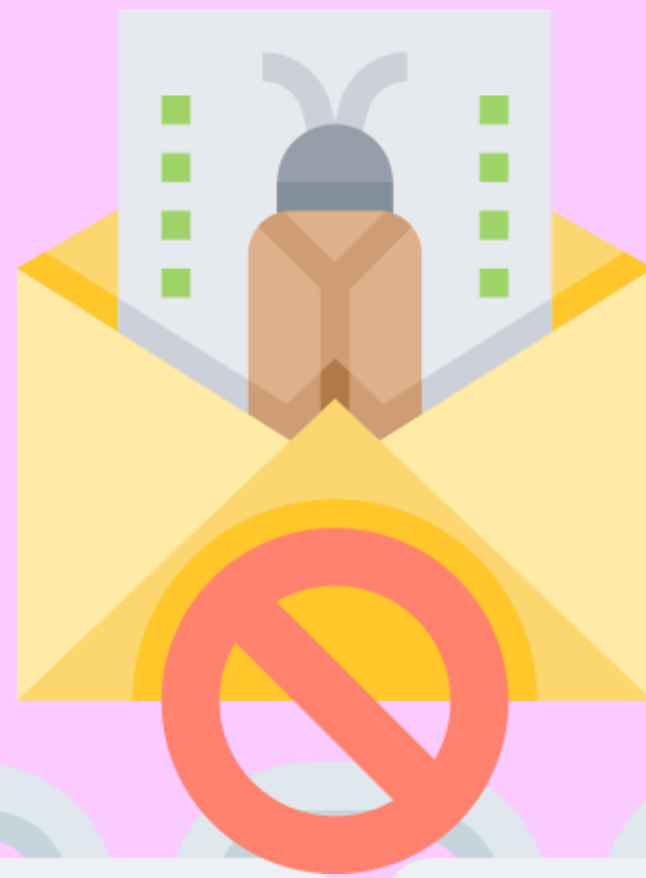
"The" Textbook on Bayes



BAYES BEGINNINGS



BAYES NOW



AI Stories: Recommender Systems





HALO

Knowledge: Gaming Stats
Uncertainty: TrueSkill Ranking
Search: Player Matchmaking

projects



BAYES THEOREM

A handwritten diagram illustrating Bayes' Theorem. The central equation is $P(A|B) = \frac{P(B|A) P(A)}{P(B)}$. Annotations include: an arrow from 'THE PROBABILITY OF "B" BEING TRUE GIVEN THAT "A" IS TRUE' to $P(B|A)$; an arrow from 'THE PROBABILITY OF "A" BEING TRUE' to $P(A)$; an arrow from 'THE PROBABILITY OF "A" BEING TRUE GIVEN THAT "B" IS TRUE' to $P(A|B)$; and an arrow from 'THE PROBABILITY OF "B" BEING TRUE' to $P(B)$.

THE PROBABILITY OF "B"
BEING TRUE GIVEN THAT
"A" IS TRUE

THE PROBABILITY
OF "A" BEING
TRUE

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

THE PROBABILITY
OF "A" BEING TRUE
GIVEN THAT "B" IS
TRUE

THE PROBABILITY
OF "B" BEING
TRUE

PROBABILITY CHAIN RULE

$$\begin{array}{ccccc} & & \text{conditional} & & \\ & & \text{probability} & & \\ P(B \text{ and } A) & = & P(A \mid B) & * & P(B) \\ \text{joint probability} & & & & \text{marginal} \\ & & & & \text{probability} \end{array}$$

CHAIN RULE TO BAYES

$$P(B \text{ and } A) = P(A \text{ and } B)$$

$$P(A | B) * P(B) = P(B | A) * P(A)$$

$$P(A | B) = \frac{P(B | A) * P(A)}{P(B)}$$

“SCIENTIFIC” BAYES

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B|A) * P(A) + P(B|A') * P(A')}$$

$$P(H_i|data) = \frac{P(data|H_i) * P(H_i)}{\sum_{i=1}^n P(data|H_i) * P(H_i)}$$

BAYESIAN INFERENCE

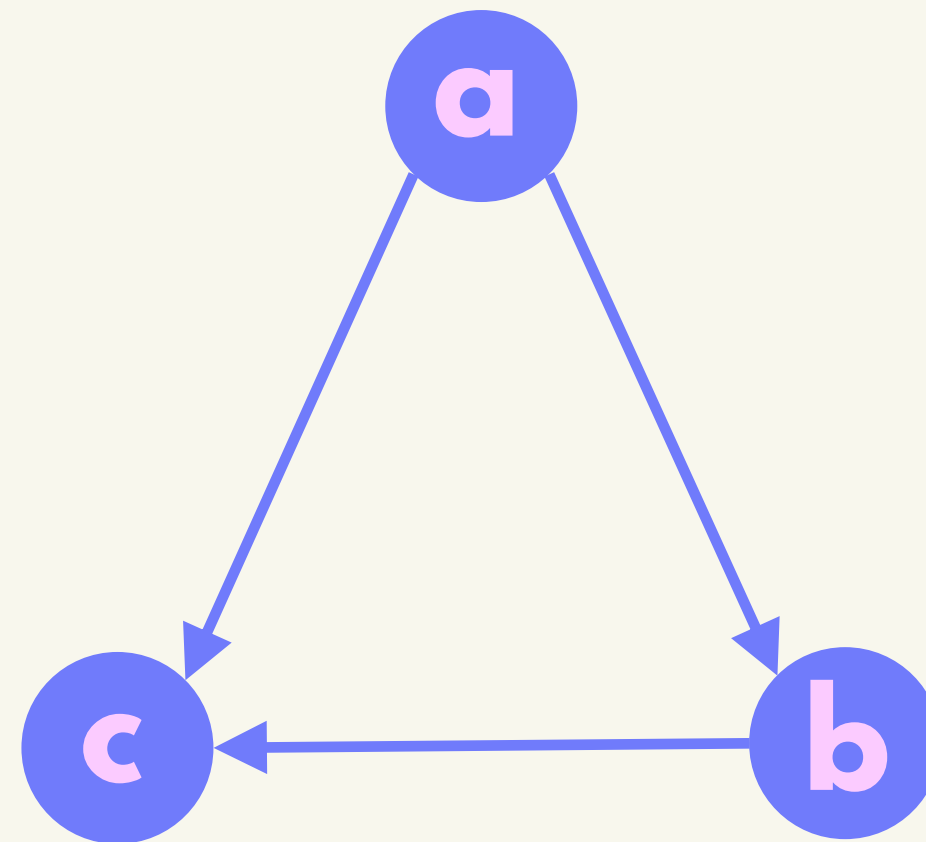
$P(A)$ = a prior, $P(B)$ = normalization

$P(B | A)$ = a likelihood

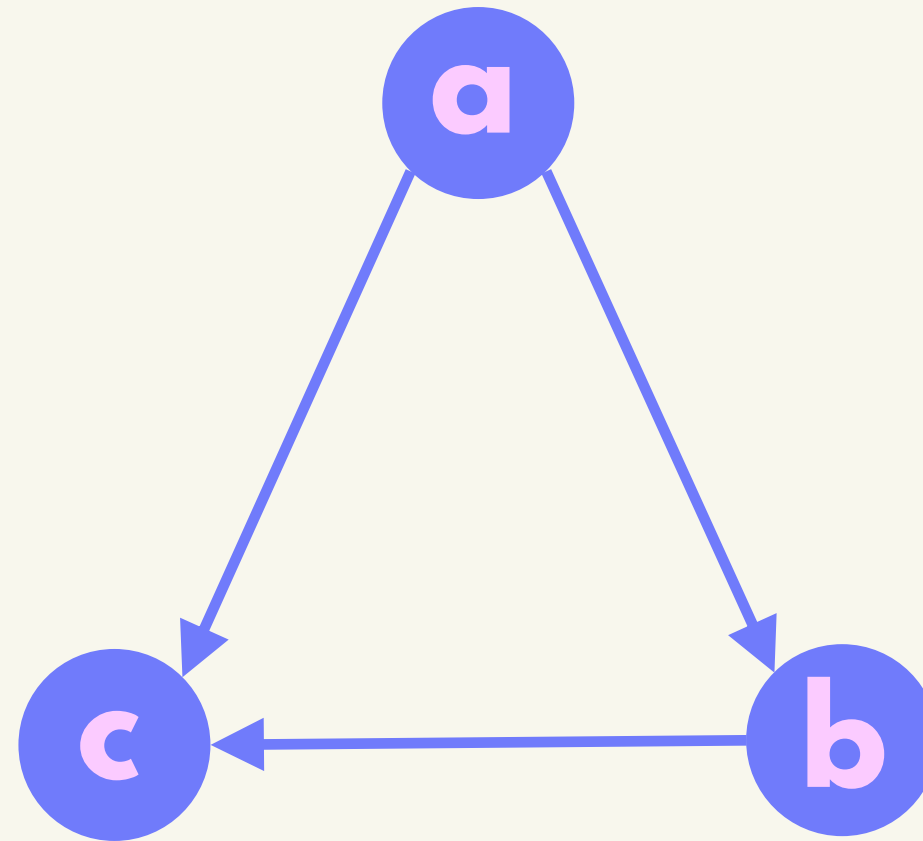
$P(A | B)$ = a posterior

$$\text{posterior} = \frac{\text{likelihood} * \text{prior}}{\text{normalization}}$$

BAYESIAN NETWORK

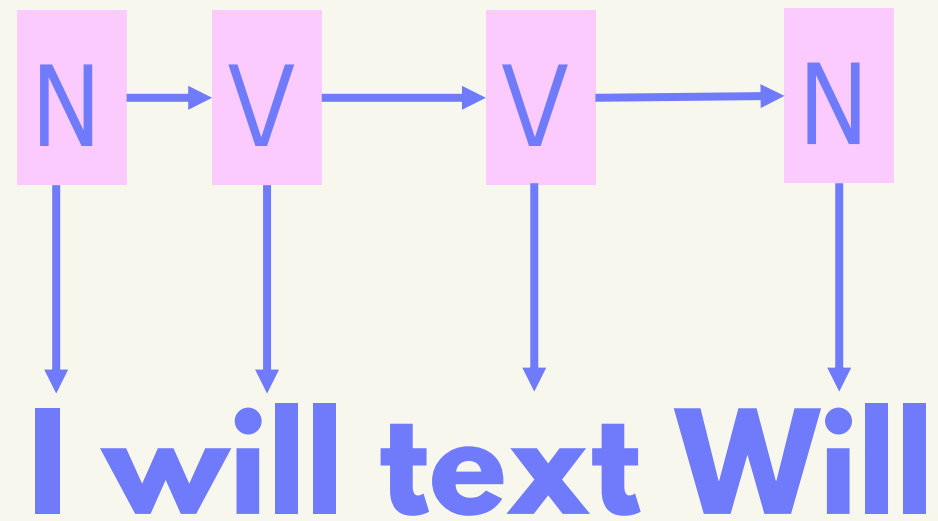


MARKOV CHAIN



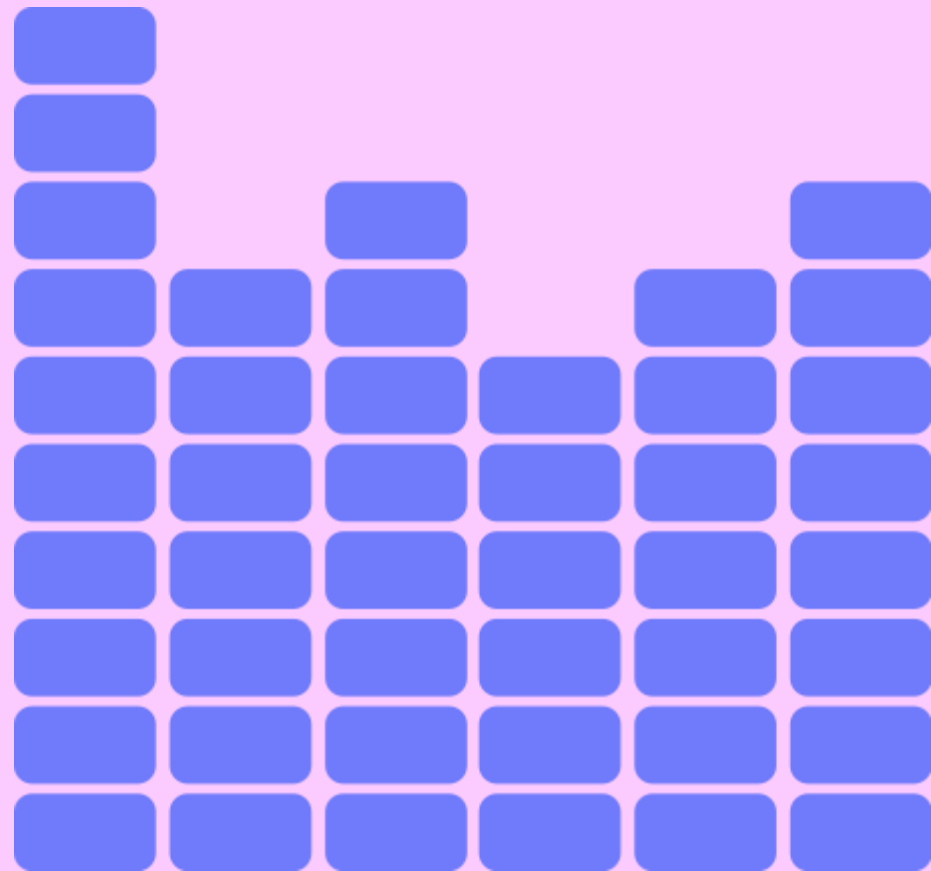
$$p(a, b, c) = p(a) \cdot p(b \mid a) \cdot p(c \mid a, b)$$

HIDDEN MARKOV MODELS

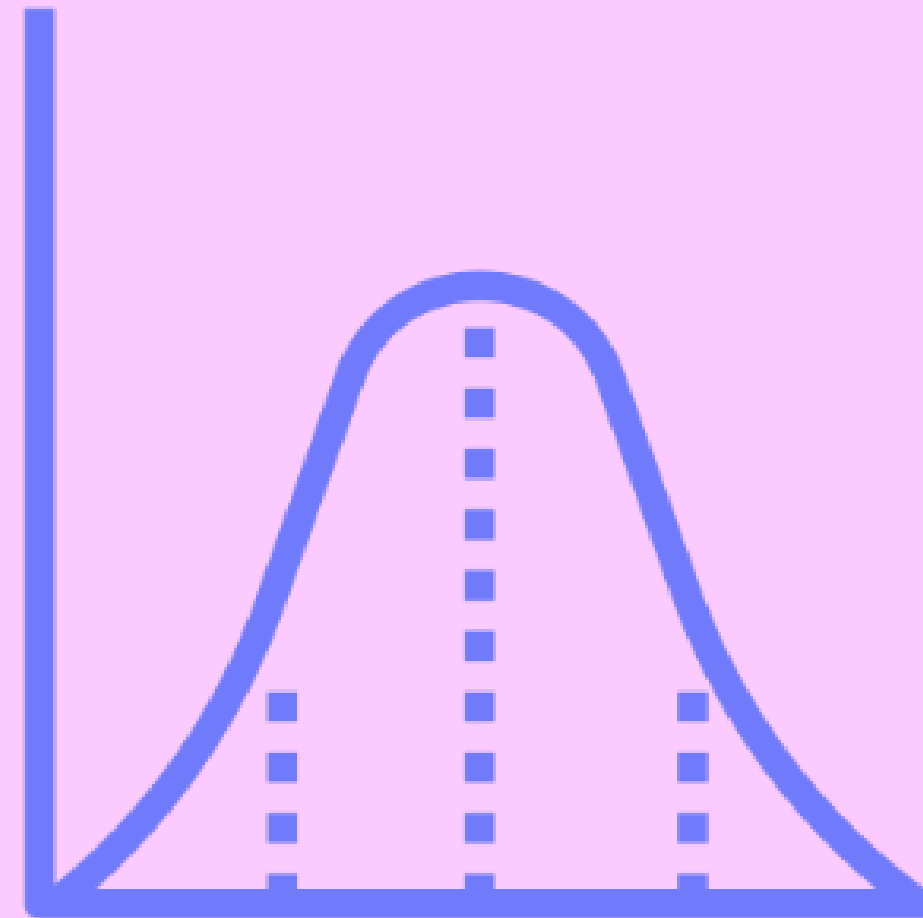


What is the prior? What is the posterior?

PROBABILITY DISTRIBUTIONS

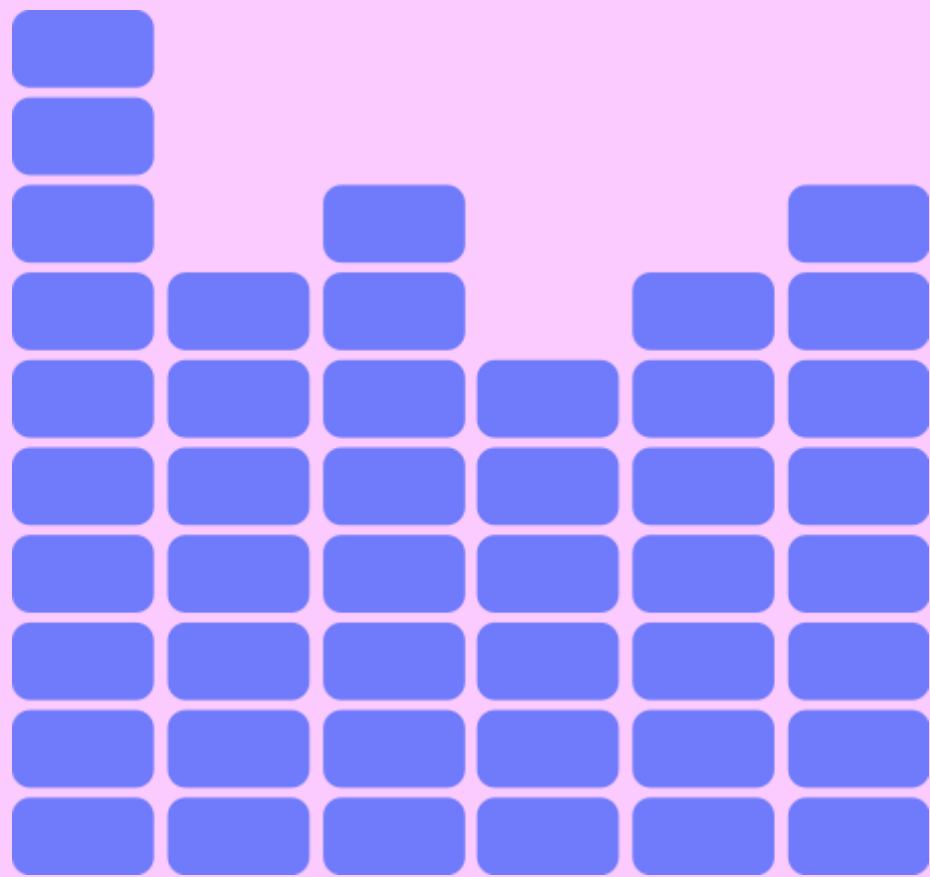


discrete



continuous

DISCRETE DISTRIBUTIONS



$$\begin{cases} P(A = a) \geq 0 \\ \sum P(A = a) = 1 \end{cases}$$

JOINT LIKELIHOODS



		Genes		
		0	1	2
Trait	Yes			
	No			



CONDITIONAL LIKELIHOODS



child

Parent		0	1	2
child				
	Has Gene			



CONDITIONAL LIKELIHOODS



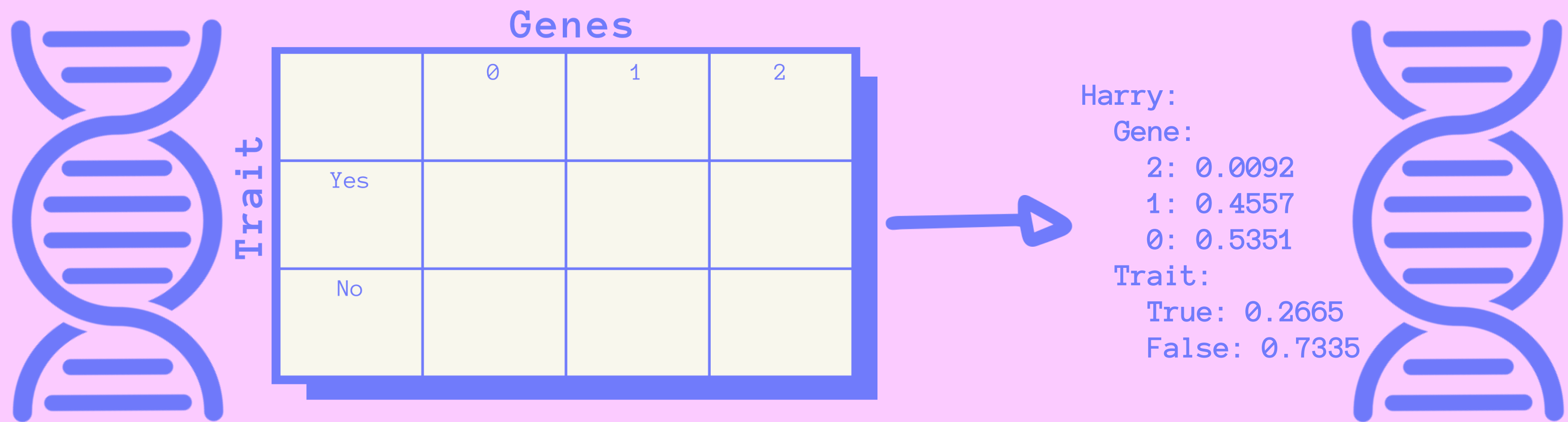
$P(\text{Child Genes} = 0 \mid \text{Mom Genes}, \text{Dad Genes})$

$P(\text{Child Genes} = 1 \mid \text{Mom Genes}, \text{Dad Genes})$

$P(\text{Child Genes} = 2 \mid \text{Mom Genes}, \text{Dad Genes})$



MARGINAL PROBABILITY



LIKELIHOOD TO PROBABILITY



Trait	Genes		
	0	1	2
Yes			
No			

$\rightarrow L(\theta) \rightarrow P(\theta) \rightarrow$

Harry:

Gene:

2: 0.0092

1: 0.4557

0: 0.5351

Trait:

True: 0.2665

False: 0.7335



pagerank

```
def transition_model(corpus, page, damping_factor):  
    """  
    Return a probability distribution over which page to visit next,  
    given a current page.  
    With probability `damping_factor`, choose a link at random  
    linked to by `page`. With probability `1 - damping_factor`,  
    choose a link at random chosen from all pages in the corpus.  
    """
```

What is the probability of visiting a page from a link? What is the probability of visiting a page randomly?



pagerank

```
def sample_pagerank(corpus, damping_factor, n):  
    """  
    Return PageRank values for each page by sampling `n` pages  
    according to transition model, starting with a page at random.  
    Return a dictionary where keys are page names, and values are  
    their estimated PageRank value (a value between 0 and 1). All  
    PageRank values should sum to 1.  
    """
```

Check out `random.choice` from `numpy` – it will be helpful here.



pagerank

```
def iterate_pagerank(corpus, damping_factor):  
    """  
    Return PageRank values for each page by iteratively updating  
    PageRank values until convergence.  
    Return a dictionary where keys are page names, and values are  
    their estimated PageRank value (a value between 0 and 1). All  
    PageRank values should sum to 1.  
    """
```

Make sure you understand generally how the Pagerank equation works in the project specs



Are you still confused about Bayes?

Let's do an exercise!