

# Application of Portfolio Theory and Risk Management

## Airline and Beer Industries

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### Executive summary:

- We began this study by deeply and thoroughly analyzing the returns of each asset by testing for normality and computing their equity curves. The returns were concluded to be stationary and distributions were fitted accordingly.
- We have successfully created an Efficient Frontier for the two industries. We observe that combining both industries and allowing short assets, creates a more vertical Efficient Frontier. In addition, adding a risk-free asset makes the CML and the tangent portfolio. The tangent portfolio with the highest Sharpe Ratio is when both industries are combined with a ratio of 1.46.
- We have constructed 6 unique portfolios that yield a return of 6%. Each portfolio is characterized by different risks. The portfolio with the lowest risk combines the risk-free asset and both industries - providing a Sharpe Ratio of 0.9.
- We examined the VaR and ES of the 15 stocks and 18 portfolios using the parametric and non-parametric methods. We conclude that during the observed period, airlines stocks carried significantly more risk than beer stocks.
- We further determined that the t-copula is the best fit for the stock returns, implying heavy tail dependence among big losses.

**Keywords:** Portfolio Theory, Copulas, Asset Allocation, Risk Management, Principal Component Analysis

**Supervisor:** Hammou El Barmi

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# 1 Introduction

The purpose of this paper is to identify, analyze and compare potential companies for a portfolio. We choose 15 companies in two different industries: the Airline industry and the Beer industry. The reasoning behind choosing two different industries is to study the benefits of diversification within an industry and a combination of sectors. To explore the benefits of diversification and to ensure correct methodology, we apply several methods. Firstly, we employ both visual and algebraic methods to affirm that the companies' returns are all stationary. Secondly, in order to create an efficient portfolio, we apply the efficient portfolio theory, theorized by Markowitz, by creating efficient portfolios both within industries and in combination. Thirdly, we implement an analysis of the Principal Components, a method of data dimension reduction which increases our understanding of the relationships between the companies. The risk measurements such as the Value-at-Risk and Expected Shortfall are computed in order to analyse the potential risks associated with the individual companies and the constructed portfolios. Lastly, we determined the best fitting copula to measure the co-dependency among the risks.

## 2 Descriptive Statistics

### 2.1 Summary Statistics

As previously mentioned, 15 companies are chosen; 7 airline companies and 8 beer companies. The returns of the stocks were collected from 2011 to 2021 via the Yahoo!Finance website. The risk-free rate used in this paper is the average interest rate for the 3-Month Treasury Bill over the past 10 years<sup>1</sup>. The 3 month T-Bill is a fairly good proxy for the risk-free asset, as the short time horizon lowers risk. The average interest annual yield is at 0.45%. The stocks' monthly means, annual means, standard deviations, skewness, kurtosis, betas and Sharpe's ratios are computed.

Stocks	Monthly Means	Annual Means	Standard Deviations	Skewness	Kurtosis	Betas	Sharpe Ratios
Lufthansa	-0.00059	-0.007	0.112	0.087	0.149	1.194	-0.009
Southwest	0.017	0.206	0.093	0.215	0.568	1.004	0.181
American	0.013	0.154	0.140	0.928	3.371	1.299	0.089
Spirit	0.013	0.155	0.128	-0.271	3.089	1.258	0.098
Heineken	0.0069	0.083	0.062	-0.113	0.335	1.009	0.105
Harboe	-0.00045	-0.005	0.0605	0.656	0.698	0.4499	-0.014
Guinness	0.0102	0.122	0.046	0.246	1.209	0.678	0.213
Asahi	0.0084	0.100	0.072	-0.418	4.165	0.801	0.111

Table 1: Summary of the stock returns. Full table can be seen in Appendix A.

We can observe that the stocks with the lowest means are Lufthansa and Harboe, and the stocks with the highest means are Southwest Airlines and Guinness. A kurtosis value of 3 can be used to identify if the data follows a normal distribution. We can see that two stocks possibly follow a normal distribution, American Airlines and Spirit Airlines. Interestingly, there is only one stock with a kurtosis value greater than 3, Asahi, which indicates that the returns have heavier tails than a normal distribution. We will later test the returns' normality with Q-Q plots and by computing the Shapiro test. The above betas were computed following the Security Market Line formula, with a 0.0375% monthly risk free rate (0.45% annual). It appears that all the airlines' betas are larger than 1, implying that they are all aggressive assets, with a risk higher than the market. Heineken has beta close to 1, which implies that Heineken has very similar risk to the market. Two other beer assets have betas larger than 1 but the remaining have betas less than 1, meaning that the assets are non-aggressive.

<sup>1</sup><https://fred.stlouisfed.org/series/TB3MS>

The Sharpe ratio is a method of calculating the performance of an investment. Hence the bigger the ratio, the better the investment. Above, the stocks with the highest Sharpe ratios appear to be Guinness and Southwest Airlines. This result will be graphically displayed in section (2.3) by computing the equity curves of the stocks.

## 2.2 Returns

Next, we chose two arbitrary companies from each industry and plot their returns.

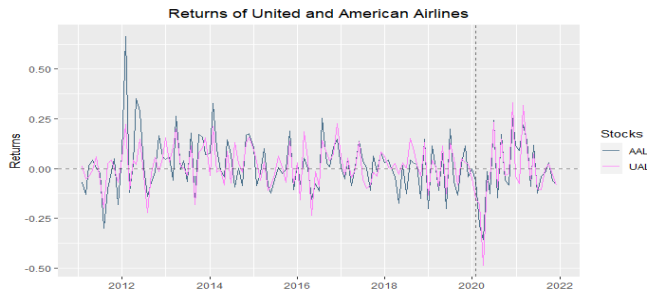


Figure 1: Returns of two Airlines.

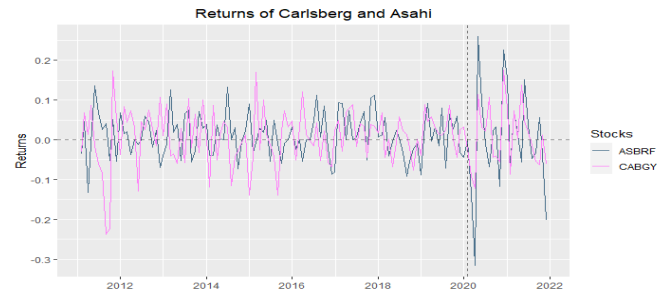


Figure 2: Returns of two Beers.

In Figure 1 the returns of American Airlines (AAL) were mainly negative before 2012 as its parent company, AMR Corporation, filed for bankruptcy in November 2011. It can be seen that in the beginning of 2012 the returns of AAL reach a high peak. This may be due to AAL's stage of its bankruptcy reorganization of cutting 13,000 jobs. Later in 2020, we can see both AAL and United Airlines (UAL) suffer tremendous losses in returns, this being a direct result of the Covid-19 pandemic.

In Figure 2, in the summer of 2011, the returns from Carlsberg fall tremendously. This might be caused by the financial crisis in Russia leading to increased prices, which led to an extended period of consumption decline. In 2020, surprisingly, Carlsberg's returns didn't suffer as big of a loss like the remaining stocks, which plummeted severely. This of course again, being a consequence of enforced lockdown due to the pandemic.

## 2.3 Equity Curves

In this section, the equity curves of all the stocks plus the SP500 index are shown.

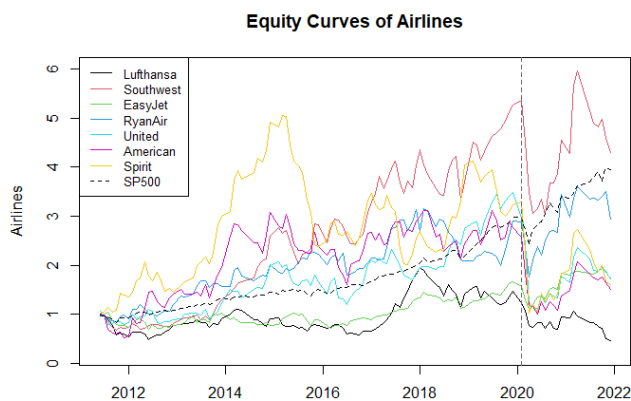


Figure 3: Equity curve of the 7 Airline stocks.

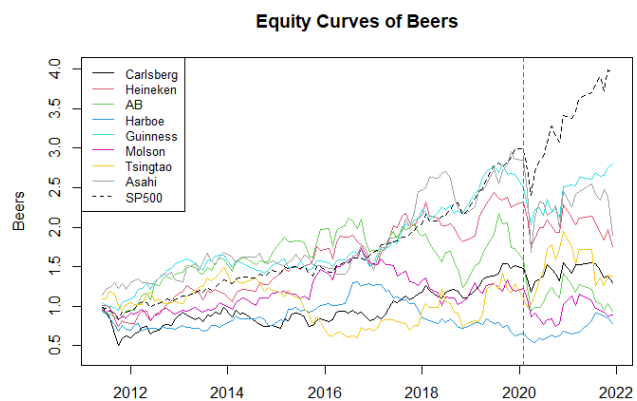


Figure 4: Equity curve of the 8 Beer stocks.

In Figure 3, a significant difference can be seen from 2014 to 2016 between Spirit Airlines and the remaining

stocks. Overall, all the stocks except Lufthansa, show an increasing trend which indicates the companies were being more profitable. Here, the SP500 index seems to represent the average increasing trend for all the stocks.

In Figure 4, there is more variability compared to Figure 3. Out of the 8 beer stocks, only 5 have constant increasing trends. We can also observe Tsingtao's contrasting oscillating trend. Lastly, the SP500 index has the most constant and increasing trend of all the beer stocks.

## 2.4 Distributions of the Returns

Here, we are interested in investigating what distribution each of the returns follow. We begin by plotting histograms and boxplots for each asset. By looking at the histograms, which can be found in Appendix A, we can observe that most of the returns appear approximately unimodal. The returns of American Airlines, United Airlines, Spirit Airlines, Asahi, Carlsberg, Harboe and Molson Coors seem to be skewed.

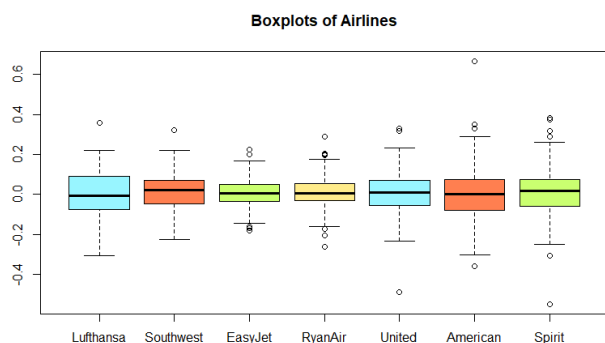


Figure 5: Boxplot of Airline stocks.

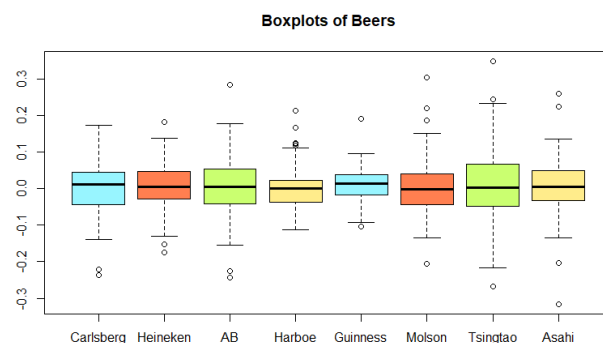


Figure 6: Boxplot of Beer stocks.

When we examine the boxplots, it appears that the means are fairly similar, but it is clear to see which assets have the highest means, which can be seen in the means table that was computed previously. In order for the returns to follow a normal distribution, the box should look symmetrical, where the mean and median lie in the same value, and there should exist only a few outliers. In both of the industries, we can see that some assets have these attributes. Next, Q-Q plots are considered to proceed with the normality analysis. The results are that the assets that look reasonably normal are: Lufthansa, Southwest Airlines, United Airlines, Carlsberg, Heineken, Anheuser-Busch, Guinness, Molson Coors, Tsingtao and Asahi. The remaining assets had several values outside the Q-Q plot's confidence bounds.

Moreover, the Shapiro test for normality is computed and returned that the assets Lufthansa, Southwest Airlines, EasyJet, RyanAir, Carlsberg (barely with a p-value of 0.05991), Heineken, Anheuser-Busch, Guinness and Tsingtao did not significantly deviate from the normal distribution.

We have investigated the normality of the assets using multiple tests, numerical and graphical, in which some of the tests have given the same results, and others haven't. However, when testing normality, it is not uncommon for some tests to not give the same answers. This is due to the tests having different methods. A stationarity test is also executed and determined that all the returns are stationary, which is consistent with the return plots, where no trends or major variability is seen.

Finally, we fit the returns with the following distributions: t, skewed student t, generalized error distribution (ged) and normal distribution, and compare their Akaike's Information Criteria (AIC) and Bayesian Information Criteria (BIC) values to determine the best model fit. The table with the results can be found in Appendix B. Generally, the AIC and BIC tend to agree on the chosen distribution, however, in the Table 10, we can observe some differences. Specifically, Lufthansa, EasyJet, RyanAir, Carlsberg, Guinness and Molson Coors have disagreeing results. This can

happen due to the difference between the estimators. The AIC gives more importance in model performance and hence it is more inclined to select more complex models. Whereas, the BIC has a severe complexity penalty, with increasing probability of finding the true model as the size of the dataset increases. Therefore, BIC is more consistent than the AIC so we favor its estimation.

### 3 Portfolio Theory

#### 3.1 Modern Portfolio Theory (MPT)

Within portfolio theory, various models are applied to construct portfolios and analyze risk. This paper focuses on the practical application of Modern Portfolio Theory (MPT). MPT was developed by Markowitz in 1952 and is one of the most dominant theories introduced for students around the world. The theory attempts to create an efficient portfolio given a specific return; it thereby uses the empirical moments to proxy the stocks' returns, variances, and covariances to construct a portfolio that minimizes risk. The base idea of MPT is that adding multiple stocks to a portfolio can create a diversification premium that enables a lower risk, given the same return.

The portfolio construction of this paper uses data from two different industries: the Airline industry and the Beer industry. By analyzing the industries separately and in combination within the MPT framework, we hope to demonstrate the advantage of diversification. Constructing a portfolio can be done both algebraically and computationally. The algebraic approach minimizes the Lagrange yielding standard formulas, which are easily applied to a wide range of stocks. The computational approach iterates through different expected returns while minimizing the expected risk. This approach is, therefore, in some cases, a more lengthy process — especially within a universe of a large number of stocks. The purpose of this paper is two-folded: calculate the different portfolios using MPT and showcase the nature of MPT. For the second purpose, a computational approach can be beneficial. Therefore, we will use this approach.

#### 3.2 MPT with no risk free asset

MPT with no risk free asset is the baseline of the model. In this case, minimizing risk for different assets will yield a horizontal u-shape in a standard deviation graph. This demonstrates the diversification gain, i.e. we can gain lower standard deviations without compromising our expected return by combining stocks. The portfolio with the lowest standard deviation is the minimum variance portfolio. The line above the minimum variance portfolio is called the efficient frontier. This line constitutes all efficient possible combinations of standard deviation and return in the specific set of stocks. An efficient portfolio is a portfolio that yields the lowest standard deviation for a given return.

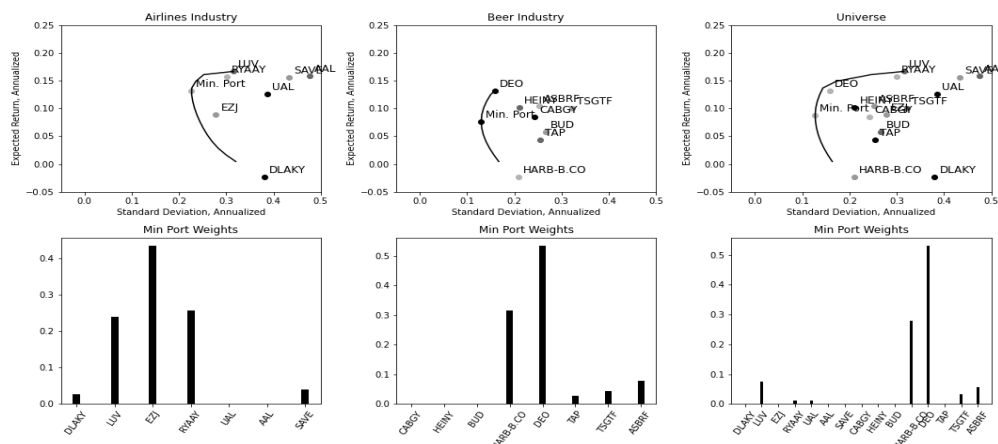


Figure 7: Efficient Frontier, no Short and no Risk Free.

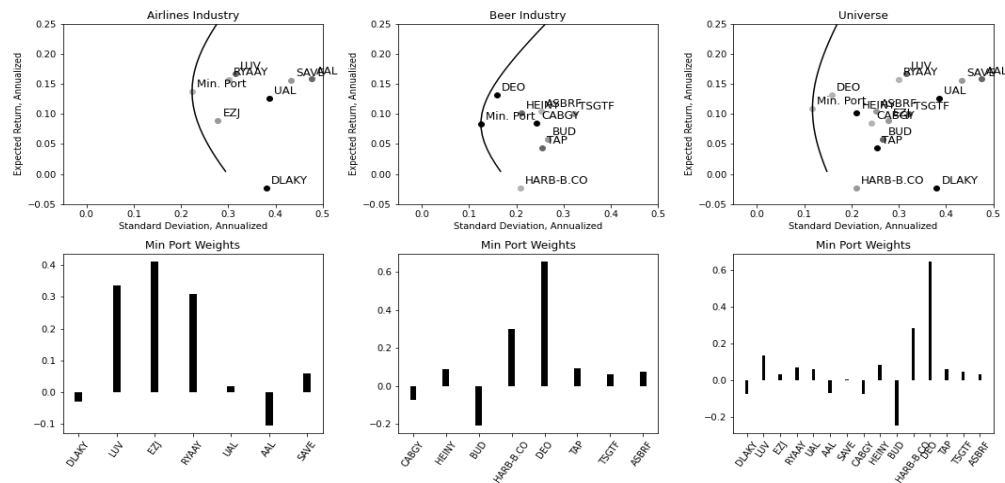


Figure 8: Efficient Frontier, Short and no Risk Free.

The figures above show the efficient frontier with and without shorting for both industries individually and in combination. As can be seen by the efficient frontier on figure 7 the airlines' efficient frontier has a higher standard deviation than beers, this is due to the general lower level of standard deviation for beer companies. In addition, the combined efficient frontier shows an even lower standard deviation than the beers. But as can be seen by the weight of the combined minimum variance portfolio, it is primarily the beer sector that contributes to the low volatility. On figure 8, we have the possibility to short the stocks. The weights of the minimum variance portfolio have a much broader range of stocks, as compared to the no shorting portfolio. This minimum variance portfolio utilizes diversification even more than a no shorting portfolio, providing lower risk. Additionally, the efficient frontier is more vertical compared to no shorting — indicating that we are able to reduce risk for a broad range of expected returns.

Without Short and Risk free			
	Std.	Expected Return	Sharpe Ratio
Airline Industry	0.22	0.13	0.56
Beer Industry	0.13	0.076	0.54
Both	0.12	0.087	0.67
With Short and no Risk free			
	Std.	Expected Return	Sharpe Ratio
Airline Industry	0.22	0.13	0.56
Beer Industry	0.125	0.08	0.59
Both	0.11	0.10	0.85
With Short and Risk free			
	Std.	Expected Return	Sharpe Ratio
Airline Industry	0	0.0054	-
Beer Industry	0	0.0054	-
Both	0	0.0054	-

Table 2: Minimum Variance Portfolio.

Note: All values for std. and return are annualized. VaR and ES will be discussed in Risk Management Part.

Without Short and with Risk free			
	Std.	Expected Return	Sharpe Ratio
Airline Industry	0.24	0.15	0.6
Beer Industry	0.15	0.13	0.82
Both	0.14	0.13	0.88
With Short and Risk free			
	Std.	Expected Return	Sharpe Ratio
Airline Industry	0.33	0.31	0.92
Beer Industry	0.18	0.18	0.96
Both	0.18	0.27	1.46

Table 3: Tangent Portfolio.

### 3.3 MPT with risk free asset

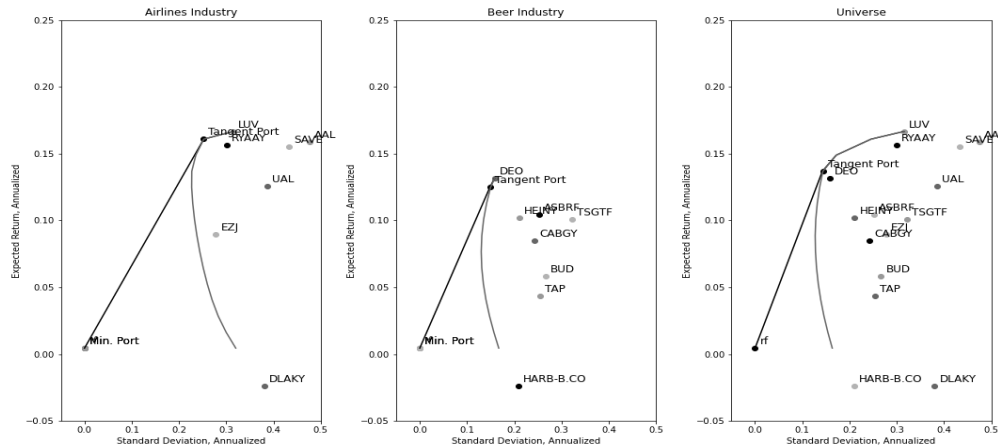


Figure 9: Tangent Portfolio, no Short and Risk Free

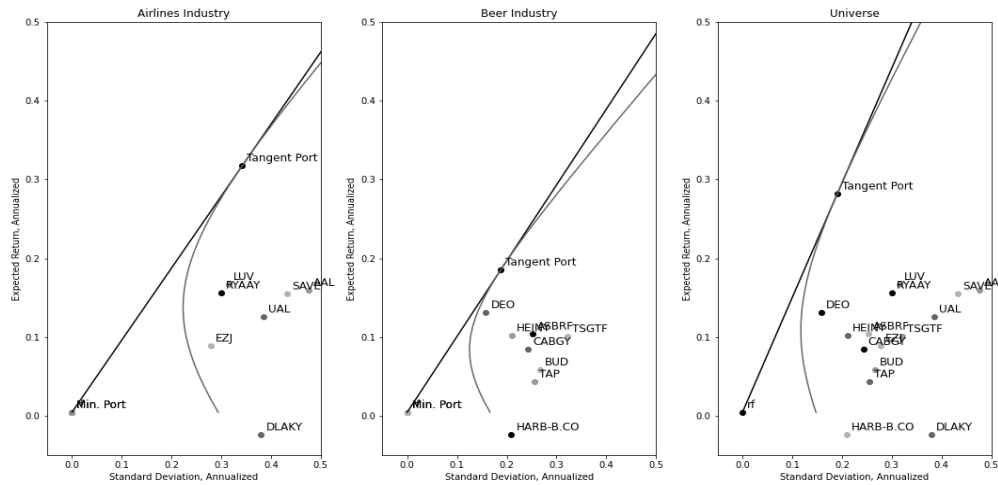


Figure 10: Tangent Portfolio, Short and Risk Free

We expand the universe by adding a risk-free asset that yields a return of 0.45% annually and has a zero covariance with the risky assets. In this case, we get the Capital Market Line (CML) instead of the efficient frontier. CML is a linear combination of the Tangent portfolio (Market Portfolio) and the risk-free asset, and these portfolios dominates all portfolios from the prior efficient frontier. The tangent portfolio is when the old efficient frontier hits the CML and must therefore be the portfolio from the efficient frontier with the highest Sharpe ratio. Figure 9 and 10 illustrate the newly formed CML and the old efficient frontier for a universe where you can and cannot short, respectively. As noted, the CML is a linear combination of the risk-free and the tangent portfolio. Therefore, all portfolios beyond the tangent portfolio on the CML are portfolios where the risk-free asset is shorted.

If we compare the two figures, we see a very clear difference in the properties of the tangent portfolios generated with and without shorting. The ‘Short Allowed’ tangent portfolio distinguishes itself by having a much higher return, and slightly higher standard deviation. This is due to the fact that you are able to short individual assets. Additionally, the tangent portfolios for the ‘no short scenario’, in the case of airline and universe, are both below the highest achievable return. This is because the portfolios above the tangent portfolio have higher standard deviations, resulting in lower Sharpe ratios.

To conclude, we find the efficient frontier, CML and tangent portfolios for the two sectors: Airline and Beer, and



a combination of both. The lower risk within the beer industry enables a low volatility for the efficient frontier. Whereas, the higher return for the Airline industry enables the tangent portfolio to have a higher return.

## 4 Asset Allocation

In this section, we construct six different portfolios in groups of two. The first group is without the risk-free rate, and the latter is with a risk free rate yielding an annual return of 0.45%. In addition, we make three other portfolios for each group, one for each of the two industries and one combined. By constructing a portfolio for each sector, we can analyze the consequences of the industry attributes noted in section 2.

Without Short and Risk free			
	Std.	Expected Return	Sharpe Ratio
Airline Industry	0.25	0.06	0.21
Beer Industry	0.13	0.06	0.41
Both	0.13	0.06	0.41

Table 4: Portfolio, Return 6%

With Risk free and without Short			
	Std.	Expected Return	Sharpe Ratio
Airline Industry	0.09	0.06	0.60
Beer Industry	0.07	0.06	0.77
Both	0.06	0.06	0.90

Table 5: Portfolio, Return 6%

Note: All values for std. and return are annualized. VaR and ES will be discussed in Risk Management Part.

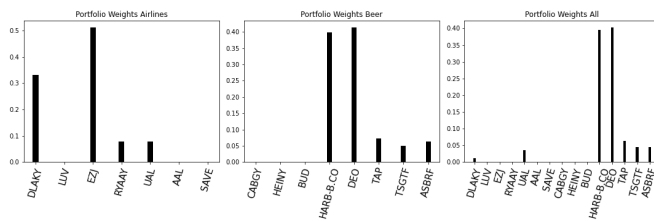


Figure 11: Portfolio weights, no short and no risk free

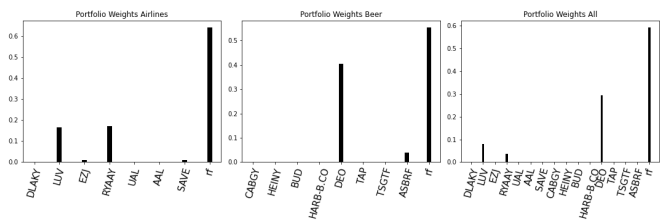


Figure 12: Portfolio weights, risk free but no short

Figure 11 and 12 illustrate the weights for each of the 6 portfolios that generate annual returns of exactly 6%. Figure 11 is the group of portfolios with no risk-free asset. These portfolios seem to have high concentrations, not surprisingly, in stocks with low standard deviations. This is very costly for the airline industry, as the high levels of risk in the industry creates a portfolio that has a standard deviation of 25%. The same high concentration can be found in figure 12, but in these portfolios it is the risk-free asset. These portfolios seem to favor the risk-free asset for the diversification and lowering of risk, and then add the risky assets with high returns to achieve the 6% target. The risk is more than halved for the high-risk airline industry and roughly halved for Beer and All.

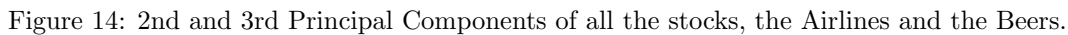
To conclude, we were able to create 6 portfolios that hit the 6% target return. The portfolios with a risk free asset have a significantly lower risk. This is because of the zero correlation between the risk free asset and the risky assets creates a larger diversification gain.

## 5 Principal Component Analysis

Before analyzing the Principal Components of the assets, we must first examine the correlations between the companies. Hence the correlation matrix is computed and, not surprisingly, the assets from the airline and beer industry are found to be more correlated with their own industry. The assets that are most correlated with each other are United Airlines and American Airlines with 0.768, and Heineken and Anheuser-Busch with 0.701. The least correlated assets within their own industry are EazyJet and American Airlines with a weak 0.259 correlation, and Harboe and Tsingtao with a low 0.034, with essentially no correlation.

Looking at the correlations between the industries, the assets have mostly weak correlations, a few moderate and no

The goal of Principal Component analysis is to reduce the data's dimensions, minimize information loss and achieve better understanding of the data. Above, the 1<sup>st</sup> and 2<sup>nd</sup> Principal Components (PC) of the stocks are computed. The 1<sup>st</sup> PC contains most of the variability of data, where all the stocks are positively correlated with the 1<sup>st</sup> PC. This is expected as the 1<sup>st</sup> PC is generally a representation of the market. We note that in the leftmost plot, containing all the stocks, half of the variance is explained by the two PC. In the airlines plot, 68.83 percent of the variance is explained by the two PC and in the beers plot 60.63 percent of the variance is explained by the two PC. We separate the assets further by computing the 3<sup>rd</sup> PC.



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## 6 Risk Management

Investments always carry different degrees of risks, and as financial products become increasingly more diverse and complex, the need to quantify risk becomes as important as it is difficult. The most popular and traditional measure of risk is volatility. However, volatility does not reflect the direction of return movements. Investors, on the other hand, are not distressed by gains, only by losses. To address this need, more advanced measures of risk were invented. Value-at-risk, or VaR, is a bound calculated with respect to an investment time horizon and a confidence level. For example, if the time horizon is one month, and the confidence interval is 99%, and VaR is \$1 million, then we can say that: “There is only a 1% chance that we will lose more than \$1 million over the next week.” A disadvantage of VaR is that it may discourage diversification. A very desirable criteria for a risk measurement is subadditivity, meaning that the risk of a portfolio comprising of assets A and B, is smaller than or equal to, the sum of their individual risks. This stems from the idea that diversification can reduce risk. VaR does not have this property, hence, the need for a risk measure that does. Expected Shortfall, or ES, has this edge over VaR. Given that the loss does exceed VaR, ES measures that expected loss. VaR and ES can be measured using two approaches, parametric, and non-parametric. Parametric, as opposed to non-parametric, assumes that the returns come from a distribution.

In this section, we look at the Value-at-risk (VaR), and Expected Shortfall (ES), of an investment of \$100,000 into each of our 15 individual assets and our 18 portfolios over a one-month investment horizon, using the parametric and non-parametric approach. The results are as follows, where MV = minimum variance, TG = tangent — the full table is in Appendix B:

Asset	Param. VaR	Param. ES	Non-param. VaR	Non-param. ES
Lufthansa	18244.93	11169.92	17257.03	21970.9
American	21252.48	21252.48	16932.27	24351.87
Spirit	19265.91	11205.06	14628.33	25358.96
Guinness	6416.28	3470.6	6591.676	8284.902
TG Airline with short	13417.91	7135.692	11778.66	17021.53
Port. 6% Airline no risk free	11577.25	6842.453	12648.91	15683.11
Port. 6% Beer + risk free	2874.874	1551.778	2765.414	3709.894

Table 6: Summary table of Parametric and Non-parametric VaR and ES for 15 stocks and 18 portfolios.

The highest and lowest VaR and ES are highlighted in the table. Overall, stocks from airline companies had higher VaR and ES while stocks from beer companies had lower VaR and ES, indicating that it would have been riskier to invest in airlines during the period observed. Similarly, portfolios with only airline stocks are the most risky and those with only beer stocks are the least risky. Adding the risk free asset also decreases the riskiness of portfolios. Since the VaR and ES values that we have are only estimates, it is helpful to find their confidence intervals, easily done through bootstrapping. The results are as follows — the full table is in Appendix B:

Asset	Std Err. VaR	Std Err. ES	95% CI VaR	95% CI ES
Spirit	2558.811	6706.345	(13781.62, 22987.59)	(20304.74, 48076.54)
Guinness	831.214	804.1956	(5782.76, 7394.468)	(7664.868, 9683.016)
TG All no short	665.2673	8983.914	(3927.320, 5321.367)	(5932.947, 12071.857)
Port. 6% Beer + risk free	335.1922	586.812	(2682.825, 3637.695)	(3326.616, 4966.736)
Port. 6% All + risk free	2533.0612	810.3197	(1739.580, 2500.666)	(2591.730, 5027.722)

Table 7: Summary table for Standard errors and 95% Confidence intervals for VaR and ES.

Of all the stocks, Spirit Airlines experienced the most variance in its returns, whereas for the portfolios, the tangent portfolios of all the stocks with no shorting and all stocks with no risk-free had the highest standard errors for ES

and VaR, respectively. This implies that the estimate for VaR and ES of these assets were the most volatile.

## 7 Copulas

Since our aim is to combine these stocks in portfolios, to measure their combined risk, it is important that we study their co-movements in extreme behavior. For this purpose, we use copula, a popular framework that characterizes the dependence between the components of a multivariate distribution. In this section, we attempt to fit a number of copulas to the 15 stock returns and use the AIC and BIC values to determine the best-fitting copula. The results are as follows:

Copula	AIC	BIC
Frank	-281.1542	-278.2709
Clayton	-370.9711	-368.0959
Gumbel	-320.2901	-317.4149
Joe	-212.8627	-209.9875
Student's t	-712.9468	-408.1759
Normal	-703.7758	-401.8801

Table 8: Copulas and their AIC, BIC values on the data

The lowest AIC and BIC values indicate that the t-copula fits our data the best, although the normal copula also fits reasonably well. The t-copula has heavier tails than the normal copula, so the fact that it has a better fit confirms our suspicion that there is stronger dependence among extreme values of the data. In terms of risks, this implies dependence among big losses.

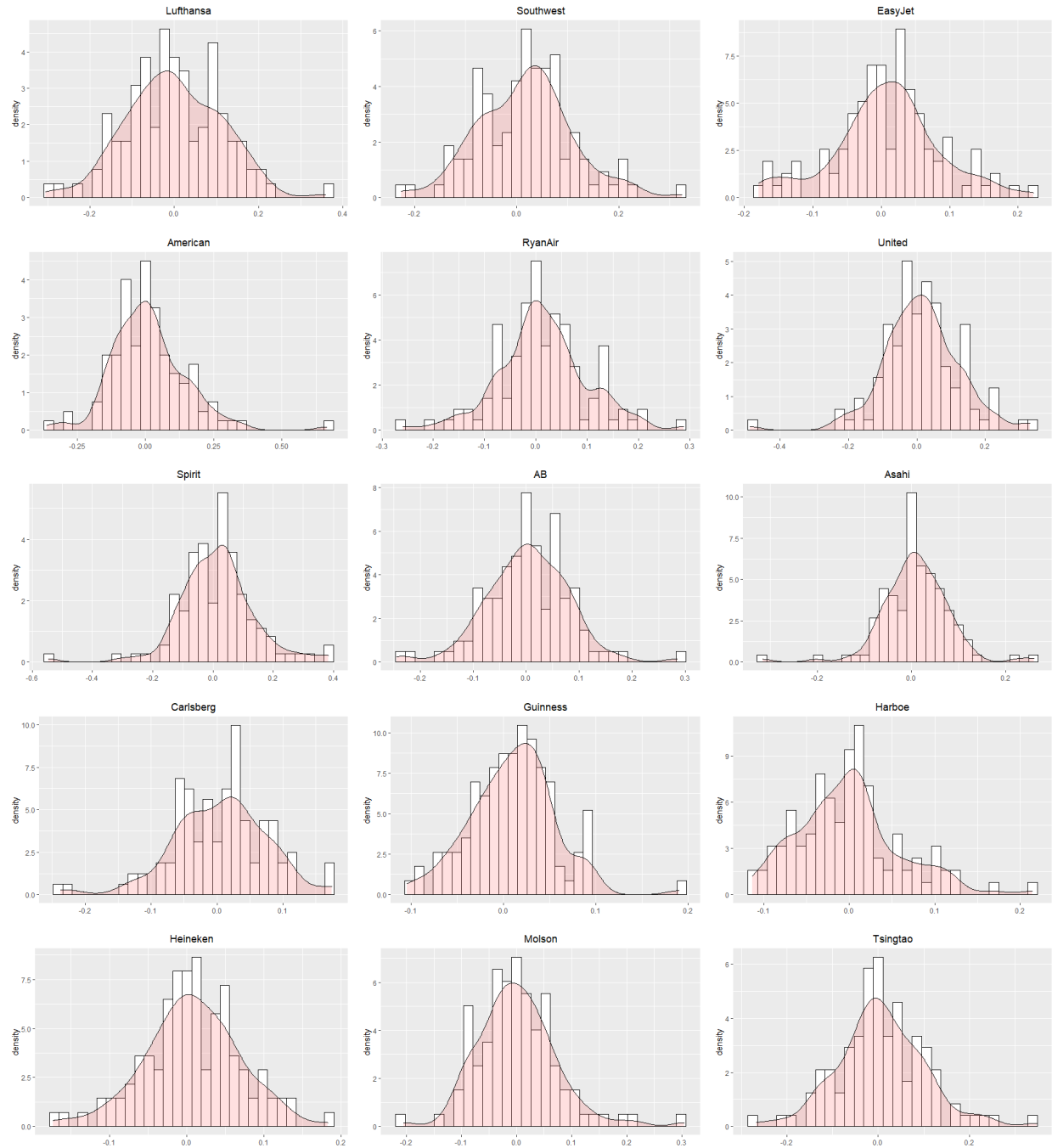
## 8 Conclusion

In this paper, we have set out to study the returns of 15 stocks from the airline and beer industries. The returns of each asset were examined by plotting their histograms, boxplots and equity curves. The assets were tested for normality by computing the Shapiro test and by plotting their respective Q-Q plots, where more than half of the assets demonstrated to follow normality, and all of the assets were concluded to be stationary. Next, the returns were fit with the t, skewed t, ged and normal distributions, and the best fit was chosen based on their respective AIC and BIC.

Using Modern Portfolio Theory, we constructed the minimum variance portfolio, the efficient frontier, the Capital Market Line and tangent portfolios for each industry and with the risk free asset. In addition, we constructed 6 distinctive efficient portfolios separated by different industries and short availability, each portfolio had a 6% annual expected return.

We analyzed VaR and ES as risk measurements for the stock and portfolio returns. The results imply that the airline industry was significantly more risky than the beer industry during the period observed. Furthermore, we determined that the t-copula best represents the co-movement of the stock returns, implying that big losses correlate with each other.

## A Appendix: Histograms



## B Appendix: Tables

Stocks	Monthly Means	Annual Means	Standard Deviations	Skewness	Kurtosis	Betas	Sharpe Ratios
Lufthansa	-0.00059	-0.007	0.112	0.087	0.149	1.194	-0.009
Southwest	0.017	0.206	0.093	0.215	0.568	1.004	0.181
EasyJet	0.008	0.095	0.078	-0.054	0.373	1.254	0.097
RyanAir	0.013	0.162	0.088	0.038	0.784	1.273	0.140
United	0.012	0.141	0.115	-0.4005	2.606	1.110	0.098
American	0.013	0.154	0.140	0.928	3.371	1.299	0.089
Spirit	0.013	0.155	0.128	-0.271	3.089	1.258	0.098
Carlsberg	0.0049	0.059	0.072	-0.383	0.958	1.281	0.063
Heineken	0.0069	0.083	0.062	-0.113	0.335	1.009	0.105
AB	0.0025	0.030	0.078	-0.034	1.409	1.283	0.027
Harboe	-0.00045	-0.005	0.0605	0.656	0.698	0.4499	-0.014
Guinness	0.0102	0.122	0.046	0.246	1.209	0.678	0.213
Molson	0.0015	0.018	0.073	0.774	2.302	0.873	0.016
Tsingtao	0.0073	0.088	0.097	0.292	1.047	0.862	0.072
Asahi	0.0084	0.100	0.072	-0.418	4.165	0.801	0.111

Table 9: Summary of the stock returns.

Stocks	Best AIC	Best BIC
Lufthansa	Skewed Sd t	t Dist
Southwest	t Dist	t Dist
EasyJet	Ged	t Dist
RyanAir	Skewed Sd t	t Dist
United	t Dist	t Dist
American	t Dist	t Dist
Spirit	t Dist	t Dist
Carlsberg	Skewed Sd t	t Dist
Heineken	t Dist	t Dist
AB	t Dist	t Dist
Harboe	Normal	Normal
Guinness	Skewed Sd t	t Dist
Molson	Skewed Sd t	t Dist
Tsingtao	Normal	Normal
Asahi	t Dist	t Dist

Table 10: Fitting different distributions to the returns.

Asset	Param. VaR	Param. ES	Non-param. VaR	Non-param. ES
Lufthansa	18244.93	11169.92	17257.03	21970.9
Southwest	13577.26	7709.64	12709.06	16344.49
EasyJet	12468.99	7288.445	14983.15	17316.64
RyanAir	12953.99	7363.967	13671.47	17681.32
United	17276.36	10092.61	15414.27	23753014
American	21252.48	21252.48	16932.27	24351.87
Spirit	19265.91	11205.06	14628.33	25358.96
Carlsberg	10821.82	6302.567	11668.34	15817.89
Heineken	9170.03	5242.139	9511.631	12450.04
AB	12175.26	7211.263	11755.09	16653.04
Harboe	10133.59	6237.924	8863.142	10498.42
Guinness	6416.28	3470.6	6591.676	8284.902
Molson	11738.09	6994.969	10112.46	13600.19
Tsingtao	14438.8	8448.289	13336.66	17879.13
Asahi	11113.12	6415.329	8781.621	15465.34
MV Airline no short	9624.625	5418.554	10440.79	15088.42
MV Airline with short	9492.84	5338.71	10591.14	14626.92
MV Beer no short	5434.154	3064.544	3668.999	6139.709
MV Beer with short	5119.817	2834.191	4615.985	6091.028
MV All no short	5213.395	2891.24	3663.698	6214.32
MV All with short	4563.874	2417.648	4185.553	5760.29
TG Airline no short	10352.51	5782.155	10564.58	15691.61
TG Airline with short	13417.91	7135.692	11778.66	17021.53
TG Beer no short	6416.28	3470.6	6591.676	8284.902
TG Beer with short	7203.078	1889.143	7158.573	9712.955
TG All no short	5580.638	2959.132	4175.251	7155.917
TG All with short	6588.397	3100.83	6777.745	7840.509
Port. 6% Airline no risk free	11577.25	6842.453	12648.91	15683.11
Port. 6% Airline + risk free	3939.9964	2199.497	3987.925	5936.578
Port. 6% Beer no risk free	5589.733	3199.663	4137.043	6398.093
Port. 6% Beer + risk free	2874.874	1551.778	2765.414	3709.894
Port. 6% All no risk free	5545.904	3172.387	4055.861	6443.849
Port. 6% All + risk free	2455.237	1296.656	1839.216	3125.593

Table 11: Parametric and Non-parametric VaR and ES for 15 stocks and 18 portfolios.

Asset	Std Err. VaR	Std Err. ES	95% CI VaR	95% CI ES
Lufthansa	1541.463	2708.64	(16339.66, 20905.05)	(20179.42, 27951.62)
EasyJet	1893.681	1240.261	(13090.56, 16828.39)	(16426.88, 19409.99)
RyanAir	2278.774	2349.771	(11916.53, 1653029)	(16047.15, 22622.42)
United	2913.479	5608.773	(13013.98, 20367.40)	(19696.71, 37.457.44)
American	2488.361	3870.27	(15410.87, 24403.19)	(21768.57, 32152.72)
Spirit	2558.811	6706.345	(13781.62, 22987.59)	(20304.74, 48076.54)
Carlsberg	1976.501	2617.028	(9050.357, 19917.562)	(14121.69, 21441.40)
Heineken	1286.024	1641.107	(8554.09, 11965.30)	(11316.80, 15707.55)
AB	2002.534	2695.756	(10128.09, 15316.00)	(15025.95, 22805.04)
Harboe	866.9745	573.4761	(86688.833, 10784.34)	(10050.77, 11148.49)
Guinness	831.214	804.1956	(5782.76, 7394.468)	(7664.868, 9683.016)
Molson	1206.253	1791.54	(9355.318, 13461.517)	(12440.36, 17530.50)
Tsingtao	1222.502	2540.807	(12412.49, 16685.26)	(16133.05, 23439.23)
Asahi	1788.398	4045.541	(8039.664, 13288.867)	(12900.57, 24758.72)
MV Airline no short	2227.221	2339.373	(8489.428, 14426.794)	(13452.53, 19233.07)
MV Airline with short	1783.7295	2143.127	(8930.947, 13814.412)	(1307.27, 18485.82)
MV Beer no short	665.8754	1431.238	(3516.196, 5826.853)	(5234.095, 9691.107)
MV Beer with short	659.5311	720.127	(4099.208, 6033.949)	(5602.308, 7543.522)
MV All no short	423.7295	1590.442	(3499.960, 5070.296)	(5147.304, 9856.441)
MV All with short	563.0657	911.5496	(3753.311, 5031.890)	(5037.937, 7812.951)
TG Airline no short	2144.805	2582.053	(9370.582, 15300.355)	(14024.66, 21333.08)
TG Airline with short	2378.596	2032.891	(70707.31, 18729.53)	(15590.36, 20071.79)
TG Beer no short	902.1355	864.6005	(5782.760, 8339.615)	(7642.915, 9872.335)
TG Beer with short	873.4374	1278.852	(6943.325, 9555.144)	(8900.549, 12425.804)
TG All no short	665.2673	8983.914	(3927.320, 5321.367)	(5932.947, 12071.857)
TG All with short	643.8685	798.5206	(6004.606, 7292.348)	(7346.438, 9616.499)
Port. 6% Airline no risk free	1414.459	1838.773	(11571.43, 15084.46)	(14394.24, 19651.11)
Port. 6% Airline + risk free	796.4276	919.0456	(3591.423, 5765.276)	(5308.958, 7868.103)
Port. 6% Beer no risk free	524.4646	1294.809	(3974.821, 48883.155)	(5541.863, 9110.936)
Port. 6% Beer + risk free	335.1922	586.812	(2682.825, 3637.695)	(3326.616, 4966.736)
Port. 6% All no risk free	592.6639	1496.985	(3892.511, 5726.735)	(5434.982, 10100.832)
Port. 6% All + risk free	2533.0612	810.3197	(1739.580, 2500.666)	(2591.730, 5027.722)

Table 12: Standard errors and 95% Confidence intervals for VaR and ES of 15 stocks and 18 portfolios.