Tarefa Básica - Coeficientes Binomiais

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(1) $\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8.7.6.\cancel{5!}}{3!\cancel{5!}} = \frac{336}{3.2.1} = \frac{336}{6} = \boxed{56}$ → alternativa (b)

(2) $\binom{200}{198} = \frac{200!}{198!(200-198)!} = \frac{200.199.\cancel{198!}}{\cancel{198!}\,2!} = \frac{39800}{2.1} = \boxed{19900}$ → alternativa (a)

(3) $\binom{n-1}{2} = \binom{n+1}{4}$

↳ 1º caso: $\frac{(n-1)!}{2!(n-1-2)!} = \frac{(n+1)!}{4!(n+1-4)!}$

$\frac{(n-1)!\,4!\,\cancel{(n-3)!}}{2!\,\cancel{(n-3)!}} = (n+1)n!$

$\frac{(n-1)!\,4.3.\cancel{2!}}{\cancel{2!}} = (n+1).n.(n-1)!$

→ $12 = \frac{(n^2+n).\cancel{(n-1)!}}{\cancel{(n-1)!}}$

$n^2+n-12=0$

$\underline{-4} + \underline{3} = -1$

$\underline{-4} \cdot \underline{3} = -12$

$n_I = -4$ (negativo = não convém)

$n_{II} = \boxed{3}$

↳ 2º caso: $\binom{n-1}{2} = \binom{n+1}{4} = 0$

↳ $n < K$:

$n-1<2$

$n<3$

$n+1<4$

$n<3$

$\boxed{n<3}$ → $\boxed{V = \{1, 2, 3\}}$

4) $\binom{20}{13}+\binom{20}{14}$ = Soma de 2 consecutivos: $\binom{n}{k}+\binom{n}{k+1}=\binom{n+1}{k+1}$

$\hookrightarrow \binom{20}{13}+\binom{20}{14}=\binom{21}{14}$

* Complementares: $\binom{n}{k}=\binom{n}{n-k} \rightarrow \binom{21}{14}=\binom{21}{21-14}=\boxed{\binom{21}{7}}$

alternativo c

5) $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\dots+\binom{n}{n}=\boxed{2^n}$

6) a) $\sum_{p=0}^{10}\binom{10}{p}=\binom{10}{0}+\binom{10}{1}+\dots+\binom{10}{10}=2^{10}=\boxed{1024}$

b) $\sum_{p=0}^{9}\binom{10}{p}=\binom{10}{0}+\binom{10}{1}+\dots+\binom{10}{9}=2^{10}-\binom{10}{10}=1024-1=\boxed{1023}$

c) $\sum_{p=2}^{9}\binom{9}{p}=\binom{9}{2}+\binom{9}{3}+\dots+\binom{9}{9}=2^9-\binom{9}{1}-\binom{9}{0}=512-9-1=$

$\hookrightarrow =\boxed{502}$

d) $\sum_{p=4}^{10}\binom{p}{4}=\binom{4}{4}+\binom{5}{4}+\dots\binom{10}{4}=\binom{11}{5}$ → soma na coluna 4

$\binom{11}{5}=\frac{11!}{5!(11-5)!}=\frac{11.10.9.8.7.\cancel{6!}}{5!\,\cancel{6!}}=\frac{11.\cancel{10}^{2}.\cancel{9}^{3}.\cancel{8}^{2}.7}{\cancel{5}.\cancel{4}.\cancel{3}.2.1}=\frac{924}{2}=\boxed{462}$

e) $\sum_{p=5}^{10}\binom{p}{5}=\binom{5}{5}+\binom{6}{5}+\dots+\binom{10}{5}=\binom{11}{6}$ → soma na coluna 5

$\binom{11}{6}=\frac{11!}{6!(11-6)!}=\frac{11.10.9.8.7.\cancel{6!}}{\cancel{6!}\,5!}=\frac{11.\cancel{10}^{2}.\cancel{9}^{3}.\cancel{8}^{2}.7}{\cancel{5}.\cancel{4}.\cancel{3}.2.1}=\frac{924}{2}=\boxed{462}$

7) $\sum_{K=0}^{m} \binom{m}{K} = 512$

$$\sum_{K=0}^{m} \binom{m}{K} = \binom{m}{0} + \dots + \binom{m}{m} = 512$$

$$2^m = 512$$

$$\cancel{2}^m = \cancel{2}^9$$

$$\boxed{m = 9}$$

alternativa

(e)

512	2
256	2
128	2
64	2
32	2
16	2
8	2
4	2
2	2
	2^9