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Tarefa Básica - Teorema do Binômio

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1 $(1+2x^2)^6 \rightarrow$ coeficiente de $x^8 = ?$

↳ Linha 6 : $\binom{6}{K} \cdot 1^{6-K} \cdot (2x^2)^K \ldots$

$\ldots \binom{6}{K} 1^{6-K} \cdot 2^K \cdot x^{2K}$ → $2K = 8$

$K = \frac{8}{2} = 4$

* valor de K para o x ter expoente 8

$\binom{6}{4} 1^{6-4} \cdot 2^4 \cdot x^{2 \cdot 4}$ → $\frac{\cancel{6}^{3} \cdot 5 \cdot 4!}{4! \cdot \cancel{2} \cdot 1} \cdot 16x^8$

$\frac{6!}{4!(6-4)!} \cdot 1^2 \cdot 16 \cdot x^8$ → $15 \cdot 16x^8 = \boxed{240}x^8$

alternativa (c)

(2) $(14x-13y)^{237}$ → soma de todos os coeficientes = ?

$(x=1, y=1)$

→ $(14-13)^{237}$

$1^{237} = \boxed{1}$ → alternativa (b)

(3) $(x+a)^{11}$, $1386x^5$, $a = ?$

→ linha 11: $\binom{11}{K} \cdot x^{11-K} \cdot a^K$

* valor de K para o x ter expoente 5: $11-K=5$

$K = 11-5 = 6$

→ $\binom{11}{6} \cdot x^{11-6} \cdot a^6 = 1386x^5$

$\frac{11!}{6!(11-6)!} \cdot x^5 \cdot a^6 = 1386x^5$

$\frac{11 \cdot \cancel{10}^{2} \cdot \cancel{9}^{3} \cdot \cancel{8}^{4} \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot 5 \cdot 4 \cdot 3 \cdot 2} \cdot \frac{\cancel{x^5}}{\cancel{x^5}} \cdot a^6 = 1386$

$462a^6 = 1386 \ldots$

$462a^6 = 1386 \rightarrow a^6 = 3$

$a^6 = \frac{1386}{462}$ $\quad a = \sqrt[6]{3}$ → alternativa (a)

4) $\left(x+\frac{1}{x^2}\right)^9$ → termo independente = ? (x^0)

$(x+x^{-2})^9$

$$\binom{9}{0}x^{9-0}\cdot(x^{-2})^0+\binom{9}{1}x^{9-1}\cdot(x^{-2})^1+\binom{9}{2}x^{9-2}\cdot(x^{-2})^2+\binom{9}{3}x^{9-3}\cdot(x^{-2})^3+\dots$$

$$\binom{9}{0}x^9\cdot x^0+\binom{9}{1}x^8\cdot x^{-2}+\binom{9}{2}x^7\cdot x^{-4}+\binom{9}{3}x^6\cdot x^{-6}+\dots$$

$$\binom{9}{0}x^9+\binom{9}{1}x^6+\binom{9}{2}x^3+\binom{9}{3}x^0$$ → termo independente (x^0)

Resposta: $\binom{9}{3}$ → alternativa (d)

5) $\left(x+\frac{1}{x^2}\right)^n$ tem termo independente, $n=?$

$(x+x^{-2})^n$

↳ Linha n: $\binom{n}{k}\cdot x^{n-k}\cdot(x^{-2})^k$

$\binom{n}{k}x^{n-k}\cdot x^{-2k}$

Termo independente (x^0)

↳ $n-k-2k=0$

$n-3k=0$

$n=3k$

$\frac{n}{3}=k$

* n dividido por 3 é igual ao número natural k, então n é divisível por 3.

↳ alternativa (c)

6 $K = \left(3x^3 + \frac{2}{x^2}\right)^5 - \left(243x^{15} + 810x^{10} + 1080x^5 + \frac{240}{x^5} + \frac{32}{x^{10}}\right)$

$(3x^3 + 2x^{-2})^5 = \binom{5}{0} \cdot (3x^3)^5 (2x^{-2})^0 + \binom{5}{1} (3x^3)^4 (2x^{-2})^1 + \binom{5}{2} (3x^3)^3 (2x^{-2})^2 + \binom{5}{3} (3x^3)^2 (2x^{-2})^3 + \binom{5}{4} (3x^3)^1 (2x^{-2})^4 + \binom{5}{5} (3x^3)^0 (2x^{-2})^5$

$= 243x^{15} + 810x^{10} + 1080x^5 + 720 + \frac{240}{x^5} + \frac{32}{x^{10}}$

$K = 243x^{15} + 810x^{10} + 1080x^5 + 720 + \frac{240}{x^5} + \frac{32}{x^{10}} - \left(243x^{15} + 810x^{10} + 1080x^5 + \frac{240}{x^5} + \frac{32}{x^{10}}\right)$

$K = \cancel{243x^{15}} + \cancel{810x^{10}} + \cancel{1080x^5} + 720 + \cancel{\frac{240}{x^5}} + \cancel{\frac{32}{x^{10}}} - \cancel{243x^{15}} - \cancel{810x^{10}} - \cancel{1080x^5} - \cancel{\frac{240}{x^5}} - \cancel{\frac{32}{x^{10}}}$

$K = \boxed{720}$ → alternativa a

7) $(2x+y)^5 \rightarrow$ soma coeficientes $= ?$

$\hookrightarrow (x=1, y=1) : (2+1)^5 = 3^5 = \boxed{243} \rightarrow$ alternativa c