Tarefa básica: Determinantes

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1) a) $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ $\quad$ det = 10 − 3 = $\boxed{7}$

3 $\quad$ 10

b) $\begin{bmatrix} -2 & -4 \\ 3 & 6 \end{bmatrix}$ $\quad$ det = −12 − (−12) = −12 + 12 = $\boxed{0}$

−12 $\quad$ −12

c) $\begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 4 & -2 \end{bmatrix} \begin{matrix} 3 & -1 \\ 2 & 1 \\ 1 & 4 \end{matrix}$ $\quad$ $\begin{matrix} 3 & -1 & 1 & 3 & -1 \\ 2 & 1 & -1 & 2 & 1 \\ 1 & 4 & -2 & 1 & 4 \end{matrix}$

−6 + 1 + 8 = 3 $\quad$ 1 − 12 + 4 = −7

det = 3 − (−7) = 3 + 7 = $\boxed{10}$

d) $\begin{bmatrix} 3 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix} \begin{matrix} 3 & 2 \\ 2 & 3 \\ 1 & 1 \end{matrix}$ $\quad$ $\begin{matrix} 3 & 2 & -1 & 3 & 2 \\ 2 & 3 & 1 & 2 & 3 \\ 1 & 1 & 4 & 1 & 1 \end{matrix}$

36 + 2 − 2 = 36 $\quad$ −3 + 3 + 16 = 16

$det = 36 - 16 = \boxed{20}$

② $A_{3\times3}$, $a_{ij} = \begin{cases} -3 \to i = j \\ 0 \to i \neq j \end{cases}$, $det = ?$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$i = j$

↳ $a_{11}, a_{22}, a_{33} = -3$

$i \neq j$

↳ o resto $= 0$

$-27 + 0 + 0 = -27$

$$\left[\begin{array}{ccc|cc} -3 & 0 & 0 & -3 & 0 \\ 0 & -3 & 0 & 0 & -3 \\ 0 & 0 & -3 & 0 & 0 \end{array}\right]$$

$0 + 0 + 0 = 0$

$det = -27 - 0 = \boxed{-27}$

↳ alternativa a

③ $\begin{vmatrix} x & 1 & x \\ 3 & x & 4 \\ 1 & 3 & 3 \end{vmatrix} = -3 \quad (det = -3)$

$3x^2 + 4 + 9x$

$$\left|\begin{array}{ccc|cc} x & 1 & x & x & 1 \\ 3 & x & 4 & 3 & x \\ 1 & 3 & 3 & 1 & 3 \end{array}\right|$$

$x^2 + 12x + 9$

$det = 3x^2 + 4 + 9x - (x^2 + 12x + 9)$

$det = 3x^2 + 4 + 9x - x^2 - 12x - 9$

$det = 2x^2 - 3x - 5$

$-3 = 2x^2 - 3x - 5$

$2x^2 - 3x - 5 + 3 = 0$

$2x^2 - 3x - 2 = 0$ *

→ $a = 2$

→ $b = -3$

→ $c = -2$

* $\Delta = (-3)^2 - 4.(2).(-2)$

$\Delta = 9 + 16 = 25$

$x_1 = \dfrac{-(-3) + \sqrt{25}}{2.2} \Rightarrow \dfrac{3+5}{4} = \dfrac{8}{4} = \boxed{2}$

$x_{11} = \dfrac{-(-3) - \sqrt{25}}{2.2} \Rightarrow \dfrac{3-5}{4}$

↳ $x_{11} = \dfrac{-2^{:2}}{4_{:2}} = \boxed{-\dfrac{1}{2}}$

Resposta: $x_I = 2$, $x_{II} = -\frac{1}{2}$

↳ $\{-1/2; 2\}$ → alternativa a

4) $x_I + x_{II} + x_{III} = ?$, $\begin{vmatrix} x-1 & -1 & 0 \\ 0 & x+1 & -1 \\ 2 & -1 & x+1 \end{vmatrix} = 2$

$(x+1)^2.(x-1)+2+0 = (x+1).(x+1).(x-1)+2$

$\begin{vmatrix} x-1 & -1 & 0 \\ 0 & x+1 & -1 \\ 2 & -1 & x+1 \end{vmatrix} \begin{matrix} x-1 & -1 \\ 0 & x+1 \\ 2 & -1 \end{matrix}$

$0+(x-1)+0 = x-1$

$det = 2$

$det = (x-1).(x+1).(x+1)+2-(x-1)$

$det = (x^2-1^2).(x+1)+2-x+1$

$det = x^3+x^2-x-\cancel{1}+2-x+\cancel{1}$

$\cancel{2} = x^3+x^2-2x+\cancel{2}$

$x^3+x^2-2x=0$

→ * relações de Girard

$(a=1, b=1, c=-2, d=0)$

↳ $x_I + x_{II} + x_{III} = -\frac{b}{a}$

↳ $x_I + x_{II} + x_{III} = -\frac{1}{1} = \boxed{-1}$

alternativa c

5) $A_{3\times2} \rightarrow a_{ij} = 2i-3j$, $B_{2\times3} \rightarrow b_{jk} = k-j$, $det\ A.B = ?$

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \rightarrow \begin{bmatrix} -1 & -4 \\ 1 & -2 \\ 3 & 0 \end{bmatrix}$

$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$

$a_{11} = 2.1-3.1 = -1$

$a_{12} = 2.1-3.2 = -4$

$a_{21} = 2.2-3.1 = 1$

$a_{22} = 2.2-3.2 = -2$

$a_{31} = 2.3-3.1 = 3$

$a_{32} = 2.3-3.2 = 0$

$b_{11} = 1-1 = 0$

$b_{12} = 2-1 = 1$

$b_{13} = 3-1 = 2$

$b_{21} = 1-2 = -1$

...

$\ldots\ b_{22} = 2-2=0$

$b_{23} = 3-2=1$ $\longrightarrow$ $B = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

$A.B = \begin{bmatrix} (0+4) & (-1-0) & (-2-4) \\ (0+2) & (1-0) & (2-2) \\ (0-0) & (3+0) & (6+0) \end{bmatrix} \rightarrow A.B = \begin{bmatrix} 4 & -1 & -6 \\ 2 & 1 & 0 \\ 0 & 3 & 6 \end{bmatrix}$

$24-0-36=-12$

$\left|\begin{matrix} 4 & -1 & -6 \\ 2 & 1 & 0 \\ 0 & 3 & 6 \end{matrix}\right|\begin{matrix} 4 & -1 \\ 2 & 1 \\ 0 & 3 \end{matrix}$

$0+0-12=-12$

$\det A.B = -12-(-12)$

$\det A.B = -12+12=0$

alternativa (c)

(6) $A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$ e $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}$, $\det A.B = ?$

$A.B = \begin{bmatrix} (2+0+0) & (-2+0-2) \\ (-1-1+0) & (1+1+0) \end{bmatrix}$

$\begin{array}{cc} 4 & 8 \end{array}$

$\hookrightarrow A.B = \begin{bmatrix} 2 & -4 \\ -2 & 2 \end{bmatrix}$ $\det A.B = 4-8 = \boxed{-4}$

alternativa (d)