Tarefa Básica - Cálculo Geral de determinantes

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(1) $\det A = ?$, $\det B = ?$

$$A = \begin{vmatrix} 1 & a & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{vmatrix} \rightarrow \det A = \begin{vmatrix} 1 & a & 0 & 1 & a \\ 0 & 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 & -1 \end{vmatrix}$$

$0 - 1 + 0 = -1$

$1 + 0 + 0 = 1$

$\det A = 1 - (-1) = 1 + 1 = \boxed{2}$

$$B = \begin{vmatrix} 1 & 0 & 0 & 3 \\ a & 1 & -1 & 4 \\ 0 & 0 & 0 & 3 \\ 0 & 1 & 1 & 4 \end{vmatrix}$$

$0 \cdot \not{a}(B_{12}) + 1 \cdot a(B_{22}) + 0 \cdot \not{a}(B_{32}) + 1 \cdot a(B_{42})$

$\hookrightarrow 1 \cdot \begin{vmatrix} 1 & 0 & 3 \\ 0 & 0 & 3 \\ 0 & 1 & 4 \end{vmatrix} + \hookrightarrow 1 \cdot \begin{vmatrix} 1 & 0 & 3 \\ a & -1 & 4 \\ 0 & 0 & 3 \end{vmatrix}$

$0+3+0=3$

$$1.\begin{vmatrix} 1 & 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 \end{vmatrix} + 1.\begin{vmatrix} 1 & 0 & 3 & 1 & 0 \\ a & -1 & 4 & 0 & -1 \\ 0 & 0 & 3 & 0 & 0 \end{vmatrix}$$

$0+0+0=0$

$0+0+0=0$ $\qquad$ $-3+0+0=-3$

$0-3=-3$ $\qquad$ $-3-0=-3$

$i+j=2+2=$ par $\neg -3$ // $\qquad$ $i+j=4+2=$ par $\neg -3$

$\det B = -3+(-3)$

$\det B = -3-3 = \boxed{-6}$

(2) $x=?$, $\det = 0$

$$\begin{vmatrix} x^2 & 0 & x & -\frac{1}{10} \\ 7{,}5 & 0 & 5 & 2 \\ 10 & 0 & 4 & 2 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0$$

$\rightarrow 0.\cancel{cof(a12)} + 0.\cancel{cof(a22)} + 0.\cancel{cof(a32)} + 1.cof(a42)$

$-5+8x^2+15x$

$$1.\begin{vmatrix} x^2 & x & -\frac{1}{10} & x^2 & x \\ 7{,}5 & 5 & 2 & 7{,}5 & 5 \\ 10 & 4 & 2 & 10 & 4 \end{vmatrix}$$

$10x^2+20x-3$

$\det = 10x^2+20x-3-(-5+8x^2+15x) = 0$

$10x^2+20x-3+5-8x^2-15x=0$

$2x^2+5x+2=0$

$\Delta = 5^2-4.2.2$

$\Delta = 25-16=9$

$$x_I = \frac{-5+\sqrt{9}}{2.2} = \frac{-5+3}{4} = \frac{-2^{:2}}{4^{:2}} = \boxed{-\frac{1}{2}}$$

$$x_{II} = \frac{-5-\sqrt{9}}{2.2} = \frac{-5-3}{4} = \frac{-8}{4} = \boxed{-2}$$

$\boxed{x=-\frac{1}{2} \text{ ou } x=-2}$

(3) det = ?

$\rightarrow 0 \cdot \text{adj}(a_{12}) + x \cdot \text{adj}(a_{22}) + (-1) \cdot \text{adj}(a_{32}) + 0 \cdot \text{adj}(a_{42})$

$$\begin{vmatrix} x & 0 & 0 & 3 \\ -1 & x & 0 & 0 \\ 0 & -1 & x & 1 \\ 0 & 0 & -1 & -2 \end{vmatrix}$$

$$x \cdot \begin{vmatrix} x & 0 & 3 \\ 0 & x & 1 \\ 0 & -1 & -2 \end{vmatrix} + -1 \cdot \begin{vmatrix} x & 0 & 3 \\ -1 & 0 & 0 \\ 0 & -1 & -2 \end{vmatrix}$$

$0 - x + 0 = -x$

$0 + 0 + 0 = 0$

$x \cdot \begin{vmatrix} x & 0 & 3 \\ 0 & x & 1 \\ 0 & -1 & -2 \end{vmatrix} \quad -1 \cdot \begin{vmatrix} x & 0 & 3 \\ -1 & 0 & 0 \\ 0 & -1 & -2 \end{vmatrix}$

$-2x^2 + 0 + 0 = -2x^2$

$0 + 0 + 3 = 3$

$\rightarrow x \cdot (-2x^2 - (-x))$

$x \cdot (-2x^2 + x)$

$-2x^3 + x^2$

$i + j = 2 + 2 =$ par $\rightarrow -2x^3 + x^2$

$\rightarrow -1 \cdot (3 - 0)$

$-1 \cdot 3 = -3$

$i + j = 3 + 2 =$ ímpar $\rightarrow +3$

$\det = \boxed{-2x^3 + x^2 + 3} \rightarrow$ alternativa (a)

(4) $A = \begin{vmatrix} x & 1 & 0 & 0 & 0 \\ 0 & x & 1 & 0 & 0 \\ 0 & 0 & x & 1 & 0 \\ 0 & 0 & 0 & x & K \\ 0 & 0 & 0 & 1 & x \end{vmatrix}$, $f(x) = \det A$ e $f(-2) = 8$, $K = ?$

$\rightarrow x \cdot \text{adj}(a_{11}) + 0 \cdot \text{adj}(a_{21}) + 0 \cdot \text{adj}(a_{31}) + 0 \cdot \text{adj}(a_{41}) + 0 \cdot \text{adj}(a_{51})$

B →

$$x \cdot \begin{vmatrix} x & 1 & 0 & 0 \\ 0 & x & 1 & 0 \\ 0 & 0 & x & k \\ 0 & 0 & 1 & x \end{vmatrix}$$

$\rightarrow x \cdot \text{cof}(b_{11}) + 0 \cdot \text{cof}(b_{21}) + 0 \cdot \text{cof}(b_{31}) + 0 \cdot \text{cof}(b_{41})$

$0 + kx + 0 = kx$

$$x \cdot \left(x \cdot \begin{vmatrix} x & 1 & 0 \\ 0 & x & k \\ 0 & 1 & x \end{vmatrix} \right)$$

$x^3 + 0 + 0 = x^3$

$\det A = x \cdot (x \cdot (x^3 - kx))$

$\det A = x^2 \cdot (x^3 - kx)$

$\det A = x^5 - kx^3$ //

$f(x) = \det A$

$f(x) = x^5 - kx^3$

$f(-2) = 8$

$f(-2) = -2^5 - k \cdot (-2)^3 = 8$

$-32 - k \cdot (-8) = 8$

$-32 + 8k = 8$

$8k = 8 + 32$

$8k = 40$

$k = \frac{40}{8} = \boxed{5}$

→ alternativa (d)