

Tarefa Básico - Sistemas Lineares: Regra de Cramer

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① a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$ $D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$

$$D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3, \quad \left\{ x = \frac{D_x}{D} = \frac{3}{5} \right\}$$

$$D_y = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6 - (-2) = -4, \quad \left\{ y = \frac{D_y}{D} = \frac{-4}{5} \right\}$$

$$V = \left\{ \left(\frac{3}{5}, -\frac{4}{5} \right) \right\}$$

AFAPEL

$$0+9+4=13$$

$$\text{D}) \begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases} \quad D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} \quad 0-12+2=-10$$

$$0+3-2=1 \quad D = -10-13=-23,$$

$$Dx = \begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & -1 \\ 4 & 1 & -2 \end{vmatrix} \rightarrow Dx = -22-1=-23 \quad x = \frac{Dx}{D}$$

$$0-21-1=-22$$

$$-4+63-4=55$$

$$Dy = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 1 & -2 \end{vmatrix} \rightarrow Dy = 32-55=-23 \quad y = \frac{Dy}{D} = \frac{-23}{-23} = 1$$

$$+6+12+14=32$$

$$0-3-14=-17$$

$$Dz = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 1 \end{vmatrix} \rightarrow Dz = 6-(-17)=23 \quad z = \frac{Dz}{D} = \frac{23}{-23} = -1$$

$$0+4+2=6$$

$$V = \{ (1, 1, -1) \}$$

$$-5-12+48=31$$

$$\textcircled{2}) y=? \quad \begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases} \quad D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix} \quad 45+8+8=61$$

$$Dy = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 1 & 2 \\ 1 & 8 & 3 \end{vmatrix} \quad D = 6!-31=30$$

$$108+2-32=78$$

$$\rightarrow Dy = 78-48=30,$$

$$\rightarrow y = \frac{Dy}{D} = \frac{30}{30} = 1 \rightarrow \text{nichts} \textcircled{2})$$

$$\textcircled{3} \quad a+b+c=? \quad \left\{ \begin{array}{l} \left\{ \begin{array}{l} x+2y+z=1 \\ 3x+y-11z=-2 \\ 2x+3y-z=1 \end{array} \right. \\ \left. \begin{array}{l} 1-33+4 = -28 \\ -1-44+9 = -36 \end{array} \right. \end{array} \right. \quad D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix} \quad \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix}$$

$$Dx = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ -2 & 1 & -11 & -2 & 1 \\ 1 & 3 & -1 & 1 & 3 \end{vmatrix} \quad D = -36 - (-37) = 1 \quad \rightarrow Dx = -29 - (-28) = -1 \quad x = \frac{Dx}{D} = \frac{-1}{1} = \boxed{1} \quad (\text{a})$$

$-1-22-6 = -29$
 $-4-11-3 = -18$

$$Dy = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & -2 & -11 & 3 & -2 \\ 2 & 1 & 2 & 1 & 1 \end{vmatrix} \quad \rightarrow Dy = -17 - (-18) = 1 \quad \rightarrow y = \frac{Dy}{D} = \frac{1}{1} = \boxed{1} \quad (\text{b})$$

$2-22+3 = -17$

$$2-6+6 = 2$$

$$Dz = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -2 & 3 & 1 \\ 2 & 3 & 1 & 2 & 3 \end{vmatrix} \quad \rightarrow Dz = 2-2 = 0 \quad \rightarrow z = \frac{Dz}{D} = \frac{0}{1} = \boxed{0} \quad (\text{c})$$

$1-8+9 = 2$

$$a+b+c = -1 + 1 + 0 = \boxed{0} \rightarrow \text{alternative } \textcircled{C}$$

$$-9-2-4 = -15$$

$$\textcircled{4} \quad x+y+z=? \quad \left\{ \begin{array}{l} \left\{ \begin{array}{l} x+2y-3z=29 \\ x+3y+2z=4 \\ x-y-2z=8 \end{array} \right. \\ \left. \begin{array}{l} -42-58-16 = -146 \\ -6+4+3 = 1 \end{array} \right. \end{array} \right. \quad D = \begin{vmatrix} 1 & 2 & -3 & 1 & 1 \\ 1 & 3 & 2 & 1 & 3 \\ 1 & -1 & -2 & 1 & -1 \end{vmatrix}$$

$$Dx = \begin{vmatrix} 29 & 2 & -3 & 29 & 1 \\ 4 & 3 & 2 & 1 & 3 \\ 8 & -1 & -2 & 8 & -1 \end{vmatrix} \quad \rightarrow Dx = -130 - (-346) = 16 \quad \rightarrow x = \frac{Dx}{D} = \frac{16}{16} = \boxed{1}$$

$-174+32+12 = -130$

$$-12 + 16 - 58 = -54$$

$$Dy = \begin{vmatrix} 1 & 2 & 9 & -3 & 1 & 2 & 9 \\ 1 & 4 & 2 & 1 & 4 \\ 1 & 8 & -2 & 1 & 8 \end{vmatrix} \rightarrow Dy = 26 - (-54) = 80, \\ -8 + 58 - 24 = 26 \quad \left\{ \begin{array}{l} y = \frac{Dy}{D} = \frac{80}{16} = 5 \\ D \end{array} \right.$$

$$87 - 4 + 16 = 99$$

$$Dz = \begin{vmatrix} 1 & 2 & 9 & 1 & 2 \\ 1 & 3 & 4 & 1 & 3 \\ 1 & -1 & 8 & 1 & -1 \end{vmatrix} \rightarrow Dz = 3 - 99 = -96, \\ 24 + 8 - 29 = 3 \quad \left\{ \begin{array}{l} z = \frac{Dz}{D} = \frac{-96}{16} = -6 \\ D \end{array} \right.$$

$$x + y + z = 1 + 5 - 6 = 0 \rightarrow \text{alternativa ①}$$

$$0 + 4 + 0 = 4$$

$$\textcircled{5} \quad \begin{cases} x = ? \\ y = ? \\ z = ? \end{cases} \quad \left\{ \begin{array}{l} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{array} \right. \quad \left\{ \begin{array}{l} D = \begin{vmatrix} 2 & 1 & 0 & 1 & 2 & 1 \\ 0 & 2 & 1 & 0 & 2 & 1 \\ 3 & 2 & 1 & 3 & 2 & 1 \end{vmatrix} \rightarrow D = 7 - 4 \\ D = 3 \end{array} \right. \\ 0 + 10 + 3 = 13 \quad 4 + 3 + 0 = 7$$

$$Dx = \begin{vmatrix} 5 & 1 & 0 & 5 & 1 \\ 3 & 2 & 1 & 3 & 2 \\ 1 & 2 & 3 & 1 & 2 \end{vmatrix} \rightarrow Dx = 17 - 13 = 4, \\ 10 + 7 + 0 = 17 \quad \left\{ \begin{array}{l} x = \frac{Dx}{D} = \frac{4}{3} \\ D \end{array} \right.$$

$$0 + 14 + 0 = 14$$

$$Dy = \begin{vmatrix} 2 & 5 & 0 & 2 & 3 \\ 0 & 3 & 1 & 0 & 3 \\ 3 & 4 & 1 & 3 & 1 \end{vmatrix} \rightarrow Dy = 21 - 14 = 7, \\ 6 + 15 + 0 = 21 \quad \left\{ \begin{array}{l} y = \frac{Dy}{D} = \frac{7}{3} \\ D \end{array} \right.$$

$$30 + 12 + 0 = 42$$

$$Dz = \begin{vmatrix} 2 & 1 & 5 & 2 & 1 \\ 0 & 2 & 3 & 0 & 2 \\ 3 & 2 & 1 & 3 & 2 \end{vmatrix} \rightarrow Dz = 37 - 42 = -5 \\ 28 + 9 + 0 = 37 \quad \left\{ \begin{array}{l} z = \frac{Dz}{D} = \frac{-5}{3} \\ D \end{array} \right.$$

$$V = \left\{ \begin{pmatrix} \frac{4}{3}, \frac{7}{3}, -\frac{5}{3} \end{pmatrix} \right\} \rightarrow \text{alternativa d)}$$

(6) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & x \\ 2 & 1 & 0 & y \\ -1 & 2 & 2 & z \end{array} \right] \xrightarrow{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 2 & 2 & -1 \end{array} \right] \rightarrow \begin{cases} x = 3 \\ 2x + y = 7 \\ -x + 2y + 2z = -1 \end{cases}$

$$\left[\begin{array}{ccc|c} (x+0+0) & 3 \\ (2x+y+0) & 7 \\ (-x+2y+2z) & -1 \end{array} \right] \sim \left[\begin{array}{ccc|c} 2x+y = 7 \\ 2 \cdot 3 + y = 7 \\ 6 + y = 7 \end{array} \right] \rightarrow \begin{cases} 2x+y = 7 \\ 6 + y = 7 \\ y = 7 - 6 = 1 \end{cases}$$

* $-x + 2y + 2z = -1$
 $-3 + 2 \cdot 1 + 2z = -1$ \rightarrow alternativa e)
 $2z = -1 - 2 + 3$
 $2z = 0$
 $z = \frac{0}{2} = 0$

Exemplo Básico - Escalonamento

① $S = \left\{ \begin{array}{l} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{array} \right.$

$$25z = -25$$

$$z = -\frac{25}{25} = -1$$

$$\left(\begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 1 & 3 & -1 & 11 \\ 1 & 0 & -5 & 3 \end{array} \right) \xrightarrow{\begin{array}{c} \\ \oplus \\ (-1) \end{array}} \left(\begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 0 & 4 & -4 & 16 \\ 0 & 0 & -5 & 3 \end{array} \right) \xrightarrow{\begin{array}{c} \\ \div 4 \\ \end{array}} \left(\begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & -5 & 3 \end{array} \right)$$

* $7x - 10z = -4 \rightarrow 7x - 10 \cdot (-1) = -4$

$$\begin{array}{l} 7x + 10 = -4 \\ 7x = -4 - 10 \\ 7x = -14 \\ x = \frac{-14}{7} = -2 \end{array} \quad \left\{ \begin{array}{l} * x + 3y - z = 11 \\ -2 + 3y - (-1) = 11 \\ 3y = 11 + 2 - 1 \\ 3y = 12 \\ y = \frac{12}{3} = 4 \end{array} \right.$$

② $x + 2y + 3z = ?$

$$\left\{ \begin{array}{l} x - 2y = 0 \\ 2y - 3z = 0 \\ x + y + z = 11 \end{array} \right. \rightarrow \left| \begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & 0 \\ 1 & 1 & 1 & 11 \end{array} \right| \sim$$

$$\sim \left| \begin{array}{ccc|c} 0 & 2 & -3 & 0 \\ 1 & 0 & 3 & 11 \end{array} \right| \sim \left| \begin{array}{ccc|c} 0 & 11 & 0 & 33 \end{array} \right|$$

$$\begin{array}{l} * 2y - 3z = 0 \\ 2 \cdot 3 = 3z \\ z = \frac{6}{3} = 2 \end{array} \quad \begin{array}{l} 11y = 33 \\ y = \frac{33}{11} = 3 \end{array}$$

$$\left\{ \begin{array}{l} x - 2y = 0 \\ x - 2 \cdot 3 = 0 \\ x = 6 \end{array} \right. \quad \left\{ \begin{array}{l} x + 2y + 3z = ? \\ 6 + 2 \cdot 3 + 3 \cdot 2 = \\ 6 + 6 + 6 = 18 \end{array} \right. \quad \text{alternativen b)$$

$$\textcircled{3} \quad z = ? \quad \left\{ \begin{array}{l} \left\{ \begin{array}{l} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{array} \right. \rightarrow \left| \begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right| \sim \end{array} \right.$$

$$\text{④) } \begin{array}{l} \sim \left(\begin{array}{cccc|c} 0 & -3 & -4 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \sim \left(\begin{array}{cccc|c} & & & & \\ 0 & 0 & -5 & -10 \end{array} \right) \end{array} \quad \begin{array}{l} -5z = -10 \\ z = \frac{-10}{-5} = 2 \end{array}$$

$$\text{④) } A = \text{Ali}, B = \text{Bio}, C = \text{Loco}$$

↳ alternativa ④

$$\left\{ \begin{array}{l} A + B + C = 68 \\ B + 2C = A \\ C + 2A = 3B \end{array} \right. \quad \rightarrow \quad \left\{ \begin{array}{l} A + B + C = 68 \\ A - B - \frac{1}{5}C = 0 \\ \frac{1}{5}A - 3B + C = 0 \end{array} \right. \quad \left. \begin{array}{l} \xrightarrow{\frac{1}{5}} \\ \xrightarrow{\frac{1}{5}} \end{array} \right.$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 68 \\ 0 & -2 & -\frac{6}{5} & -68 \\ 0 & -\frac{16}{5} & \frac{4}{5} & -68 \end{array} \right) \sim \left(\begin{array}{ccc|c} & & & \\ 0 & -2 & -\frac{6}{5} & -68 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|c} & & & \\ 0 & 0 & \frac{68}{25} & \frac{476}{5} \end{array} \right) \quad \begin{array}{l} \xrightarrow{\frac{68}{25}C = \frac{476}{5}} \\ \xrightarrow{C = \frac{476}{5} \cdot \frac{25}{68} = 35} \end{array}$$

$$\text{*} -2B - \frac{6}{5}C = -68 \quad \left\{ \begin{array}{l} \text{*} A + B + C = 68 \quad \rightarrow A = 68 - 13 - 35 \\ A + 13 + 35 = 68 \quad \rightarrow A = 20 \end{array} \right.$$

$$-2B - \frac{6}{5} \cdot 35 = -68$$

$$-2B = -68 + 42$$

$$B = \frac{-26}{-2} = 13 //$$

$$\text{Ali} \rightarrow \text{R\$} 20,00$$

$$\text{Bio} \rightarrow \text{R\$} 13,00$$

$$\text{Loco} \rightarrow \text{R\$} 35,00$$

↳ alternativa A

$$\textcircled{5} \quad \left. \begin{array}{l} \text{Alfau: R\$ 134,00} \\ \text{Benta: R\$ 115,00} \\ \text{Líntio: R\$ 48,00} \end{array} \right\} A(\text{quantidade}) = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix} \rightarrow \begin{array}{l} \text{Alfau} \\ \text{Benta} \\ \text{Líntio} \end{array}$$

$$x \text{ (preços unitários)} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad x + y + z = ?$$

*Temporar: (multiplicar matrizes A e x)

$$\begin{array}{l} \text{Alfau: } 3y + 4z = 134 \\ \text{Benta: } x + 5z = 115 \\ \text{Líntio: } 2x + 1y = 48 \end{array} \rightarrow \begin{cases} 3y + 4z = 134 \\ x + 5z = 115 \\ 2x + 1y = 48 \end{cases}$$

$$\xrightarrow{-3} \left(\begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 2 & 1 & 0 & 48 \end{array} \right) \sim \left(\begin{array}{ccc|c} -6 & 0 & 4 & -10 \\ 1 & 0 & 5 & 115 \\ 0 & 0 & 0 & 0 \end{array} \right) (6)$$

$$\sim \left(\begin{array}{ccc|c} 0 & 0 & 34 & 680 \\ & & \vdots & \vdots \\ & & 0 & 0 \end{array} \right) \rightarrow 34z = 680 \\ z = \frac{680}{34} = 20$$

$$\begin{array}{l} * x + 5z = 115 \\ x + 5 \cdot 20 = 115 \\ x = 115 - 100 \end{array} \quad \begin{array}{l} x + y + z = ? \\ 15 + 18 + 20 = 53 \end{array}$$

$$x = 15,$$

(alternativa ②)

$$* 2x + y = 48$$

$$2 \cdot 15 + y = 48$$

$$y = 48 - 30 = 18,$$