Tarefa Básica – Matriz Inversa

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1) $A = B^{-1}$, $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$, $x + y = ?$

$* A \cdot A' = I_n$

↳ $B \cdot A = I_n$

$$\begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} \cdot \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{cases} 3x - 5 = 1 & \text{I} \\ xy + 10 = 0 & \text{II} \end{cases}$$

I) $3x - 5 = 1$

$3x = 1 + 5$

$3x = 6$

$x = \frac{6}{3} = \boxed{2}$

II) $x \cdot y + 10 = 0$

$2y = -10$

$y = \frac{-10}{2} = \boxed{-5}$

$x + y = 2 + (-5)$

$= 2 - 5 = \boxed{-3}$

↳ alternativa c

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2) $K = ?$, $A = \begin{bmatrix} 1 & 0 & 1 \\ K & 1 & 3 \\ 1 & K & 3 \end{bmatrix}$ → não tem inverso (det = 0)

$1+3K+0 = 3K+1$

$\det A = \begin{bmatrix} 1 & 0 & 1 \\ K & 1 & 3 \\ 1 & K & 3 \end{bmatrix} \begin{matrix} 1 & 0 \\ K & 1 \\ 1 & K \end{matrix}$

$3+0+K^2 = K^2+3$

$= K^2+3-(3K+1)$
$= K^2+3-3K-1$
$= K^2-3K+2 = 0$

$\Delta = 9-4.1.2$
$\Delta = 9-8 = 1$

$K_1 = \dfrac{3+\sqrt{1}}{2.1} = \dfrac{3+1}{2} = \dfrac{4}{2} = \boxed{2}$

$K_{11} = \dfrac{3-\sqrt{1}}{2.1}$

$K_{11} = \dfrac{3-1}{2} = \dfrac{2}{2} = \boxed{1}$

* Alternativa (C).

(3) $B = A^{-1}$, $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \rightarrow B = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \div 2$

* Regra matriz de ordem 2

$\rightarrow B = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix} \rightarrow$ alternativa (c)

$\det A = 12 - 10 = 2$

(4) * matriz é inversível ($\det \neq 0$)

$\begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix} \begin{matrix} x & 1 \\ 3 & 1 \\ 10 & 1 \end{matrix}$

$\det = x^2 + 26 - (20 + 5x)$

$x^2 + 26 - 20 - 5x \neq 0$

$x^2 - 5x + 6 \neq 0$

$20 + 2x + 3x = 20 + 5x$ { $x^2 + 20 + 6 = x^2 + 26$

$\rightarrow \Delta = 25 - 4.1.6$

$\Delta = 25 - 24 = 1$

$x_1 \neq \frac{5 + \sqrt{1}}{2.1} \neq \frac{5+1}{2} \neq \frac{6}{2} = \boxed{3}$

$x_1 \neq \frac{5 - \sqrt{1}}{2.1} \neq \frac{5-1}{2} \neq \frac{4}{2} = \boxed{2}$

$\rightarrow \{x \neq 3$ e $x \neq 2\}$

alternativa (a)

(5) $A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}$, $A + A^{-1} = ?$

$\det A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{matrix} -1 & -1 \\ 2 & 1 \\ 1 & 1 \end{matrix} = 7 - 6 = 1$

$2 + 2 + 2 = 6$ $\quad 1 + 2 + 4 = 7$

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$$A = \begin{bmatrix} -1_{11} & -1_{12} & 2_{13} \\ 2_{21} & 1_{22} & -2_{23} \\ 1_{31} & 1_{32} & -1_{33} \end{bmatrix} \qquad A' = \begin{bmatrix} (-1-(-2)) & (-2-(-2)) & (2-1) \\ (1-2) & (1-2) & (-1-(-1)) \\ (2-2) & (2-4) & (-1-(-2)) \end{bmatrix}$$

$i+j$ = ímpar → mudar sinal:

$$A' = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \quad \Big\{ \; \bar{A} = (A')^t = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\bar{A}}{1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

alternativa (b)

$$A + A^{-1} = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

6) $(X.A)^t = B$

$((X.A)^t)^t = B^t$ → matriz transposta de uma transposta é a matriz normal

$X.A = B^t$

$X.\underbrace{A.A^{-1}}_{1} = B^t.A^{-1} \rightarrow \boxed{X = B^t.A^{-1}}$ → alternativa (b)

7) $B = \begin{bmatrix} x \\ y \end{bmatrix}$, $C = \begin{bmatrix} 4x+5y \\ 5x+6y \end{bmatrix}$, $A^{-1} = ?$, $AB = C$

$$A.\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x+5y \\ 5x+6y \end{bmatrix} \longrightarrow A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} \quad \Big\{ \begin{array}{l} \det A = 24-25 \\ \det A = -1 \end{array}$$

* Regra matriz de ordem 2:

$$A^{-1} = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \div -1 \Rightarrow A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix} \rightarrow \text{alternativa } d$$

8) $A = \begin{bmatrix} 2 & K \\ -2 & 1 \end{bmatrix}$, soma valores $K = ?$, $\det A = \det A^{-1}$

$\det A = 2 - (-2K)$
$\det A = 2 + 2K$

$\det A \cdot \det A^{-1} = 1$
$(2+2K) \cdot (2+2K) = 1$
$4 + 4K + 4K + 4K^2 = 1$

$4K^2 + 8K + 4 - 1 = 0$
$4K^2 + 8K + 3 = 0$

$\Delta = 64 - 4 \cdot 4 \cdot 3$
$\Delta = 64 - 48 = 16$

$K_1 = \dfrac{-8 + \sqrt{16}}{2 \cdot 4} = \dfrac{-8+4}{8} = \dfrac{-4 :4}{8 :4} = \dfrac{-1}{2}$

$K_{II} = \dfrac{-8 - \sqrt{16}}{2 \cdot 4} = \dfrac{-8-4}{8} = \dfrac{-12 :4}{8 :4}$

$K_{II} = \dfrac{-3}{2}$

soma valores de K: $-\dfrac{1}{2} - \dfrac{3}{2} = \dfrac{-4}{2} = \boxed{-2}$ → alternativa b

9) $A_{2\times2}$, $B_{2\times2}$, $\det A \neq 0$ e $\det B \neq 0$

a) $(A+B) \cdot (A-B) = \boxed{A^2 - AB + BA - B^2}$ $(AB \neq BA)$

b) $(A+B)^2 = A^2 + 2 \cdot A \cdot B + B^2$ → $\boxed{AB = BA}$

c) $\dfrac{\det(A)}{\det(-A)}$ → $\det(-A) = (-1)^2 \cdot \det A = \det A$

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$$\frac{\det A}{\det(-A)} = \boxed{1}$$

d) $B = A^{-1} \rightarrow \det A \cdot \det B = 1 \rightarrow \boxed{\det B = \frac{1}{\det A}}$