

# NETWORK THEORY

## UNIT-1:

Review of DC Network basics, Nodal and Mesh Analysis

## NETWORK TOPOLOGY

- ① Terminology ✓
- ② Cut-set and Tie-set matrices for planar networks with DC sources ✓
- ③ Related definitions and problems ✓
- ④ Source Transformation ✓
- ⑤ Duality ✓

## NETWORK THEOREMS

- ⑥ Thévenin's Theorem ✓
- ⑦ Norton's Theorem ✓
- ⑧ Maximum Power Transfer Theorem ✓
- ⑨ Superposition Theorem ✓
- ⑩ Tellegen's " ✓
- ⑪ Reciprocity " ✓
- ⑫ Millers " ✓
- ⑬ Fettman's " ✓
- ⑭ Compensation Theorems ✓  
with DC and AC excitations

## OHM'S LAW:

At a constant temperature, the current flowing through a resistor is directly proportional to the potential difference across the resistor.

$$I \propto V \quad (T = \text{const})$$

$$I = \frac{V}{R} \quad \text{where } V = \text{potential difference across } R$$

Potential drop in resistor,  $V = IR$

$$R = \frac{V}{I}$$

$I = \text{Amperes (A)}$

$V = \text{Volts (V)}$

$R = \text{ohms (\Omega)}$

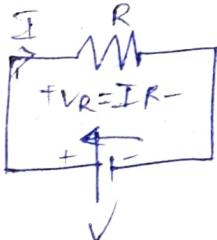
$$R = \frac{PL}{A}$$

resistor: dissipates energy.

$$\vartheta(t) = R i(t) \quad \text{Time Invariant}$$

$$\vartheta(t) = R(t) i(t) \times \text{Time Variant}$$

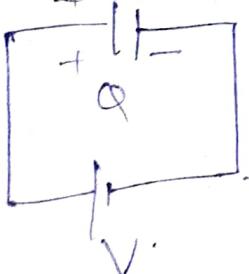
$$R_t = R_{\text{ref}} (1 + \alpha \Delta t)$$



$$P = VI = I^2 R = \frac{V^2}{R} \text{ watts (W)}$$

$$E = P \times t \text{ Joules (J)}$$

C charge principle



$$C = \frac{\epsilon_0 A}{d} \quad (A = \text{overlapping plate area})$$

$$V = \frac{Q}{C} ; V = \frac{It}{C} = \frac{1}{C} \int_{-\infty}^t q(t') dt'$$

$$\dot{q}(t) = C \frac{dv}{dt}$$

Capacitor stores static energy.

$$\text{Power, } P(t) = \vartheta(t) i(t)$$

$$E = \int P(t) dt = \int \vartheta(t) C \frac{dv}{dt} dt = C \int v dv = \frac{Cv^2}{2} = \frac{1}{2} CV^2$$

Inductor  $\rightarrow$  Flux principle

$$i = \frac{\Phi}{L}$$

$$V = L \frac{di}{dt}$$

Faraday's  
second law.

$$V = N \frac{d\Phi}{dt}$$

$$P = \frac{1}{L} \int_{-\infty}^t V dt$$

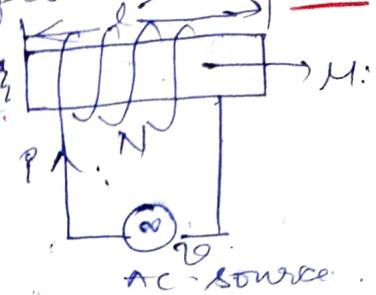
$$E = \frac{1}{2} L i^2$$

$\Phi = N\phi$  = Total flux.  
 permeability  $\rightarrow$  cross sectional area  
 $L = \mu A N^2$   $\rightarrow$  No. of coils  $\rightarrow$  length of core.  
 length of core.

$\mu$  = permeability of the material.

ability of the material to allow the flow of flux.

Inductor stores dynamic energy.



$V \rightarrow$  AC At

$I \rightarrow$  AC At

flux is  $\uparrow$   
clockwise  
anticlockwise

$$\Phi \propto i$$

$$\Phi = Li$$

$$\frac{d\Phi}{di} = \frac{Li}{dt}$$

$$V = L \frac{di}{dt}$$

$$\Phi = N\phi$$

$$\frac{d\Phi}{dt} = N \frac{d\phi}{dt}$$

$$= V..$$

$$Nd\frac{\phi}{dt} = \frac{li}{dt}$$

$$Nd\frac{\phi}{di} = L ; L = N \frac{d\phi}{di} = N\mu \cdot \frac{A}{l}$$

(as long as core is not saturated with linearity)

pressure developed  
 $P = \rho g H$

water level  
 $\rightarrow$  pot diff.  
source drive current

$$L = \frac{N\phi}{l} = \frac{\Phi}{l}$$



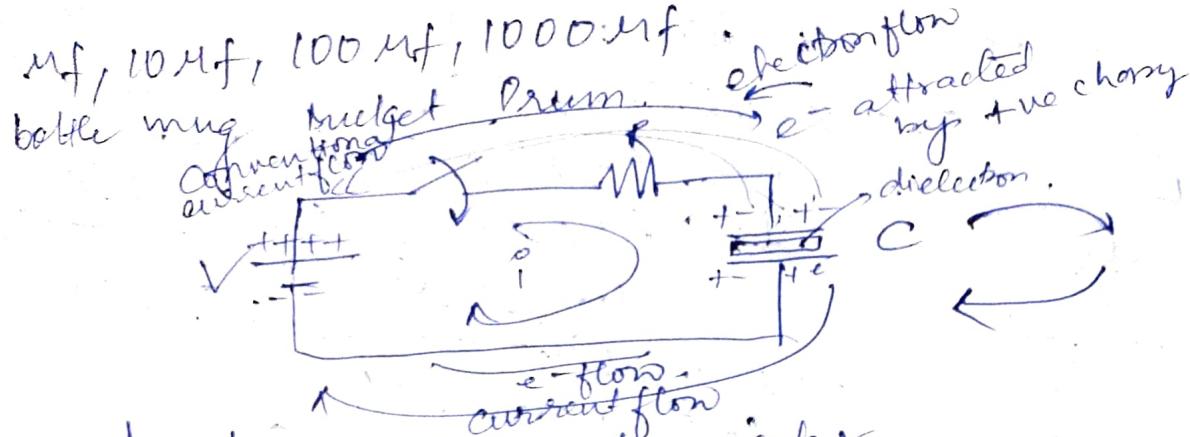
Water level =  $\frac{\text{Quantity}}{\text{dia. of bkt}}$

$$V = \frac{Q}{C}$$

diameter of pipe

$\frac{D_p}{R}$

Quantity  $\Delta Q$ .  
water flows  
water level  
 $= l$   
pipe dia.  
water level  
 $= L$   
diameter  
of bucket.



$\sqrt{Q}$  bucket to store water  
★ Capacitor to store charge.

charging Capacitor = filling bucket ( $\propto t$ )

water flowing through bucket  $\Rightarrow$  bucket will not fill.  
Hence, place dielectric material.

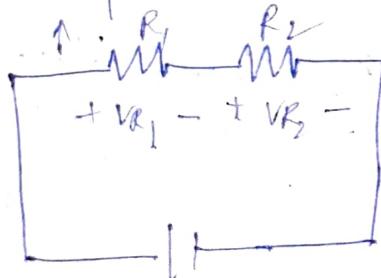
(Capacitor slowly becomes another source)

$C = \frac{\epsilon_0 A}{d} \rightarrow$  Overlapping plate area.

(Insulators)  
Content between the plates

$d$ : distance b/w two plates.

Voltage Division Formulae.

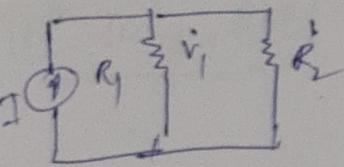


$$VR_1 = \frac{V \times R_1}{R_1 + R_2}, \quad VR_2 = \frac{V \times R_2}{R_1 + R_2}$$

$$VR_1 + VR_2 = V$$

$$V_{C1} = \frac{V \times C_2}{C_1 + C_2}, \quad V_{C2} = \frac{V \times C_1}{C_1 + C_2}$$

$$\psi_L = \frac{\psi_T \times L_1}{L_1 + L_2}, \quad \psi_{T2} = \frac{\psi_T \times L_2}{L_1 + L_2}$$



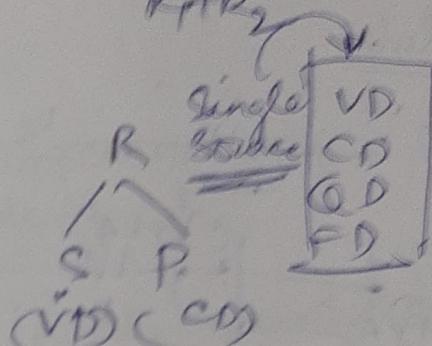
$$I_{R_1} = \frac{I \times R_2}{R_1 + R_2}$$

$$I_{R_2} = \frac{I \times R_1}{R_1 + R_2}$$

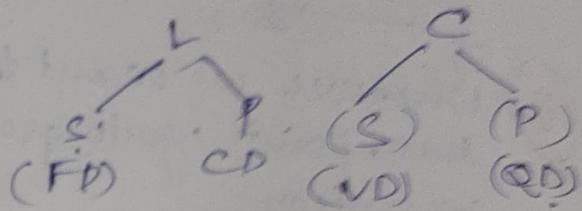
$$Q_1 = \frac{Q_1 \times Q}{Q_1 + Q_2}$$

$$Q_2 = \frac{Q_2 \times Q_1}{Q_1 + Q_2}$$

$$I_L = \frac{I \times L_2}{L_1 + L_2}, \quad I_R = \frac{I \times L_1}{L_1 + L_2}$$



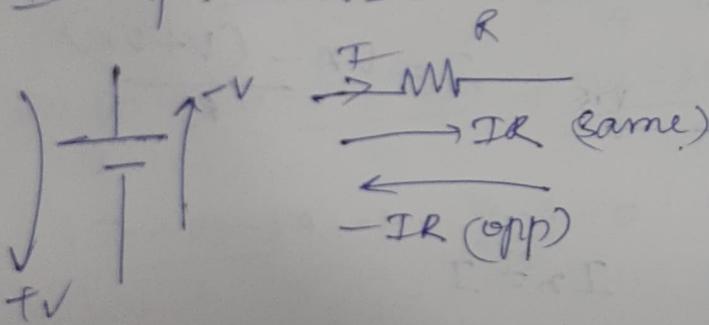
Single Source problems  
Series & Parallel



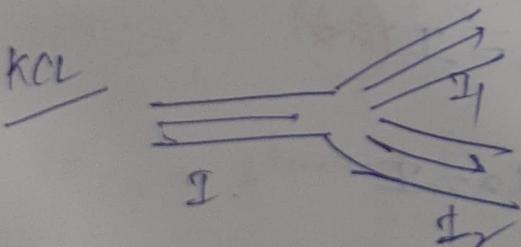
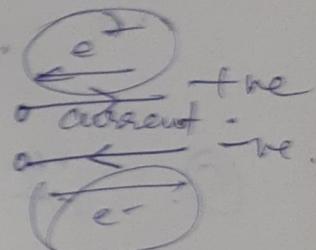
K-Laws

$$\begin{array}{c} \text{KVL} \\ \text{KCL} \end{array} \rightarrow \sum I = 0$$

$$\sum \text{emf} - \sum IR = 0$$



$$-IR_1$$



$$\frac{\text{KVL}}{CR_1} \quad CR_2$$

$$60$$

$$80$$

$$40$$

$$20$$

$$100 R_1$$

Ohms Law

$$V = IR$$

$$KVL$$

loop/mesh

Mesh analysis

$$I = \frac{V}{R}$$

$$KCL$$

of Node

Nodal analysis

① Mesh / loop current

② Branch current

③ Branch voltage

④ Node voltage

$\leftrightarrow$

dual

each other

① Node voltage

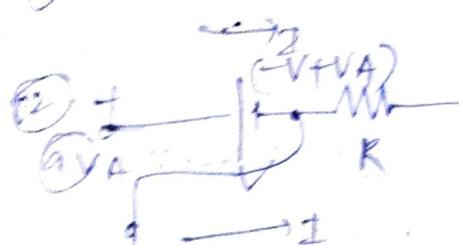
② Branch voltage

③ Branch current

# NODAL ANALYSIS

## Rules

(i)   $I = \frac{V_A - V_B}{R}$

(ii)   $I = \frac{V_A - V_B - V}{R} = \frac{(-V + V_A) - V_B}{R}$

Whenever we want to know the voltage, start at the same point, go to ground following C.G.R, KVL sign conventions.

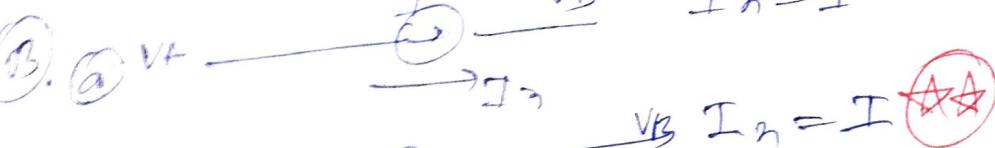
$\leftarrow I \quad I = V_B - \frac{(-V + V_A)}{R}$

(iii)   $I = \frac{V + V_A - V_B}{R}$

$$I = \frac{V_A - (V + V_B)}{R}$$

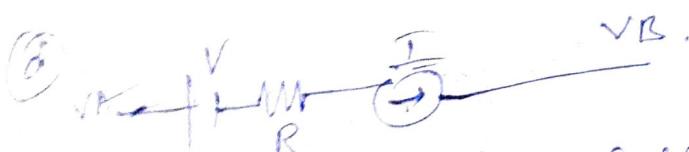


$$I_n = I$$

(v)   $I_n = I$  ★★

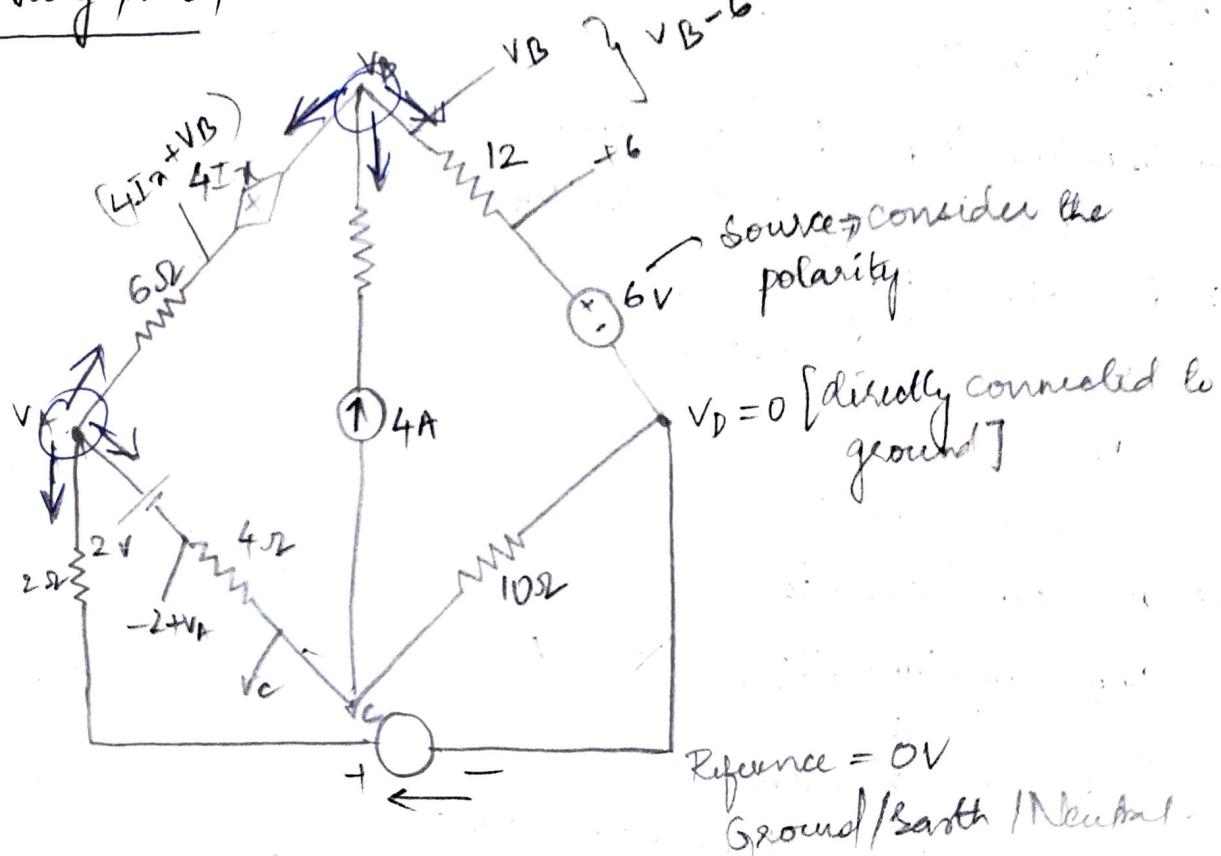
(vi)   $I_n = I$

(vii) 

(viii) 

If branch contains a current source, branch current can be decided which is independent of voltage source, voltage drop.

17<sup>th</sup> July, 2019



$$\frac{V_A - 10}{2} + \frac{V_A - (V_B + 4I_2)}{6} + \frac{(V_A - 2) - V_C}{4} = 0$$

$$\frac{6V_A - 60}{12} + \frac{2V_A - 2V_B - 8I_3}{12} + \frac{3V_A - 6 - 3V_C}{12} \rightarrow (i)$$

$$\frac{4I_2 + V_B - V_A}{6} - 4 + \frac{V_B - 6}{12} = 0$$

$$\frac{V_C - (V_A - 2)}{4} + 4 + \frac{V_C - 0}{10} = 0$$

~~I<sub>2</sub>~~ is flowing from D to B.

$$I_2 = 6 - \frac{V_B}{12}$$

$$V_A = 12 - 2.10V$$

$$V_B = 58.207V$$

$$V_C = -4.063$$

$$I_2 = -4.3505$$

①  $V_A, V_B, V_C$  are node voltages wrt ground (by default)

Find

(i)  $V_A \ V_B \ V_C \ I_A$

(ii)  $V_{AB} \ V_{BC} \ V_{CA}$  - Branch voltage  
(Voltage at A wrt B ( $V_A - V_B$ ))

(iii) Branch voltages.

$$V_{AB} = V_A - V_B = -45.897 \text{ V}$$

$$V_{BC} = 62.24 \text{ V}$$

$$V_{CA} = -16.373 \text{ V}$$

(iv) Branch currents

$$I_{AB} = \frac{V_A - V_B - 4I_A}{6} = -10.54 \text{ A}$$

$$I_{BD} = 3.593, I_{DC} = -4$$

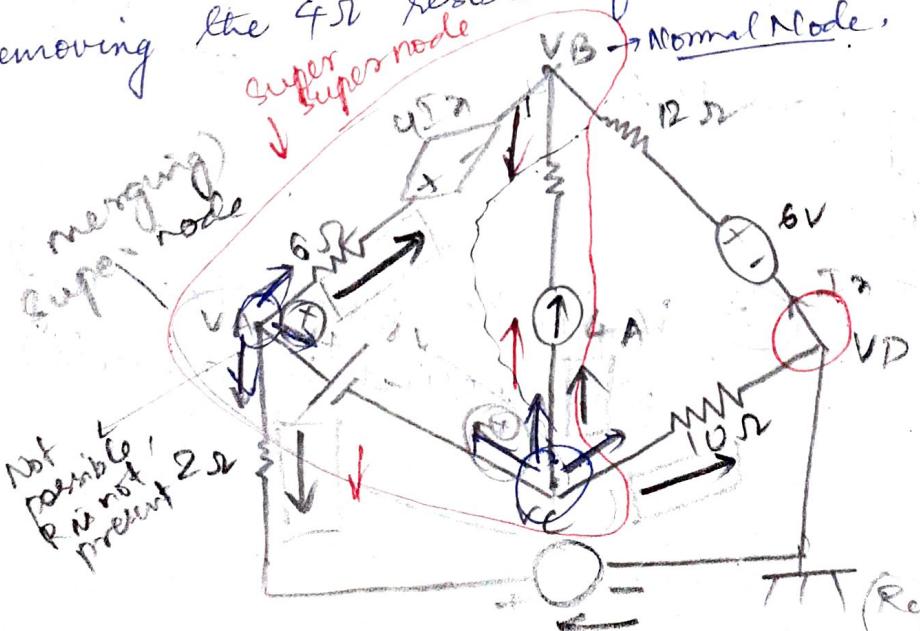
$$I_{CA} = -4.593, I_{EP} = -0.0406$$

Apply Ohm's law, where there is a resistor  
\* Don't apply it, where there is no R

Voltmeter  $\rightarrow \infty$  Resistance, Ammeter  $\rightarrow 0$  Resistance

Voltage & current sources resistance is not known and should  
not apply Ohm's law to voltage source & current source

(v) Removing the  $4\Omega$  resistor from the above circuit diagram



Super super node  
→ normal node

super super node  
↳ normal node  
super node  
↳ principal node  
node  
↳ least  
branch

It is not possible to do nodal analysis at  $V_A$  and  $V_C$ , so we will merge them known as SUPER NODE.

$$\therefore \frac{V_A - 10}{2} + \frac{V_C}{10} + 4 + \frac{V_A - (4i_2 + V_B)}{6} = 0 \quad \left. \begin{array}{l} \text{super Node} \\ \text{Equation} \end{array} \right\} \quad \text{Equation 1}$$

18<sup>th</sup> July, 2019  
when there is ONLY VOLTAGE SOURCE, between two principal nodes, go for the SUPER NODE.

$$\frac{(4i_2 + V_B) - V_A - 4 + \frac{V_B - 6}{12}}{6} = 0 \rightarrow \text{Equation 2}$$

$$i_2 = \frac{6 - V_B}{12} \rightarrow \text{Equation 3}$$

$$V_{AC} = V_A - V_C = +2V \rightarrow \text{Equation 4}$$

(4) To find  $V_{AB}$ , start at A, go to B. (or) go to red + terminal (or) start at voltage red button, go to black button following CGR KVL convention

Solve for  $V_A, V_B, V_C, i_2$

- ① N.V
- ② B. Voltage
- ③ B. Current

$$\frac{V_A - 10}{2} + \frac{V_C}{10} + 4 + \frac{V_A - 4i_2 - V_B}{6} = 0.$$

$$15V_A - 150 + 3V_C + 120 + \frac{6}{6}V_A - 20i_2 - 5V_B = 0,$$

30

$$20V_A - 5V_B + 3V_C - 30 - 20i_2 = 0 \rightarrow \text{Equation 1}$$

$$20V_A - 5V_B + 3V_C - 30 - \frac{20}{12}V_B (6 - V_B) = 0.$$

$$60V_A - 15V_B + 9V_C - 30 - 30 + 5V_B = 0,$$

$$60V_A - 10V_B + 9V_C - 60 = 0. \rightarrow \text{Equation 1},$$

$$-6V_A + 7V_B - 150 = 0 \rightarrow \text{Equation 2},$$

$$V_A - V_C = 2 \rightarrow \text{Equation 3},$$

$$F_y = \frac{V_C}{10} + 4$$

$$\frac{12}{15}V_B$$

$$i_2 = \frac{6 - V_B}{12}$$

$$V_A = V_C + 2$$

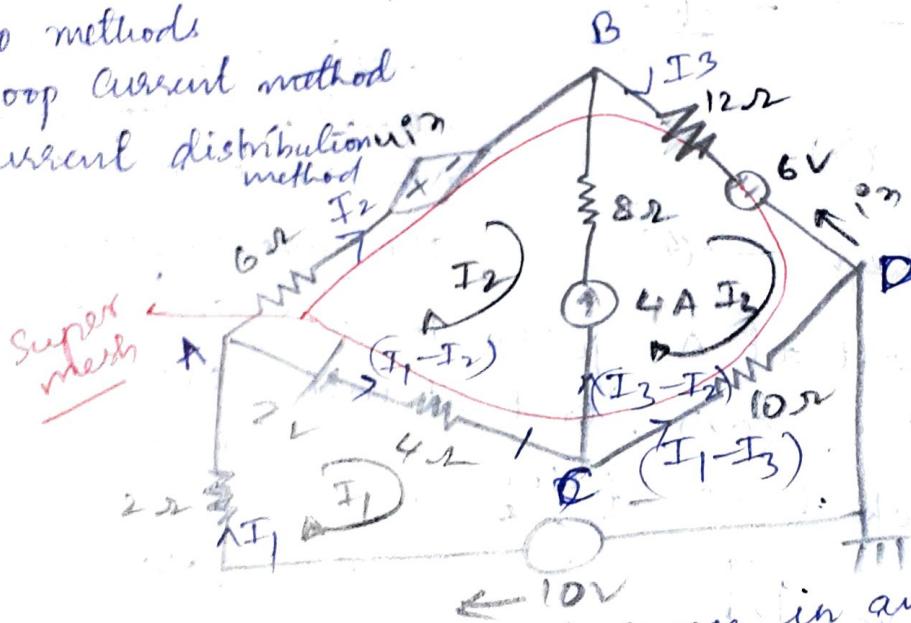
$$60V_A - 15V_B = 0.$$

## MESH ANALYSIS%

Two methods

(i) Loop Current method

(ii) Current distribution method



steps

- ① Loop currents
- ② Branch current
- ③ 4 equations

\* Mesh eq is not possible for loop containing  $I_2, I_3$  because voltage across 4A current source is not known

When there is a current source in any branch go for super mesh.

$$-10 + 2I_1 + 2 + 4(I_1 - I_2) + 10(I_1 - I_3) = 0 \rightarrow ①$$

$$(app^{\text{mesh eq}}) -4(I_1 - I_2) - 2 + 6I_2 + 4I_3 + 12I_3 + 6 - 10(I_1 - I_3) = 0 \rightarrow ②$$

$$in = -I_3 \rightarrow ③$$

$$I_3 - I_2 = +4 \rightarrow ④$$

Solve for ①)  $I_1 =$

$$I_2 =$$

$$I_3 =$$

$$in =$$

} LOOP CURRENTS

## II BRANCH CURRENTS

### III BRANCH VOLTAGE:

voltage at A w.r.t B

$$V_{AB} = 6I_2 + 4I_3.$$

$$V_{BC} = 12I_3 + 6 - 10(I_1 - I_3)$$

$$V_{CA} = -4(I_1 - I_2) - 2$$

### IV NODE VOLTAGE:

$$V_A = -8I_1 + 10$$

$$V_B =$$

$$V_C =$$

$\times$  takes lot of time

$$V_{AB} = V_A - V_B$$

$$V_{BC} = V_B - V_C$$

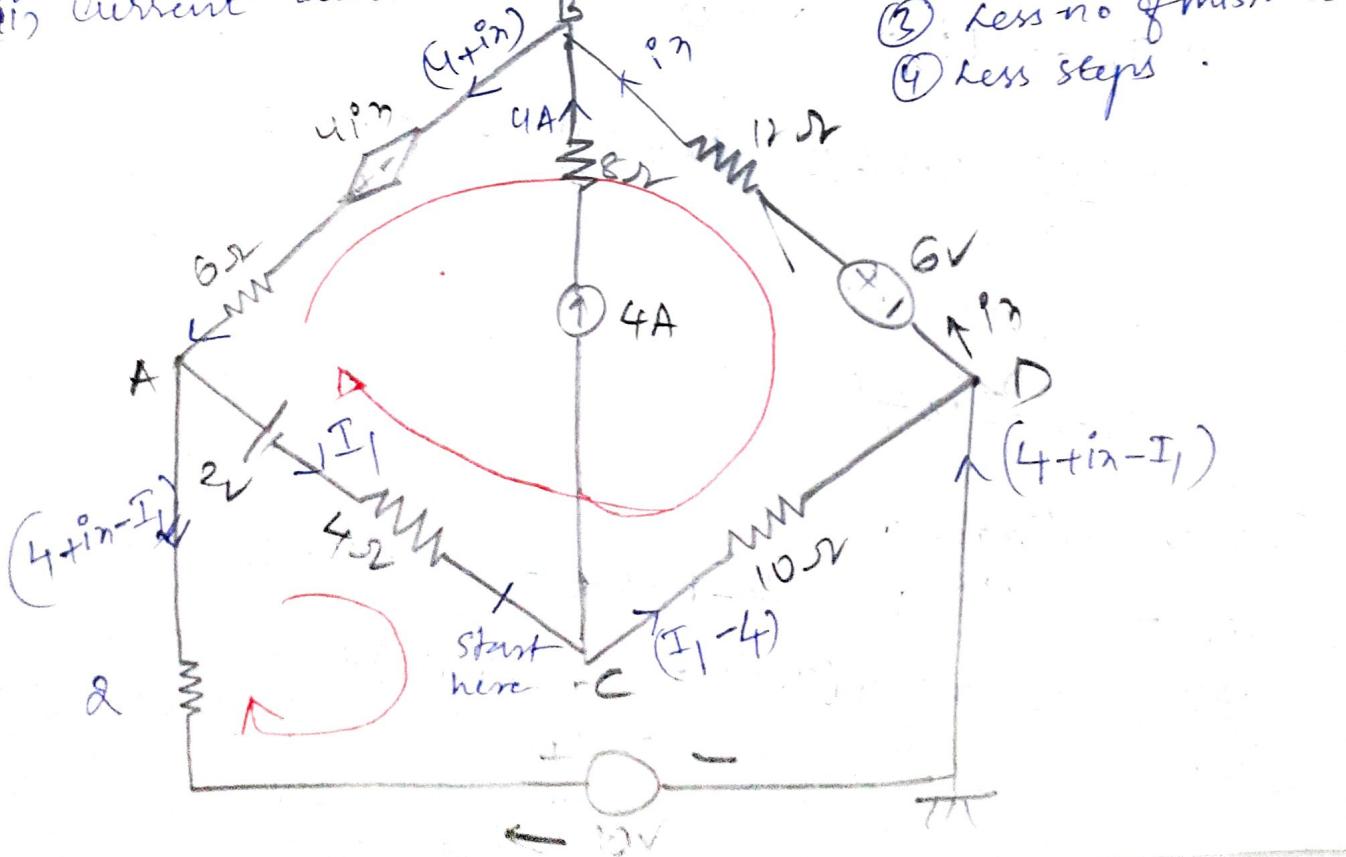
$$V_{CA} = V_C - V_A$$

(don't go via current source, preferably shorter path)

### Disadvantages

- (1) More number of unknowns
- (2) More no. of equations
- (3) More equations
- (4) More mistakes
- (5) More steps

(ii) Current distribution method.



### Advantages

- ① Less no. of unknowns
- ② Less no. of eqns
- ③ Less no. of mistakes
- ④ Less steps

$$-10 - 2(4 + i_2 - i_1) + 2 + 4i_1 + 10(i_1 - 4) = 0 \quad \rightarrow ①$$

$$-4i_1 - 2 - 6(4 + i_2) + 4i_2 - 12i_2 + 6 - 10(i_1 - 4) = 0 \quad \rightarrow ②$$

(H.W)

Solve I)  $I_1 =$   
 $i_2 =$

Branch currents

II) B.V.

III) N.N.

$$-10 - 8 - 2i_2 + 2i_1 + 2 + 4i_1 + 10i_1 - 40 = 0$$

$$-18 - 2i_2 + 2i_1 + 2 + 14i_1 - 40 = 0$$

$$16i_1 - 16 - 2i_2 - 40 = 0$$

$$16i_1 - 2i_2 - 56 = 0 \quad \rightarrow ①$$

$$-4i_1 - 2 - 24 - 6i_2 + 4i_2 - 12i_2 + 6 - 10i_1 + 40 = 0$$

$$-4i_1 - 26 - 12i_2 + 46 - 10i_1 = 0$$

$$-14i_1 - 14i_2 + 20 = 0 \quad \rightarrow ②$$

$$I_1 = \frac{206}{63} = 3.2698 \quad i_2 = \frac{-116}{63} = -1.8412$$

$$I_1 = 3.27 \text{ A}$$

$$i_2 = -1.84 \text{ A}$$

Branch Voltages

$$V_{AB} = (4 + i_2) 6 = (4 - 1.84) 6 = 2.16 \times 6 = 12.96 \text{ V}$$

$$V_{BC} = -12i_2 + 6 - 10(i_1 - 4) = -12 \times (-1.84) + 6 - 10(3.27 - 4) = 12 \times 1.84 + 6 - 10(-0.23) = 12 \times 1.84 + 6 + 7.3$$

$V_{CA}$

$$V_{BC} = 25.38$$

$$V_{CA} = -4I_1 - 2.$$

$$= -4 \times 3.27 - 2$$

$$= -13.08 - 2$$

$$V_{CA} = -15.08$$

Node Voltage:

$$V_A = 2(4 + i\pi - I_1) + 10$$

$$= 2(4 + 1.84 - 3.27) + 10$$

$$V_B = 12 + b =$$

$$V_C = 10(I_1 - 4)$$

$$R_a = \frac{R_{ab} R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_b = \frac{R_{bc} R_{ab}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_c = \frac{R_{ac} R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$

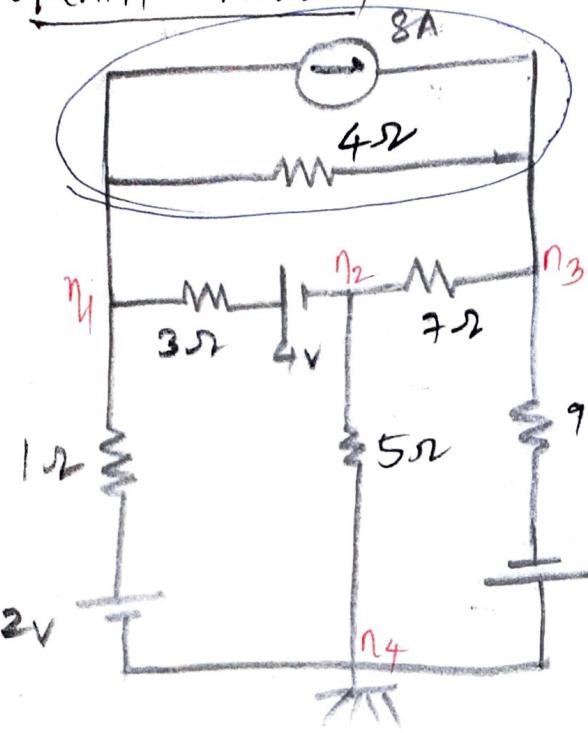
$$R_{ab} = R_a + R_b + \frac{R_a R_b}{R_c}$$

$$R_{bc} = R_b + R_c + \frac{R_b R_c}{R_a}$$

$$R_{ca} = R_c + R_a + \frac{R_c R_a}{R_b}$$

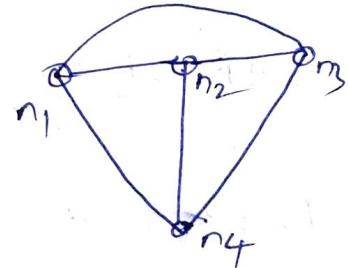
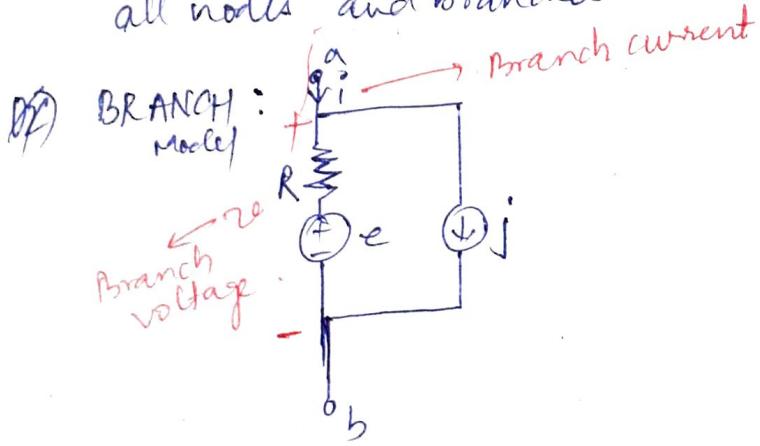
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## GRAPH THEORY

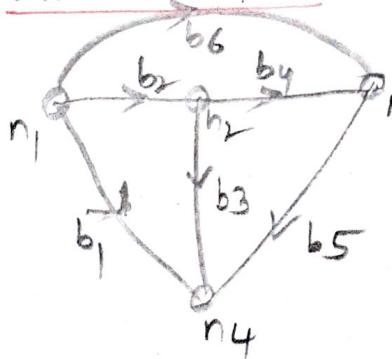


Find branch currents using tie-set schedule?

(1) GRAPH: It is the skeleton of given ckt containing all nodes and branches.



(2) ORIENTED GRAPH:

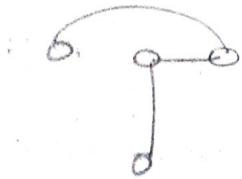
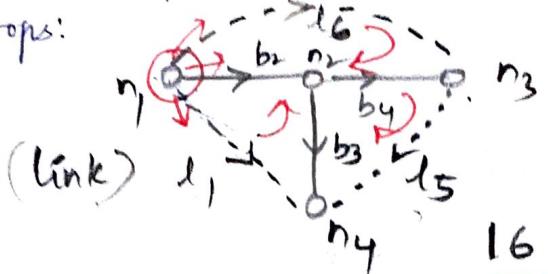


After we assign current direction, and labelling each branch then graph changes to oriented graph.  
Note: The current directions are not fixed (your choice)

(3) TREE:

Remove minimum number of branches such that there should not be any closed loops.

Possible loops:



16 POSSIBLE TREES.

### ④ TIE-SET MATRIX: $[M]$

	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$
$b_1$	1	-1	-1	0	0	0
$b_5$	0	0	-1	1	1	0
$b_6$	0	-1	0	-1	0	1

(ii) Go along with link current direction  
 same +1  
 opp -1  
 else 0.

(i) Corresponding to each link, one loop is closed.

### ⑤ PROPERTIES:

(a)  $M\vartheta = 0$ ;  $M = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{bmatrix}$   $\begin{bmatrix} \vartheta_1 \\ \vartheta_2 \\ \vartheta_3 \\ \vartheta_4 \\ \vartheta_5 \\ \vartheta_6 \end{bmatrix}$

(KVL/mesh equations)

$$\Rightarrow \vartheta_1 - \vartheta_2 - \vartheta_3 = 0$$

$$-\vartheta_3 + \vartheta_4 + \vartheta_5 = 0$$

$$-\vartheta_2 - \vartheta_4 + \vartheta_6 = 0.$$

(b)  $M^T I = \overset{\text{Link currents}}{\underset{\text{current in the link}}{\underset{\text{KCL / Nodal equations}}{I}}} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = [i]$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = [i]$$

$$v = R(i - j) + e$$

Ohm's Law Voltage Equation

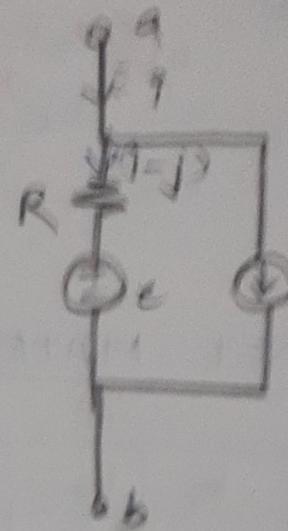
06) Combine:

$$M[v] = 0$$

$$M[R(i - j) + e] = 0$$

$$M[R(M^T I - j) + e] = 0$$

$$\boxed{MRM^T I = M(Rj - e)}$$



$M \rightarrow$  Tie Set Matrix

R, j, e - definition

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$6 \times 6$   
 $6 \times 6$

$$j = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$6 \times 1$

Orientation is compatible  $\Rightarrow +8$ )

$\Rightarrow$  voltage source

$$e = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \\ -6 \\ 0 \end{bmatrix}$$

In accordance with  
Orientation, it is opp  
 $\Rightarrow$  It has to take +)

Q7) Solve for  $I$ :

$$[ ]_{3 \times 3} [I]_{3 \times 1} = [ ]_{3 \times 1}$$

use  
Cramer's Rule

Q8)  $M^T I = \text{Branch currents (solution to given question)}$ .

$$M = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \end{bmatrix}_{3 \times 6} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}_{6 \times 6} \begin{bmatrix} 100 \\ -10-1 \\ -1-10 \\ 0-2-1 \\ 0-10 \\ 0-0-1 \end{bmatrix}_{6 \times 3}$$

$$\begin{bmatrix} 1 & -1 & 5 & 0 & 0 & 0 \\ 0 & 0 & -5 & 7 & 9 & 0 \\ 0 & -3 & 0 & -7 & 0 & 4 \end{bmatrix}_{3 \times 6} \begin{bmatrix} 100 \\ 10-1 \\ 1-10 \\ 0-10 \\ 0-10 \\ 0-0-1 \end{bmatrix}_{6 \times 6} \begin{bmatrix} I \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{pmatrix} 100 & 0 & 2 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & -6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+1-5 & -5 & 1 \\ 5 & +5+7+9 & -7 \\ 3 & -7 & 3+7+4 \end{bmatrix}_{3 \times 3} = \begin{bmatrix} 1+1-4 & 0 & 0 \\ 0 & 0-1 & 110 \\ 0 & -10 & -101 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 32 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} -3 & -5 & 1 \\ 5 & 2 & -7 \\ 3 & -7 & 14 \end{bmatrix} = \begin{bmatrix} 1-1-1 & 0 & 0 & 0 \\ 0 & 0-1 & 110 \\ 0 & -10 & -101 \end{bmatrix}_{3 \times 6} \begin{pmatrix} -2 \\ -4 \\ 0 \\ 0 \\ 6 \\ 32 \end{pmatrix}_{6 \times 1}$$

$$\begin{bmatrix} -3 & -5 & +1 \\ 5 & 21 & -7 \\ 3 & -7 & 14 \end{bmatrix} \begin{bmatrix} I_1 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 36 \end{bmatrix}_{3 \times 1}$$

$$\begin{bmatrix} -3 & -5 & +1 \\ 5 & 21 & -7 \\ 3 & -7 & 14 \end{bmatrix} \begin{bmatrix} I_1 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 36 \end{bmatrix}$$

$$-3I_1 - 5I_5 + I_6 = 2$$

$$5I_1 + 21I_5 - 7I_6 = -6$$

$$3I_1 - 7I_5 + 14I_6 = 36$$

$$I_1 = -1.63 \text{ A}, \quad I_5 = 1.3 \text{ A}, \quad I_6 = 3.566 \text{ A}$$

$$\begin{bmatrix} 100 \\ -402 \\ -140 \\ 011 \\ 010 \\ 001 \end{bmatrix} \begin{bmatrix} I_1 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}$$

$$\begin{bmatrix} -1.63 \\ -1.63 - 3.56 \\ -1.63 - 1.3 \\ 1.3 - 3.56 \\ 1.3 \\ 3.56 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}$$

$$i_1 = -1.63 \text{ A}$$

$$i_2 = -1.93 \text{ A}$$

$$i_3 = 0.33 \text{ A}$$

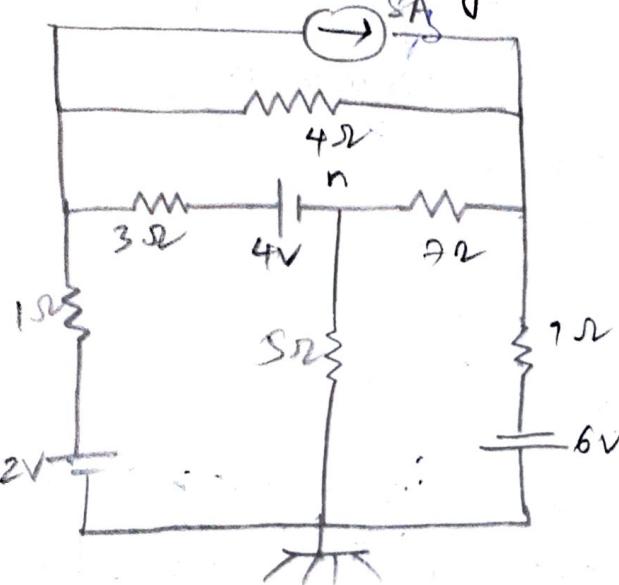
$$i_4 = -2.26 \text{ A}$$

$$i_5 = 1.3 \text{ A}$$

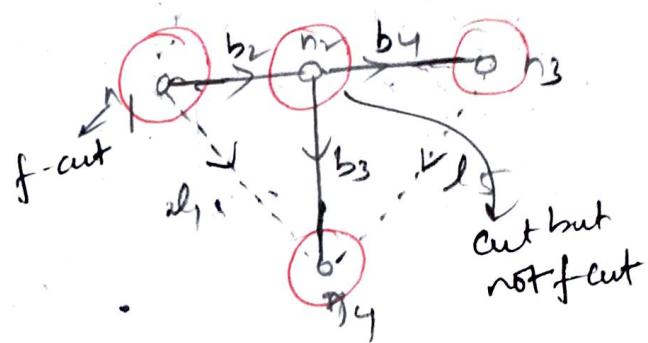
$$i_6 = 3.56 \text{ A}$$

25<sup>th</sup> July 2019

Find branch voltages using cut set schedule?



Since problem is same,  
graph oriented graph and  
tree are same.



Q1) Cut-set Matrix ( $A_a$ )

Incident matrix.

$$\begin{array}{c} \rightarrow \\ 0 \end{array} \rightarrow +1$$

$$\begin{array}{c} \leftarrow \\ 0 \end{array} \leftarrow -1$$

$$0 \quad 0$$

	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$
$n_1$	1	1	0	0	0	1
$n_2$	0	-1	1	1	0	0
$n_3$	0	0	0	-1	1	-1
$n_4$	-1	0	-1	0	-1	0

Q2) Properties:

$M_{12} = 0$  (Mesh equation)

$M^T I = i$  (Nodal)

$$\vartheta = RCI - j + e$$

$$A^T I = 0$$

$$A^T V = \vartheta$$

$$i = G(\vartheta - e) + j$$

(\* Add vertically get zero  $\Rightarrow$  correct)

Consider the node  
with single line.

$$M R M^T I = M (Rj - e)$$

loop impedance      Loop voltage  
matrix.              matrix.

Source vector.

$$AGA^T V = A(Ge - j)$$

Node Admittance  
matrix.

$A$  = Reduced I.M.

$$[G] = \begin{bmatrix} 1/1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/4 \end{bmatrix}$$

07) Solve for  $v = \begin{bmatrix} v_1 \\ v_3 \\ v_5 \end{bmatrix}$

08)  $A^T v = 29$

$$I = \begin{bmatrix} I_1 \\ I_5 \\ I_6 \end{bmatrix}$$

Node voltage

$$A^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \\ -1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix} \quad 3 \times 6 \quad 6 \times 6 \quad 6 \times 3$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \\ -6 \\ 6 \end{bmatrix} \quad 6 \times 6 \quad 6 \times 6 \quad 6 \times 1$$

$$\Rightarrow \begin{bmatrix} 1 & 1/3 & 0 & 0 & 1 & 0 & 1/4 \\ 0 & 0 & 0 & -1/3 & 1/9 & -1/4 \\ -1 & 0 & -1/5 & 0 & -1/9 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \\ -6 \\ 6 \end{bmatrix} \quad 3 \times 6 \quad 6 \times 3$$

$$\Rightarrow \begin{bmatrix} 2+1/3+1/4 & -1/4 & -1 \\ -1/4 & 1/7+1/9+1/4 & -1/9 \\ -1 & -1/9 & 1+1/5+1/9 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \left( \begin{bmatrix} 2 \\ 4/3 \\ 0 \\ 0 \\ -6/9 \\ 6 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 8 \end{bmatrix} \right)$$

$$\Rightarrow \begin{bmatrix} 19/12 & -1/4 & -1 \\ -1/4 & 12/252 & -1/9 \\ -1 & -1/9 & 59/45 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 4/3 \\ 0 \\ 0 \\ -6/9 \\ 6 \end{bmatrix} \quad 3 \times 6 \quad 6 \times 1$$

$$\begin{bmatrix} 19/12 & -1/4 & -1 \\ -1/4 & 12/252 & -1/9 \\ -1 & -1/9 & 59/45 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \\ v_5 \end{bmatrix} = \begin{bmatrix} 2+4/3-8 \\ 0-6/9+8 \\ -2+6/9+0 \end{bmatrix} \quad 3 \times 1$$

$$\frac{19}{12}v_1 - \frac{v_3}{4} - v_5 = -\frac{14}{3}$$

$$-\frac{v_1}{4} + \frac{12.7}{12}v_3 - \frac{v_5}{9} = \frac{22}{3}$$

$$-v_1 - \frac{v_3}{9} + \frac{5.9v_5}{45} = -\frac{4}{3}$$

$$v_1 = -1.3406, \quad v_3 = 13.6923, \quad v_5 = -0.8791$$

$$A^T V = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}_{6 \times 3} \begin{bmatrix} -1.3406 \\ v_1 \\ 13.6923 \\ -0.8791 \\ v_5 \\ 3x_1 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}$$

$$\begin{bmatrix} -1.3406 + 0.87 \\ -1.3406 \\ 0.87 \\ -13.7 \\ 13.7 + 0.87 \\ -1.3406 - 13.7 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} \Rightarrow \begin{bmatrix} -0.4706 \\ -1.3406 \\ 0.87 \\ -13.7 \\ 14.57 \\ -15.0406 \end{bmatrix}$$

Athore	V. V. Beeg	P.G & L CB, PGSS
$n_1, n_2, n_3, n_4$	vertices	Nodes (N)
$b_1, b_2, b_3, b_4, b_5, b_6$	edges (e)	Branches (b)
$t_1, t_5, t_6$	links (l)	kinkes (e)
$b_2, b_3, b_4$	branches	-
		twigs
	$l = e - (v - 1)$	$l = b - (N - 1)$
		$c = b - (N - 1)$

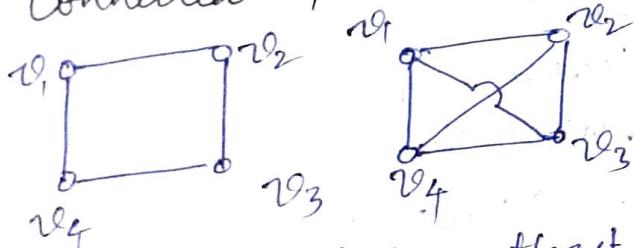
- 1) Nodes  
 2) C/W graph  
 3) T/B.  
 4) Previous Question Bank.

1) Sub-graph: part of the graph is called sub-graph.  
 2) Spanning sub-graph: If subgraph contain all the vertices, it is called as spanning subgraph

Ex: All Trees.

All Trees are spanning subgraphs. All spanning subgraphs may not be trees.

3) Connected v/s Complete



Not Connected.

If path contains atleast one edge, it is connected graph.  
 If path contains only one edge, it is called complete graph.  
 Complete graph is connected graph, but connected graph is not complete graph.

4) No of edges in complete graph =  $\frac{n(n-1)}{2}$

$$\frac{4 \times 3}{2} = 6$$

5). Degree of vertex  $\Rightarrow$  No of edges connected to vertex is called as degree.

$$\sum_{i=1}^n d(v_i) = 2e$$

$$8(v_1) + 8(v_2) + 8(v_3) + 8(v_4)$$

$$3+3+3+3 = 2 \times 6.$$

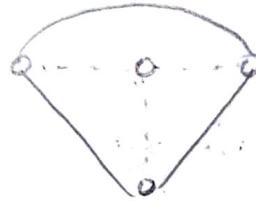
6) Total possible number of trees given by  $(2)^{n-2} = 4^{4-2} = 4^2 = 16$ .  
 (valid for complete graph)

For connected graph,  $|AAT|$

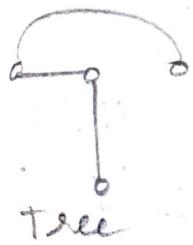
7) COMPLEMENT OF TREE (Co-tree):



Tree



Co-tree



Tree



All co-trees are not trees. Some co-trees are trees.

8) Rank of Incidence Matrix is almost  $n-1$ .

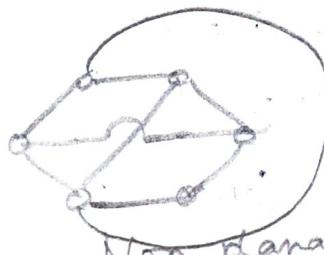
9) Proper tree if voltage sources and capacitors included in the branch and current sources and inductors included in links. Proper trees are useful to write state equations.

10) Planar v/s Non planar:



Planar  $\rightarrow$  (2D)

(Dual Network)



Non planar

3D

No dual

11) f-ckt (fundamental circuit):

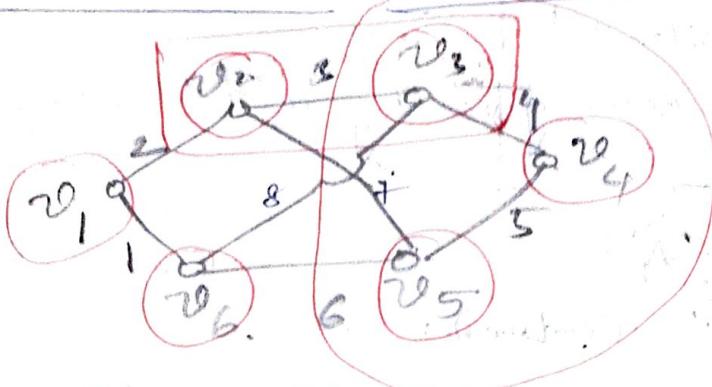
Corresponding to each link, one loop is closed, that loop is called as fundamental circuit

$$l = b - (N - 1)$$

No of links = No of f-ckts.

= No of mesh equations needed to analyse the problem

12) Cut, Valid cut, Not a cut & Invalid cut, f-cut:



$\checkmark$  (1,2)  $\checkmark$  (2,3,4)  $\checkmark$  (4,5)  $\checkmark$  (2,3,8,4) etc

Not a cut (1), (2,3), (7,3), (4) (5)

Invalid cut (2,3,4,6)

### DUALITY

Two networks are dual to each other when the mesh equations of one circuit are in the same mathematical form of nodal equations of other circuit.

### Properties

$$V_C(0) \leftrightarrow I_L(0)$$

$$R \leftrightarrow G$$

$$L \leftrightarrow C$$

$$\text{Voltage} \leftrightarrow \text{Current}$$

$$\text{Voltage source} \leftrightarrow \text{Current source}$$

$$\text{Thévenin} \leftrightarrow \text{Norton}$$

$$\begin{matrix} \text{Voltage} & \leftrightarrow \text{Current} \\ \text{Division} & \text{Division} \end{matrix}$$

$$\text{KVL} \leftrightarrow \text{KCL}$$

$$\text{Nodal} \leftrightarrow \text{mesh/loop}$$

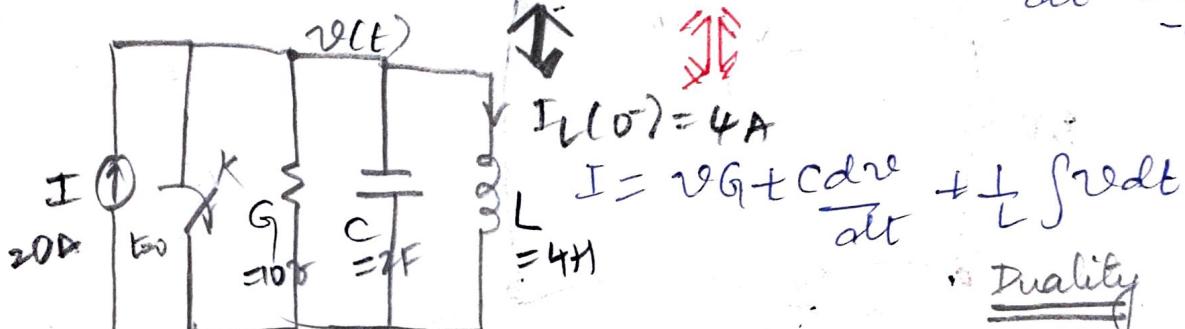
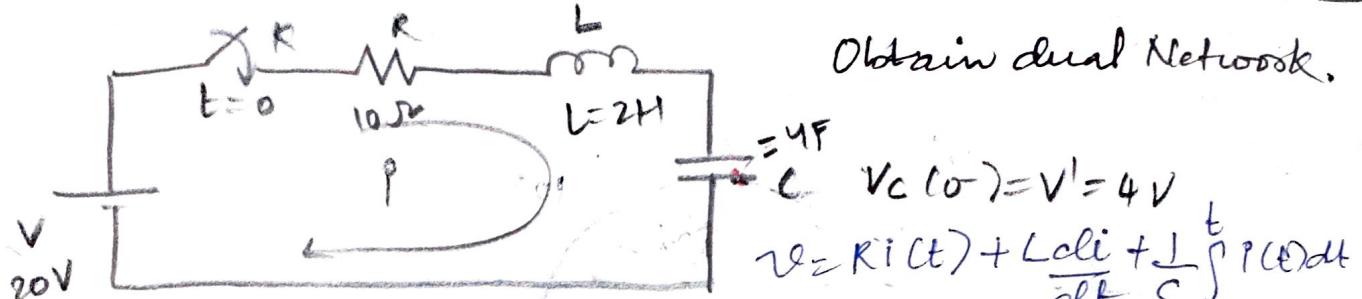
$$\text{Tie Set} \leftrightarrow \text{cut-set}$$

$$\begin{matrix} \text{Voltage Dependent} & \leftrightarrow \text{Current Dependent} \\ \text{Current Source} & \text{Voltage Source} \end{matrix}$$

$$\text{Series} \leftrightarrow \text{parallel}$$

$$\text{Open} \leftrightarrow \text{Close}$$

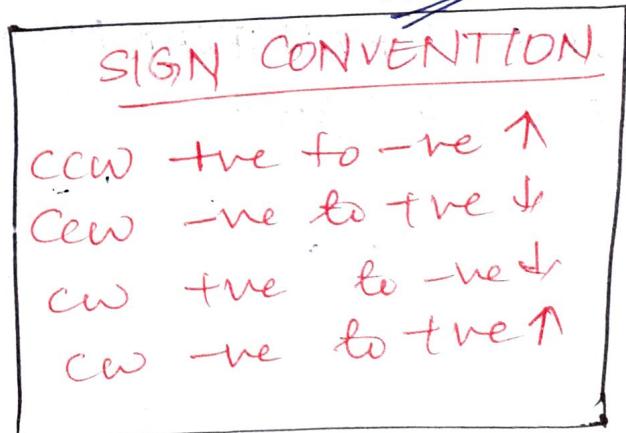
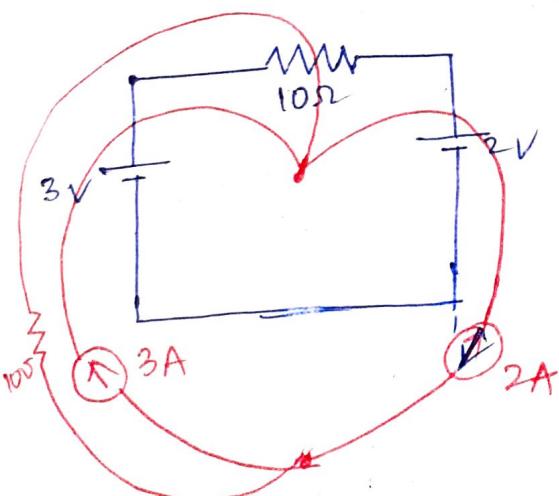
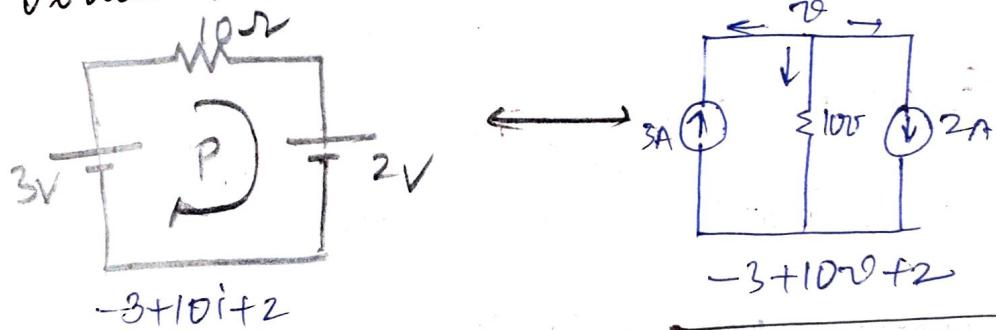
$$\psi(\text{charge}) \leftrightarrow \Phi(\text{Flux})$$

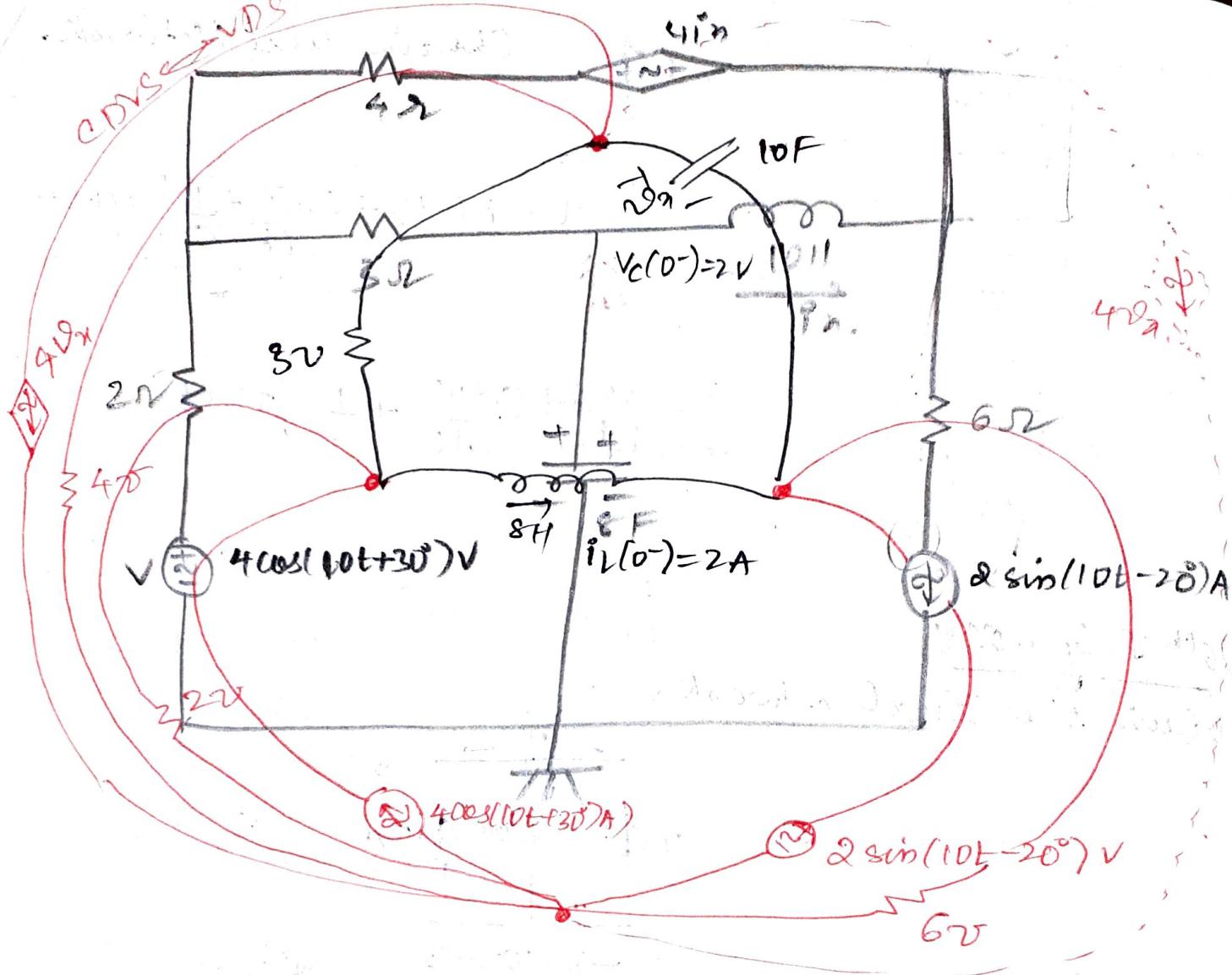


It is not equivalent

26<sup>th</sup> July 2019

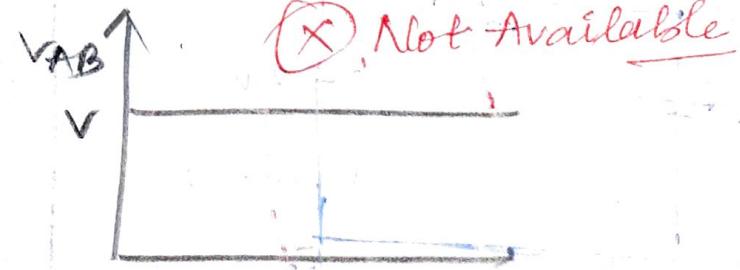
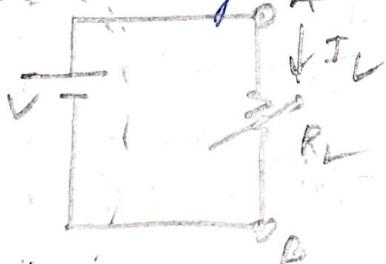
Obtain the dual network.





## SOURCE TRANSFORMATION: (S.T.):

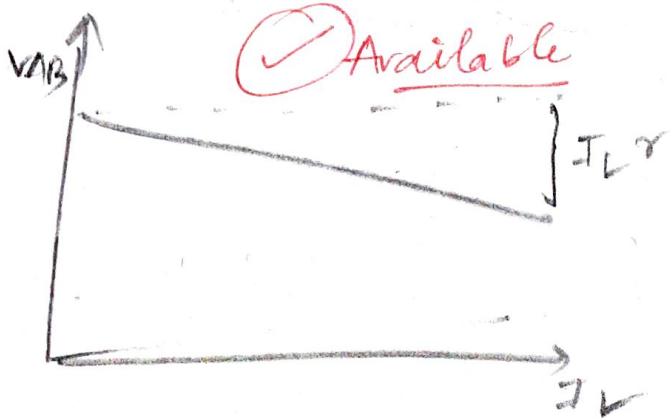
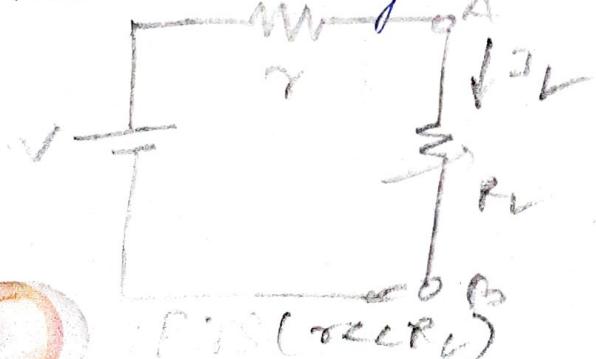
Ideal voltage source



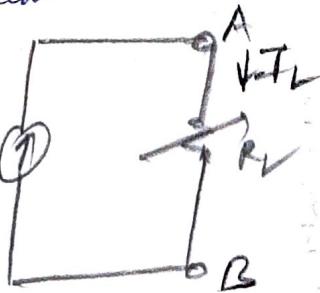
Ideal voltage

source ( $r = 0$ )

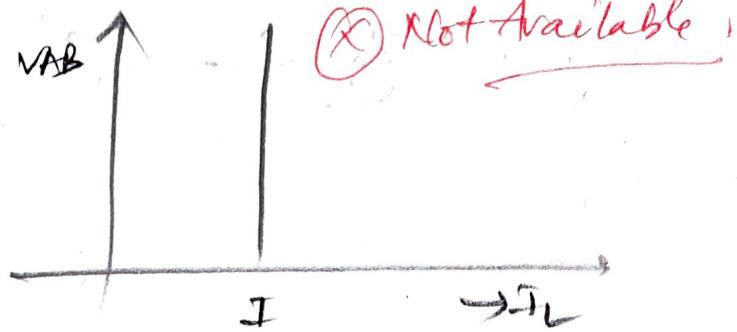
Practical voltage source



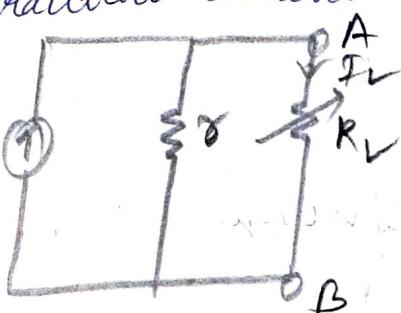
Ideal Current Source:



I.S.C. ( $r = \infty$ )



Practical Current Source:



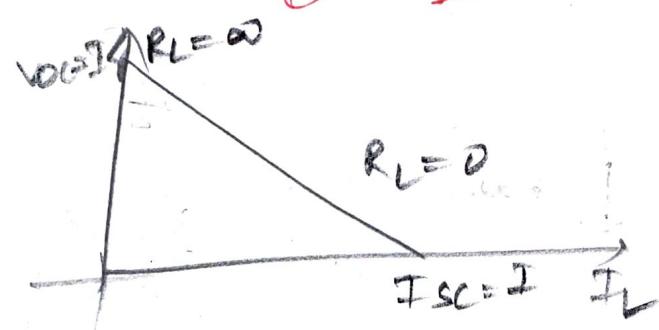
( $\beta \gg R_L$ )  
P.C.S.

31<sup>st</sup> July 2019

Q1.  $V_{AB} = V_{DC} = V = IR$

Q2.  $I_{SC} = \frac{V}{r}$

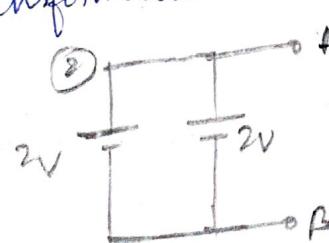
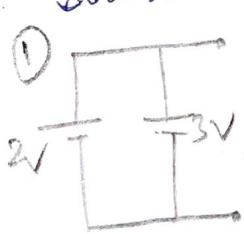
Q3.  $R_{AB} = r$



What is it?  
How to use it  
Where to apply  
Where not to use?

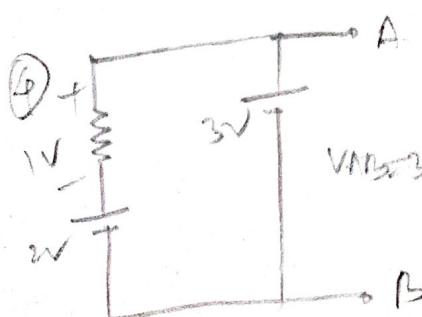
Points

Wherever we are finding response (current, voltage, power, phase, frequency etc), that element ( $R, C, L, V_S, C_S, AC, DC$ , dependent, independent) should not be disturbed or modified using any transformation techniques such as source transformation & star-delta transformation.

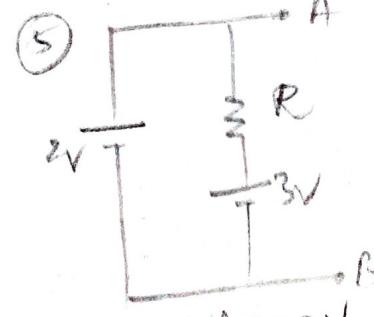


$V_{AB} = \text{Not possible}$      $V_{AB} = 2V$

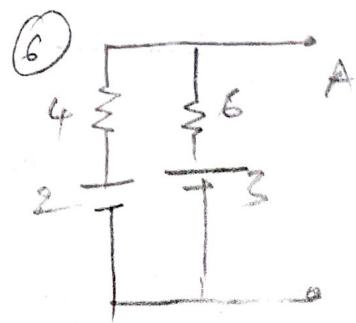
$V_{AB} = \text{Not possible}$

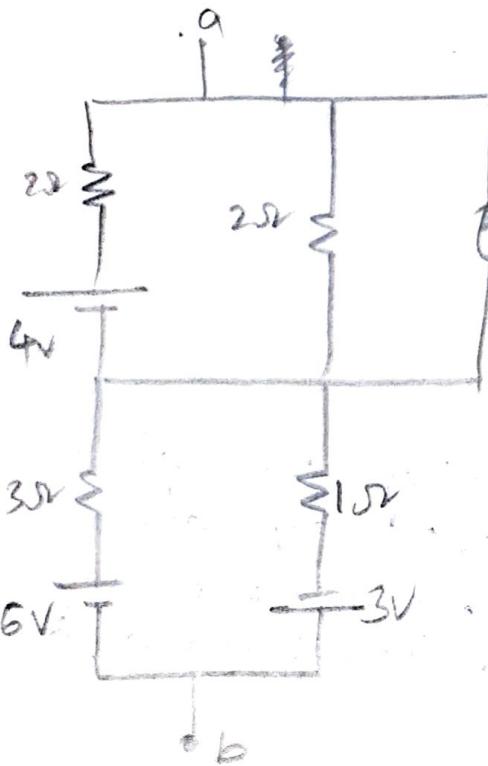
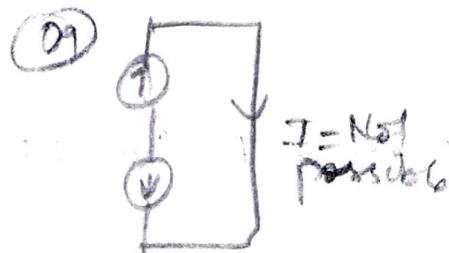
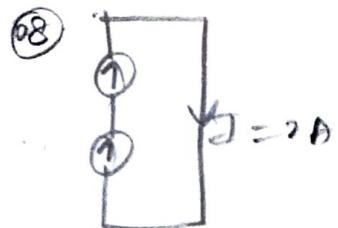
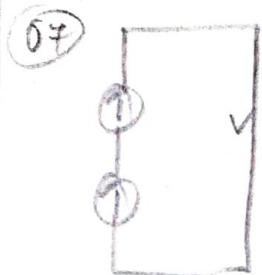
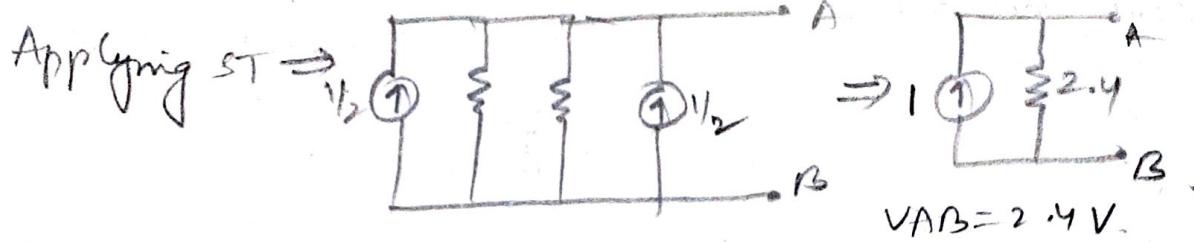


$V_{AB} = 3V$



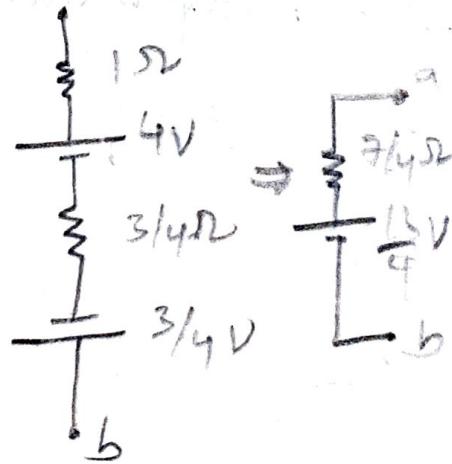
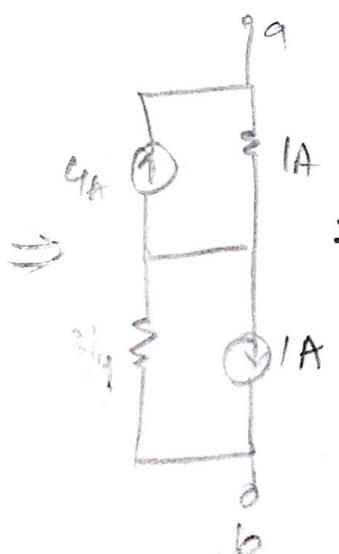
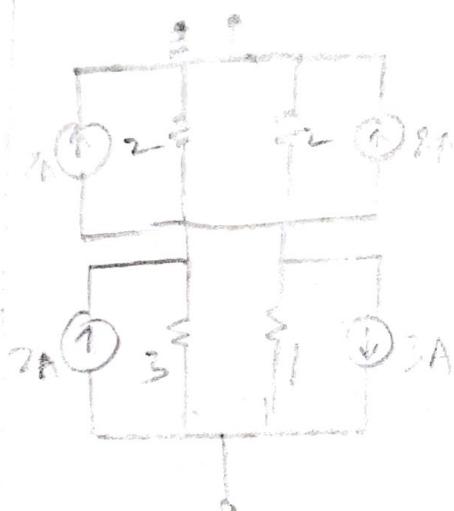
$V_{AB} = 2V$





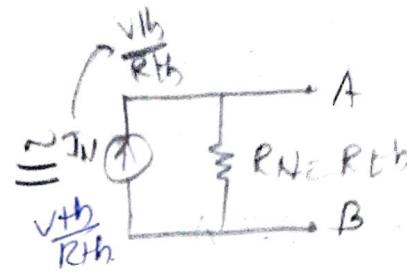
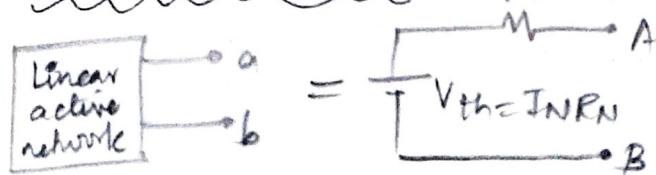
Convert into single voltage source equivalent

We can add CS in series, voltage source in ~~parallel~~ parallel



## THEOREMS:-

Thevenin and Norton



$$V_{th} = V_{oc} = V_{AB} \text{ (Open circuit voltage across AB)}$$

$I_{sc} = I_N$  = Current flowing from A to B.

$R_{th}$  → Resistance viewed from AB Keeping all voltage sources short and current sources open.

01<sup>st</sup> August 2029

Thevenins + Norton

01) All sources are Independent:

$$V_{th}, I_{NOR}, R_{th}, R_{NOR}$$

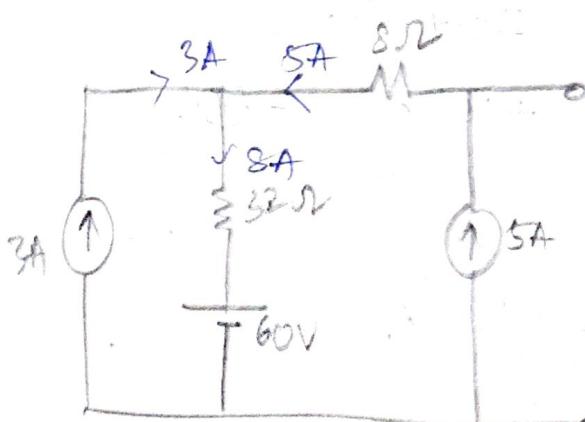
02) Some sources are dependent and others are independent.

$$V_{th}, I_N, R_{th} \quad (R_{th} = \frac{V_{th}}{I_N})$$

03) All sources are dependent (PASSIVE Network)

$$V_{th}=0; I_{NOR}=0 \quad R_{th} \text{ (Indirectly)}$$

$$\text{Apply } V \text{ across AB; find } I \Rightarrow R_{th} = \frac{IV}{I} \text{ (or) } \frac{V}{IA} \cdot \frac{V}{I}$$



Find Thevenin's equivalent.

\* All are independent sources.

① VD, CD, QD, FD.

② N.Mash → can't apply if sources are having diff frequencies.

③ S.T

④ Y-Δ

05) superposition principle  $f(0+0) = f(x_1) + f(x_2)$

Five statements given earlier

$$V_{OC} = V_{AB} = V_{th} = 5 \times 8 + 8 \times 32 + 60 = 356 V$$

$$R_{th} = 8 + 32 = 40 \Omega$$

$$R_N = 40 \Omega$$

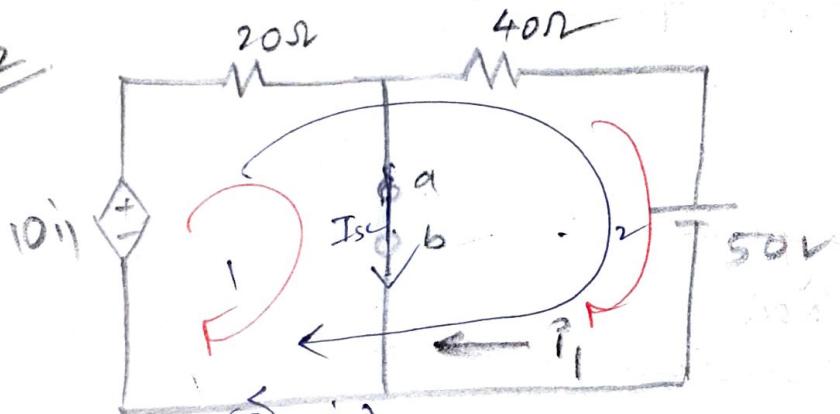
$$I_N = \frac{356}{40} = 8.9 A$$

$$I_{SC} = 5 + \frac{60}{40} + \frac{8 \times 32}{40} = 5 + 1.5 + 6.4 = 12.9 A$$

$$\begin{array}{r} 89 \\ 4) 356 \\ -32 \\ \hline 36 \end{array}$$

Ass  
- Thursday

Q2



$$-10i_1 + 10i_1 + 40i_1 + 50 = 0 \quad \text{(open def condition)}$$

$$50i_1 + 50 = 0 \quad \boxed{i_1 = -1}$$

$$V_{th} = 40 \times i_1 + 50 = 80 - 40 = 10 V. \quad \boxed{V_{th} = 10}$$

$$I_{SC} = ?$$

$$-10i_1 + 20I_{SC} = 0. \quad -10i_1 + 20(I_{SC} + i_1) = 0$$

$$20I_{SC} + 10i_1 = 0 \rightarrow \textcircled{1}.$$

$$40i_1 + 50 = 0.$$

$$i_1 = \frac{-5}{4} = -1.25 A$$

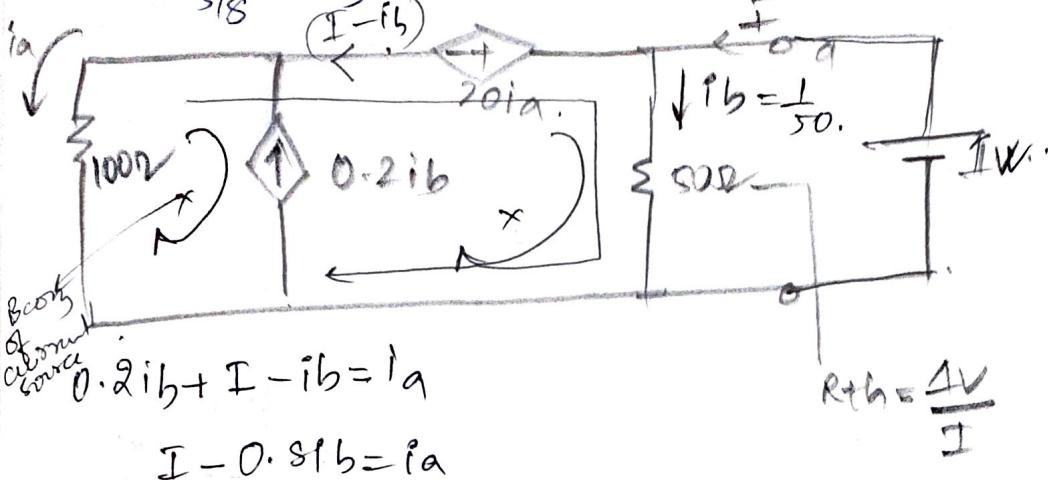
$$\boxed{i_1 = -1.25 A}$$

$$\begin{array}{r} 1.25 \\ 4) 12.5 \\ -12 \\ \hline 5 \end{array}$$

$$20I_{SC} + 12.5 = 0$$

$$I_{SC} = \frac{12.5}{20} = \frac{1.25}{2} \quad \boxed{I_{SC} = 0.625 A}$$

$$R_{th} = \frac{10}{50} = \frac{80}{5} = 16 \Omega$$



$$I - \frac{0.8}{50} = \frac{1}{120}$$

$$I = \frac{0.8}{50} + \frac{1}{120}$$

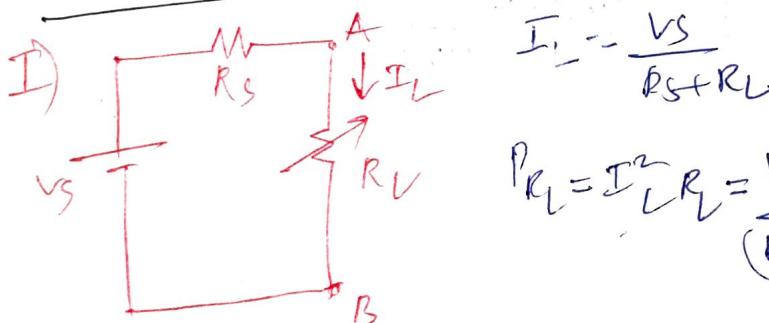
$$= \frac{1}{10} \left[ \frac{8}{50} + \frac{1}{12} \right]$$

$$I = 0.024$$

2(13/80)

### MAXIMUM POWER TRANSFER THEOREM:

Mtech  
DGate  
2) MBA  
CAT



$$P_{RL} = I_L^2 R_L = \frac{V_S^2}{(R_S + R_L)^2} \times R_L$$

$$\frac{dP_{RL}}{dR_L} \equiv 0 \Rightarrow R_L = R_S \text{ condition}$$

$$P_{max} = \frac{V_S^2}{4 \times R_L}$$

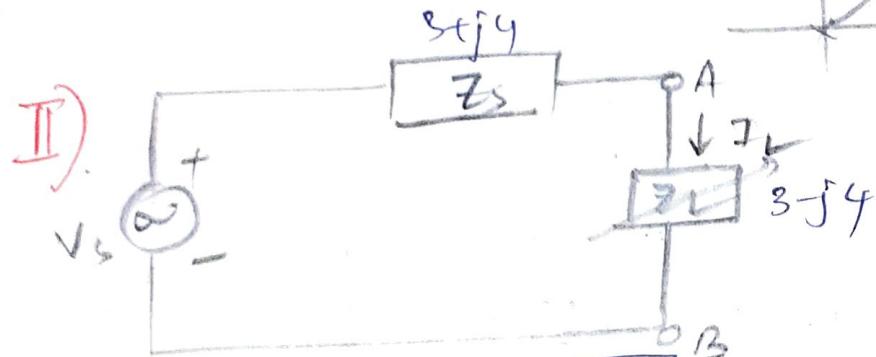
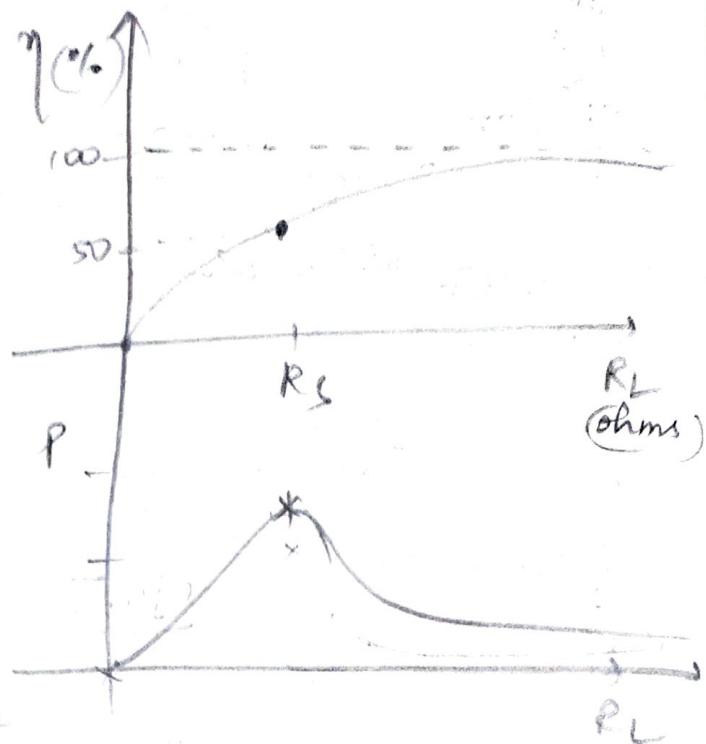
$$\text{Efficiency } \eta = 50\% \text{ if } R_L = R_S$$

$$\eta = 100\% \text{ if } R_L = \infty$$

$$\eta = 0\% \text{ if } R_L = 0$$

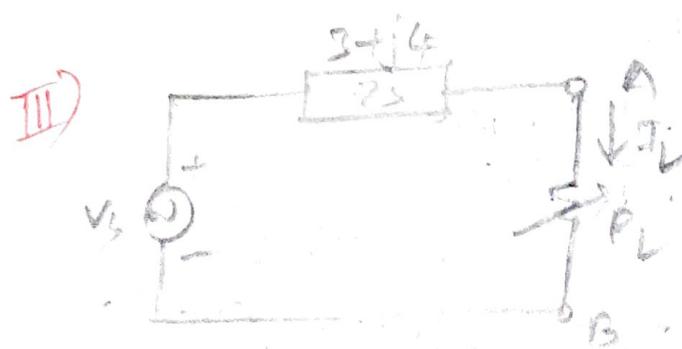
4 possibilities

- ①  $R_L = 0$
- ②  $R_S = 0$
- ③  $R_L = R_S$
- ④  $R_S = R_L$



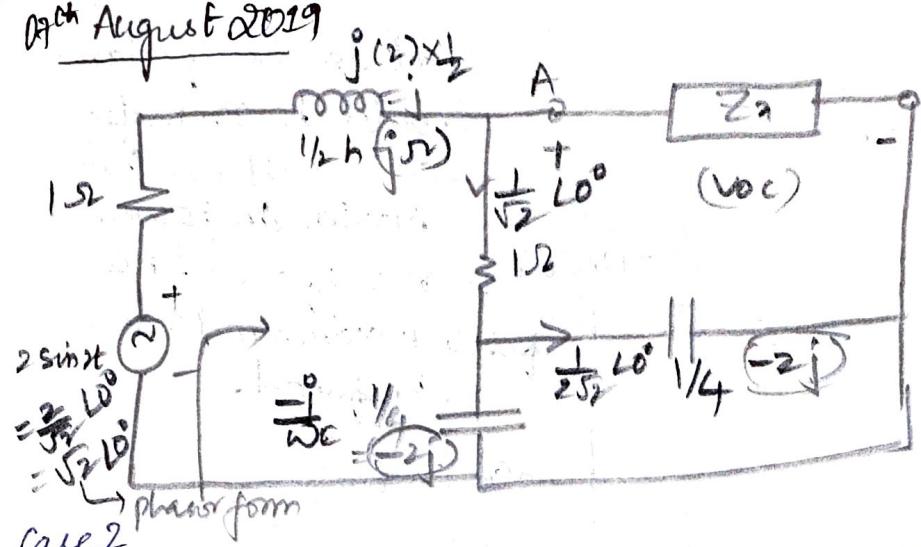
Condition  $Z_L = Z_s^*$

$$P_{max} = \frac{V_s^2}{4 \times R_L}$$



$$R_L = |Z_s| = |3+j6| = 5 \quad P_{max} = \frac{V_s^2}{2(R_S + R_L)}$$

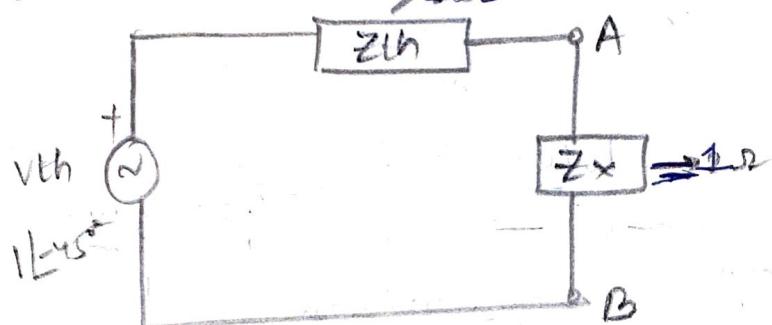
09<sup>th</sup> August 2019



For what value of  $Z_L$ , Maximum Power Transferred to  $Z_L$  & also find Max Power Transfer

But PPT theorem is applied on thevenin equivalent.

$\frac{1}{2}L$



$$X_L = j\omega L = j \times 2 \times \frac{1}{2}$$

$$X_L = j$$

$$X_C = \frac{j}{\omega C}$$

$$= \frac{j}{2 \times 4} = -2j$$

$Z_T$  (total impedance looking from the source)

$$Z_T = 1+j + 1-j = 2\Omega$$

$$I \cdot T = \frac{\sqrt{2} L^0}{2}$$

$$= \frac{1}{\sqrt{2}} L^0$$

$$V_{AB} = 1 \times \frac{1}{\sqrt{2}} L^0 - 2j \times \frac{1}{2\sqrt{2}} L^0$$

$$= \frac{1}{\sqrt{2}} L^0 - j \frac{1}{\sqrt{2}} L^0 = \frac{1}{\sqrt{2}} (1-j) L^0 = \frac{1}{\sqrt{2}} \times \sqrt{2} (-45^\circ)$$

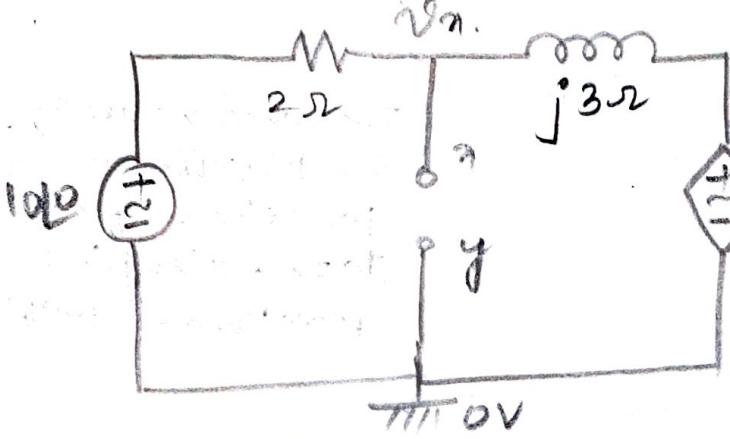
$$v_{th} = 1 \angle -45^\circ$$

$Z_{th} \Rightarrow$  total impedance viewed from A, B,  $\rightarrow$  RT's.  
keeping the <sup>all</sup> independent sources constant.

$$Z_{th} = (1+j) || (1-j)$$

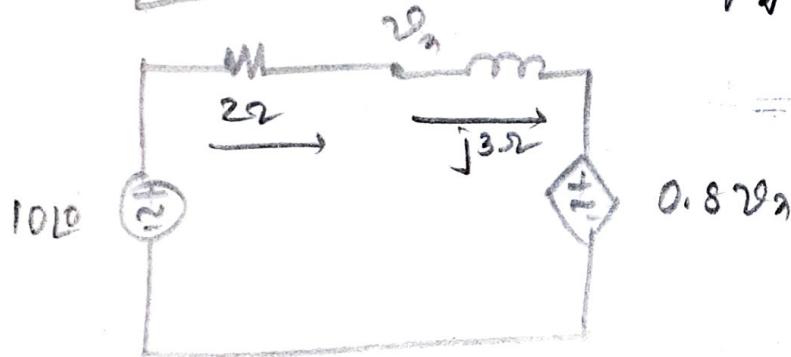
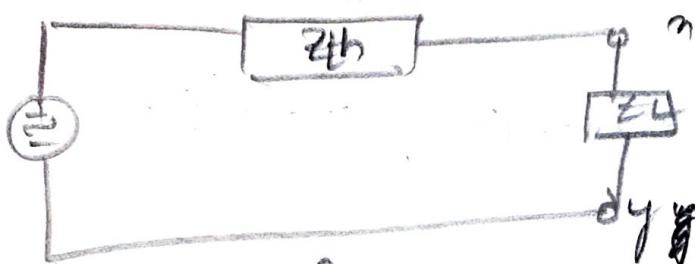
$$= 1\Omega$$

$$P_{max} = \frac{V_s^2}{4 \times R_L} = \frac{1^2}{4 \times 1} = 0.25 \text{ Watts}$$



Q) what load impedance should be connected across  $n, y$  so that maximum power is transferred? Also, find Maximum Power Transferred.

$$Z_{\text{load}} = R_n$$



$$\frac{10V - 2V_2}{2} = \frac{2V_n - 0.8V_2}{j3} = 3190^\circ$$

$$2V_n = 10 \angle 20^\circ$$

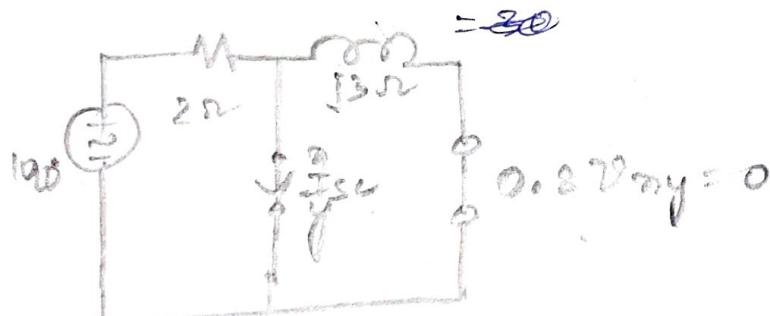
$$3190^\circ = 30^\circ$$

$$30^\circ - 3190^\circ V_n = 2V_n - 1.6V_2$$

$$30^\circ = 0.4V_2 + 3190^\circ V_2$$

$$V_n = \frac{30^\circ}{0.4 + 3190^\circ} = \frac{30^\circ}{0.4 + 3j} = \frac{30j}{0.4 + 3j} = \frac{30^\circ 90^\circ}{3.026182.4}$$

$$V_{\text{th}} = 10 \angle 20^\circ$$



$I_{\text{SC}}$  = short-circuit current

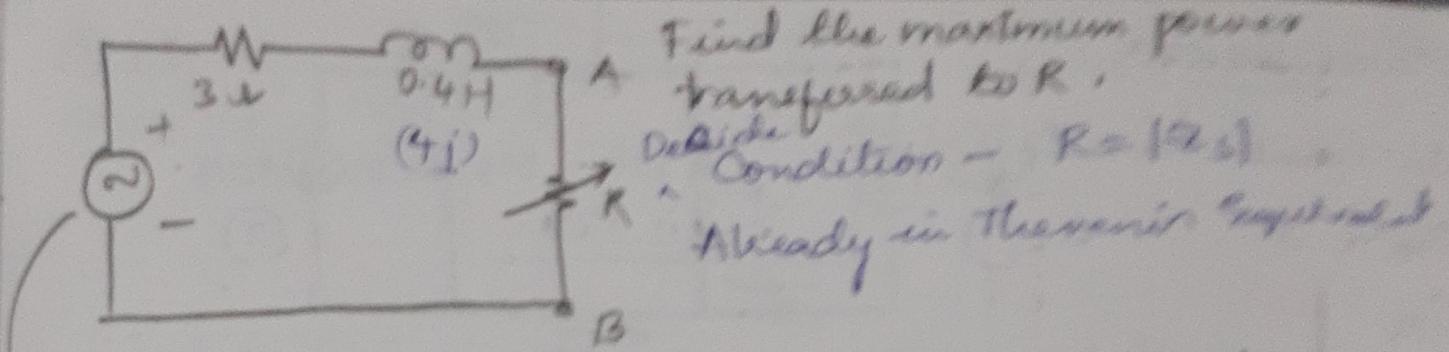
$$I_{\text{SC}} = \frac{10V}{2} = 5A$$

$$Z_{\text{th}} = \frac{10 \angle 20^\circ}{2} = 2 \angle 20^\circ$$

$$Z_{\text{th}} = 2 \angle 20^\circ$$

$$Z_L = Z_{\text{th}}^* = 2 \angle -20^\circ \Rightarrow Z_L = 2 \angle -20^\circ$$

$$P_{\text{max}} = \frac{V^2}{4 \times R_L} = \frac{10^2}{4 \times 2 \cos 20^\circ} = \frac{100}{4 \times 1.98} = \frac{100}{7.92} = 12.626 \text{ watts}$$



$$10\sqrt{2} \cos(10t - 20^\circ)$$

$$Z_T = 3 + 4j \quad R = \sqrt{12^2 + 4^2} = 5$$

$$P_{max} = \frac{V_s^2}{2(R_s + R_L)} = \frac{10^2}{2 \times (3 + 5)} = 10W$$

$$P = \frac{100}{27.2} = 3.67 \text{ W}$$

## SUPER POSITION THEOREM.

Definition: Any linear active network containing at least one independent source, then the final response is equal to addition of individual responses.

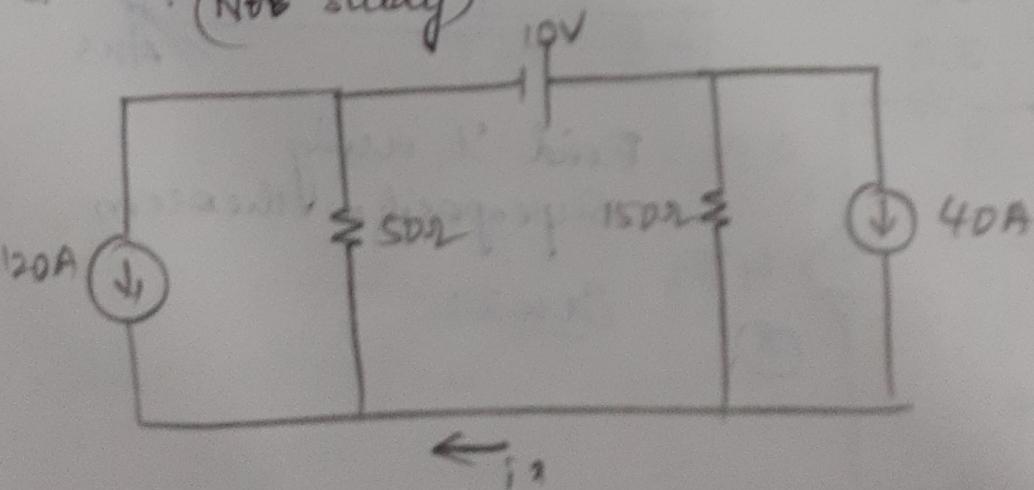
Case 1: All sources are independent

$$f(z_1 + z_2 + z_3) = f(z_1) + f(z_2) + f(z_3)$$

Case 2: Some sources are dependent

$$\begin{aligned} & \text{" " " Independent} \\ & f(z_1 + z_2 + z_3) = f(z_1 + z_2) + f(z_2 + z_3) \\ & \text{Independent} \end{aligned}$$

Case 3: All are dependent  $\rightarrow$  passive  $\rightarrow$  no response  
 (Not Study)



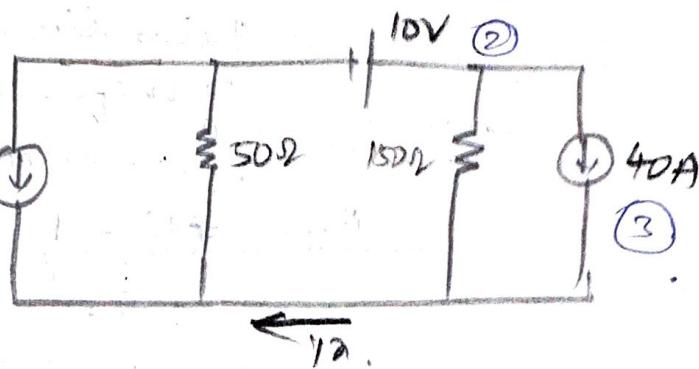
Find  $i_R$  using Superposition theorem.

### Three sources

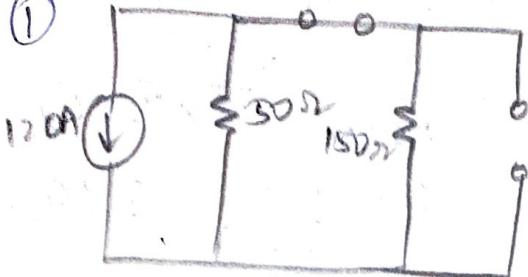
Ex:- Consider Source 1  
 $I_{n1} = 12 \text{ A}$  ①

$I_{n2} = 10 \text{ V}$  ②

$I_{n3}$



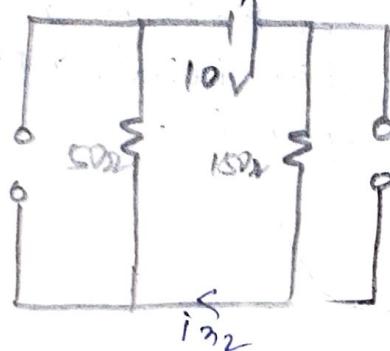
①



$$I_{n1} = -\frac{12 \times 50}{200} = -30$$

$$\boxed{I_{n1} = -30 \text{ A}}$$

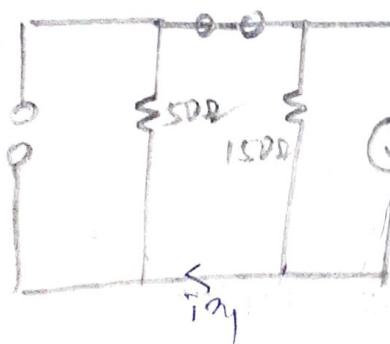
②



$$I_{n2} = \frac{10}{200} = \frac{1}{20} = 0.05 \text{ A}$$

$$\boxed{I_{n2} = 0.05 \text{ A}}$$

③

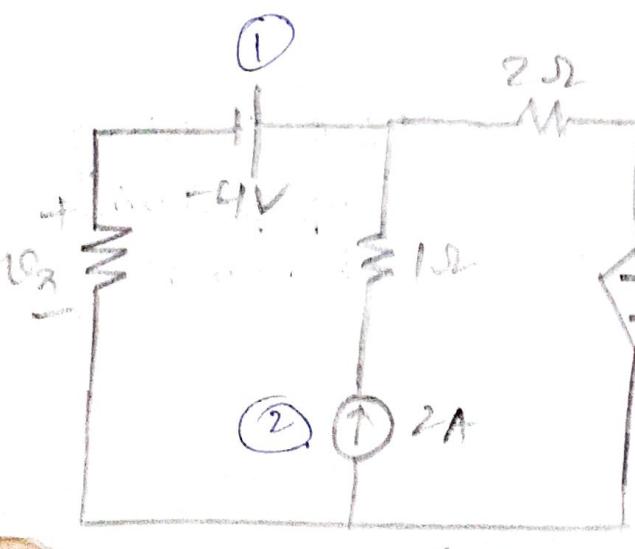


$$I_{n3} = \frac{40 \times 150}{200} = +30 \text{ A}$$

$$\boxed{I_{n3} = +30 \text{ A}}$$

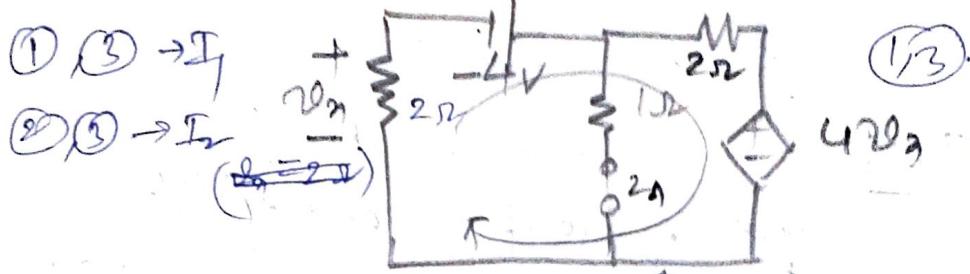
$$I_n = I_{n1} + I_{n2} + I_{n3} = -30 + 0.05 + 30$$

$$\boxed{I_n = 0.05 \text{ A}} \quad \text{Ans}$$



Find  $I$  using  
Superposition Theorem

Case 2:



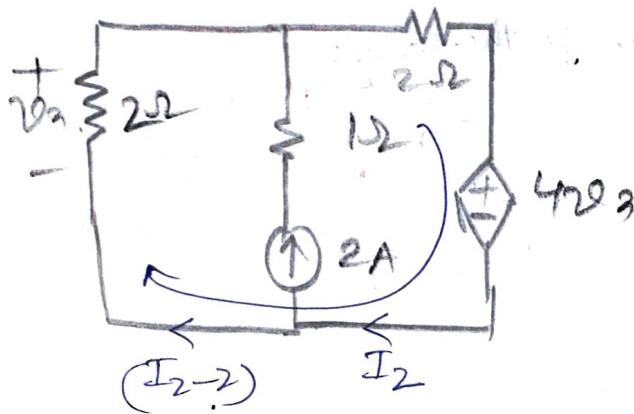
$$2I_1 + 4 + 2I_1 + 4V_2 = 0 \quad | \quad V_2 = -2I_1$$

$$4I_1 + 4V_2 + 4 = 0$$

$$4I_1 - 8I_1 + 4 = 0$$

$$4I_1 = 4$$

$$\underline{I_1 = 1 \text{ A}}$$



$$2I_2 + 4V_2 + 2(I_2 - 2) = 0 \quad | \quad \textcircled{1}$$

$$V_2 = -2(I_2 - 2) \quad | \quad \textcircled{2}$$

$$2I_2 - 8(I_2 - 2) + 2(I_2 - 2) = 0$$

$$2I_2 - 6(I_2 - 2) = 0$$

$$-4I_2 + 12 = 0$$

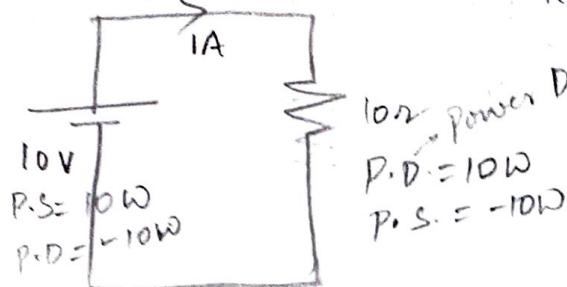
$$\underline{\boxed{I_2 = 3 \text{ A}}}$$

$$I = I_1 + I_2 = 1 + 3 = 4 \text{ A} \quad | \quad \boxed{I_1 = 4 \text{ A}}$$

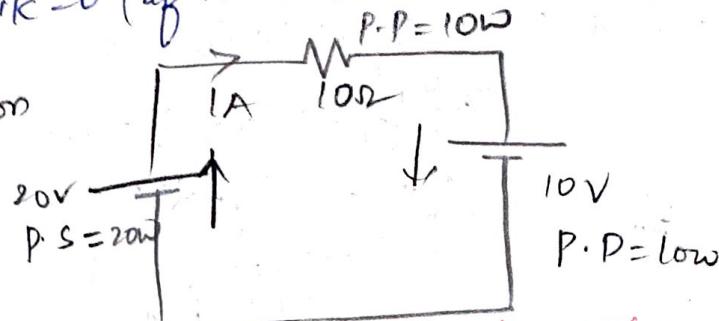
08<sup>th</sup> August 2019

## TELEGAN'S MILLER'S THEOREM

Total Power in the network is zero if KVL and KCL are satisfied. Mathematically,  $\sum_{k=1}^n v_k i_k = 0$  (if KVL & KCL are satisfied)



Net Power supplied = 0



Point this source will supply everytime

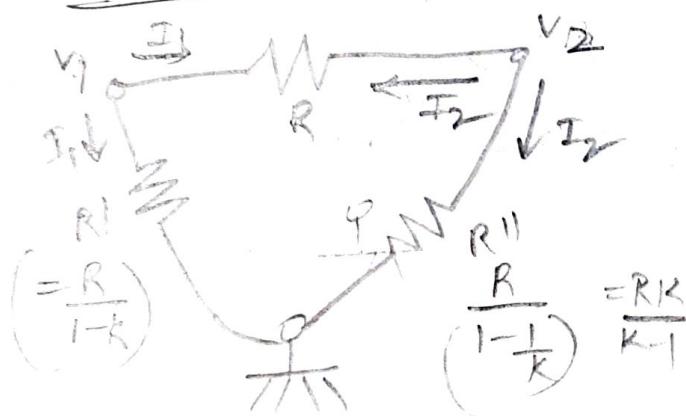
Sign Convention for the Source:

$\frac{I}{T}$  ↗ Dissipation = +ve  
 $\frac{I}{T}$  ↓ P.supply = -ve

$\frac{I}{T}$  ↑ P.S. = +ve  
 $\frac{I}{T}$  ↓ P.S. = -ve.

Independent of the current direction, R always dissipates

## MILLER'S THEOREM

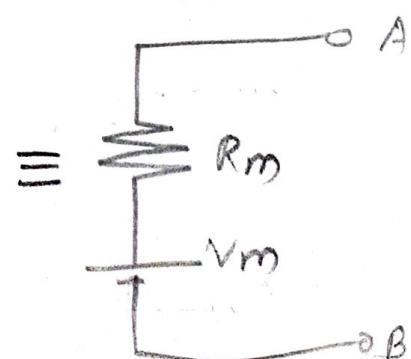
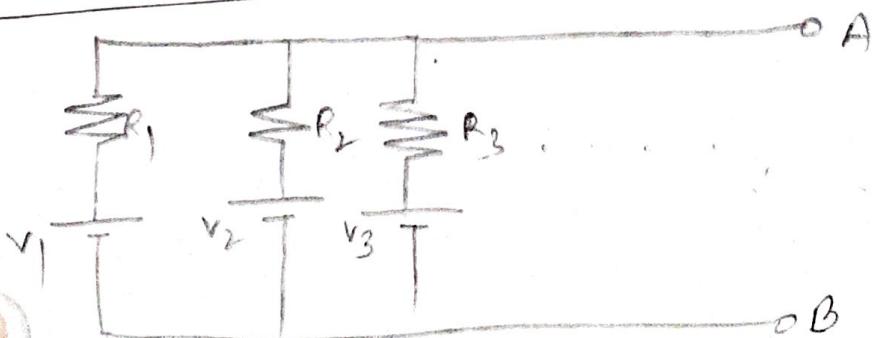


$$I_1 = \frac{V_1 - V_2}{R} = \frac{V_1}{R} \left[ 1 - \frac{V_2}{V_1} \right]$$

$$\text{let } \frac{V_2}{V_1} = k$$

$$I_1 = \frac{V_1}{R} (1-k) = \frac{V_1}{\frac{R}{1-k}} = \frac{V_1}{k}$$

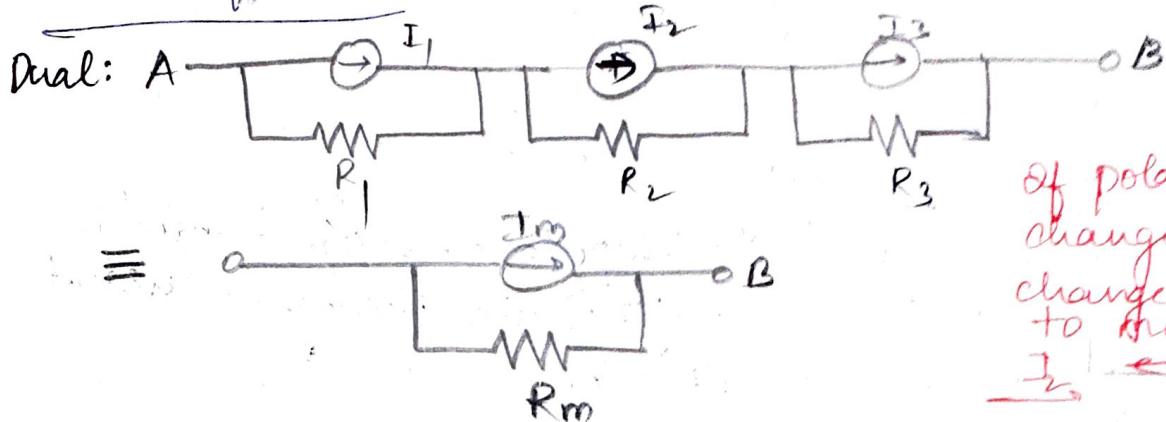
## MILLMAN'S THEOREM:



$$V_m = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3 + \dots}{G_1 + G_2 + G_3 + \dots}$$

$$R_m = \frac{1}{G_1 + G_2 + G_3 + \dots}$$

Source Transformation:  $V_S \rightarrow C.S.$  Add them  $\rightarrow V_S$ .

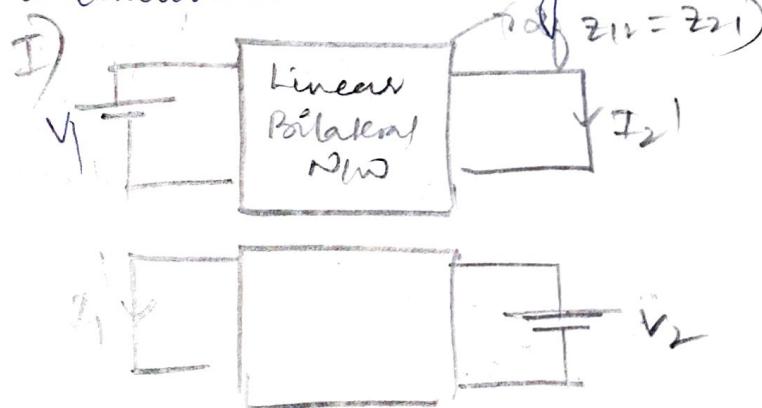


If polarities are changed, just change the sign to minus

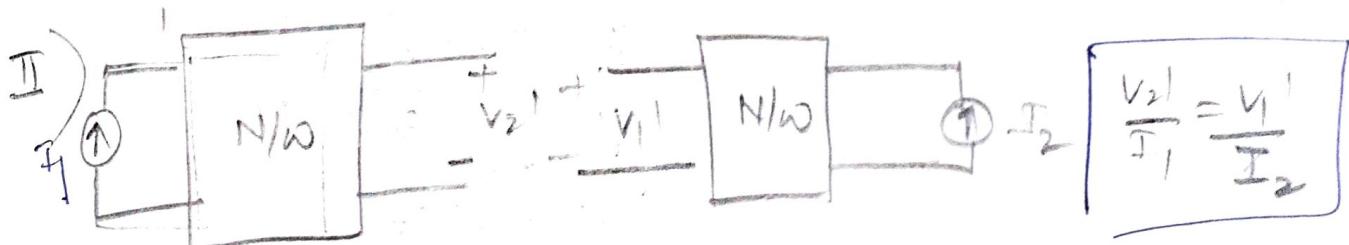
$I_1 \leftarrow -I_1$

$$I_m = \frac{I_1 R_1 + I_2 R_2 + I_3 R_3 + \dots}{R_1 + R_2 + R_3 + \dots} \quad R_m = R_1 + R_2 + R_3 + \dots$$

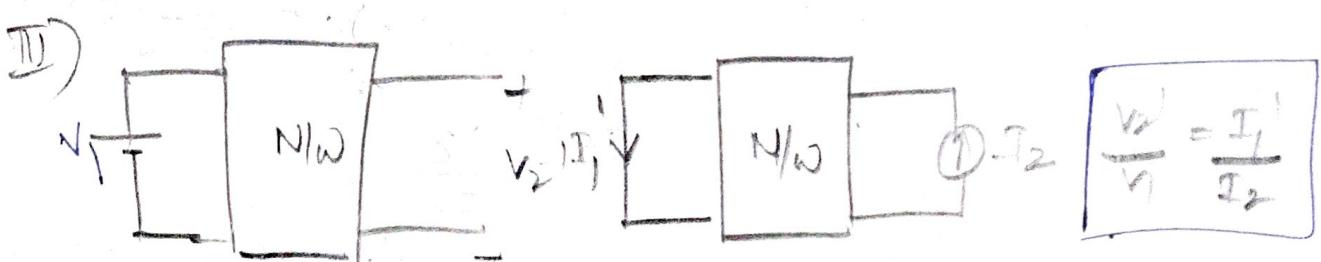
RECIPROCITY THEOREM: In any linear bilateral network, the ratio of response to excitation is constant even if response and excitation interchange.



$$\frac{I_2}{V_1} = \frac{I_1}{V_2}$$

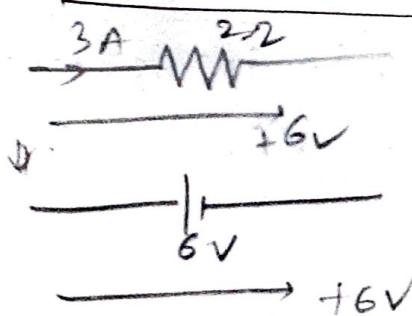


$$\frac{V_2}{I_1} = \frac{V_1}{I_2}$$

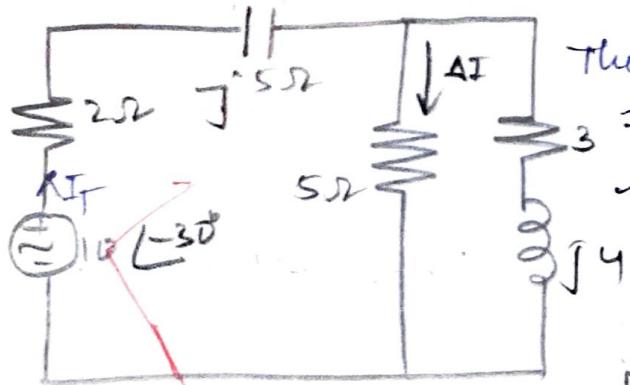


$$\frac{V_2}{V_1} = \frac{I_1}{I_2}$$

# COMPENSATION THEOREM: Substitution Theorem



This theorem is useful to find change in current in any branch due to change in impedance in any one branch.



The impedance  $3+j4$  changed to  $3+7+j12$ . Find change in current in  $5\Omega$ .

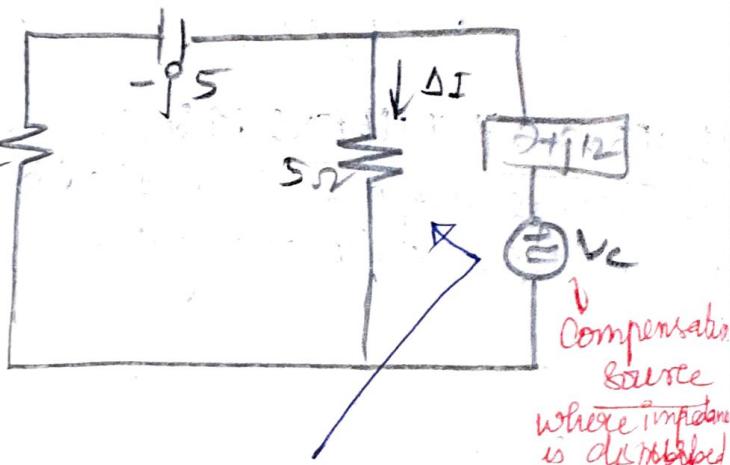
$I$  is current before change where we want to find change in current.

$$\textcircled{3} \quad V_C = I \cdot Z$$

$$\Delta Z = (7+j12) - (3+j4) \\ = (4+j8)$$

$$\textcircled{1} \quad Z_T = (2-j5) + (5)(3+j4) \quad \textcircled{2} \quad I_T = \frac{10(-30)}{Z_T}$$

looking from base



Compensation source where impedance is distributed

$$\textcircled{3} \quad I_T = \frac{I_T \times (2-j5)}{5+(3+j4)}$$

$$\textcircled{4} \quad \text{Substitute in } V_C = I \cdot Z$$

$$Z'_T = (7+j12) + 5(2-j5)$$

$$I'_T = \frac{V_C}{Z'_T}$$

$$\Delta I = \frac{I_T - I'_T}{(2-j5) + (5)}$$