POORNIMA

Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

Unit-I (Sets, Function & Relation)

Sets.

→ Sets are the fundamental discrete ~~not~~ structure on which most of the discrete structures are build, Sets are used to Group objects together

⇒ Definitions :-> A Set is a Collection of Well-defined objects (Called its Element.)

→ Generally, Set are denoted by Capital Letters.

Representation of Sets.

Tabular / Roaster.

↓

{1, 2, 3, 4}

{2, 8, 9}

Set builder.

↓

$A = \{x \mid x = 2^n ; n \in N\}$

ϕ = Empty Set, Null

$x \in A$ = x belongs to A or x is Element of A

N → Natural Number.
W → Whole Number.
R → Real Number

Z → Integer.
Q → Rational Number.
C → Complex Number.

Main Ideas, Questions & Summary:

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Types of Sets.

① **Finite Sets** :→ A set is said to be finite when there are finite Elements in the set

Ex:- $A = \{x \mid x \in N \ \& \ x \le 5\}$

② **Infinite Sets** :→ If No. of Elements are infinite then set is said to be infinite.

Ex:- $A = \{x \mid x \in N \ \& \ x \ge 5\}$

③ **Equal Sets** :→ Contains the same no. of Elements or Elements are same in both sets.

Ex:- $A = \{1, 2, 3, 4\}$

$B = \{1, 1, 2, 3, 3, 4\}$

④ **Null Sets (ϕ)** :→ Empty set are set which has no Element

Ex:- $\{x \mid x^2 = 9 \ ; x \text{ is Even number}\}$

⑤ **Disjoint Sets** :→ If no elements are Common in both the sets.

Ex:- $A = \{1, 2, 3, 4\}$

$B = \{8, 9\}$

⑥ **Comparability** :→ Two Sets A & B are Comparabel if $A \subseteq B$ and $B \subseteq A$ then $A = B$

$x \in A \Rightarrow x \in B$

$A \subseteq B$ — (1)

$x \in B \Rightarrow x \in A$

$B \subseteq A$ — (2)

$A = B$

$\therefore 2^n$ is Total Subsets We Can make from a set where n is no. of Elements.

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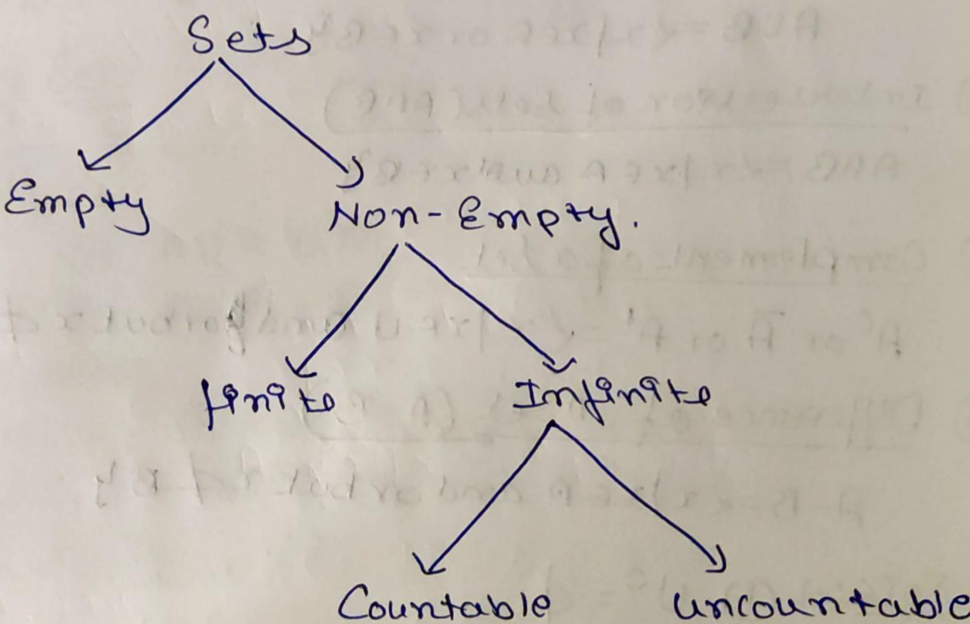
∴ A finite set having 'n' elements has "2^n" subsets.

⑦ Power Set :-> All Subset of a set is Called Power set.

⑧ Cardinality of Set :-> The Total no. of Elements in a set is called its Cardinality. & it is denoted by "n(A)" or "|A|".

Ex:- $A = \{0, 1, 2\}$.
$n(A) = 3$



⑨ Countable Set :-> If there is one to one Mapping between the Element of a set and natural number.

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(10) Uncountable sets :- If there cannot be defined one to one Mapping between the Element of a set & natural number.

(11) Singleton sets :-

(12) Universal set (U) :-

Operations of Set

If A and B are two sets.

(1) Union of Sets $(A \cup B)$

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

(2) Intersection of sets $(A \cap B)$

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

(3) Complement of a set

A^c or $\bar{A}$ or $A' = \{x \mid x \in U$ and / or but $x \notin A\}$

(4) Difference of a Set $(A - B)$

$A - B = \{x \mid x \in A$ and or but $x \notin B\}$.

NOTE :-
(1) $U^c = \phi$
(2) $\phi^c = U$
(3) $A \cup A^c = U$
(4) $A \cap A^c = \phi$

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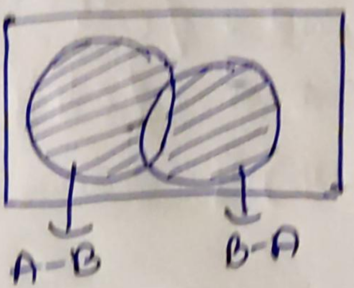
Symmetric Difference in Two Sets (⊕)

$A \oplus B = (A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

Venn diagram.



Properties of Set

① Idempotent Law.

ⓐ $A \cup A = A$

ⓑ $A \cap A = A$

② Associative Law.

ⓐ $(A \cup B) \cup C = A \cup (B \cup C)$

ⓑ $(A \cap B) \cap C = A \cap (B \cap C)$

③ Communicative Law

ⓐ $A \cup B = B \cup A$

ⓑ $A \cap B = B \cap A$

④ Distributive Law

ⓐ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ⓑ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

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(5) Demorgan Law

(a) $(A \cup B)^c = A^c \cap B^c$

(b) $(A \cap C)^c = A^c \cup B^c$.

Proof's

(1) $A \cup A = A$

L.H.S Let $x \in A \cup A$.

$\Rightarrow \{x \in A$ or $x \in A\}$.

$\Rightarrow x \in A$.

$\Rightarrow A \cup A \subset A$ — (1)

Similarly.

$A \subset A \cup A$ — (2)

from (1) & (2)

$\therefore A \cup A = A$.

H.P

(2) $(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S $x \in (A \cup B) \cup C$

$\Rightarrow \{x \in (A \cup B)$ or $x \in C\}$

$\Rightarrow \{\{x \in A$ or $x \in B\}$ or $x \in C\}$.

$\Rightarrow \{x \in A$ or $\{x \in B$ or $x \in C\}$.

$\Rightarrow \{x \in A$ or $x \in (B \cup C)\}$

$\Rightarrow x \in A \cup (B \cup C)$

$(A \cup B) \cup C \subset A \cup (B \cup C)$ — (1)

Similarly

$A \cup (B \cup C) \subset (A \cup B) \cup C$ — (2)

therefore from (1) & (2).

$(A \cup B) \cup C = A \cup (B \cup C)$.

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③ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Let $x \in A \cup (B \cap C)$

$\Rightarrow \{x \in A \text{ or } x \in (B \cap C)\}$

$\Rightarrow \{x \in A \text{ or } \{x \in B \text{ and } x \in C\}\}$

$\Rightarrow \{\{x \in A \text{ or } x \in B\} \text{ and } \{x \in A \text{ or } x \in C\}\}.$

$\Rightarrow \{x \in (A \cup B) \text{ and } x \in (A \cup C)\}.$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C$

Similarly

$\Rightarrow (A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$

$\therefore A \cup (B \cap C = (A \cup B) \cap (A \cup C.$

④ $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

Let $x \in (A-B) \cup (B-A)$

$x \in (A-B) \cup (B-A)\}.$

$\Rightarrow \{x \in (A-B) \text{ or } x \in (B-A)\}.$

$\Rightarrow \{x \in (A-B) \text{ or } \{x \in B \text{ and } x \notin A\}\}.$

$\Rightarrow \{\{x \in A \text{ and } x \notin B\} \text{ or } \{x \in B \text{ and } x \notin A\}\}.$

$\Rightarrow \{\{x \in A \text{ or } x \in B\} \text{ and } \{x \notin B \text{ or } x \notin A\}\}.$

$\Rightarrow \{\{x \in A \text{ or } x \in B\} \text{ and } \{x \notin A \text{ or } x \notin B\}\}$

$\Rightarrow \{x \in (A \cup B) \text{ and } x \notin (A \cap B)\}$

$\Rightarrow x \in (A \cup B) - (A \cap B)$

$(A-B) \cup (B-A) \subset (A \cup B) - (A \cap B.$

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Similarly

$$(A \cup B) - (A \cap B) \subset (A-B) \cup (B-A)$$

$$\therefore (A \cup B) - (A \cap B) = (A-B) \cup (B-A)$$

(5) $(A-B) - C = A - (B \cup C)$

$\Rightarrow x \in (A-B) - C$

$\Rightarrow \{x \in (A-B) \text{ and } x \notin C\}$

$\Rightarrow \{\{x \in A \text{ and } x \notin B\} \text{ and } x \notin C\}$

$\Rightarrow \{x \in A \text{ and } \{x \notin B \text{ and } x \notin C\}\}$

$\Rightarrow \{x \in A \text{ and } x \notin (B \cup C)\}$

$\Rightarrow x \in A - (B \cup C)$

$\therefore (A-B) - C \subset A - (B \cup C)$

Similarly

$$A - (B \cup C) \subset (A-B) - C$$

therefore

$$(A-B) - C = A - (B \cup C)$$

H.P.

Duality :→ For Any Equation in Algebra of set, involving the set 'U', 'ϕ' and operation of Union & intersection in the dual Equation which will also be true is obtained by Replacing 'U' by 'ϕ' the Union by intersection and intersection by Union

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for Ex:-

Q. Write the dual of the following Equation

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

dual $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $U^c = \phi$

dual $\phi^c = U$.

(iii) $A \cup (B \cap A) = A$

also written as

$(U \cap A) \cup (B \cap A) = A$

dual $(\phi \cup A) \cap (B \cup A) = A$

Partition :-> A Set's having A, B, C, D ------- of Non-Empty subsets then.

A, B, C, D ----- is called Partition of set.

(i) $A \cup B \cup C \cup D \cup$ ------- $= S$

(ii) The intersection of Every pair of different Subset $A \cap B \cap C \cap D \cap$ ------- $= \phi$ is the empty set (disjoint sets)

Main Ideas, Questions & Summary:

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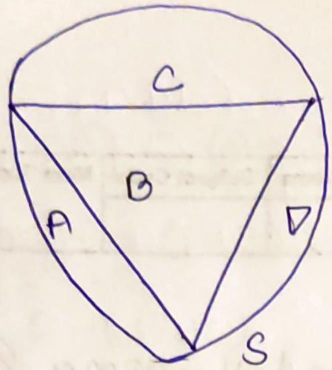

Example:→ Let $A = \{1, 2, 3, 4\}$
$B = \{5, 7\}$
$C = \{8\}$
$D = \{6, 9\}$

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Addition Principle

Theorm:→ for two finite sets A & B which are disjoint (No Common Element) i.e. $A \cap B = \phi$ then $\boxed{n(A \cup B) = n(A) + n(B)}$

Proof Let A have 'm_1' elements then
$n(A) = m_1$
Let B have 'm_2' Elements then
$n(B) = m_2$

Here A & B are disjoint sets therefore $A \cup B$ will have all the elements of A & all the Elements of set B therefore the number of Element in $A \cup B$ is $m_1 + m_2$

Thus $A \cup B = n(A) = n(B)$
$= m_1 + m_2$

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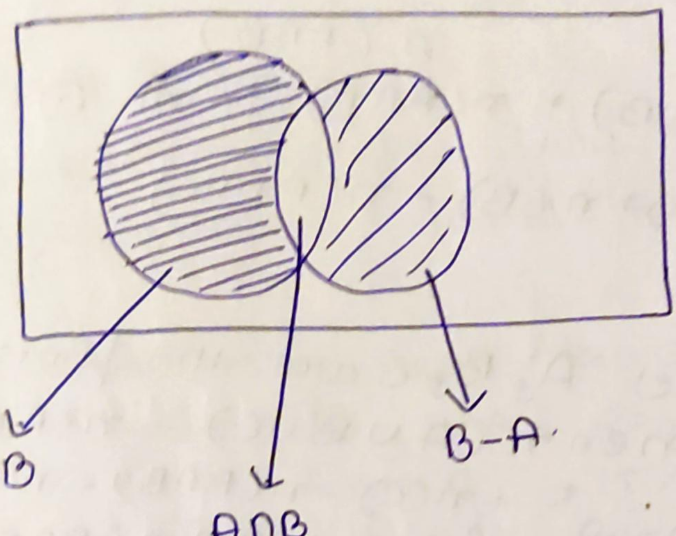
Prove for finite sets A & B

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$



We know that

$(A-B) \cup (A\cap B) \cup (B-A) = A\cup B$.

Here $(A-B)$, $(A\cap B)$ and $(B-A)$ are pairwise disjoint.

therefore by addition principle

$n(A\cup B) = n(A-B) + n(A\cap B) + n(B-A)$ — (1)

Now

$A = (A-B) \cup (A\cap B)$

& $(A-B) \cap (A\cap B) = \phi$

$n(A) = n(A-B) + n(A\cap B)$ — (2)

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Now

$$B = (B-A) \cup (A \cap B)$$
$$\& \ (B-A) \cap (A \cap B) = \phi$$
$$\therefore \ n(B) = n(B-A) + n(A \cap B) \quad \text{—} \ (3)$$

Add (2) & (3)

$$n(A) + n(B) = \{ n(A-B) + n(A \cap B) + n(B-A) \} + n(A \cap B)$$

$$n(A) + n(B) = n(A \cup B) + n(A \cap B) \text{ from — } (1).$$

$$\Rightarrow n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Theorm :→ Let A, B, C are the finite sets then $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Proof :→ Let $A \cup B = S$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) \quad \text{—} \ (1)$$

$$n(A \cup B \cup C) = n(S \cup C)$$
$$= n(S) + n(C) - n(S \cap C)$$
$$= n(S) + n(C) - n((A \cup B) \cap C)$$
$$= n(S) + n(C) - n((A \cap C) \cup (B \cap C))$$
$$= n(S) + n(C) - (n(A \cap C) + n(B \cap C) - n(A \cap B \cap C))]$$
$$= n(A \cup B) + n(C) - (n(A \cap C) + n(B \cap C) - n(A \cap B \cap C))$$
$$\Rightarrow n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

H.P

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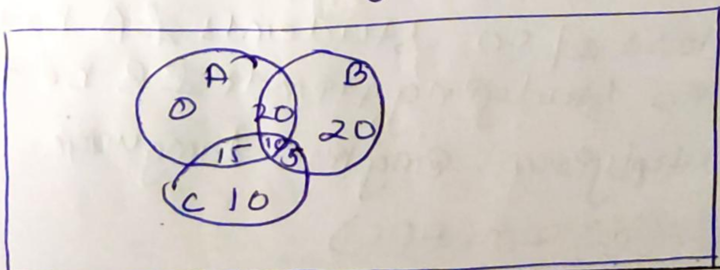
Principle of Inclusion & Exclusion

If A & B be any non-disjoint sets then
$|A \cup B| = |A| + |B| - |A \cap B|$
If A, B, C are the finite sets then.
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

Q. In a town 45% read Magazine 'A', 55% Read Magazine (B)', 40% Read Magazine 'C', 30% Read 'A and B', 15% Read '(B & C)', 25% Read (A & C), 10% Read All the three Magazines. find what percentage
1. Do not Read any Magazine
2. What percentage two Exactly two of Magazine.

Solution
Let A, B, C denote the set that of All. those read Magazine A, B, C

U=100



45% - A
55% - B
40% - C
30% - A & B
15% - B & C
25% - A & C
10% - All

Main Ideas, Questions & Summary:

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Here

$n(A) = 45$
$n(B) = 55$
$n(C) = 40$
$n(A \cap B) = 30$
$n(B \cap C) = 15$
$n(A \cap C) = 25$
$n(A \cap B \cap C) = 10$
$n(U) = 100$

No. of persons who read atleast one Magzine.
$= 20 + 10 + 15 + 5 + 20 + 10 = 80.$

(1) Persons who donot Read Any Magzine $= 100 - 80$
$= 20.$

i.e. 20% Persons donot Read Any Magzine.

(2) Percentage Exactly Reading two Magzine
$= 20 + 15 + 5$
$= 45.$

Q. In A class of 50 students, 30 are studing Hindi & 25 English language & 10 are studying both languages How Many students are studying either language.

Solun Let U be the class of 50 students & A be the set of those who studying Hindi & B be the set those who studying English language

$n(A \cup B) = n(A) + n(B) - n(A \cap C)$
$= 30 + 25 - 10$
$= 45$ Ans

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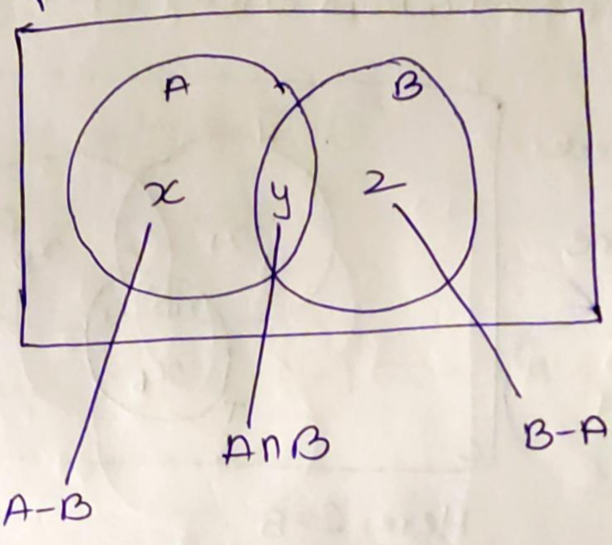
Q. In a group of 52 persons 16 drink tea but not coffee & 33 drink tea find

(i) How many drink tea & coffee both

(ii) How many drink coffee but not tea.

Solution.

Let U be the set of 52 person & A be the set of those who drink Tea & B be the set of those who drink coffee.



$n(A-B) = x = 16$

$n(A \cap B) = y$.

$n(B-A) = z$.

(i) $x+y = 33$

$y = 33-16$

$= 17$. ∴ Persons who drink tea & coffee are 17.

Main Ideas, Questions & Summary:

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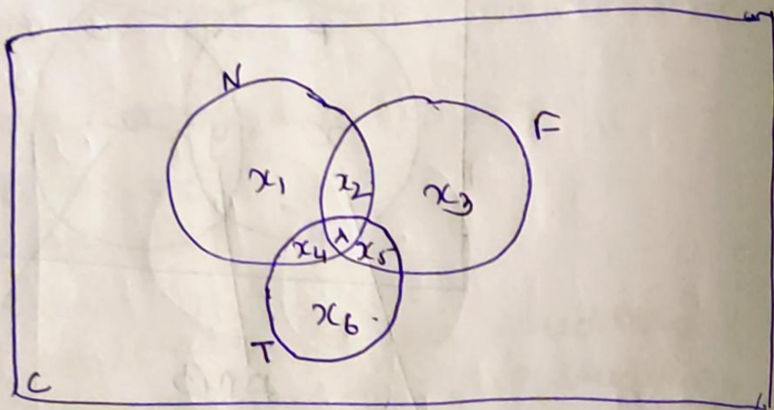
(ii) Persons who drink Coffee but not tea are.

$$Z = U - x - y$$
$$= 52 - 16 - 17$$
$$= 19 \text{ Ans}$$

Qo. Im a survey of 60 peoples It was found that 25 Read 'New Week' ~~Read~~ 26 Read 'Time' & 26 Read 'Magzine fortune' also 9 Reads both "NewWeek & fortune", 11 Read "NewWeek & Time" and 8 Read both "Time & fortune". If 8 Read No of three Magzine determine the No of people who Read Exactly one Magzine

Solun

U – 60
25 – NewWeek
26 – Time
26 – Fortune.
9 – N&F
11 – N&T
8 – T&F.
8 – None



Here $C = 8$.

$$(x_1 + x_3 + x_6) + (x_2 + x_4 + x_5) + \lambda = 60 - 8$$
$$= 52 \quad - (1)$$

$$x_1 + (x_2 + x_4) + \lambda = 25 \quad - (2)$$
$$x_6 + (x_4 + x_5) + \lambda = 26 \quad - (3)$$
$$x_3 + (x_2 + x_5) + \lambda = 26 \quad - (4)$$
$$x_2 + \lambda = 9 \quad - (5)$$
$$x_4 + \lambda = 11 \quad - (6)$$
$$x_5 + \lambda = 8 \quad - (7)$$

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Add Eq^n (2) + (3) + (4)

$(x_1+x_6+x_3)+2(x_2+x_4+x_5)+3\lambda = 77$ — (8)

Add Eq^n (5) + (6) + (7)

$(x_2+x_4+x_5)+3\lambda = 28$ — (9)

Putting Eq^n (9) in (8)

$(x_1+x_6+x_3)+(x_2+x_4+x_5)+28 = 77$

$(x_1+x_6+x_3)+(x_2+x_4+x_5) = 49$ — (ii)

$49+\lambda = 52$

$\boxed{\lambda = 3}$

Putting value of λ in Eq^n - (9)

$(x_2+x_4+x_5)+9 = 28$

$(x_2+x_4+x_5) = 19$

Put value of $(x_2+x_4+x_5) = 19$ in Eq^n (ii)

$(x_1+x_6+x_3)+19 = 49$

$\boxed{x_1+x_6+x_3 = 30}$

Main Ideas, Questions & Summary:

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Q. A Survey on a Sample of 25 New Cars being sold At a local auto dealer Conducted to see which of three popular options. Air Condition (A), Radio (R), Power Window (W) were already installed in a Car. The Survey found 15 had A, 12 had R, 11 had W, 5 had A & W, 4 had R and W, 9 had A and R and 3 had All the Three find the No. of Cars that had.

(i) only Power Window

(ii) only A

(iii) only R.

(iv) R & W.

(v) A & R but Not W.

(vi) only one option

(vii) Atleast one option.

(ix) None of the Three.

Solution

15 – A

12 – R

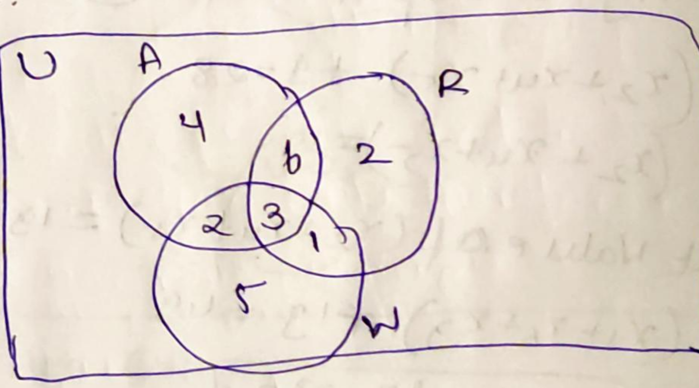
11 – W.

5 – A & W

4 – R & W

9 – A & R

3 – All



(i) Only W

$\Rightarrow$ 5 Ans

(ii) Only A

$\Rightarrow$ 4 Ans

(iii) only R

$\Rightarrow$ 2 Ans

(iv) R & W

$\Rightarrow 6 + 3 = 9$ Ans

(v) A & R

$\Rightarrow$ 6 Ans

(vi) only one option

$\Rightarrow 4 + 5 + 2$

$\Rightarrow$ 11 Ans

(vii) 23 Ans

(ix) $U - 23$

$= 2$ Ans

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Pigenhole Principle

If 'N' pigeons are assigned to 'M' pigeon holes and 'm<n' then atleast one pigeon hole contains 2 or more pigeons.

Proof

Suppose Each pigeon hole contains atmost one pigeon then atmost m pigeon have been assigned. Since m<n, Not all pigeons have been assigned. pigeon holes this is the contradiction therefore. atleast one pigeon hole contains two or more pigeon.

Generalised Pigeonhole Principle:

Let 'n' pigeons are assigned to 'm' pigeonsholes then one of the pigeon hole must contain atleast $\left\lfloor \frac{n-1}{m} \right\rfloor + 1$ where $\left\lfloor \frac{n-1}{m} \right\rfloor$ is the floor that is greatest integer i.e. less than or Equal $\frac{n-1}{m}$.

Main Ideas, Questions & Summary:

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Proof:→ We have 'n' pigeons and 'm' pigeonholes such that n>m assuming that each of 'm' pigeonhole contain Not more than $\lfloor \frac{n-1}{m} \rfloor$ pigeons, the total no of pigeons in the 'm' pigeonholes must less or equal to $m \times \lfloor \frac{n-1}{m} \rfloor \le m \times \frac{n-1}{m}$

$$= n-1$$

But there are 'n' pigeons so this is Contradict our assumption that a pigeon holes contain not more than $\lfloor \frac{n-1}{m} \rfloor$ pigeons. So one of the pigeon holes must contain atleast $\lfloor \frac{n-1}{m} \rfloor + 1$.

Q. Show that if 30 dictionaries in a library Contain a total of 61327 Pages then one of the dictionary must have atleast 2045 pages

Soln Let the pages be the pigeons and dictionary in a library be pigeon halls therefore by generalized pigeon hole principle

$$= \frac{61326}{30} + 1$$

$$= 2045$$

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Function

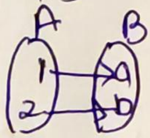
A function is a type of Relation. A function is a Rule such that (1) one manner, corresponding mapping define from set A to set B such that Each & Every Element of set A has a Unique image in the set B.

Mathematically $f: A \to B$.

$y = f(x)$

for $x \in A, y \in B$.

NOTE:- ① f is the collection of ordered pairs.



$f: \{(1,a), (2,b)\}$

② f is a subset of $A \times B$

$\boxed{f \subset A \times B}$

Domain, Codomain & Range of the function

Domain :- If $f: A \to B$ then set A is known as the domain of f and set B is known as Codomain of the f.

Main Ideas, Questions & Summary:

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Range:- The Range of the function is the set of images of its domain or it is a subset of its Codomain

$f: A \to B$

Range of $f = \{ b \mid b \in B ; (a, b) \in f \}$

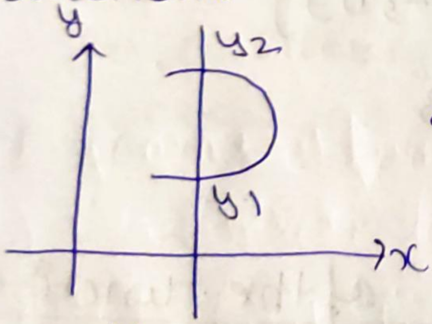
Ex:- $A = \{1, 2, 3, 4\}$.

$B = \{a, b, c\}$

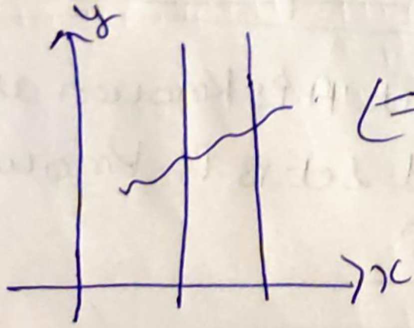
$f: \{(2,a), (3,b), (1,a)\}$.

Graphical Representation of a function

If a line parallel to Y-axis cut the graph at more than or Equal to two points then it is not a function.



⇐ This is not a function b/c element does not have the unique image.



⇐ This is a function b/c element have unique image.

POORNIMA

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NOTE:- (1) domain will consist of those value of x for which
(i) $f(x)$ does not tends to infinity.
(ii) $f(x)$ does not become imaginary
(iii) $f(x)$ does not become indeterminant.

Q. Find the domain of the following function.

(i) $f(x) = \frac{x}{x-5} \Rightarrow$ domain $f = R - \{5\}$

(ii) $f(x) = \frac{x}{x^2+1} \Rightarrow$ domain $f = R$

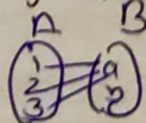
(iii) $f(x) = \sqrt{x-4} \Rightarrow$ domain $f = [4, \infty)$

(iv) $f(x) = \frac{\sqrt{x+2}}{x-2} \Rightarrow$ domain $f = [2, \infty) - \{2\}$

Types of function

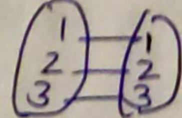
(i) Constant function :- $f: A \to B$

Range $f(a) = k$



(ii) Identity function :- $f: A \to A$

Range $f(x) = x$



Main Ideas, Questions & Summary:

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Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

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Q. Find the domain of the following function.

(i) $f(x) = \frac{x}{x-5} \Rightarrow$ domain $f = R - \{5\}$

(ii) $f(x) = \frac{x}{x^2+4} \Rightarrow$ domain $f = R$

(iii) $f(x) = \sqrt{x-4} \Rightarrow$ domain $f = [4, \infty)$

(iv) $f(x) = \frac{\sqrt{x+2}}{x-2} \Rightarrow$ domain $f = [2, \infty) - \{2\}$

Types of function

(i) Constant function :- $f: A \to B$

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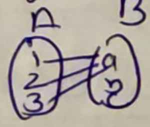
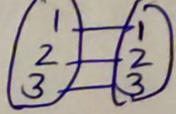


(ii) Identity function :- $f: A \to A$

Range $f(x) = x$



Main Ideas, Questions & Summary:

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(iii) Equal function :-> Two function f & g are said to be Equal if and only if.

(i) domain of f = domain of g.

(ii) Codomain of f = Codomain of g.

(iv) $f(x) = g(x)$ for Every $x \in$ domain of f = domain of g.

Example :-> Let $A = \{1, 2\}$ & $B \{3, 4\}$

& $f: A \to B$, $f(x) = x^2 + 2$

$g: A \to B$ $g(x) = 3x$

(iv) Floor function :-> $\lfloor 9.8 \rfloor = [9 + 0.8] = 9$

$[\frac{1}{2}] = 0$

(v) Ceiling function :-> $\lceil 9.8 \rceil = [10 - 0.2] = 10$

(vi) Modulus function :-> $f(x) = |x|$

$= x, x > 0$

$= -x, x < 0$

(vii) Exponential function

(viii) logarithmic function

(ix) Polynomial function

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Properties of function

① **One-One function or Injection** :-> A function $f: A \to B$ is said to be one to one or one one function if each element of A have different images in set B that is $\forall a_1, a_2 \in A$

$$a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2$$

$$\text{or } f(a_1) = f(a_2) \Rightarrow a_1 = a_2$$

for Example :-> let $f: R \to R$
where $f(x) = x^3+1, \forall x \in R$.

$= \text{let } x_1, x_2 \in R$

$= \& \; f(x_1) = f(x_2)$

$= x^3+1 = x_2^3+1$

$= x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

② **Many One function** :-> A function $f: A \to B$ which maps two or more element of set A to the same element of set B i.e. two or more elements of set A have same image in B.

$$\text{i.e. } a \neq b \Rightarrow f(a) = f(b)$$

$$\forall a, b \in A.$$

Main Ideas, Questions & Summary:

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Example:- let $f: R \to R$

$f(x) = x^2 ; \forall x \in R$

Let $x_1, x_2 \in R$

& $f(x_1) = f(x_2)$

$x_1^2 = x_2^2$

$x_1^2 - x_2^2 = 0$

$(x_1 + x_2)(x_1 - x_2) = 0$

$x_1 = -x_2$

$x_1 = x_2$.

(3) Onto function or Subjection :- If the Mapping $f: A \to B$ is such that Each Element of set B is the f image of atleast one element of set A then f is called onto Mapping in this case.

$\boxed{f(A) = B}$ or $\boxed{\text{Codomain} = \text{Range}}$

Example:- $f: N \to N$

$f(x) = x^2 ; \forall x \in N$

Every Element of B has pre image in A

$f(N) = N \ \forall x \in N$

(4) Into function :- If the function $f: A \to B$ such that At least one element of B is not the f image of Any element of set 'A'. then f is called into function and for this

$f(A) \neq B$

& $f(A) \subset B$

or

Codomain $\neq$ Range

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Example:-> let $f: R \to R$

$f(x) = x^2 \;\forall\; x \in R$

let $x_1, x_2 \in R$

& $f(x_1) = f(x_2)$

Here Negative Numbers donot appear in the Range therefore it is into function.

Properties of function

1. one-one

$f: R \to R$

$f(x) = 2x + 3$

if $f(x_1) = f(x_2)$ then $x_1 = x_2$

2. onto

Codomain = Range

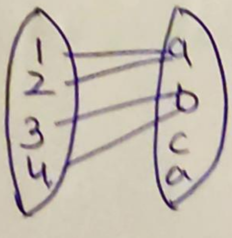
$f(A) = B$

3. Into

$f(A) \neq B$

Codomain $\neq$ Range

4. Many one



Main Ideas, Questions & Summary:

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⇒ <u>Bijection Mapping</u> :→ If A Mapping $f: A \to B$ is said to be Bijection if it is one-one and onto Mapping.

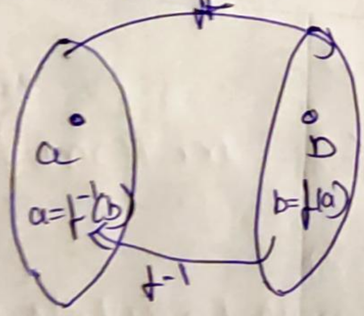
for ex :→ $f: Z \to Z$

$f(x) = x+1 \; \forall x \in Z$

$y = x+1$

$x = 1-y$.

⇒ <u>Inverse of a function</u> :→ If $f: A \to B$ be one-one, onto Mapping. then $f^{-1}: B \to A$. Which associates to Each element $b \in B$, the Unique Element $a \in A$ is Called Inverse of the Mapping $(f: A \to B)$



<u>Q:</u> check f is bijection or not

(i) $f: R \to R$
where $f(x) = -5x+4$

(iv) $f(x) = x^5+1$

(ii) $f: R \to R$
where $f(x) = x^2+1$

(iii) $f: R \to R$
$f(x) = \dfrac{x+r}{x+b}$

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Sol-1 for One-One

Let $f(x_1) = f(x_2)$; $\forall x_1, x_2 \in R$

$= -5x_1+4 = -5x_2+4$

$= -5x_1 = -5x_2$

$\Rightarrow x_1 = x_2$ f is one-one — (1)

for Onto

we have to show that

$f(A) = B$

$f(R) = R$

Let $y = f(x) = -5x+4$

$= x = \frac{4-y}{5}$ $\forall y \in R$, x exists.

therefore f is onto — (2)

from (1) & (2)

It is bijection

Sol-2 $f: R \to R$

$f(x) = x^2+1$

Let $f(x_1) = f(x_2)$

$x_1^2+1 = x_2^2+1$

$x_1^2 = x_2^2$.

It is not one-one function.

Main Ideas, Questions & Summary:

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Sol-3 $f: R \to R$

$$f(x) = \frac{x+8}{x+b}.$$

There is Not a function

Sol-4

Hashing function :-> A modular function $f: N \to B$ where $B = \{1, 2, \ldots\ldots 100\}$

& N is set of Natural Numbers defined by $h(n) = x \{\text{mod } 101\}$ is called the Hashing function.

Qo Let An Account number 2473871 belongs to the file number 78.

Solun $h(2473871) = 247831 \text{ mod } \{101\}$.

$= 78$

$2473871 = 24493 \times 101 + 78$

$= 78 \{\text{mod } 101\}$

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Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

Composition of functions :-

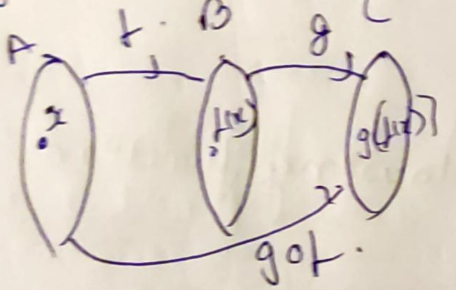
Product of Mapping or Composite of function

Let $f: A \to B$

$g: B \to C$.

then the Composite of the function f & g denoted by gof (or gf) is a mapping.

$gof: A \to C$ such that $gof(x) = g[f(x)] \ \forall x \in A$.



Q. find gof and fog when $f: R \to R$ & $g: R \to R$ define by $g(x) = 1 + \frac{1}{1-x}$ and $f(x) = x^2 - 1$

$fog(x) = f[g(x)]$

$= f\left[1 + \frac{1}{1-x}\right]$

$= f\left[\frac{1-x+1}{1-x}\right]$

$= f\left[\frac{2-x}{1-x}\right]$

$= f\left[\frac{2-x}{1-x}\right]^2 - 1$

Main Ideas, Questions & Summary:

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$gof(x) = g[f(x)]$

Properties

Q. find the Range of the following function

(i) $y = \frac{1}{x-5}$

(ii) $f(x) = x^2$

(iii) $f(x) = \sqrt{9-x^2}$

(iv) $y = \frac{2+x}{2-x}$

(v) $y = \frac{x}{1+x^2}$.

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(i) $y = \frac{1}{x-5}$

$\frac{1}{y} = x-5$

$\frac{1}{y} + 5 = x$. $R = \{0\}$

$x = \frac{1+5y}{y}$

(ii) $y = x^2$

$x = \pm\sqrt{y}$ $[0, \infty]$

(iii) $f(x) = \sqrt{4-x^2}$

$y = \sqrt{9-x^2}$

$9 - y^2 = x^2$

$x = \pm\sqrt{9-y^2}$

Range $0 \le y \le 3$ Ans

(iv) $y = \frac{2+x}{2-x}$

(v) $y = \frac{x}{1+x^2}$

$x = +1 \pm \sqrt{-4y^2}$

Range $[-\frac{1}{2}, \frac{1}{2}] = \{0\}$

Main Ideas, Questions & Summary:

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Q. The total cost of 13 refrigerator at a department store is Rs 1,23,050, show that one refrigerator must cost atleast 94,66/-

Solun

Properties.

(i) Associative Law

If $f: X \to Y$; $g: Y \to Z$; $h: Z \to T$ are functions such that $(hog)of$ & $ho(gof)$ are defined then.

$$(hog)of = ho(gof)$$

Let $g: Y \to Z$; $h: Z \to T$

$f: X \to Y$; $(hog): Y \to T$

$(hog)of: X \to T$

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$f: X \to Y \quad g: Y \to Z$

$(gof)(x) = g[f(x)] \Rightarrow X \to Z$

$ho(gof)(x) = \quad X \to T$

$(hog)of$ & $ho(gof)$ define on $X \to T$

Now $[(hog\,of](x) = [hog][f(x)]$

$= h[g\,f(x)]$

$= ho(gof)(x)$

$= ho(gof)(x)$

$\Rightarrow (hog)of = ho(gof)$

(ii) Identity Law

Let $f: X \to Y$ be a function if I_X & I_Y are the Identity function on the set 'X' & 'Y' respectively

(i) $F o I_X = F$

(ii) $I_Y o F = F$

Main Ideas, Questions & Summary:

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Theorm:→ If $f: X \to Y$ & $g: Y \to X$ are the functions such that $g \circ f = I_X$ & $f \circ g = I_Y$ then. show that f & g are one-one & onto and. $g = f^{-1}$

Proof: Let $x_1, x_2 \in X$

$f(x_1) = f(x_2)$

$\Rightarrow g[f(x_1)] = g[f(x_2)]$

$\Rightarrow (g \circ f)(x_1) = (g \circ f)(x_2)$

$\Rightarrow I_X(x_1) = I_X(x_2)$

$\Rightarrow x_1 = x_2$.

$\therefore$ f is one-one

For onto

Let $y \in Y$ & $g(y) = x$.

$\Rightarrow f[g(y)] = f(x)$

$\Rightarrow (f \circ g)(y) = f(x)$

$\Rightarrow I_Y = f(x)$

$= \boxed{y = f(x)}$

$\therefore$ invertable when one-one & onto.

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Now $f \circ g = I_Y$.

$\Rightarrow f^{-1}(f \circ g) = f^{-1}(I_Y)$

$\Rightarrow (f^{-1} \circ f) \circ g = f^{-1}$

$\Rightarrow I_Y \circ g = f^{-1}$

$\Rightarrow g = f^{-1}$

Theorm :- Let $f: R \to R$ define by $f(x) = ax+b$. Where $a, b, x \in R$ & $a \neq 0$ show that f is invertable.

Proof Here to show invertable show one-one & onto.

Let $x_1, x_2 \in R$.

$\Rightarrow f(x_1) = f(x_2)$

$\Rightarrow ax_1 + \cancel{b} = ax_2 + \cancel{b}$

$\Rightarrow \cancel{a}x_1 = \cancel{a}x_2$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one

Main Ideas, Questions & Summary:

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for onto

$$y = ax + b$$

$$y - b = ax$$

$$x = \frac{y-b}{a}.$$

for every value of y. x is Real Number

$\therefore$ f is onto.

$\therefore$ As f is one-one & onto therefore f is invertable.

Q. Let $f: Z \to Z$ such that $f(x) = x+2 \ \forall x \in Z$ and $G: Z \to Z$ be the functions then find G such that $g \circ f = I_Z$

Soln

$$g \circ f = I_Z$$

$$g$$

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Theorm:- The inverse map $f^{-1}: B \to A$ is also one-one onto or if f is one-one onto map then f^{-1} is also one-one onto map.

Proof Let $f^{-1}(b_1) = a_1$

& $f^{-1}(b_2) = a_2$ $\forall$ $b_1, b_2 \in B$
$a_1, a_2 \in A$.

Now let $f^{-1}(b_1) = f^{-1}(b_2)$

$\Rightarrow a_1 = a_2$

$\Rightarrow f(a_1) = f(a_2)$; because f is one-one.

$\Rightarrow b_1 = b_2$

This implies f^{-1} is one-one.

Since any $a \in A$ is the f^{-1} image of the element $b \in B$ where $b = f(a)$ so mapping is onto.

Theorm:- If $f: A \to B$ be one-one, onto then the inverse Map of f is unique.

Main Ideas, Questions & Summary:

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Proof

$g: B \to A$ &

$h: B \to A$.

are two inverse mapping of f. Here we have to proof that $g = h$

Let $b \in B$, $g(b) = a_1$ & $h(b) = a_2$ $\forall a_1, a_2 \in A$.

Since g & h are inverse mapping of f

therefore

$$g(b) = a_1 \Rightarrow f(a_1) = b$$

$$h(b) = a_2 \Rightarrow f(a_2) = b$$

Here f is one-one

$$f(a_1) = b$$

$$\& \ f(a_2) = b$$

$$\Rightarrow a_1 = a_2$$

$$\Rightarrow g(b) = h(b)$$

This implies mapping of f is unique.

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Relation

⇒ **Ordered pair** :- Let a & b be any two objects 'a' is assigned as a first position and 'b' is assigned as the second position then (a, b) is called ordered of a & b.

✱ If $(a, b) = (c, d)$

$\Rightarrow a = c \;\&\; b = d$.

⇒ **Cartesian product** :- It is the basic binary operation applied between two sets.

Represented as '$A \times B$' - collection of all ordered pair of set A to set B is called cartesian product.

$$A \times B = \{ (a, b) \mid a \in A \text{ and } b \in B \}.$$

NOTE :- $A \times B \neq B \times A$ unless $A = B$.

A = {a, b, c} B = {1, 2, 3}

$$A \times B = \{ (a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1), (c,2), (c,3) \}$$

⇓

This is cartesian product

Main Ideas, Questions & Summary:

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If

$\{(a,1),(a,2)\ (b,3)\ (c,3)\}$

$\Downarrow$

This is the Relation Not the cartesian product.

NOTE:- In cartesian product we have 'm' 'n' elements in total elements are mn

Q. for non-void sets A, B, C, D prove

(i) $A\times(B\cup C) = (A\times B)\cup(A\times C)$

Let $(x,y) \in A\times(B\cup C)$.

$\Rightarrow x\in A$ and $y\in(B\cup C)$

$\Rightarrow x\in A$ and $\{y\in B$ or $y\in C\}$

$\Rightarrow \{x\in A$ and $y\in B\}$ or $\{x\in A$ and $y\in C\}$

$\Rightarrow (x,y)\in A\times B$ or $(x,y)\in A\times C$

$\Rightarrow (x,y)\in(A\times B)\cup(A\times C)$

therefore $A\times(B\cup C) \subset (A\times B)\cup(A\times C)$

Similarly

Let $(x,y)\in(A\times B)\cup(A\times C)$

$\Rightarrow (x,y)\in A\times B$ or $(x,y)\in A\times C$.

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$\Rightarrow \{x \in A$ and $y \in B\}$ or $\{x \in A$ and $y \in C\}$.

$\Rightarrow x \in A$ and $\{y \in B$ or $y \in C\}$.

$\Rightarrow x \in A$ and $y \in (B \cup C)$

$\Rightarrow x, y \in A \times (B \cup C)$

$\Rightarrow (A \times B) \cup (A \times C) \subset A \times (B \cup C)$

therefore $A \times (B \cup C) = (A \times B) \cup (A \times C)$

H.P

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

Let $x, y \in (A \times (B - C))$

$\Rightarrow x \in A$ and $y \in (B - C)$

$\Rightarrow x \in A$ and $\{y \in B$ and $y \notin C\}$

$= (x \in A$ and $y \in B)$ and $(x \in A$ and $y \notin C)$

$\Rightarrow (x, y) \in (A \times B)$ and $(x, y) \notin (A \times C)$

Main Ideas, Questions & Summary:

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$\Leftrightarrow (x,y) \in (A\times B)-(A\times C)$

$\Rightarrow A\times(B-C) \subset (A\times B)-(A\times C)$

Similarly

$\Rightarrow (x,y) \in (A\times B)-(A\times C)$

$\Rightarrow (x,y) \in (A\times B)$ and $y \notin (A\times C)$

$\Rightarrow \{x\in A$ and $x \in B\}$ and $\{y \notin A$ and $y \notin C\}$

(iii) $A\times(B\cap C) = (A\times B)\cap(A\times C)$

Let $x,y \in A\times B\cap C)$

$\Rightarrow x\in A$ and $y \in (B\cap C)$

$\Rightarrow x \in A$ and $y\in B$ and $y \in C)$

$\Rightarrow (x\in A$ and $y\in B)$ and $(x\in A$ and $y \in C)$

$\Rightarrow x,y \in A\times B$ and $x,y \in A\times C$.

$\Rightarrow (x,y \in (A\times B)\cap(A\times C)$

Similarly.

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$x, y \in (A \times B) \cap (A \times C)$

$\Rightarrow x \in (A \times B)$ and $y \in (A \times C)$

$\Rightarrow \{x \in A$ and $y \in B\}$ and $\{y \in A$ and $y \in C\}$.

=

(iv) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D$

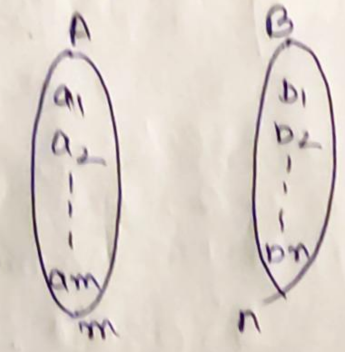
Main Ideas, Questions & Summary:

Library / Website Ref.:-

Relation :-

If A & B be non-empty sets a Relation from A to B is the subset of $A \times B$. If $R \subset A \times B$ & $(a,b) \in R$ we say that a is Related to b and denoted by 'aRb'; If a is not Related to b then we write '$a \not R b$'

NOTE:- ~~No of element~~



→ No. of element in $A \times B$ is mn

→ No. of subset in $A \times B$ is 2^{mn}

→ No. of Relation from A to B is 2^{mn}

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Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

Complement of Relation ($\bar{R}$ | R^c | R')

* $\bar{R} = \{a,b \mid a,b \in A \times B \text{ and } a,b \notin R\}$.

* $\bar{R} = (A \times B) - R$

* $R \cup \bar{R} = A \times B$.

* $R \cap \bar{R} = \phi$

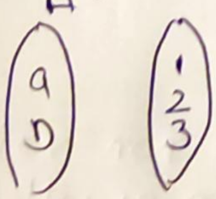
Example:- If $R = \{(a,2), (b,1), (b,3)\}$.

$A \times B = \{(a,1) (b,1) (b,2) (a,2) (a,3) (b,3)\}$

$\bar{R} = ?$

Solu^n

A B



$\bar{R} = \{(a,1), (b,2), (a,3)\}$.

Main Ideas, Questions & Summary:

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Inverse of a Relation (R^{-1})

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

Example :-

A: a, b

B: 1, 2, 3

If $R = \{(a,2), (b,1), (b,3)\}$

$R^{-1} = \{(2,a), (1,b), (3,b)\}$

Types / Properties of Relation :

Reflexive	Symmetric	Transitive.
Irreflexive.	Anti Symmetric	
	Asymmetric	

Reflexive Relation :- A relation are on a non-void set. A is known as reflexive relation. If each member of A is R-related to itself i.e $\forall a \in A;\ (a,a) \in R$

$A \times A$	a	b	c
a	aa	ab	ac
b	ba	bb	bc
c	ca	cb	cc

$A = \{a, b, c\}$

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(i) $R = \phi \longrightarrow$ No

(ii) $R = A \times A \longrightarrow$ Yes.

(iii) $R = \{(a,a), (b,b), (c,c), (a,b), (b,c)\} \longrightarrow$ Yes.

(iv) $R = \{(a,a), (b,b), (c,c)\} \longrightarrow$ Yes.

(v) $R = \{(a,b), (b,a), (a,a)\} \longrightarrow$ No

(vi) $R = \{(a,b), (b,c), (c,a)\} \longrightarrow$ No.

<u>Example</u> :- Let A be the set of all straight lines in a plane. The Relation 'R' in A defined by "x is || to y". Since every straight line is parallel ~~to~~ itself so reflexive relation is hold.

⇒ <u>Irreflexive Relation</u> :- A relation 'R' on a set 'A' is known as Irreflexive Relation if $\forall a \in A, (a,a) \notin R$.

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⇒ Symmetric Relation :- A Relation R on a non empty set 'A' is said to symmetric if aRb ⇒ bRa. i.e. whenever (a,b) ∈ R then (b,a) ∈ R ∀ a,b ∈ A

A = {a, b, c}

	a	b	c
a	aa	ab	ac
b	ba	bb	bc
c	ca	cb	cc

(i) $R = \{(a,b), (b,a)\}$ ⟶ Yes

(ii) $R = \{(b,c), (c,b), (b,b), (c,c)\}$ → Yes

(iii) $R = \{(a,a), (b,b), (c,c)\}$ → Yes

(iv) $R = A \times A$ → Yes

(v) $R = \phi$ ⟶ Yes.

(vi) $R = \{(a,b), (b,c), (a,c)\}$ → NO

(vii) $R = \{(a,b), (b,a), (a,c)\}$ → NO

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Example :- Let 'A' be the set of all straight lines in a plane the Relation 'R' is defined by.

x to y is Symmetric Relation because

$x \perp y \Rightarrow y \perp x \ \forall x, y \in A$.

$\Rightarrow$ Anti Symmetric Relation :- A Relation 'R' in a Set 'A' is said to be Antisymmetric Relation if aRb & bRa $\forall a,b \in A$. may both be true only when $a = b$.

for Example :- Let 'N' be the set of natural numbers & R be the Relation define by.

"a divides b" $\forall a,b \in N$ then R is an Antisymmetric Relation as "a divides b" and "b divides a" $\Rightarrow a = b$

$\Rightarrow$ Asymmetric Relation :- A Relation 'R' on a Set 'A' is said to be Asymmetric.

$\forall (a,b) \in A$. if $(a,b) \in R$ $(b,a) \in R$.

* In above Symmetric Example Relation (v) & (vi) are assymmetric Relation

Main Ideas, Questions & Summary:

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Transitive Relation :- A Relation R on a set A is to be transitive if ∀ a,b,c ∈ A and aRb, bRc ⇒ aRc i.e. if (a,b) ∈ R, (b,c) ∈ R ⇒ (a,c) ∈ R

A = {1,2,3}

	1	2	3
1	11	12	13
2	21	22	23
3	31	32	33

(i) $\phi \longrightarrow$ Yes

(ii) $A \times A \longrightarrow$ Yes

(iii) $\{(1,1), (2,2), (3,3)\} \longrightarrow$ Yes.

(iv) $\{(1,2), (1,3)\} \longrightarrow$ Yes.

(v) $\{(1,2), (3,3) (1,3)\} \longrightarrow$ Yes.

(vi) $\{(2,3)\} \longrightarrow$ Yes.

(vii) $\{(1,2), (3,1)\} \longrightarrow$ No

For Example:- The Relation greater than (>) define on a set of natural number 'N' is transitive because $x, y, z \in N$ if $x > y, y > z \Rightarrow x > z$

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Equivalance Relation

If a Relation R in a set A then R is called an equivalance Relation in set A if:-

(i) R is Reflexive

(ii) R is Symmetric

(iii) R is Transitive

Q. Prove that in the set I of integers, the Relation R define by aRb if $a \cong b \mod 2$ or a-b is multiple of 2 is an Equivalance Relation

Soln

Given

$$aRb \equiv a \cong b \mod 2$$

$$= (a-b) = 2 \cdot n, \ n \in I.$$

(i) Reflexive:

$$aRa \equiv a \cong a \mod 2.$$

$$= (a-a) = 0 = 2 \cdot 0$$

$\Rightarrow$ This is True.

$\Rightarrow$ This implies R is Reflexive.

(ii) Symmetric:-

$$aRb = (a-b) = 2 \cdot n$$

$$\Rightarrow (b-a) = 2 \cdot (-n), \ n \in I.$$

$$= \text{multiple of } 2$$

$$\Rightarrow bRa$$

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⇒ R is symmetric.

(iii) Transitive :-

aRb & bRc then prove aRc

Let
$aRb = (a-b) = 2n$ — (1)
$bRc = (b-c) = 2m$ — (2) $n, m \in I$

Add Eqn (1) + (2)
$= (a-c)\ 2(n+m)$, $n+m \in I$
⇒ aRc.

⇒ R is Transitive.

Here R is Reflexive, symmetric & Transitive therefore R is Equivalance Relation.

Q. Prove that the Relation R_1 & R_2 in the set N×N the set of natural numbers defined as follows :-

(i) $(a,b)\ R_1\ (c,d) \iff a+d = b+c$

(ii) $(a,b)\ R_2\ (c,d) \iff a \cdot d = b \cdot c$

are Equivalence $\forall\ a, b, c, d \in N$.

Soln (i)

Reflexive :-

Let $(a,b) \in N \times N$ $\forall a, b \in N$

therefore

$(a,b)\ R_1\ (a,b) = a+b = b+a$
$= a+b = a+b$
= is True.

⇒ R_1 is Reflexive.

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Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

Symmetric. Let (a,b) & $(c,d) \in N \times N$

We have to Prove that $(a,b) R (c,d)$ then $(c,d) R (a,b)$

$(a,b) R (c,d) \Rightarrow a+d = b+c.$

$\Rightarrow c+b = d+a$

$\Rightarrow (c,d) R (a,b)$

$\Rightarrow$ ~~[illegible]~~ R_1 is Symmetric

Transitive

Let (a,b) & (b,c) ~~[illegible]~~ & $(e,f) \in N \times N)$

We have to Prove that $(a,b) R (c,d)$ & $(c,d) R (e,f)$

then $(a,b) R (e,f)$

$(a,b) R (c,d) = a+d = b+c$ — ①

$(c,d) R (e,f) = (c+f) = (d+e)$ — ② $a,b,c,d,e,f \in N$.

Add ① & ②

$= a+f+\cancel{d}+\cancel{c} = b+e+\cancel{d}+\cancel{c}$

$= a+f = b+e$

$\Rightarrow (a,b) R (e,f)$ is True

$\Rightarrow R_1$ is Transitive.

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ere R, is Reflexive, Symmetric & Transitive therefore
R_1 is Equivalance Relation.

)

Reflexive :-

Let $(a,b) \in N \times N \quad \forall a,b \in N$.

therefore $(a,b) R_2 (a,b) = \quad a \cdot b = b \cdot a$
$a \cdot b = b \cdot a$
$=$ is True

$\Rightarrow R_2$ is Reflexive

Symmetric :-

Let (a,b) & $(c,d) \in N, N \quad \forall a,b,c,d \in N$.

$(a,b) R (c,d)$ then Prove $(c,d) R (a,b)$

$(a,b) R (c,d) = a \cdot d = b \cdot c$
$= c \cdot b = d \cdot a$
$\Rightarrow (c,d) R (a,b)$

$\Rightarrow$ R is Symmetric

Transitive :-

Let (a,b) & (c,d) & $(e,f) \in N \times N \; \forall a,b,c,d,e,f \in N$.

$(a,b) R (c,d)$ & $(c,d) R (e,f)$ then Prove
$(a,b) R (e,f)$

$(a,b) R_2 (c,d) = a \cdot d = b \cdot c$ — (1)

$(c,d) R_2 (e,f) = c \cdot f = d \cdot e$ — (2)

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Multiply (1) & (2)

$(a \cdot d) \cdot (c \cdot f) = (b \cdot c) \cdot (d \cdot e)$

$= a \cdot f = b \cdot e$

$\Rightarrow (a, b)\ R_2 \cdot (e, f)$

$\Rightarrow R_2$ is Transitive

$\Rightarrow R_2$ is Reflexive, Symmetric & Transitive therefore R_2 is Equivalence Relation.

Partial Order Relation :-

A Relation R defined on a set 'A' is said to be partial Order Relation in 'A' if

(i) R is Reflexive

(ii) R is Anti Symmetric

(iii) R is Transitive

then the Relation is called Partial Order Relation

Q. Prove that the Relation R Given by $aRb \Longleftrightarrow a \leq b$, is a Partial Order Relation in the Set of Real Numbers.

Solun (i) Reflexive

Let $a \in R$ $\forall a \in R$

$aRa \Rightarrow a \leq a$

= This True

$\Rightarrow R$ is Reflexive

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(ii) <u>AntiSymmetric</u>

Let $a, b \in R$

$aRb \Rightarrow a \leq b$

$\Rightarrow b \leq a$ is True only $a = b$

$\Rightarrow$ R is Antisymmetric.

(iii) <u>Transitive</u>

Let $a, b, c \in R$

~~Then prove~~ aRb & bRc then prove aRc.

$aRb = a \leq b$ — (1)

$bRc = b \leq c$. — (2)

from (1) & (2)

$= a \leq c$

$\Rightarrow aRc$

$\Rightarrow$ R is Transitive.

$\Rightarrow$ R is Reflexive, Antisymmetric & Transitive. therefore it is Partial Order Relation