RISK AND RATES OF RETURN

Investment Returns and Risk

With most investments, an individual or business spends money today, expecting to earn even more in the future. However, most investments are risky.

Return on Investments

The concept of return provides investors with a convenient way to express the financial performance of an investment.

Illustration: Suppose an investor bought 10 shares of a stock for P1,000. The stock pays no dividends, but at the end of one (1) year, the investor sells his stock for P1,100. How much is the return on the P1,000 investment?

$$Return = Amount\ to\ be\ received - amount\ invested = P1{,}100 - P1{,}000 = P100$$

If the stock was sold for only P900 at the end of the year, the return would be -P100.

It is easy expressing returns in numbers but making a meaningful judgment and timing the return can be challenging.

1. To make a meaningful judgment about the return, the scale (size) of the investment must be known.

   A P100 return on a P100 investment is a great return (assuming the investment is held for one [1] year), but a P100 return on a P10,000 investment would be a poor return.

2. To know the timing of the return.

   P100 return on a P100 investment is a great return if it occurs after one (1) year, but the same amount of return after 100 years is not very good.

The solution for the scale and timing problems is to express investment results as *rates of return*, or *percentage returns*. For example, the rate of return on the 1-year stock investment, when P1,100 is received after one (1) year, is 10%:

$$Rate\ of\ Return = \frac{Amount\ to\ be\ received - Amount\ invested}{Amount\ invested} = \frac{Return}{Amount\ invested} = \frac{P100}{P1{,}000} = \mathbf{0.10}\ or\ \mathbf{10}\%$$

The rate of return calculation "standardizes" the return by considering the annual return per unit of investment. Although this example has only one outflow and one inflow, the annualized rate of return can easily be calculated in situations where multiple cash flows occur over time by using the time value of money concepts (Brigham & Houston, 2022).

Stand-Alone Risk

Risk is a probability or threat of damage, injury, liability, loss, or any other negative occurrence caused by external or internal vulnerabilities that may be avoided through preemptive action (Emerald Insight, 2022). Thus, risk refers to the chance that some unfavorable event will occur. If an individual is engaged in skydiving, he/she is taking a chance with his/her life as skydiving is risky. If one bets on horses, he is gambling with his money. For an investment in financial assets or new projects, the unfavorable event might result in a lower return than expected. An asset's risk can be analyzed in two (2) ways: (a) on a **stand-alone basis**, where the asset is considered in isolation; and (b) as **part of a portfolio**, which is a collection of assets. Thus, an asset's stand-alone risk is the risk an investor would face if she held only this one asset. Most assets are held in portfolios, but it is necessary to understand stand-alone risk to recognize risk in a portfolio context (Brigham & Houston, 2022).

Illustration: Suppose an investor buys P100,000 of short-term Treasury bills with an expected return of 5%.
- In this case, the investment's return, 5%, can be estimated quite precisely, and the investment is defined as being essentially risk-free.
- This same investor could also invest the P100,000 in a company's stock venturing for oil business. Returns on the stock would be much harder to predict.
- In the worst case, the company would go bankrupt, and the investor would lose all his or her money, in which case the return would be -100%.
- In the best-case scenario, the company would discover huge amounts of oil, and the investor would receive a 1,000% return. When evaluating this investment, the investor might analyze the situation and conclude that the expected rate of return, in a statistical sense, is 20%. Still, the actual rate of return could range from, +1,000% to -100%. Because there is a significant danger of earning much less than the expected return, such a stock would be relatively risky.

No investment should be undertaken unless the expected rate of return is high enough to compensate for the perceived risk. In the example, few investors would be willing to buy the oil exploration stock if its expected return didn't exceed that of the Treasury bill, which can be considered an extreme example. An asset's risk is different when iis held by itself versus when it is held as a part of a group, or portfolio, of assets (Brigham & Houston, 2022).

Statistical Measures of Stand-Alone Risk

1. *Probability Distributions* – It is a listing of possible outcomes or events with a probability assigned to each outcome.

   The table below gives a sample probability distribution for Mac Products, which makes engines for long-haul trucks (18-wheelers) and Phil. Water supplies an essential product and thus has very stable sales and profits.

	Mac Products			**Phil. Water**		
Economy which affects demand (1)	*Probability of this Demand Occurring (2)*	*Rate of Return if this demand occurs (3)*	*Product (2) x (3) (4)*	*Probability of this Demand Occurring (5)*	*Rate of Return if this demand occurs (6)*	*Product (5) x (6) (7)*
Strong	0.30	80%	24.00%	0.30	15%	4.50%
Normal	0.40	10%	4.00%	0.40	10%	4.00%
Weak	0.30	-60%	-18.00%	0.30	5%	1.50%
	1.00	Expected Return =	**10.00%**	**1.00**	Expected Return =	**10.00%**

   - Three (3) possible states of the economy are shown in column 1, and the probabilities of these outcomes, expressed as decimals rather than percentages, are given in column 2 and then repeated in column 5. There is a 30% chance of a strong economy and thus strong demand, a 40% probability of normal demand, and a 30% probability of weak demand.
   - Columns 3 and 6 show the returns for the two (2) companies under each state of the economy. Returns are relatively high when demand is strong and low when demand is weak. Notice, though, that Mac Product's rate of return could vary far more widely than Phil. Water. Indeed, there is a high probability that Mac's stock will suffer a 60% loss, though at worst, Phil. Water should have a 5% return.
   - Columns 4 and 7 show the products of the probabilities times the returns under the different demand levels. When each product is totaled, the expected rate of return is obtained for each stock. Both stocks have an expected return of 10%.

   Although the sample companies are illustrative, it is also somewhat unrealistic. In reality, most stocks have at least some chance of producing a negative return.

2. *Expected Rates of Return, $\hat{r}$ ("r hat")* – This is the rate of return expected to be realized from an investment; the weighted average of the probability distribution of possible results.

   The expected return can also be calculated with an equation instead of a table:

$$Expected\ Rate\ of\ Return = \hat{r} = P_1r_1 + P_2r_2 + \cdots + P_Nr_N \ \ OR\ \ \hat{r} = \sum_{i=1}^{N} P_1r_1$$

   The expression with sigma (Σ) means "sum up," or add the values of N factors. If $i = 1$, then $P_ir_i = P_1r_1$; if $i = 2$, then $P_ir_i = P_2r_2$; and so forth, until $i = N$, the last possible outcome. The symbol $\sum_{i=1}^{N}$ says, 'Go through the following process: First, let $i = 1$ and find the first product; then let $i = 2$ and find the second product; then continue until each individual product up to N has been found. Add these individual products to find the expected rate of return.' (Brigham & Houston, Fundamentals of Financial Management, 2022)

3. *Standard Deviation, σ (sigma)* – This is a statistical measure of the variability of a set of observations.

   It is useful to measure risk for comparative purposes, but the risk can be defined and measured in several ways. A common definition that is satisfactory for our purpose is based on probability distributions, such as those shown in *Figure 1.*

   The tighter the probability distribution of expected future returns, the smaller the risk of a given investment. Accordingly, Phil. Water is less risky than Mac Products because there is a smaller chance of Phil's accrual return. Water will end up far below its expected return.

   The standard deviation (σ) can be used to quantify the tightness of the probability distribution. The smaller the standard deviation, the tighter the probability distribution, and the lower the risk.

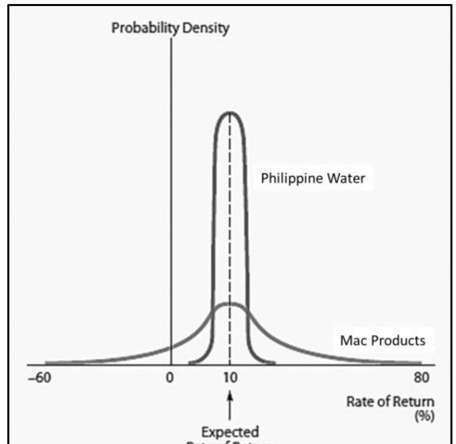

Figure 1. Continuous Probability Distributions of Mac Products and Phil. Water
Source: Fundamentals of Financial Management, 2022 p. 277

   Illustration: Using the previous data from Mac Products, the standard deviation can be computed as follows:

Economy which affects demand (1)	*Probability of this Demand Occurring (2)*	*Rate of Return if this demand occurs (3)*	*Deviation: Actual - 10% Expected Return (4)*	*Squared Deviation (5)*	*Squared Deviation x Probability (6)*
Strong	0.30	80%	70.00%	0.4900	0.1470
Normal	0.40	10%	0.00%	0.0000	0.0000
Weak	0.30	-60%	-70.00%	0.4900	0.1470
	1.00			∑ = Variance:	**0.2940**
				Standard deviation = Square root of variance: σ =	**0.5422**
				Standard deviation expressed as a percentage: σ =	**54.22%**

   Columns 1, 2, and 3 are from the previous data of Mac Products, then in column 4, the deviation of the return was computed for each demand state from the expected return: $Actual\ return\ -$

$Expected\ return$ (10%). The computed deviations are squared in column 5. Each squared deviation is then multiplied by the relevant probability and is shown in column 6. The sum of the products in column 6 is the variance of the distribution. Finally, the square root of the variance is computed, which is the standard deviation, and is shown as a fraction and a percentage (Brigham & Houston, 2022).

The *standard deviation* measures how far the actual return is likely to deviate from the expected return. Mac Products' standard deviation is 54.22%, so its actual return is likely to be quite different from the expected 10% (Brigham & Houston, Fundamentals of Financial Management, 2022).

Economy which affects demand (1)	*Probability of this Demand Occurring (2)*	*Rate of Return if this demand occurs (3)*	*Deviation: Actual - 10% Expected Return (4)*	*Squared Deviation (5)*	*Squared Deviation x Probability (6)*
Strong	0.30	15%	5.00%	0.0025	0.0008
Normal	0.40	10%	0.00%	0.0000	0.0000
Weak	0.30	5%	-5.00%	0.0025	0.0008
	1.00			∑ = Variance:	**0.0015**
		Standard deviation = Square root of variance: σ =			**0.0387**
		Standard deviation expressed as a percentage: σ =			**3.87%**

Phil. Water's standard deviation is 3.87%, so its actual return should be much closer to the expected return of 10%.

The average publicly traded firm's σ has been 20% to 30% in recent years, so Mac Products is riskier than most stocks, and Phil. Water is less risky.

4. *Historical, or Past Realized, Rates of Return,* $\bar{r}$ *("r bar")*

The mean and standard deviation can be computed based on a subjective probability distribution. Assuming actual historical data was given, the deviation of returns could be shown:

Year (1)	*Return (2)*	*Deviation from Average (3)*	*Squared Deviation (4)*	
20x6	30.00%	19.75%	0.0390	
20x7	-10.00%	-20.25%	0.0410	
20x8	-19.00%	-29.25%	0.0856	
20x9	40.00%	29.75%	0.0885	
Average	10.25%		0.2541	Sum of Squared Deviations (SSDevs)
			0.0847	SSDevs/(N-1) = SSDevs/3
			29.10%	Standard Deviation = Square root of SSDevs/3: σ

Because past results are often repeated in the future, the historical σ is often used as an estimate of future risk. The average return for the past period (10.3%) may also be used to estimate future returns, but this is problematic because the average historical return varies widely depending on the period examined. In the illustration, if we went from 20x6 to 20x7, a different average from 10.3% would have resulted. The average historical return stabilizes with more years of data, but that brings into question whether data from many years ago are still relevant today (Brigham & Houston, 2022).

5. *Coefficient of Variation (CV)*

If a choice were made between two (2) investments with the same expected returns but different standard deviations, most people would choose the one with the lower standard deviation and, therefore, the lower risk. Similarly, given a choice between two (2) investments with the same risk (standard deviation) but different expected returns, investors would generally prefer the investment with the higher expected return. To most people, this is common sense — return is "good" and risk is "bad"; consequently, investors want as much return and as little risk as possible. Other measures of risk were used by analysts to choose between two (2) investments if one has the higher expected return. Still, the other has the lower standard deviation (Brigham & Houston, Fundamentals of Financial Management, 2022).

The coefficient of variation (CV) is the standard deviation divided by the expected return:

$$Coefficient\ of\ Variation = CV = \frac{\sigma}{\hat{r}}$$

The coefficient of variation shows the risk per unit of return, and it provides a more meaningful risk measure when the expected returns on two (2) alternatives are not the same. Because of Phil. Water and Mac Products have the same expected return; the variation coefficient is unnecessary in this case. In this example, the firm with the larger standard deviation, Mac Products, must also have a larger coefficient of variation. The coefficient of variation for Mac Products is $\frac{54.22}{10} = 5.42$, and the coefficient of variation for Phil. Water is $\frac{3.87}{10} = 0.39$. Thus, Mac Products is about 14 times riskier than Phil. Water based on this criterion (Brigham & Houston, Fundamentals of Financial Management, 2022).

6. *Sharpe Ratio*

Another alternative risk measure is the *Sharpe ratio*, developed by William Sharpe, the Nobel Prize-winning economist. The Sharpe ratio compares the asset's realized excess return to its standard deviation over a specified period:

$$Sharpe\ Ratio = \frac{(Return - Risk\ free\ rate)}{\sigma}$$

Depending on the circumstances, an analyst may calculate the Sharpe ratio using historical returns and standard deviation, or the calculations may be based on forward-looking estimates of expected returns. In either case, excess returns measure the amount that investment returns are above the risk-free rate, so investments with returns equal to the risk-free rate will have a zero Sharpe ratio. It follows that over a given time period, investments with higher Sharpe ratios performed better because they generated higher excess returns per unit of risk.

Illustration: Sharpe ratio is computed on a forward-looking basis for Phil. Water and Mac Products, and the risk-free rate is assumed 4%.

Phil. Water

$$Sharpe\ Ratio = \frac{(10\% - 4\%)}{3.87\%} = 1.55$$

Mac Products

$$Sharpe\ Ratio = \frac{(10\% - 4\%)}{54.22\%} = 0.11$$

Therefore, it will be seen on a risk-adjusted basis that Phil. Water is expected to perform better, because it has the same expected excess return as Mac Products but with considerably less risk (Brigham & Houston, Fundamentals of Financial Management, 2022).

7. *Risk Aversion and Required Returns*

Risk-averse investors dislike risk and require higher rates of return as an inducement to buy riskier securities.

Illustration: Suppose a person has inherited P10 million, which he plans to invest and then retire on the income. He can buy a 5% Treasury bill, which ensures he earns P500,000 in interest. Alternatively, he can buy stock in A&B Enterprises. If A&B's research programs are successful, the purchased stocks will increase to P21,000,000. However, if the research is a failure, the value of the stock will be zero, and he will be bankrupt. The person considered A&B's chances of success or failure as 50-50, so the expected value of the stock a year from now is 0.5(P0) + 0.5(P21,000,000) = P10,500,000. Subtracting the P10,000,000 cost leaves an expected P500,000 profit and a 5% rate of return, the same as for the Treasury bill:

$$Expected\ rate\ of\ return = \frac{Expected\ Ending\ Value - Cost}{Cost} = \frac{P10{,}500{,}000 - P10{,}000{,}000}{P10{,}000{,}000} = \mathbf{5}\%$$

Given the choice of the sure P500,000 profit and the risky expected P500,000 profit and 5% return, which one would the investor choose? *If the investor chooses the less risky investment, he is a risk-averse. Most investors are risk-averse; the average investor is about his or her "serious money."*

8. *Risk Premium (RP)* - This is the difference between the expected rate of return on a given risky asset and that on a less risky asset.

   For security prices and rates of return, risk aversion implies that when other things are held constant, the higher a security's risk, the higher its required return. If this situation does not hold, prices will change to meet the required condition.

   Illustration: Assuming Phil. Water and Mac Products sell each company's stock for P100 per share, each with an expected rate of return of 10%. Investors are averse to risk; thus, under those conditions, there would be a general preference for Phil. Water. People with money to invest would bid for Phil. Water and Mac Products' stockholders would want to sell and use the money to buy stocks from Phil. Water. Buying pressure would quickly drive Phil. Water's stock price to increase, and selling pressure would simultaneously cause Mac's price to fall.

   The prices would change the expected returns of the two (2) securities. Suppose, for example, that Phil. Water's stock price was bid up from P100 to P125, and Mac's stock price declined from P100 to P77. These price changes would cause Phil. Water's expected return falls to 8%, and Mac's return rises to 13%. The difference in returns, 13% - 8% = 5%, would be a **risk premium (RP)**, representing the additional compensation investors require for bearing Mac Products' higher risk (Brigham & Houston, 2022).

Capital Asset Pricing Model (CAPM)

Developed in the 1960s, *Capital Asset Pricing Model (CAPM)* is a model based on the proposition that any stock's required rate of return is equal to the risk-free rate of return plus a risk premium that reflects only the risk remaining after diversification. The intuition of CAPM will be used to explain how risk should be considered in a world where stocks and other assets are held in portfolios.

The risk of a stock in a portfolio is typically lower than its risk when it is held alone. Because investors dislike risk and risk can be reduced by holding portfolios, most stocks are held in portfolios. Banks, pension funds, insurance companies, mutual funds, and other financial institutions must hold diversified portfolios. Most individual investors—at least those whose security holdings constitute a significant part of their total wealth—also hold portfolios. Therefore, the fact that one particular stock's price increases or decreases are not significant—what is important is the return on the portfolio and the portfolio's risk. Logically, the risk and return of an individual stock should be analyzed in terms of how the security affects the risk and return of the portfolio in which it is held (Brigham & Houston, Fundamentals of Financial Management, 2022).

Illustration: Pay Down, Inc. is a collection agency that operates nationwide through 37 offices. The company is not well known; its stock is not very liquid, and its earnings have experienced sharp fluctuations in the past. It suggests that Pay Down is risky and that its required rate of return, r, should be relatively high. However, Pay

Down's required return in 2x19 (and all other years) was quite low compared to most other companies. It indicates that investors think Pay Down is a low-risk company despite its uncertain profits.

Analysis: This counterintuitive finding concerns diversification and its effect on risk. Pay Down's earnings rise during recessions, whereas most other companies' earnings decline when the economy slumps. Thus, Pay Down's stock is like insurance; it pays off when other investments go wrong, so adding Pay Down to a portfolio of "regular" stocks stabilizes the portfolio's returns and makes it less risky (Brigham & Houston, 2022).

Expected Portfolio Returns, $\hat{r}_p$

The expected return on a portfolio $(\hat{\boldsymbol{r}}_{\boldsymbol{p}})$ is the weighted average of the expected returns of the individual assets in the portfolio, with the weights being the percentage of the total portfolio invested in each asset:

$$\hat{\boldsymbol{r}}_{\boldsymbol{p}} = w_1\hat{r}_1 + w_2\hat{r}_2 + \cdots + w_N\hat{r}_N \ \ OR \ \ \hat{r}_p = \sum_{i=1}^{N} w_i\hat{r}_i$$

Here, $\hat{r}_i$ is the expected return on the ith stock; the w_i's are the stocks' weights or the percentage of the total value of the portfolio invested in each stock; and N is the number of stocks in the portfolio (Brigham & Houston, Fundamentals of Financial Management, 2022).

Illustration: Assume the following data:

Stock (1)	*Expected Return (2)*	*Amount Invested (3)*	*Percent of Total (w_i) (4)*	*Product: (2) x (4) (5)*	
Microsoft	7.75%	25,000,000.00	25%	1.94%	
IBM	7.25%	25,000,000.00	25%	1.81%	
GE	8.75%	25,000,000.00	25%	2.19%	
Exxon Mobil	7.75%	25,000,000.00	25%	1.94%	
	7.88%	100,000,000.00	100%	**7.88%**	= Expected return on a portfolio

The estimated returns on the four (4) stocks are shown in column 1 for the coming year, in column 2. Suppose an investor had 100,000,000 and planned to invest 25,000,000, or 25% of the total, in each stock. The investor could multiply each stock's percentage weight, as shown in column 4, by its expected return and obtain the product terms in column 5. Then in column 5, the expected portfolio return was computed by totaling each product resulting in 7.88%.

If you added a fifth stock with a higher expected return, the portfolio's expected return would increase, and vice versa if you added a stock with a lower expected return.

The key point to remember is that the expected return on a portfolio is a weighted average of expected returns on the stocks in the portfolio.

Additional points to be considered (Brigham & Houston, 2022):

a. The expected returns in column 2 would be based on a study of some type. However, they would still be subjective and judgmental because different analysts could look at the same data and reach different conclusions. Therefore, this type of analysis must be viewed with a critical eye. Nevertheless, it is useful if one is to make intelligent investment decisions.

b. If other companies were added, which are relatively risky, the expected returns, as estimated by the marginal investor, would be relatively high; otherwise, investors would sell them, drive down their prices, and force the expected returns above the returns on safer stocks.

c. After the fact and a year later, the actual **realized rates of return**, $\bar{r}_i$ on the individual stocks, the $\bar{r}_i$ “r-bar” values would almost certainly be different from the initially expected values. That would cause the portfolio’s actual return, $\bar{r}_p$, to differ from the expected return, $\hat{r}_p$=7.875%.

Portfolio Risk

Although the expected return on a portfolio is simply the weighted average of the expected returns on its individual stocks, the portfolio’s risk, σ_P, is not the weighted average of the individual stocks’ standard deviations. The portfolio’s risk is generally smaller than the average of the stocks’ σs because diversification lowers the portfolio’s risk.

Illustration:

	A	B	C	D	E
1	Year	Stock W	Stock M	Portfolio WM	
2	2x15	40%	-10%	15%	
3	2x16	-10%	40%	15%	
4	2x17	40%	-10%	15%	
5	2x18	-10%	40%	15%	
6	2x19	15%	15%	15%	
7	Average Return	15.00%	15.00%	15.00%	=AVERAGE(D2:D6)
8	σ	25.00%	25.00%	0.00%	=STDEV.S(D2:D6)
9			Correlation Coefficient	-1.00	=CORREL(B2:B6,C2:C6)
10					

Consider the data on Stocks W and M, individually and on a portfolio with 25% in each stock. The table shows that the returns on individual stocks vary widely from year to year. Therefore, individual stocks are risky. However, the portfolio’s returns are constant at 15%, indicating that it is not risky. The two (2) stocks would be quite risky if they were held in isolation, but when combined to form Portfolio WM, they have no risk. If an investor invests all his money in Stock W, he would have an expected return of 15%, but he would face a great deal of risk. The same thing would hold if he invested entirely in Stock M. However, if he invested 50% in each stock, he would have the same expected return of 15%, but with no risk whatsoever. Being rational and averse to risk, he and all other rational investors would choose to hold the portfolio and not the stocks individually (Brigham & Houston, Fundamentals of Financial Management, 2022).

Stocks W and M can be combined to form a riskless portfolio because their returns move countercyclically to each other; when W’s stocks fall, M’s stocks rise, and vice versa. The tendency of two (2) variables to move together is called **correlation**, and the **correlation coefficient, $\boldsymbol{\rho}$** (pronounced “rho”), measures this tendency. In statistical terms, the returns on Stocks W and M are said to be perfectly negatively correlated, with $\boldsymbol{\rho = -1.0}$. The opposite of perfect negative correlation is a perfect positive correlation, with $\boldsymbol{\rho = +1.0}$. If returns are unrelated, they are considered independent and $\boldsymbol{\rho = 0}$ (Brigham & Houston, Fundamentals of Financial Management, 2022).

The returns on two (2) perfectly positively correlated stocks with the same expected return would move up and down together, and a portfolio consisting of these stocks would be exactly as risky as the individual stocks (Brigham & Houston, Fundamentals of Financial Management, 2022).

The Beta Coefficient

When a stock is held by itself, its risk can be measured by the standard deviation of its expected returns. However, σ is not appropriate when the stock is held in a portfolio, as stocks generally are.

Relevant Risk – This is the risk that remains once the stock is in a diversified portfolio and contributes to the portfolio’s market risk. It is measured by the extent to which the stock moves up or down with the market.

- *Diversified Risk* is that part of a security's risk associated with random events; it can be eliminated by proper diversification. This risk is also known as company-specific or unsystematic risk.
- *Market Risk* is the risk that remains in a portfolio after diversification has eliminated all company-specific risk. This risk is also known as *nondiversifiable* or *systematic, or beta risk.*

The tendency of a stock to move with the market is measured by its **beta coefficient**, **b**. Analysts often use historical data and assume that the stock's historical beta will give them a reasonable estimate of how the stock will move relative to the market in the future.

Illustration: Using historical data, consider *Figure 2*, which shows historical returns on three (3) stocks and a market index.

In Year 1, "the market," as defined by a portfolio containing all stocks, had a total return (dividend yield plus capital gains yield) of 10%, as did the three (3) individual stocks. In Year 2, the market went up sharply, and its return was 20%. Stock H (for high) soared to 30%; A (for average) returned 20%, the same as the market; and L (for low) returned 15%. In Year 3, the market dropped sharply; its return was -10%. The three (3) stocks' returns also fell, H's return was -30%, A's was -10%, and L broke even with a 0% return. In Years 4 and 5, the market returned 0% and 5%, respectively, and the three (3) stocks' returns were as shown in *Figure 2*.

Year	r_M	r_H	r_A	r_L
1	10.0%	10.0%	10.0%	10.0%
2	20.0%	30.0%	20.0%	15.0%
3	-10.0%	-30.0%	-10.0%	0.0%
4	0.0%	-10.0%	0.0%	5.0%
5	5.0%	0.0%	5.0%	7.5%

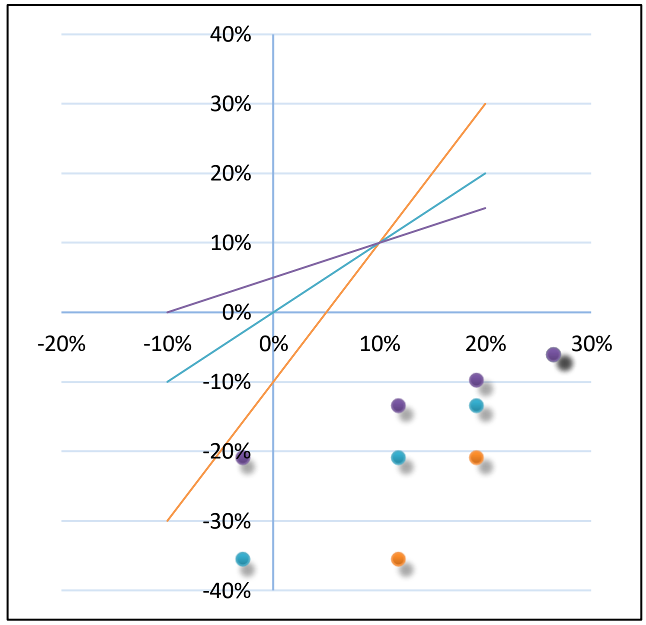

Figure 2. Betas: Relative Volatility of Stocks H, A, and L
Source: Fundamentals of Financial Management, 2019, p. 277

Computing *beta*:

1. *Rise-Over-Run.* Divided the vertical axis change that results from a given change on the horizontal axis, i.e., the change in the stock's return divided by the change in the market return. For Stock H, when the market rises from -10% to +20%, or by 30%, the stock's return goes from -30% to +30%, or by 60%. Thus, beta H by the rise-over-run method is 60/30 = 2.0. Similarly, beta A will be 1.0, and beta L will be 0.5. This procedure is easy in the illustration because all the points lie on a straight line, but if the points were scattered around the trend line, computing an exact beta is impossible using this manner.

2. *Excel*. The slope function of Excel can be used in calculating betas. Here are the functions for the three (3) illustrative stocks:

$Beta_H$	2.0	=SLOPE(C2:C6,B2:B6)
$Beta_A$	1.0	=SLOPE(D2:D6,B2:B6)
$Beta_L$	0.5	=SLOPE(E2:E6,B2:B6)

A plot of the data shows that the three (3) stocks moved up or down with the market but that H was twice as volatile as the market, A was exactly as volatile as the market, and L had only half the market's volatility. It is apparent that the steeper the line, the greater the stock's volatility and, thus, the larger its loss in a down market. The slopes of the lines are the stocks' beta coefficients. It is shown in *Figure 2* that the slope coefficient for H is 2.0; for A, it is 1.0; and for L, it is 0.5. Thus, beta measures a given stock's volatility relative to the market, and an *average stock's beta*, b_A=1.0 (Brigham & Houston, 2022).

Stock A is defined as an **average-risk stock** because it has a beta of b=1.0 and thus moves up and down in step with the general market. Thus, an average stock will generally move up by 10% when the market moves up by 10% and fall by 10% when the market falls by 10%. A large portfolio of such b=1.0 stocks would (1) have all its diversifiable risk removed but (2) still move up and down with the broad market averages and thus have a degree of risk.

Stock H, which has b=2.0, is twice as volatile as an average stock, which means that it is twice as risky. The value of a portfolio consisting of b=2.0 stocks could double—or halve—in a short time. If holding such a portfolio, an investor could quickly go from being a millionaire to being bankrupt.

On the other hand, with b=0.5, Stock L is only half as volatile as the average stock, and a portfolio of such stocks would rise and fall only half as rapidly as the market. Thus, its risk would be half that of an average-risk portfolio with b=1.0.

Betas for thousands of companies are calculated and published by Yahoo!, Google, and numerous other organizations. Most stocks have betas in the range of 0.50 to 1.50, and the average beta for all stocks is 1.0, which indicates that the average stock moves in sync with the market. If a stock whose beta is greater than 1.0 is added to a b_p=1.0 portfolio, the portfolio's beta and, consequently, its risk will increase. Conversely, if a stock whose beta is less than 1.0 is added to a b_p=1.0 portfolio, the portfolio's beta and risk will decline. Thus, because a stock's beta reflects its contribution to the riskiness of a portfolio, beta is the theoretically correct measure of the stock's riskiness (Brigham & Houston, Fundamentals of Financial Management, 2022).

The Relationship Between Risk and Rates of Return

The former section demonstrated that beta is the most appropriate measure of a stock's relevant risk under the CAPM theory. The next question is this: For a given level of risk as measured by beta, what rate of return is required to compensate investors for bearing that risk?

- $\hat{r}_i$ — Expected rate of return on the ith stock
- r_i — Required rate of return on the ith stock. Note that if $\hat{r}_i$ is less than r_i, the typical investor will not purchase this stock or will sell it if he or she owns it. If $\hat{r}_i$ is greater than r_i, the investor will purchase the stock because it looks like a bargain. Investors will be indifferent if $\hat{r}_i = r_i$. Buying and selling by investors tend to force the expected return to equal the required return, although the two (2) can differ from time to time before the adjustment is completed.
- $\bar{r}_i$ — Realized, after-the-fact return. A person does not know $\bar{r}_i$ at the time, he/she is considering the purchase of a stock.
- r_{RF} — Risk-free rate of return. In this context, r_{RF} is generally measured by the return on U.S. Treasury securities. Some analysts recommend that short-term T-bills be used; others recommend long-term T-bonds. T-bonds were usually used because their maturity is closer to the average investor's holding period for stocks.
- b_i — Beta coefficient of the ith stock. The beta of an average stock is $b_A = 1.0$.
- r_M — Required rate of return on a portfolio consisting of all stocks is called the *market portfolio*. r_M is also the required rate of return on an average $(b_A = 1.0)$ stock.
- RP_M — $(r_M - r_{RF})$ = risk premium on "the market" and the premium on an average stock. This is the additional return over the risk-free rate required to compensate an average investor for assuming an average amount of risk. Average risk means a stock, where $b_i = b_A = 1.0$.
- RP_i — $(r_M - r_{RF})b_i = (RP_M)b_i$ = risk premium on the ith stock. A stock's risk premium will be less than, equal to, or greater than the premium on an average stock, RP_M, depending on whether its beta is less than, equal to, or greater than 1.0. if $b_i = b_A = 1.0$, then $RP_i = RP_M$.

Market Risk Premium, RP_M, shows the premium investors require for bearing the risk of an average stock. The size of this premium depends on how risky investors think the stock market is and their degree of risk aversion (Brigham & Houston, 2022).

Illustration: Assuming that at the current time, Treasury bonds yield $r_{RF} = 3\%$ and an average share of stock has a required rate of return of $r_M = 8\%$. Therefore, the market risk premium is 5%, computed as follows:

$$RP_M = r_M - r_{RF} = 8\% - 3\% = 5\%$$

It should be noted that the risk premium of an average stock, $r_M - r_{RF}$, is hard to measure because it is impossible to obtain a precise estimate of the expected future return of the market, r_M. Given the difficulty of estimating future market returns, analysts often look to historical data to estimate the market risk premium. Historical data suggest that the market risk premium varies yearly due to changes in investors' risk aversion but has generally ranged from 4% to 8% (Brigham & Houston, 2022).

Although historical estimates might be a good starting point for estimating the market risk premium, those estimates would be misleading if investors' attitudes toward risk changed considerably over time (Brigham & Houston, Fundamentals of Financial Management, 2022).

The risk premium on individual stocks varies systematically from the market risk premium. For example, if one stock is twice as risky as another, measured by its beta coefficients, its risk premium should be twice as high. Therefore, if the market risk premium, RP_M, and the stock's beta, b_i, is known the risk premium as the products $(RP_M)b_i$ can be found.

Illustration: If beta for Stock L = 0.5 and RP_M=5%, RP_L will be 2.5%. The computation follows:

$$Risk\ premium\ for\ Stock\ L = RP_L = (RP_M)b_L = (5\%)(0.5) = \mathbf{2.5}\%$$

The required return for any stock can be computed with the following formula:

$$Required\ return\ on\ a\ stock = Risk\ free\ return + Premium\ for\ the\ stock's\ risk$$

Here, the risk-free return includes a premium for expected inflation; if the stocks under consideration have similar maturities and liquidity, the required return on Stock L can be found using the **Security Market Line** (**SML**) equation:

$$Required\ return\ on\ Stock\ L = Risk\ free\ return + (Market\ risk\ premium)(Stock\ L's\ beta)$$

$$r_L = r_{RF} + (r_M - r_{RF})b_L = r_{RF} + (RP_M)b_L = 3\% + (8\% - 3\%)(0.5) = 3\% + 2.5\% = \mathbf{5.5}\%$$

Stock H had $b_H = 2.0$, so its required rate of return is 13%:

$$r_H = 3\% + (8\% - 3\%)(2.0) = 3\% + 10\% = \mathbf{13}\%$$

An average stock, with $b = 1.0$, would have a required return of 8%, the same as the market return:

$$r_A = 3\% + (8\% - 3\%)(1.0) = 3\% + 5\% = \mathbf{8}\% = \boldsymbol{r_M}$$

References:

Brigham, E. F., & Houston, J. F. (2022). *Fundamentals of Financial Management.* Cengage Learning, Inc.
Emerald Insight. (2022). Retrieved from Risk: https://www.emerald.com/insight/search?q=risk&showAll=true