Microclimate and species composition shape the contribution of fuel moisture to positive fire-vegetation feedbacks

Supplementary information 2: models specification

Vapour pressure deficit model

To compare the VPD between communities within each transect we fitted a hierarchical model assuming a Log-Normal distribution of the daily VPD time-series and considering the temporal correlation between observations using a corAR1 correlation function. We included fixed effects of the community on the mean (μ) and the standard deviation (σ) at the log scale, and a site random effect on μ.

$$\log(\mathbf{y}_{cs}) \sim \text{MVN}(\boldsymbol{\mu}_{cs}, \boldsymbol{\Sigma}_c)$$
$$\Sigma_{c,ij} = \sigma_c^2 \ \rho^{\mathrm{k}_{ij}}$$

$$\mu_{cs} = \eta_c + \delta_s$$
$$\eta_c \sim \text{Normal}(0, 5)$$
$$\delta_s \sim \text{Normal}(0, \tau)$$
$$\tau \sim \text{Half-Normal}(1)$$

$$\sigma_c \sim \text{Half-Normal}(1.5)$$
$$\rho \sim \ \text{Uniform}(0, 1)$$

$\mathbf{y}_{cs}$ is the vector of length $N = 119$ maximum VPD daily values (between 2019-11-18 and 2020-03-15) at community $c \in \{1, ..., 4\}$ and site $s \in \{1, ..., 10\}$, k_{ij} is the temporal distance (in days) between observations i and j, with i and $j \in \{1, ..., N\}$,
$\boldsymbol{\mu}_{cs}$ is a vector repeating μ_{cs} N times, which is the median VPD at community c and site s in the log scale,
σ_{cs} is the standard deviation of $\log(\mathbf{y}_{cs})$,
ρ_{cs} is the temporal correlation coefficient between consecutive observations at the log scale,
and MVN stands for Multivariate Normal.
η are fixed effects for μ, which vary between communities, while δ are site-level random effects.

Normal distributions are parameterized with mean and standard deviation. The Half-Normal distribution is parameterized only with the standard deviation, with implicit mean equal to zero. As the response is assumed to follow a Log-Normal distribution, its mean is computed as

$$\text{E}(\mathbf{y}_{cs}) = \exp\left(\mu_{cs} + \frac{\sigma_{cs}^2}{2}\right),$$

while

$$\exp(\mu_{cs}) = \text{median}(\mathbf{y}_{cs}).$$

We ran 10 chains for 2000 iterations, leaving 1000 for warmup. Considering all parameters and derived quantities computed in Stan (e.g., means), the maximum $\hat{\text{R}}$ value was 1.003, indicating convergence, and the smallest effective sample size was 2408.

Fuel moisture models

As we were interested in the variation of moisture content between communities mediated through microclimate, we fitted a Generalized Linear Mixed Model (GLMM) for every fuel type, specifying a regression of moisture content as a function of VPD at the site level, using the average maximum daily VPD of each site as predictor. We modeled the mean of the response with with a log link, including random effects of site (combinations of community and transect), date, and their combination. We initially tried to fit a temporal correlation structure for the date random effects, but with only nine dates we could not estimate the correlation parameter, so we assumed independent effects. We did not include the community factor in the models because we hypothesized that the largest community effect on moisture content would be mediated by the microclimate and because community and VPD are highly correlated, which would have hindered the estimation of the VPD effect and the interpretation of results.
For live fuels we assumed a Gamma distribution of the response, as its asymmetry allowed a good fit to the data. As dead fuels showed some extreme moisture content values close to zero, a Gamma distribution needed to fit a high variance to accommodate these low values, implying also a high density in large moisture content values that were not compatible with the data. This led us to prefer Normal distributions truncated at 0 for dead fuel moisture content, which allow a higher density at values near zero without fitting an extreme variance.

We fully specify the model for the live fuel mixture, which was the most complex, and mention the differences for other fuel types below.

$$
\begin{aligned}
\mathrm{y}_{dsp} &\sim \mathrm{Gamma}(\alpha_{ds}, \beta_{dsp}) \\
\alpha_{ds} &= \frac{1}{\phi_{ds}} \\
\beta_{dsp} &= \frac{1}{\mu_{dsp}\ \phi_{ds}}.
\end{aligned}
$$

Here, y_{dsp} is the moisture content at date $d \in \{1, ..., 9\}$, site $s \in \{1, ..., 10\}$ and sampling plot $p \in \{1, ..., 50\}$ (there were 5 sampling plots by site). α and β are the shape and rate parameters, where $\mu = \mathrm{E(y)}$, and ϕ is a dispersion parameter. We modeled both μ and ϕ with a log link function.

Model for μ:

$$
\begin{aligned}
&\log(\mu_{dsp}) = \eta_\mu + \lambda\ \mathrm{VPD}_s + \varepsilon_{\mu\mathrm{S},s} + \varepsilon_{\mu\mathrm{DS},ds} + \varepsilon_{\mu\mathrm{P},p} \\
&\\
&\eta_\mu \sim \mathrm{Normal}(\log(\text{prior mean}_{\eta\mu}), 1) \\
&\lambda \sim \mathrm{Normal}(0, 1) \\
&\\
&\varepsilon_{\mu\mathrm{S},s} \sim \mathrm{Normal}(0, \sigma_{\mu\mathrm{S}}) \\
&\sigma_{\mu\mathrm{S}} \sim \text{Half-Normal}(1) \\
&\\
&\varepsilon_{\mu\mathrm{DS},ds} \sim \mathrm{Normal}(0, \sigma_{\mu\mathrm{DS,s}}) \\
&\sigma_{\mu\mathrm{DS,s}} \sim \text{Half-Normal}(0.7)
\end{aligned}
$$

The suffix $_\mu$ states that these parameters model μ and not ϕ. Lower-case subscripts (d, s and p) are variable, while upper-case subscripts (S, DS and P) are used to name the random effects and standard deviations corresponding to site, date × site, and plot, respectively. $\varepsilon_{\mu\mathrm{DS},ds}$ is a random effect for the date within each site, with every site having its own among-dates standard deviation: $\sigma_{\mu\mathrm{DS},s}$.
VPD_s is the average maximum daily VPD in each site. The value of prior $\text{mean}_{\eta\mu}$ varied between fuel types (see below).

The model for ϕ was similar, only excluding the VPD effect and estimating only one $\sigma_{\phi\mathrm{DS}}$ (this parameter varied by site only in the litter model):

$$\log(\phi_{dsp}) = \eta_\phi + \varepsilon_{\phi\mathrm{S},s} + \varepsilon_{\phi\mathrm{DS},ds}$$

$$\eta_\phi \sim \mathrm{Normal}(\log(\text{prior mean}_{\eta\phi}), 1)$$

$$\varepsilon_{\phi\mathrm{S},s} \sim \mathrm{Normal}(0, \sigma_{\phi\mathrm{S}})$$
$$\sigma_{\phi\mathrm{S}} \sim \text{Half-Normal}(1)$$

$$\varepsilon_{\phi\mathrm{DS},ds} \sim \mathrm{Normal}(0, \sigma_{\phi\mathrm{DS}})$$
$$\sigma_{\phi\mathrm{DS}} \sim \text{Half-Normal}(1)$$

The random effect for plot was only included for the live fuel mixture because the dead fuel models did not converge when this term was added, probably because the plot effect was too small for dead fuels, as residuals analysis showed. For single-species live fuels we did not record the sampled individuals, so we did not have a "sampling plot" variable.

In the case of dead fuels, the likelihood was a Truncated Normal distribution with lower limit "a" and standard deviation ϕ:

$$\mathrm{y}_{dsi} \sim \text{Truncated Normal}(\mu_{ds}, \phi_{ds}, \mathrm{a} = 0),$$

and models for $\log(\mu)$ and $\log(\phi)$ were specified as for the live fuel mixture. (Here there is no sampling plot, but there are around 5 observations by date and site, represented by the i subscript).

For dead fuels and *Schinus patagonicus* we noticed that the VPD showed a slightly non-(log)linear effect, so we modelled it with a thin plate spline with 2 basis functions (excluding the intercept). This set up allows to fit functions with quadratic shape that are smoother than a polynomial function (including VPD^2 in the model). For these fuel types the term $\lambda\ \mathrm{VPD}_s$ would then be replaced by $\mathbf{X}_s.\ \boldsymbol{\lambda}$, where $\boldsymbol{\lambda}$ is a vector of length 2, and x_{sb} is the the spline basis $b \in \{1, 2\}$ evaluated at VPD_s. The matrix $\mathbf{X}$ has dimension 10 (sites) $\times$ 2 (basis functions). We constructed the thin plate spline basis functions with the R package `mgcv` (Wood, 2017), and as they were parameterized to have a diagonal penalty matrix, we specified the following prior on its coefficients: $\lambda_b \sim \mathrm{Normal}(0, 2.5)$.

The prior means defined for the intercepts of different fuel types were the following:

Fuel type	prior $\text{mean}_{\eta\mu}$	prior $\text{mean}_{\eta\phi}$
1 h fuel sticks	50	15
10 h fuel sticks	50	15
Laura	150	0.05
Coihue	150	0.05
Caña	100	0.05
Litter	50	15
Live fuel mixture	150	0.05

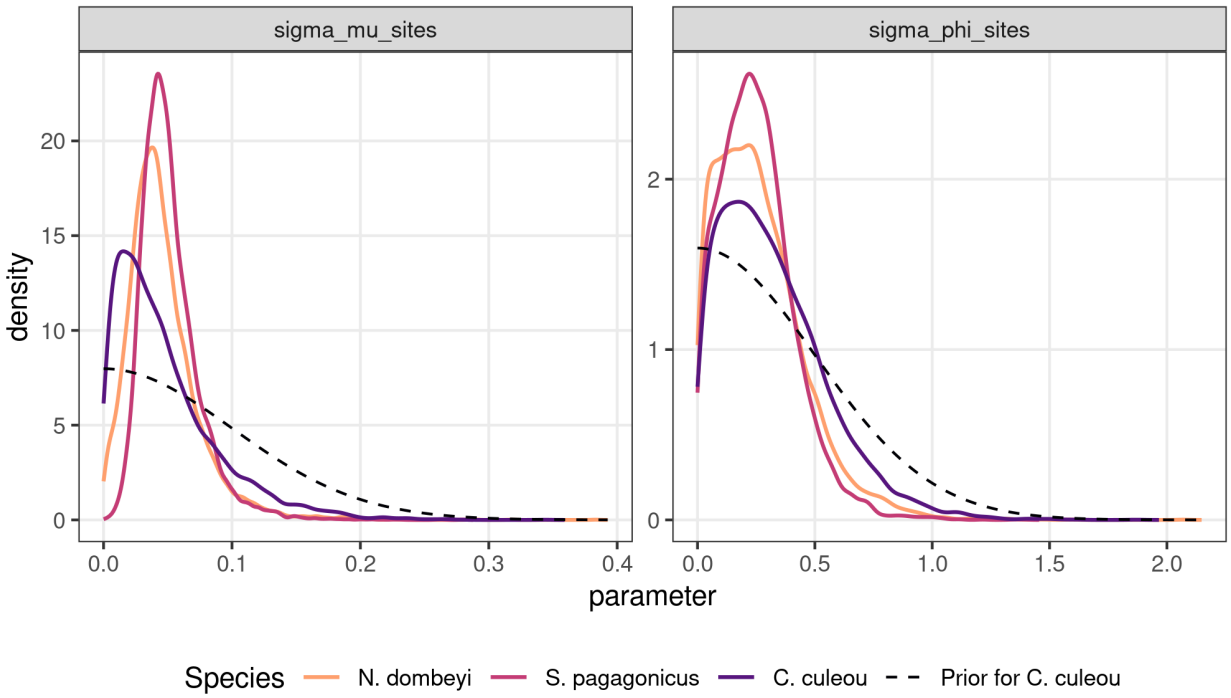
The prior distributions were chosen using simulations and keeping in mind the moisture content values reported by Bianchi & Defossé (2014), Bianchi et al. (2018), Blackhall et al. (2012) and Blackhall et al. (2019). We chose prior distributions that were wider than what previous data suggested while keeping the response values in reasonable ranges. prior $\text{mean}_{\eta\phi}$ takes smaller values for live fuels because these are dispersion parameters for Gamma distributions (see equations above), while in the case of dead fuels prior $\text{mean}_{\eta\phi}$ controls the prior for the standard deviation of a Truncated Normal distribution.

In the model for *Chusquea culeou* the parameters $\sigma_{\mu\mathrm{S}}$ and $\sigma_{\phi\mathrm{S}}$ were hard to estimate because the species was found only at four sites. With the relatively wide priors defined above the posterior copied the priors and the

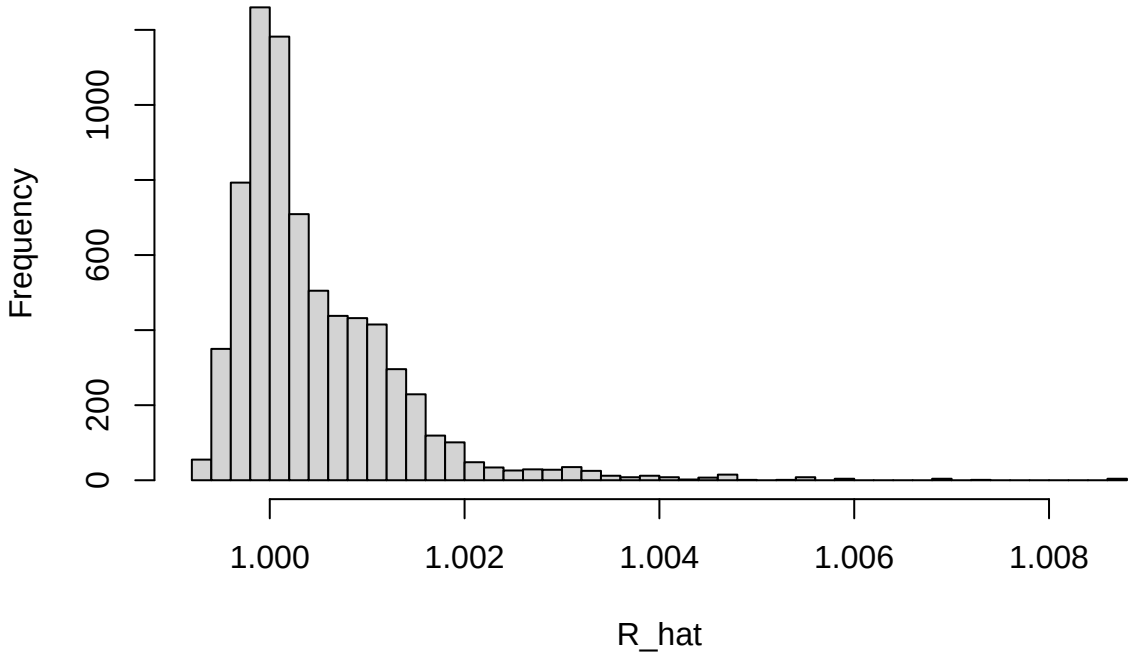
chains showed many divergent transitions. To overcome this issue, we informed these parameters' priors based on the posteriors estimated for *Schinus patagonicus* and *Nothofagus dombeyi*, defining the following priors:

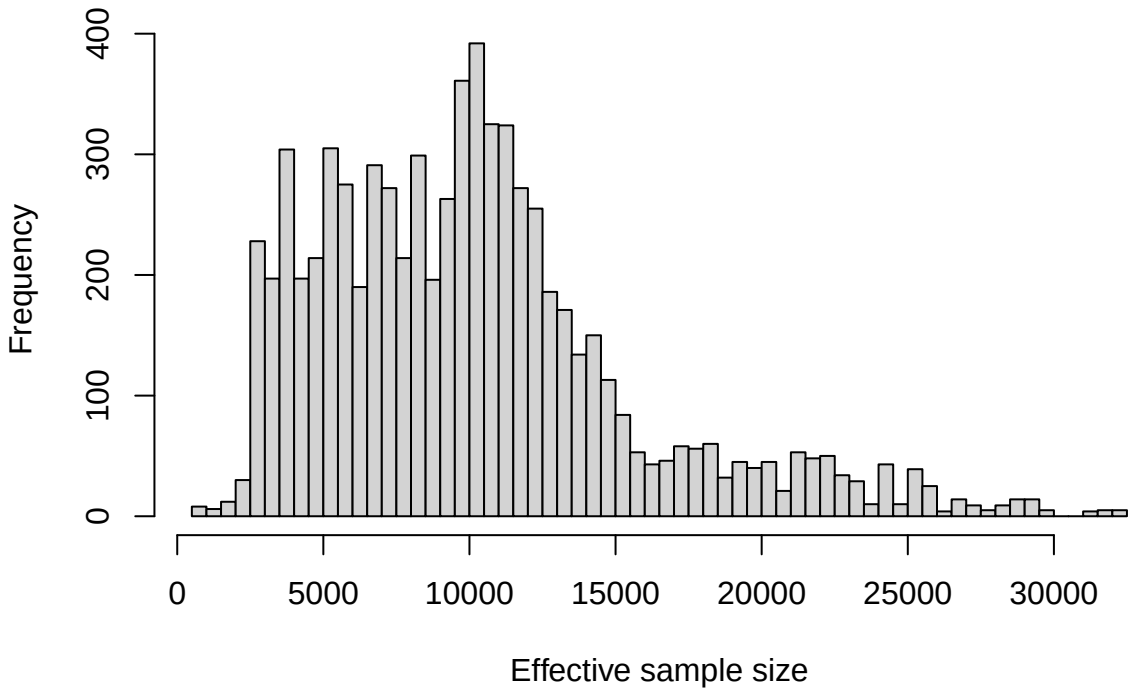
$$\sigma_{\mu S} \sim \text{Half-Normal}(0.1)$$
$$\sigma_{\phi S} \sim \text{Half-Normal}(0.5)$$

The posterior densities for these paramters and the prior used for *C. culeou* are shown in the following figure.



For all fuel moisture models we ran 10 chains for 1000 iterations after warm-up (1500 for live fuel mixture, 2500 for 1h fuel sticks), and the number of warm-up iterations per chain varied by fuel type: 12000 for *C. culeou*, 1500 for live fuel mixture and 1000 for the remaining fuel types. The longer warm-up for some models was needed to avoid divergent transitions during the sampling phase, and the larger number of samples in some fuel types was needed to reach a high effective sample size. Across all fuel moisture models and parameters (including derived quantities computed in Stan), the maximum $\hat{R}$ value was 1.0086, indicating convergence, and the smallest effective sample size was 826.7. The following figures show their distributions:


Frequency
0
200
600
1000
1.000
1.002
1.004
1.006
1.008
R_hat



Scripts and data to fit these models are available at
https://github.com/barberaivan/fmc_alternative_states.git

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