**UNIVERSIDADE FEDERAL DA GRANDE DOURADOS**
**Cálculo Diferencial e Integral III — Avaliação P2**
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Engenharia Mecânica 13/06/2019

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| Nota | |

**Aluno(a):**.................................................................................................

**Todas as respostas devem ser justificadas.**

1. Calcule a integral dupla $\iint_D \operatorname{sen}(x^2)\, dA$, onde $D$ é a região da figura 1.

2. Calcule a integral dupla $\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} \operatorname{sen}(x^2+y^2)\, dy\, dx$.

3. Sabendo que $\iiint_E dV$ calcula o volume do sólido $E$, calcule o volume do sólido na figura 2.

4. Calcule o trabalho realizado pelo campo $F(x,y,z) = (yz, xz, xy+2z)$ ao mover uma partícula de $(0,0,0)$ a $(2,1,0)$.

5. Calcule a integral de linha $W = \int_C F \cdot\ dr$, onde $F(x,y) = (x(x+y), xy)$ e $C$ é a curva composta pelos segmentos de reta de $(0,0)$ a $(1,0)$, de $(1,0)$ a $(0,1)$ e de $(0,1)$ a $(0,0)$.

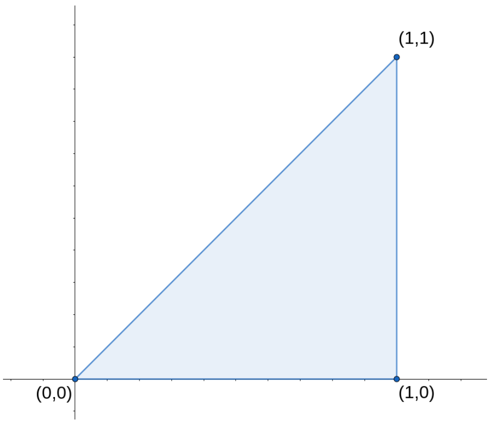



Figura 1: Exercício 1

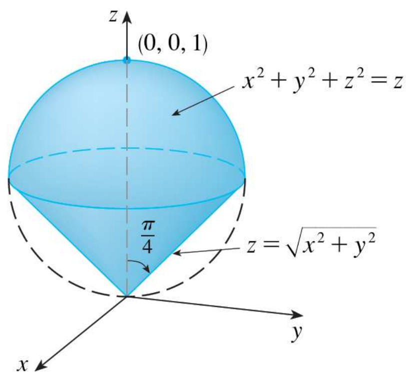
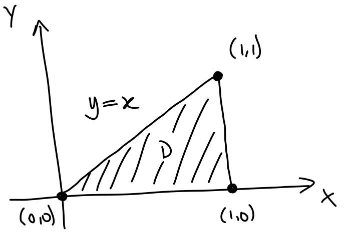



Figura 2: Exercício 3

*Boa Prova!*

① 


$D = \{(x,y) \mid 0 \leq x \leq 1,\ 0 \leq y \leq x\}$

$$\iint_D \operatorname{sen} x^2 \, dA = \int_0^1 \int_0^x \operatorname{sen} x^2 \, dy\, dx$$

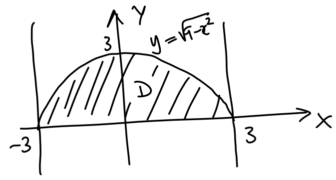
$$= \int_0^1 \operatorname{sen} x^2 \cdot \left( y\Big|_0^x \right) dx = \int_0^1 x \cdot \operatorname{sen} x^2 \, dx \overset{(u=x^2)}{=} \frac{1}{2} \int_0^1 \operatorname{sen} u \, du = \frac{1}{2}\left(-\cos u \Big|_0^1\right)$$

$$= \frac{1}{2}\left(-\cos 1 + \cos 0\right) = \frac{1 - \cos 1}{2}.$$

② A região de integração é tal que:

$-3 \leq x \leq 3$

$0 \leq y \leq \sqrt{9-x^2}$




Em coordenadas polares: $D = \{(r,\theta) \mid 0 \leq r \leq 3,\ 0 \leq \theta \leq \pi\}$

Assim,

$$\int_{-3}^{3} \int_0^{\sqrt{9-x^2}} \operatorname{sen}(x^2+y^2)\, dy\, dx = \int_0^{\pi} \int_0^{3} \operatorname{sen}(r^2) \cdot r\, dr\, d\theta = \int_0^{\pi} d\theta \cdot \int_0^3 r \operatorname{sen} r^2 \, dr$$

$$\overset{(u=r^2)}{=} \left( \theta \Big|_0^{\pi} \right) \cdot \left( \frac{1}{2} \int_0^9 \operatorname{sen} u \, du \right) = (\pi - 0) \cdot \frac{1}{2} \left( -\cos u \Big|_0^9 \right) = \frac{\pi}{2}(-\cos 9 + \cos 0) = \frac{\pi}{2}(1 - \cos 9)$$

③ Em coordenadas esféricas:

$x^2 + y^2 + z^2 = z \Rightarrow \rho^2 = \rho \cos\phi \Rightarrow \rho = \cos\phi$

$z = \sqrt{x^2+y^2} \Rightarrow \phi = \frac{\pi}{4}$ (indicado na figura)

$\therefore\ E = \{(\rho, \theta, \phi) \mid 0 \leq \rho \leq \cos\phi,\ 0 \leq \theta \leq 2\pi,\ 0 \leq \phi \leq \frac{\pi}{4}\}$

Logo,

$$\text{Volume}(E) = \iiint_E dV = \int_0^{\frac{\pi}{4}} \int_0^{2\pi} \int_0^{\cos\phi} \rho^2 \operatorname{sen}\phi \, d\rho\, d\theta\, d\phi$$

$$= \int_0^{2\pi} d\theta \cdot \int_0^{\frac{\pi}{4}} \int_0^{\cos\phi} \rho^2 \operatorname{sen}\phi \, d\rho\, d\phi = 2\pi \cdot \int_0^{\frac{\pi}{4}} \operatorname{sen}\phi \left( \frac{\rho^3}{3} \Big|_0^{\cos\phi} \right) d\phi = \frac{2\pi}{3} \int_0^{\frac{\pi}{4}} \operatorname{sen}\phi \cdot \cos^3\phi \, d\phi$$

$$\overset{(u=\cos\phi)}{=} -\frac{2}{3}\pi \int_1^{\frac{\sqrt{2}}{2}} u^3 \, du = \frac{2}{3}\pi \left( \frac{u^4}{4} \Big|_{\frac{\sqrt{2}}{2}}^{1} \right) = \frac{2}{3}\pi \cdot \frac{1}{4}\left[ 1^4 - \left(\frac{\sqrt{2}}{2}\right)^4 \right] = \frac{\pi}{6}\left[ 1 - \frac{4}{16} \right] = \frac{\pi}{8}$$

④ O campo $F$ é conservativo. De fato, se $f$ é tal que $\nabla f = F$:

$$\begin{cases} \dfrac{\partial f}{\partial x} = yz & (1) \\ \dfrac{\partial f}{\partial y} = xz & (2) \\ \dfrac{\partial f}{\partial z} = xy + 2z & (3) \end{cases}$$

De (1):

$$\int \frac{\partial f}{\partial x}\, dx = \int yz\, dx \Rightarrow f(x,y,z) = xyz + C(y,z)$$

Substituindo em (2):

$$xz \overset{(2)}{=} \frac{\partial f}{\partial y} = xz + \frac{\partial C}{\partial y} \Rightarrow \frac{\partial C}{\partial y}(y,z) = 0 \Rightarrow C(y,z) = K(z)$$

$$\Rightarrow f(x,y,z) = xyz + K(z)$$

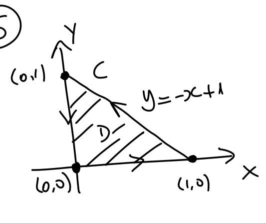
Substituindo em (3):

$$xy + 2z \overset{(3)}{=} \frac{\partial f}{\partial z} = xy + K'(z) \Rightarrow K'(z) = 2z \Rightarrow K(z) = z^2$$

Portanto, $f(x,y,z) = xyz + z^2$ é uma função potencial de $F$.

Pelo Teo. Fund. das Int. de Linha:

$$W = \int_C F \cdot dr = \int_C \nabla f \cdot dr = f(2,1,0) - f(0,0,0) = 0.$$

⑤ 


A curva $C$ é fechada, simples, orient. positivamente e suave por partes.

$F(x,y) = (x(x+y), xy) \Rightarrow \dfrac{\partial Q}{\partial x} = y$ e $\dfrac{\partial P}{\partial y} = x$ são contínuas.

Pelo Teo. de Green:

$$W = \int_C F \cdot dr = \iint_D y - x\, dA = \int_0^1 \int_0^{-x+1} y - x\, dy\, dx = \int_0^1 \frac{y^2}{2} - xy \Big|_{y=0}^{y=-x+1} dx$$

$$= \int_0^1 \frac{(-x+1)^2}{2} - x(-x+1)\, dx = \int_0^1 \frac{x^2 - 2x + 1}{2} + x^2 - x\, dx = \int_0^1 \frac{x^2 - 2x + 1 + 2x^2 - 2x}{2}\, dx$$

$$= \frac{1}{2} \int_0^1 3x^2 - 4x + 1\, dx = \frac{1}{2}\left( x^3 - 2x^2 + x \Big|_0^1 \right) = 0.$$