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Nota	

UNIVERSIDADE FEDERAL DA GRANDE DOURADOS
Cálculo 2 — Avaliação PS
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Matemática 26/10/2022

Aluno(a):...

Todas as respostas devem ser justificadas.

Avaliação P1:

1. Estime a área abaixo do gráfico de $f(x) = \text{sen}(x)$ de $x = 0$ até $x = 2\pi$ usando seis retângulos aproximantes e extremos esquerdos dos subintervalos.

2. Encontre f tal que $f''(x) = 3 - \text{sen}(x)$, $f(0) = 1$ e $f(\pi/2) = 0$.

3. Derive $F(x) = \displaystyle\int_1^{1/x} \text{sen}^4(t)\, dt$ com relação a x.

4. Calcule a áre da região delimitada pelas curvas $x = 1 - y^2$ e $x = y^2 - 1$.

5. Resolva a integral definida $\displaystyle\int_0^{\pi/2} \text{sen}(x)\cos(\cos x)\, dx$.

Avaliação P2:

1. Resolva a integral indefinida $\displaystyle\int (x^2 + 2x)\cos x\, dx$.

2. Resolva a integral pelo método das frações parciais $\displaystyle\int \frac{3x+1}{(x+1)(x-1)}\, dx$.

3. Calcule a integral $\displaystyle\int \frac{1}{\sqrt{x^2+4}}\, dx$.

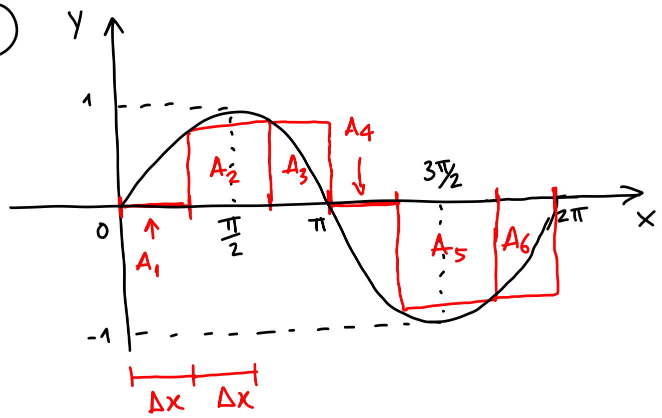
4. Determine, se possível, o valor da integral $\displaystyle\int_0^1 \frac{5}{x^5}\, dx$.

5. Determine os valores de p para os quais a integral $\displaystyle\int_1^{\infty} \frac{1}{x^p}\, dx$ é convergente.

Boa Prova!

Solução P1

(1)



$$\Delta x = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$A_1 = \Delta x \cdot \operatorname{sen}(0) = \frac{\pi}{3} \cdot 0 = 0$$

$$A_2 = \Delta x \cdot \operatorname{sen}\left(\frac{\pi}{3}\right) = \frac{\pi}{3} \cdot \frac{\sqrt{3}}{2} = \frac{\pi\sqrt{3}}{6}$$

$$A_3 = \Delta x \cdot \operatorname{sen}\left(2\frac{\pi}{3}\right) = \frac{\pi}{3} \cdot \frac{\sqrt{3}}{2} = \frac{\pi\sqrt{3}}{6}$$

$$A_4 = \Delta x \cdot \operatorname{sen}\left(3\frac{\pi}{3}\right) = \frac{\pi}{3} \cdot 0 = 0$$

$$A_5 = \Delta x \cdot \operatorname{sen}\left(4\frac{\pi}{3}\right) = \frac{\pi}{3} \cdot \left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi\sqrt{3}}{6}, \quad A_6 = \Delta x \cdot \operatorname{sen}\left(5\frac{\pi}{3}\right) = \frac{\pi}{3} \cdot \left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi\sqrt{3}}{6}$$

Portanto, $A = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 = 2\frac{\pi\sqrt{3}}{6} - 2\frac{\pi\sqrt{3}}{6} = 0.$

(2) $f''(x) = 3 - \operatorname{sen} x$

$$\Rightarrow f'(x) = 3x + \cos x + c_1 \quad \Rightarrow f(x) = \frac{3x^2}{2} + \operatorname{sen} x + c_1 x + c_2$$

Logo,

$$1 = f(0) = c_2$$

$$0 = f(\pi/2) = \frac{3}{2} \cdot \left(\frac{\pi}{2}\right)^2 + \operatorname{sen}\frac{\pi}{2} + c_1\frac{\pi}{2} + c_2 = \frac{3\pi^2}{8} + 1 + \frac{\pi}{2}c_1 + c_2$$

$$\Rightarrow \frac{3\pi^2}{8} + 1 + \frac{\pi}{2}c_1 + 1 = 0 \Rightarrow \frac{\pi}{2}c_1 = -\frac{3\pi^2}{8} - 2 = \frac{-3\pi^2 - 16}{8}$$

$$\Rightarrow c_1 = \frac{-3\pi^2 - 16}{4\pi}$$

Portanto, $f(x) = \dfrac{3x^2}{2} + \operatorname{sen} x - \dfrac{3\pi^2 + 16}{4\pi}x + 1$

③ sejam

$$f(x) = \int_1^x \operatorname{sen}^4 t \, dt \Rightarrow f'(x) = \operatorname{sen}^4 x$$

$$g(x) = \frac{1}{x} \Rightarrow g'(x) = -\frac{1}{x^2}$$

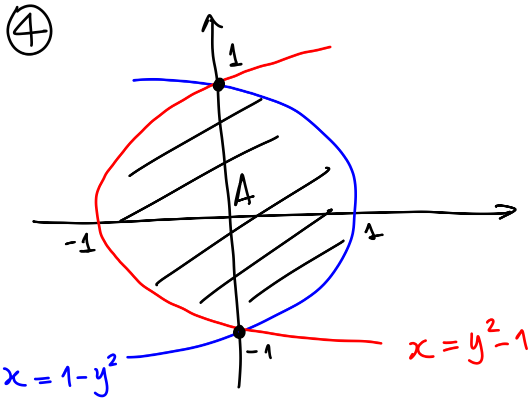
Temos que:

$$f(g(x)) = f\left(\frac{1}{x}\right) = \int_1^{1/x} \operatorname{sen}^4 t \, dt = F(x)$$

$$\Rightarrow F'(x) = f'(g(x)) \cdot g'(x).$$

$$\Rightarrow F'(x) = \operatorname{sen}^4\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) = -\frac{1}{x^2} \cdot \operatorname{sen}^4\left(\frac{1}{x}\right).$$

④



Calculando as interseções:

$$1 - y^2 = y^2 - 1 \Rightarrow 2y^2 = 2 \Rightarrow y^2 = 1$$

$$\Rightarrow y = \pm 1.$$

$$\therefore A = \int_{-1}^{1} 1 - y^2 - (y^2 - 1) \, dy = \int_{-1}^{1} 1 - y^2 - y^2 + 1 \, dy = 2 \int_{-1}^{1} 1 - y^2 dy$$

$$= 2 \left(y - \frac{y^3}{3} \Big|_{-1}^{1} \right) = 2 \left[1 - \frac{1}{3} - (-1) + \frac{(-1)^3}{3} \right] = 2 \cdot \left[2 - \frac{2}{3} \right]$$

$$= 2 \cdot \frac{4}{3} = \frac{8}{3}.$$

5 Chame $u = \cos x$, logo $du = -\operatorname{sen} x \, dx \Rightarrow \operatorname{sen} x \, dx = -du$.

Além disso, $x = 0 \Rightarrow u = \cos 0 = 1$ e $x = \frac{\pi}{2} \Rightarrow u = \cos\frac{\pi}{2} = 0$.

Assim,

$$\int_0^{\pi/2} \operatorname{sen} x \cdot \cos(\cos x)\, dx = \int_1^0 -\cos u \, du = \int_0^1 \cos u \, du = \operatorname{sen} u \Big|_0^1$$

$$= \operatorname{sen} 1 .$$

Solução P2

① Integrando por partes:

$$u = x^2+2x \quad \Rightarrow \quad du = (2x+2)\,dx$$
$$dv = \cos x\, dx \qquad v = \operatorname{sen} x$$

$$\therefore \int (x^2+2x)\cos x\, dx = (x^2+2x)\operatorname{sen} x - \int (2x+2)\operatorname{sen} x\, dx$$

Por partes novamente:

$$u = 2x+2 \quad \Rightarrow \quad du = 2\,dx$$
$$dv = \operatorname{sen} x\, dx \qquad v = -\cos x$$

$$\therefore \int (2x+2)\operatorname{sen} x\, dx = -(2x+2)\cos x - \int -2\cos x\, dx$$
$$= -(2x+2)\cos x + 2\operatorname{sen} x + C$$

Portanto,

$$\int (x^2+2x)\cos x\, dx = (x^2+2x)\operatorname{sen} x + (2x+2)\cos x - 2\operatorname{sen} x + C.$$
$$= (x^2+2x-2)\operatorname{sen} x + (2x+2)\cos x + C$$

② $$\frac{3x+1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{A(x-1)+B(x+1)}{(x+1)(x-1)}$$

$$\Rightarrow 3x+1 = A(x-1) + B(x+1), \quad \forall x \in \mathbb{R}$$

P/ $x=1$: $4 = 2B \Rightarrow B = 2$

P/ $x=-1$: $-2 = -2A \Rightarrow A = 1$

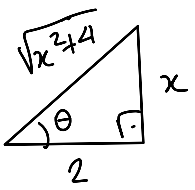
$$\therefore \frac{3x+1}{(x+1)(x-1)} = \frac{1}{x+1} + \frac{2}{x-1}$$

Assim,

$$\int \frac{3x+1}{(x+1)(x-1)}\,dx = \int \frac{1}{x+1}\,dx + \int \frac{2}{x-1}\,dx$$

$$= \ln|x+1| + 2\ln|x-1| + C$$

③ Seja $\alpha = \sqrt{x^2+4} \Rightarrow \alpha^2 = x^2+4$.



$$tg\,\theta = \frac{x}{2} \Rightarrow x = 2\,tg\,\theta \Rightarrow dx = 2\sec^2\theta\,d\theta$$

$$\cos\theta = \frac{2}{\sqrt{x^2+4}} \Rightarrow \frac{1}{\sqrt{x^2+4}} = \frac{\cos\theta}{2} \Rightarrow \sec\theta = \frac{\sqrt{x^2+4}}{2}$$

$$\therefore \int \frac{1}{\sqrt{x^2+4}}\,dx = \int \frac{\cos\theta}{2}\cdot 2\sec^2\theta\,d\theta = \int \sec\theta\,d\theta$$

$$= \int \sec\theta \cdot \frac{\sec\theta + tg\,\theta}{\sec\theta + tg\,\theta}\,d\theta = \int \frac{\sec^2\theta + \sec\theta \cdot tg\,\theta}{\sec\theta + tg\,\theta}\,d\theta$$

Tomando $u = \sec\theta + tg\,\theta$, temos $du = (\sec\theta \cdot tg\,\theta + \sec^2\theta)\,d\theta$.

$$\therefore \int \frac{1}{\sqrt{x^2+4}}\,dx = \int \frac{\sec^2\theta + \sec\theta \cdot tg\,\theta}{\sec\theta + tg\,\theta}\,d\theta = \int \frac{1}{u}\,du$$

$$= \ln|\sec\theta + tg\,\theta| + C = \ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| + C.$$

④ $f(x) = \frac{5}{x^5}$ não está definida para $x=0$, logo a integral é imprópria.

$$\int \frac{5}{x^5}\,dx = 5\int \frac{1}{x^5}\,dx = 5\int x^{-5}\,dx = 5\,\frac{x^{-4}}{-4} + c = -\frac{5}{4x^4} + c$$

$$\therefore \int_0^1 \frac{5}{x^5}\,dx = \lim_{t\to 0^-} \int_t^1 \frac{5}{x^5}\,dx = \lim_{t\to 0^-}\left(-\frac{5}{4x^4}\Big|_t^1\right)$$

$$= \lim_{t\to 0^-}\left(-\frac{5}{4} + \frac{5}{4t^4}\right) = \infty .$$

Portanto, a integral é divergente.

⑤ Temos que:

$$\int \frac{1}{x^p}\,dx = \int x^{-p}\,dx = \frac{x^{-p+1}}{-p+1} + c, \quad \forall\, p \neq 1.$$

e $\int \frac{1}{x}\,dx = \ln|x| + c$ quando $p = 1$.

Logo, se $p \neq 1$:

$$\int_1^\infty \frac{1}{x^p}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^p}\,dx = \lim_{t\to\infty}\left(\frac{x^{-p+1}}{-p+1}\Big|_1^t\right)$$

$$= \lim_{t\to\infty}\left(\frac{t^{-p+1}}{-p+1} - \frac{1}{-p+1}\right)$$

se $-p+1>0$, então o limite não existe. Se $-p+1<0$, então o limite é igual a $-\frac{1}{-p+1}$, pois $t^{-p+1} \xrightarrow{t\to\infty} 0$.

Assim, a integral é:

divergente para $-p+1>0$, ou seja, $p<1$.

convergente para $-p+1<0$, ou seja, $p>1$.

Para $p=1$:

$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{t\to\infty} \int_1^{t} \frac{1}{x}\,dx = \lim_{t\to\infty} \left(\ln x \Big|_1^t \right) = \lim_{t\to\infty} (\ln t - \ln 1)$$

$$= \lim_{t\to\infty} \ln t = \infty.$$

A integral diverge.