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UNIVERSIDADE FEDERAL DA GRANDE DOURADOS
Cálculo de Várias Variáveis — Avaliação P2
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Matemática 30/08/2023

Aluno(a):..

Todas as respostas devem ser justificadas.

1. Determine o volume do sólido definido abaixo da superfície $f(x, y) = y^3 e^{2x}$ e acima do retângulo $R = [0, 2] \times [0, 4]$.

2. Calcule a integral $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3 + 1}\, dy\, dx$ invertendo a ordem de integração.

3. Calcule a integral $\displaystyle\int_0^1 \int_y^{\sqrt{2-y^2}} x - y\, dx\, dy$.

4. Calcule a integral $\displaystyle\iiint_E \sqrt{x^2 + y^2}\, dV$, onde E é a região que está dentro do cilindro $x^2 + y^2 = 9$ e entre os planos $z = -1$ e $z = 3$.

5. Descreva o sólido cujo volume é dado pela integral $\displaystyle\int_0^{\pi/6} \int_0^{\pi/2} \int_0^3 \rho^2 \operatorname{sen}\phi\, d\rho\, d\theta\, d\phi$ e calcule seu volume.

Boa Prova!

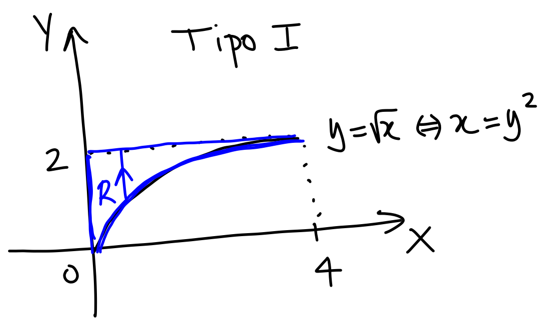
Avaliação P2

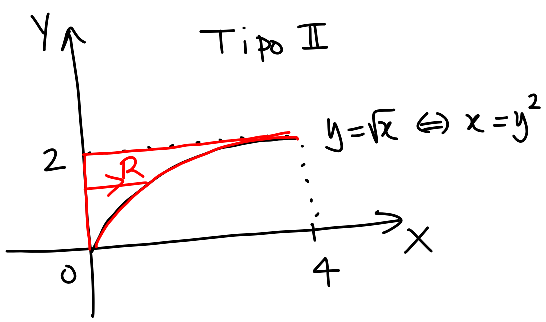
① Como $y \in [0,4]$, então $y^3 \geq 0$. Além disso, $e^{2x} > 0, \forall x \in \mathbb{R}$.
Logo, $f(x,y) = y^3 \cdot e^{2x} \geq 0$ em R e o volume do sólido é igual ao valor da integral

$$\iint_R y^3 \cdot e^{2x}\, dA = \int_0^2 \int_0^4 y^3 \cdot e^{2x}\, dy\, dx = \int_0^2 e^{2x} dx \cdot \int_0^4 y^3 dy$$

$$= \frac{e^{2x}}{2}\Big|_0^2 \cdot \frac{y^4}{4}\Big|_0^4 = \left(\frac{e^4}{2} - \frac{e^0}{2}\right)\left(\frac{256}{4} - \frac{0}{4}\right) = \left(\frac{e^4-1}{2}\right) \cdot 64 = 32(e^4-1)$$

② Descrevendo a região de integração como Tipo II:





$R = \{(x,y) \mid 0 \leq x \leq 4, \sqrt{x} \leq y \leq 2\}$

$R = \{(x,y) \mid 0 \leq y \leq 2, 0 \leq x \leq y^2\}$

Logo

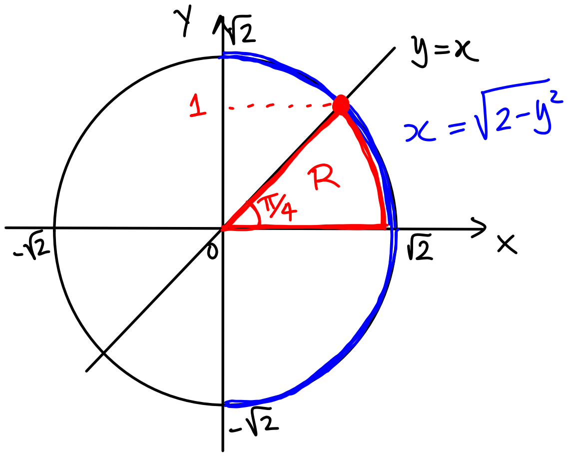
$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy\, dx = \int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\, dx\, dy = \int_0^2 \frac{1}{y^3+1}\left(\int_0^{y^2} dx\right) dy$$

$$= \int_0^2 \frac{1}{y^3+1}\left(x\Big|_0^{y^2}\right) dy = \int_0^2 \frac{y^2}{y^3+1}\, dy = \int_1^9 \frac{1}{u} \cdot \frac{1}{3}\, du = \frac{1}{3}\int_1^9 \frac{1}{u}\, du$$

$$\left(\begin{array}{l} u = y^3+1 \Rightarrow du = 3y^2 dy \\ y=0 \Rightarrow u=1 \text{ e } y=2 \Rightarrow u=9 \end{array}\right)$$

$$= \frac{1}{3}\left(\ln u\Big|_1^9\right) = \frac{1}{3}(\ln 9 - \ln 1) = \frac{\ln 9}{3}.$$

③ $R=\{(x,y)\mid 0\le y\le 1,\ y\le x\le\sqrt{2-y^2}\}$



$x=\sqrt{2-y^2}\ \Rightarrow\ x^2=2-y^2$

$\Rightarrow\ x^2+y^2=2$ (circ. raio $\sqrt{2}$)

Interseção:

$y=\sqrt{2-y^2}\ \Rightarrow\ y^2=2-y^2\Rightarrow 2y^2=2$

$\Rightarrow\ y=\pm 1$

Em coord. polares, $R=\{(r,\theta)\mid 0\le r\le\sqrt{2},\ 0\le\theta\le\frac{\pi}{4}\}$. Logo,

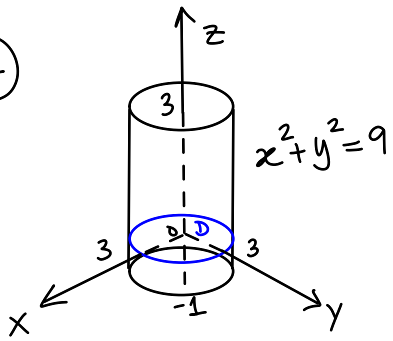
$$\int_0^{\sqrt{2}}\int_y^{\sqrt{2-y^2}} x-y\,dx\,dy=\int_0^{\sqrt{2}}\int_0^{\pi/4}(r\cos\theta-r\,\mathrm{sen}\,\theta)\,r\,d\theta\,dr$$

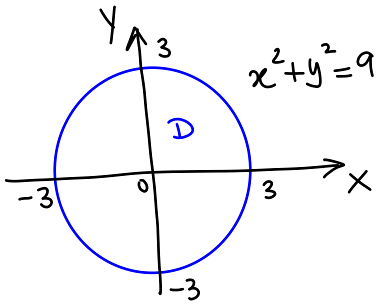
$$=\int_0^{\sqrt{2}}\int_0^{\pi/4} r^2(\cos\theta-\mathrm{sen}\,\theta)\,d\theta\,dr=\int_0^{\sqrt{2}} r^2dr\cdot\int_0^{\pi/4}\cos\theta-\mathrm{sen}\,\theta\,d\theta$$

$$=\left(\frac{r^3}{3}\Big|_0^{\sqrt{2}}\right)\cdot\left(\mathrm{sen}\,\theta+\cos\theta\Big|_0^{\pi/4}\right)=\left[\frac{(\sqrt{2})^3}{3}-0\right]\cdot\left(\mathrm{sen}\frac{\pi}{4}+\cos\frac{\pi}{4}-\mathrm{sen}\,0-\cos 0\right)$$

$$=\frac{2\sqrt{2}}{3}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-0-1\right)=\frac{2\sqrt{2}}{3}(\sqrt{2}-1)=\frac{4-2\sqrt{2}}{3}.$$

④





$E=\{(x,y,z)\mid (x,y)\in D,\ -1\le z\le 3\}$

$D=\{(r,\theta)\mid 0\le r\le 3,\ 0\le\theta\le 2\pi\}$

$\Rightarrow E=\{(r,\theta,z)\mid 0\le r\le 3,\ 0\le\theta\le 2\pi,\ -1\le z\le 3\}$

$$\therefore\ \iiint_E\sqrt{x^2+y^2}\,dV=\int_0^3\int_0^{2\pi}\int_{-1}^3\sqrt{r^2}\cdot r\,dz\,d\theta\,dr$$

$$\int_0^3\int_0^{2\pi}\int_{-1}^3 \sqrt{r^2}\cdot r\,dz\,d\theta\,dr = \int_0^3\int_0^{2\pi}\int_{-1}^3 r^2\,dz\,d\theta\,dr$$

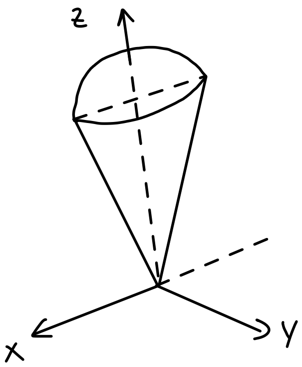
$$= \int_0^3 r^2 dr \cdot \int_0^{2\pi} d\theta \cdot \int_{-1}^3 dz = \left(\frac{r^3}{3}\Big|_0^3\right)\left(\theta\Big|_0^{2\pi}\right)\cdot\left(z\Big|_{-1}^3\right) = \frac{27}{3}\cdot 2\pi\cdot(3+1)$$

$$= 72\pi$$

(5) A região de integração é

$$E = \{(\rho,\theta,\phi) \mid 0\leq\rho\leq 3,\ 0\leq\theta\leq\frac{\pi}{2},\ 0\leq\phi\leq\frac{\pi}{6}\},$$

a metade localizada no semi-espaço $y\geq 0$ da região cônica da esfera de raio 3 de faz ângulo $\frac{\pi}{6}$ com o semi-eixo z positivo e vértice na origem.



Como o integrando é $f(x,y,z)=1$, o valor da integral é igual ao volume do sólido E.

Calculando a integral:

$$\int_0^{\pi/6}\int_0^{\pi/2}\int_0^3 \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi = \int_0^{\pi/6}\sin\phi\, d\phi\cdot\int_0^{\pi/2} d\theta\cdot\int_0^3 \rho^2 d\rho$$

$$= \left(-\cos\phi\Big|_0^{\pi/6}\right)\cdot\left(\theta\Big|_0^{\pi/2}\right)\cdot\left(\frac{\rho^3}{3}\Big|_0^3\right) = \left(-\cos\frac{\pi}{6}+\cos 0\right)\cdot\frac{\pi}{2}\cdot 9$$

$$= \frac{9\pi}{2}\left(1-\frac{\sqrt{3}}{2}\right) = \frac{9\pi}{4}\left(2-\sqrt{3}\right)$$