Nota	

UNIVERSIDADE FEDERAL DA GRANDE DOURADOS
Cálculo 2 — Avaliação P2
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Matemática 19/10/2022

Aluno(a):...

Todas as respostas devem ser justificadas.
Escolha cinco exercícios e anote suas escolhas no quadro de notas acima.

1. Calcule a integral indefinida $\int \cos\left(\sqrt{x}\right)\ dx$.

2. Encontre uma primitiva para $f(x) = \dfrac{x^3+2x}{x^4+4x^2+3}$.

3. Calcue a integral $\displaystyle\int \frac{1}{\sqrt{t^2+9}}\ dt$.
   (Use $\int \sec(x)\ dx = \ln|\sec(x)+\operatorname{tg}(x)|+c$ se achar necessário.)

4. Calcule a integral imprópria $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\ dx$.

5. (a) Encontre uma primitiva para $f(x) = \dfrac{x}{\sqrt{x^2-1}}$.

   (b) Determine o valor da integral $\displaystyle\int_1^2 f(x)\ dx$.

6. Sejam $p(x) = ax+b$, $q(x) = x^2 - cx$ e $f(x) = \dfrac{p(x)}{q(x)}$.

   (a) Fatore $q(x)$.

   (b) Escreva $f(x)$ como soma de frações parciais.

   (c) Calcule a integral $\displaystyle\int f(x)\ dx$.

7. Calcule a integral indefinida $\displaystyle\int x\ln(1+x)\ dx$.

8. Encontre uma primitiva para $f(x) = \sqrt{1-4x^2}$.
   (Use $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$ e $\operatorname{sen}(2x) = 2\operatorname{sen}(x)\cos(x)$ se achar necessário.)

Boa Prova!

Solução P2

① Chame $t=\sqrt{x}$, logo $x=t^2 \Rightarrow dx = 2t\,dt$. Assim,

$$\int \cos(\sqrt{x})\,dx = \int \cos(t)\cdot 2t\,dt = 2\int t\cdot \cos t\,dt$$

Integrando por partes:

$$u = t \Rightarrow du = dt$$
$$dv = \cos t\,dt \Rightarrow v = \operatorname{sen} t$$

$$\therefore \int t\cos t\,dt = t\operatorname{sen} t - \int \operatorname{sen} t\,dt = t\operatorname{sen} t + \cos t + C.$$

Portanto,

$$\int \cos(\sqrt{x})\,dx = 2\left[\sqrt{x}\operatorname{sen}(\sqrt{x}) + \cos(\sqrt{x}) + C\right].$$

② Sol. 1: Chame $u = x^4+4x^2+3$, logo $du = (4x^3+8x^2)dx$
$\Rightarrow \frac{1}{4}du = (x^3+2x^2)dx$. Assim,

$$\int \frac{x^3+2x^2}{x^4+4x^2+3}dx = \frac{1}{4}\int \frac{1}{u}du = \frac{1}{4}\ln|x^4+4x^2+3| + C$$

Sol. 2: seja $y = x^2$, então

$$x^4+4x^2+3 = 0 \Rightarrow y^2+4y+3 = 0 \Rightarrow y=-1 \text{ ou } y=-3.$$

Mas, $x^2=-1$ e $x^2=-3$ não têm sol. real, ou seja, $q(x)$ não tem raízes reais. Assim,

$$x^4+4x^2+3 = (x^2+1)(x^2+3).$$

$$f(x)=\frac{x^3+2x}{x^4+4x^2+3}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{x^2+3}=\frac{(Ax+B)(x^2+3)+(Cx+D)(x^2+1)}{x^4+4x^2+3}$$

$$\Rightarrow x^3+2x = Ax^3+3Ax+Bx^2+3B+Cx^3+Cx+Dx^2+D$$

$$=(A+C)x^3+(B+D)x^2+(3A+C)x+3B+D$$

$$\Rightarrow \begin{cases} A+C=1 \\ B+D=0 \\ 3A+C=2 \\ 3B+D=0 \end{cases} \Rightarrow \begin{cases} C=1-A \\ B=-D \\ 3A+1-A=2 \\ -3D+D=0 \end{cases} \Rightarrow \begin{cases} C=1-A \\ B=-D \\ A=1/2 \\ D=0 \end{cases} \Rightarrow \begin{cases} C=1/2 \\ B=0 \\ A=1/2 \\ D=0 \end{cases}$$

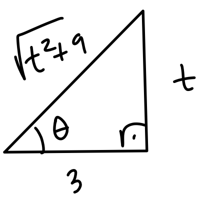
Assim, $f(x)=\frac{1}{2}\cdot\frac{x}{x^2+1}+\frac{1}{2}\cdot\frac{x}{x^2+3}$

$$\therefore \int f(x)\,dx = \frac{1}{2}\int\frac{x}{x^2+1}\,dx+\frac{1}{2}\int\frac{x}{x^2+3}\,dx$$

$$=\frac{1}{2}\int\frac{1}{u}\,du+\frac{1}{2}\int\frac{1}{u}\,du$$

$$=\frac{1}{2}\ln|x^2+1|+\frac{1}{2}\ln|x^2+3|+C.$$

③ Seja $\alpha=\sqrt{t^2+9}$, logo $\alpha^2=t^2+9$. Assim,



$$tg\theta=\frac{t}{3}\Rightarrow t=3tg\theta \Rightarrow dt=3\sec^2\theta\,d\theta$$

$$\cos\theta=\frac{3}{\sqrt{t^2+9}}\Rightarrow\frac{1}{\sqrt{t^2+9}}=\frac{\cos\theta}{3}\Rightarrow\sec\theta=\frac{\sqrt{t^2+9}}{3}$$

$$\therefore \int\frac{1}{\sqrt{t^2+9}}\,dt=\int\frac{\cos\theta}{3}\cdot 3\sec^2\theta\,d\theta=\int\sec\theta\,d\theta=\ln|\sec\theta+tg\theta|+c$$

$$=\ln\left|\frac{\sqrt{t^2+9}}{3}+\frac{t}{3}\right|+c.$$

④ Seja $u=-x^2$, logo $du=-2x\,dx \Rightarrow x\,dx = -\frac{1}{2}du$.

$$\int x e^{-x^2} dx = -\frac{1}{2}\int e^{u} du = -\frac{1}{2} e^{-x^2} + C.$$

Assim,

$$\int_0^{\infty} x e^{-x^2} dx = \lim_{t\to\infty} \int_0^{t} x e^{-x^2} dx = \lim_{t\to\infty}\left(-\frac{1}{2} e^{-x^2}\Big|_0^t\right)$$

$$= -\frac{1}{2}\lim_{t\to\infty}\left(e^{-t^2}-1\right) = \frac{1}{2}$$

$$\int_{-\infty}^{0} x e^{-x^2} dx = \lim_{t\to\infty} \int_t^{0} x e^{-x^2} dx = \lim_{t\to\infty}\left(-\frac{1}{2} e^{-x^2}\Big|_t^0\right)$$

$$= -\frac{1}{2}\lim_{t\to\infty} 1-e^{-t^2} = -\frac{1}{2}.$$

Portanto,

$$\int_{-\infty}^{\infty} x e^{-x^2} dx = \int_{-\infty}^{0} x e^{-x^2} dx + \int_0^{\infty} x e^{-x^2} dx = -\frac{1}{2}+\frac{1}{2} = 0$$

⑤ a) Seja $u = x^2-1$, logo $du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}du$.

$$\int \frac{x}{\sqrt{x^2-1}} dx = \frac{1}{2}\int \frac{1}{\sqrt{u}} du = \frac{1}{2}\int u^{-1/2} du = \frac{1}{2}\cdot 2u^{1/2} + C = \sqrt{x^2-1} + C.$$

b) A integral é imprópria:

$$\int_1^2 f(x)\,dx = \lim_{t\to 1^-}\int_t^2 f(x)\,dx = \lim_{t\to 1^-}\left(\sqrt{x^2-1}\Big|_t^2\right) = \lim_{t\to 1^-}\left(\sqrt{3}-\sqrt{t^2-1}\right) = \sqrt{3}.$$

⑥ a) $x^2 - cx = x(x-c)$

b) $$f(x) = \frac{ax+b}{x^2-cx} = \frac{A}{x} + \frac{B}{x-c} = \frac{A(x-c)+Bx}{x^2-cx}$$

$$\Rightarrow ax+b = Ax - Ac + Bx = (A+B)x - Ac$$

$$\Rightarrow \begin{cases} A+B=a \\ -Ac = b \Rightarrow A = -\frac{b}{c} \end{cases} \Rightarrow -\frac{b}{c} + B = a \Rightarrow B = a + \frac{b}{c} = \frac{ac+b}{c}.$$

$$\therefore f(x) = -\frac{b}{c} \cdot \frac{1}{x} + \frac{ac+b}{c} \cdot \frac{1}{x-c}$$

c) $$\int f(x)\,dx = -\frac{b}{c} \cdot \int \frac{1}{x}\,dx + \frac{ac+b}{c} \int \frac{1}{x-c}\,dx$$

$$= -\frac{b}{c} \ln|x| + \frac{ac+b}{c} \ln|x-c| + k.$$

⑦ Seja $t = 1+x$, logo $x = t-1 \Rightarrow dx = dt$.

$$\int x \ln(1+x)\,dx = \int (t-1)\ln t\,dt$$

Integrando por partes:

$u = \ln t \Rightarrow du = \frac{1}{t}\,dt$

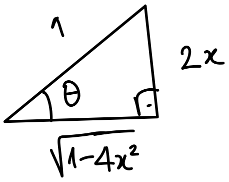
$dv = (t-1)\,dt \Rightarrow v = \frac{t^2}{2} - t$

$$\therefore \int (t-1)\ln t\,dt = \left(\frac{t^2}{2} - t\right)\ln t - \int \left(\frac{t^2}{2} - t\right) \cdot \frac{1}{t}\,dt = \left(\frac{t^2}{2} - t\right)\ln t - \int \frac{t}{2} - 1\,dt$$

$$= \left(\frac{t^2}{2} - t\right)\ln t - \frac{t^2}{4} + t + c.$$

Portanto, $$\int x \ln(1+x)\,dx = \left[\frac{(1+x)^2}{2} - (1+x)\right]\ln(1+x) - \frac{(1+x)^2}{4} + (1+x) + c.$$

⑧ Seja $\alpha = \sqrt{1-4x^2}$, logo $\alpha^2 = 1-4x^2 \Rightarrow 1 = \alpha^2 + 4x^2$. Assim,



$$\text{sen}\,\theta = 2x \Rightarrow x = \frac{\text{sen}\,\theta}{2} \Rightarrow dx = \frac{\cos\theta}{2}\,d\theta$$

$$\cos\theta = \sqrt{1-4x^2}$$

$$\therefore \int \sqrt{1-4x^2}\,dx = \int \cos\theta \cdot \frac{\cos\theta}{2}\,d\theta = \frac{1}{2}\int \cos^2\theta\,d\theta$$

$$= \frac{1}{2}\int \frac{1+\cos(2\theta)}{2}\,d\theta = \frac{1}{4}\left(\int d\theta + \int \cos(2\theta)\,d\theta\right) = \frac{1}{4}\left[\theta + \frac{\text{sen}(2\theta)}{2} + c\right]$$

$$= \frac{1}{4}\left[\theta + \frac{2\,\text{sen}\,\theta\cos\theta}{2} + c\right] = \frac{1}{4}\left[\text{arcsen}(2x) + 2x\sqrt{1-4x^2} + c\right].$$