1	
2	
3	
4	
5	
Nota	

UNIVERSIDADE FEDERAL DA GRANDE DOURADOS
Cálculo Diferencial e Integral II — Avaliação P1
Prof. Adriano Barbosa

Eng. Civil 09/11/2017

Aluno(a):...

Todas as respostas devem ser justificadas.

1. Resolva a integral definida $\displaystyle\int_0^{\pi/2} \operatorname{sen}(\theta)\, e^{\cos(\theta)}\, d\theta$.

2. Calcule a integral $\displaystyle\int_0^{a} e^{y} \cos(a-y)\, dy$, onde a é uma constante.

3. Dados $p(x) = x^2 + 8x - 3$ e $q(x) = x^3 + 3x^2$:

   (a) Fatore $q(x)$.

   (b) Escreva $\dfrac{p(x)}{q(x)}$ como soma de frações parciais.

   (c) Calcule a integral $\displaystyle\int \frac{p(x)}{q(x)}\, dx$.

4. Determine se a integral imprópria $\displaystyle\int_2^{\infty} \frac{dx}{x \ln x}$ é convergente ou divergente e calcule seu valor, se possível.

5. Resolva a integral indefinida $\displaystyle\int e^{\sqrt{x}}\, dx$.

Boa Prova!

① Chamando $u = \cos\theta$, temos $du = -\operatorname{sen}\theta \, d\theta$, $x = 0 \Rightarrow u = 1$ e $x = \frac{\pi}{2} \Rightarrow u = 0$, logo

$$\int_0^{\pi/2} \operatorname{sen}\theta \, e^{\cos\theta} \, d\theta = -\int_1^0 e^u \, du = \int_0^1 e^u \, du = e^u \Big|_0^1 = e^1 - e^0 = e - 1.$$

② Integrando por partes, temos

$$u = \cos(a-y) \Rightarrow du = -\operatorname{sen}(a-y) \cdot (-1) \, dy = \operatorname{sen}(a-y) \, dy$$
$$dv = e^y \, dy \Rightarrow v = e^y$$

logo

$$\int e^y \cos(a-y) \, dy = e^y \cos(a-y) - \underbrace{\int e^y \operatorname{sen}(a-y) \, dy}_{I} \qquad (*)$$

Resolvendo I por partes:

$$u = \operatorname{sen}(a-y) \Rightarrow du = \cos(a-y) \cdot (-1) \, dy = -\cos(a-y) \, dy$$
$$dv = e^y \, dy \Rightarrow v = e^y$$

$$\therefore \; I = e^y \operatorname{sen}(a-y) - \int e^y \cdot [-\cos(a-y)] \, dy = e^y \operatorname{sen}(a-y) + \int e^y \cos(a-y) \, dy.$$

Assim, a eq. $(*)$ pode ser escrita como

$$\int e^y \cos(a-y) \, dy = e^y \cos(a-y) - \left[e^y \operatorname{sen}(a-y) + \int e^y \cos(a-y) \, dy \right]$$
$$= e^y \cos(a-y) - e^y \operatorname{sen}(a-y) - \int e^y \cos(a-y) \, dy$$

$$\Rightarrow 2 \int e^y \cos(a-y) \, dy = e^y \left[\cos(a-y) - \operatorname{sen}(a-y) \right]$$

$$\Rightarrow \int e^y \cos(a-y) \, dy = \frac{e^y}{2} \left[\cos(a-y) - \operatorname{sen}(a-y) \right].$$

Portanto,

$$\int_0^a e^y \cos(a-y) \, dy = \frac{e^y}{2} \left[\cos(a-y) - \operatorname{sen}(a-y) \right] \Big|_0^a$$
$$= \frac{e^a}{2} \left[\cos 0 - \operatorname{sen} 0 \right] - \frac{e^0}{2} \left[\cos a - \operatorname{sen} a \right] = \frac{e^a - \cos a + \operatorname{sen} a}{2}.$$

③ a) Fatorando $q(x)$:

$$q(x) = x^3 + 3x^2 = x^2(x+3)$$

Logo, as raízes de $q(x)$ são 0 e -3.

b) $$\frac{x^2+8x-3}{x^3+3x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3} = \frac{Ax(x+3) + B(x+3) + Cx^2}{x^2(x+3)}$$

$$\Rightarrow x^2+8x-3 = Ax(x+3) + B(x+3) + Cx^2$$

$$= Ax^2 + 3Ax + Bx + 3B + Cx^2$$

$$= (A+C)x^2 + (3A+B)x + 3B$$

$$\Rightarrow \begin{cases} A + C = 1 & \Rightarrow C = -2 \\ 3A + B = 8 & \Rightarrow A = 3 \\ 3B = -3 & \Rightarrow B = -1 \end{cases}$$

Logo,

$$\frac{p(x)}{q(x)} = \frac{3}{x} - \frac{1}{x^2} - \frac{2}{x+3}$$

c) Pelo item (b),

$$\int \frac{p(x)}{q(x)}\,dx = 3\int \frac{1}{x}\,dx - \int \frac{1}{x^2}\,dx - 2\int \frac{1}{x+3}\,dx$$

$$= 3\ln|x| + \frac{1}{x} - 2\ln|x+3| + C.$$

④ Pela definição de integral imprópria, temos

$$\int_2^\infty \frac{1}{x\ln x}\,dx = \lim_{t\to\infty} \int_2^t \frac{1}{x\ln x}\,dx$$

Resolvendo a integral, chame $u = \ln x \Rightarrow du = \frac{1}{x}\,dx$, logo

$$\int \frac{1}{x\ln x}\,dx = \int \frac{1}{\ln x}\cdot\frac{1}{x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\ln x| + C$$

Assim,

$$\int_2^t \frac{1}{x \ln x}\,dx = \ln|\ln x|\Big|_2^t = \ln|\ln t| - \ln|\ln 2|$$

e portanto,

$$\int_2^\infty \frac{1}{x\ln x}\,dx = \lim_{t\to\infty}\left(\ln|\ln t| - \ln|\ln 2|\right)$$

Como $\ln t \xrightarrow{t\to\infty} \infty$, $\ln|\ln t| \xrightarrow{t\to\infty} \infty$, logo a integral é divergente.

⑤ Chamando $z = \sqrt{x}$, temos $dz = \frac{1}{2\sqrt{x}}dx = \frac{1}{2z}dx \Rightarrow dx = 2z\,dz$, logo

$$\int e^{\sqrt{x}}\,dx = \int e^{z}\cdot(2z)\,dz = 2\underbrace{\int z e^{z}\,dz}_{I}$$

Integrando I por partes, temos:

$u = z \Rightarrow du = dz$
$dv = e^z dz \Rightarrow v = e^z$

$$\therefore\ I = z e^z - \int e^z\,dz = ze^z - e^z + c_1 = e^z(z-1) + c_1$$

Portanto,

$$\int e^{\sqrt{x}}\,dx = 2\left[e^z(z-1) + c_1\right] = 2e^z(z-1) + C$$

$$= 2e^{\sqrt{x}}(\sqrt{x}-1) + C$$