UNIVERSIDADE FEDERAL DA GRANDE DOURADOS
Cálculo de Várias Variáveis — Avaliação P2
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Matemática 19/04/2023

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Nota	

Aluno(a):...

Todas as respostas devem ser justificadas.

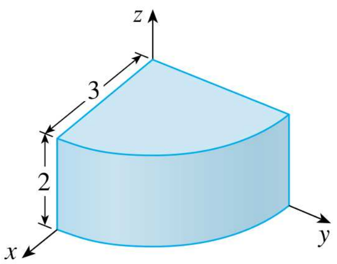
1. Calcule a integral dupla $\displaystyle\iint_R y \operatorname{sen}(xy)\, dA$, $R = [1,2] \times [0,\pi]$.

2. Esboce ou descreva a região cuja área é dada pela integral $\displaystyle\int_{\pi/4}^{3\pi/4} \int_1^2 r\, dr\, d\theta$ e calcule-a.

3. Calcule a integral $\displaystyle\iint_D \operatorname{sen}(y^2)\, dA$, $D = \{(x,y) \mid 0 \leq x \leq 1, x \leq y \leq 1\}$.

4. (a) Escreva a integral tripla de uma função contínua $f(x,y,z)$ sobre o sólido abaixo determinando seus limites de integração.

   (b) Calcule o volume do sólido utilizando a integral tripla encontrada no item anterior.



5. Calcule $\displaystyle\iiint_E x^2 + y^2\, dV$, onde E está entre as superfícies $x^2 + y^2 + z^2 = 4$ e $x^2 + y^2 + z^2 = 9$.

Boa Prova!

Solução P2

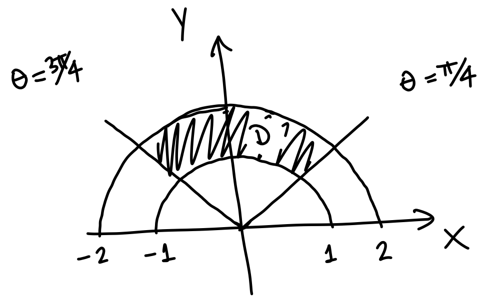
① $\iint y \operatorname{sen}(xy)\, dA = \int_0^{\pi}\int_1^2 y\operatorname{sen}(xy)\,dx\,dy = \int_0^{\pi} y\left[\frac{-\cos(xy)}{y}\Big|_1^2\right]dy$

$= \int_0^{\pi} -\cos(2y) + \cos(y)\, dy = -\frac{\operatorname{sen}(2y)}{2} + \operatorname{sen}(y)\Big|_0^{\pi}$

$= -\frac{\operatorname{sen}(2\pi)}{2} + \operatorname{sen}(\pi) + \frac{\operatorname{sen}(0)}{2} - \operatorname{sen}(0) = 0.$

② A região de integração é

$D = \{(r,\theta) \mid 1 \le r \le 2,\ \frac{\pi}{4} \le \theta \le \frac{3\pi}{4}\}$

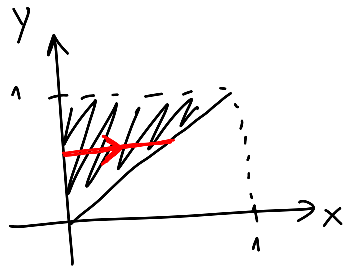


Calculando a integral:

$\int_{\pi/4}^{3\pi/4}\int_1^2 r\,dr\,d\theta = \int_{\pi/4}^{3\pi/4} d\theta \cdot \int_1^2 r\,dr = \left(\theta\Big|_{\pi/4}^{3\pi/4}\right)\cdot\left(\frac{r^2}{2}\Big|_1^2\right)$

$= \left(\frac{3\pi}{4} - \frac{\pi}{4}\right)\left(2 - \frac{1}{2}\right) = \frac{\pi}{2}\cdot\frac{3}{2} = \frac{3\pi}{4}.$

③



$$D = \{(x,y) \mid 0 \le x \le 1,\ x \le y \le 1\}$$

$$= \{(x,y) \mid 0 \le y \le 1,\ 0 \le x \ \ y\}$$

$$\iint_D \operatorname{sen}(y^2)\, dA = \int_0^1 \int_0^y \operatorname{sen}(y^2)\, dx\, dy = \int_0^1 \operatorname{sen}(y^2)\left(x\Big|_0^y\right) dy$$

$$= \int_0^1 \operatorname{sen}(y^2)\cdot y\, dy = \int_0^1 \operatorname{sen}(u)\cdot \frac{1}{2}\, du = -\frac{1}{2}\left(\cos u \Big|_0^1\right) = -\frac{1}{2}\left[\cos(1) - 1\right]$$

$$= \frac{1-\cos(1)}{2}.$$

$(u = y^2 \Rightarrow du = 2y\,dy)$

④ a) O sólido pode ser descrito em coord. cilíndricas como

$$E = \{(r,\theta,z) \mid 0 \le r \le 3,\ 0 \le \theta \le \tfrac{\pi}{2},\ 0 \le z \le 2\}.$$

Assim,

$$\iiint_E f(x,y,z)\, dV = \int_0^2 \int_0^{\pi/2} \int_0^3 f(r\cos\theta, r\operatorname{sen}\theta, z)\cdot r\, dr\, d\theta\, dz$$

b) $$V = \int_0^2 \int_0^{\pi/2} \int_0^3 r\, dr\, d\theta\, dz = \int_0^2 dz \cdot \int_0^{\pi/2} d\theta \cdot \int_0^3 r\, dr$$

$$= 2\cdot\frac{\pi}{2}\cdot\left(\frac{r^2}{2}\Big|_0^3\right) = \frac{9}{2}\pi.$$

⑤ A região de integração pode ser descrita em coord. esféricas como

$$E = \{(\rho, \theta, \phi) \mid 2 \leq \rho \leq 3,\ 0 \leq \theta \leq 2\pi,\ 0 \leq \phi \leq \pi\}.$$

Assim,

$$\iiint_E x^2+y^2\, dV = \int_0^{\pi}\int_0^{2\pi}\int_2^3 (\rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\phi \sin^2\theta)\rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi}\int_2^3 \rho^4 \sin^3\phi \, d\rho\, d\theta\, d\phi = \int_0^{2\pi} d\theta \cdot \int_2^3 \rho^4 d\rho \cdot \int_0^{\pi} \sin^3\phi \, d\phi$$

$$= 2\pi \cdot \left(\frac{\rho^5}{5}\Big|_2^3 \right) \cdot \int_0^{\pi} \sin\phi \cdot \sin^2\phi \, d\phi = \frac{422}{5}\pi \cdot \int_0^{\pi} \sin\phi (1-\cos^2\phi)\, d\phi$$

$$(u = \cos\phi \Rightarrow du = -\sin\phi\, d\phi)$$

$$= \frac{422}{5}\pi \left(-\int_1^{-1} 1-u^2\, du \right) = \frac{422}{5}\pi \cdot \int_{-1}^{1} 1-u^2\, du = \frac{422}{5}\pi \left(u - \frac{u^3}{3}\Big|_{-1}^{1} \right)$$

$$= \frac{422}{5}\pi \left[1 - \frac{1}{3} - (-1) + \frac{(-1)^3}{3} \right] = \frac{422}{5}\pi \cdot \left(2 - \frac{2}{3} \right) = \frac{422}{5}\pi \cdot \frac{4}{3} = \frac{1688}{15}\pi.$$