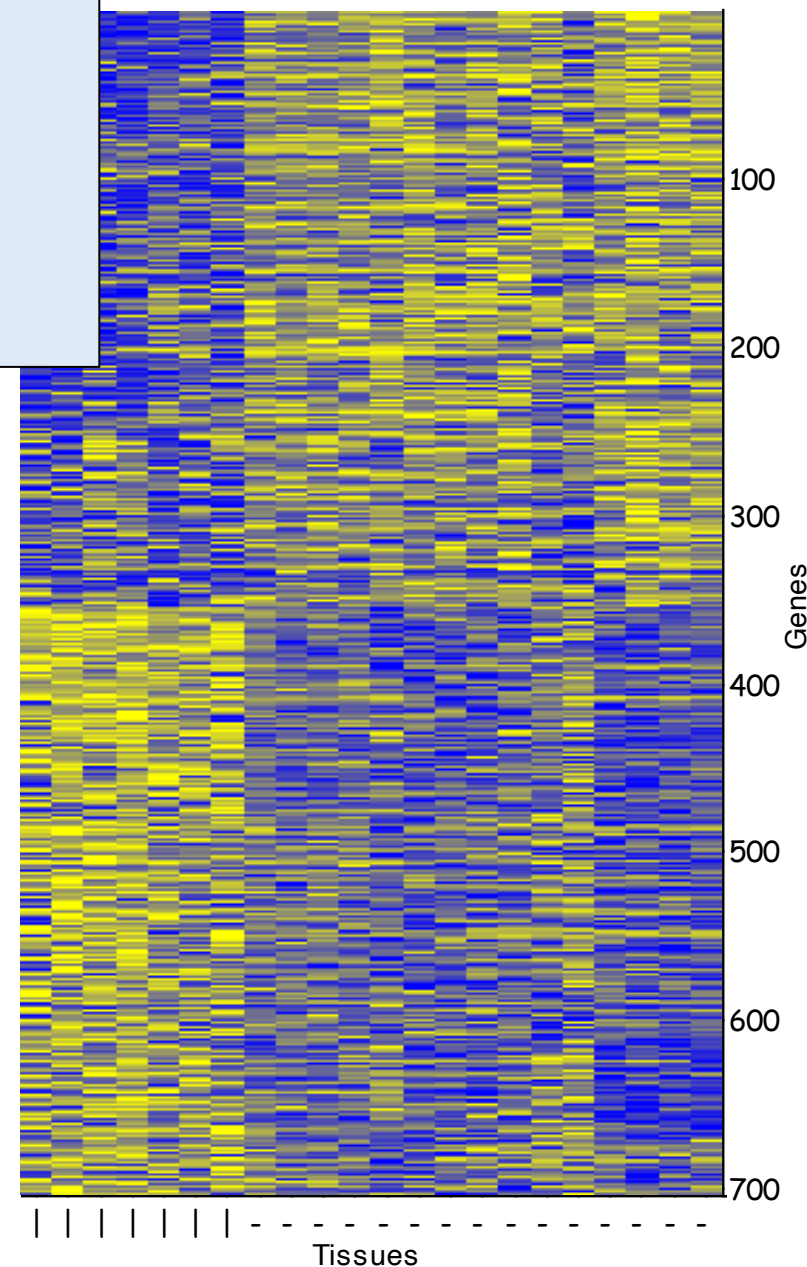


A brief guide to multiple testing and FDR

Zohar Yakhini,
Leon Anavy,
Ben Galili
– RUNI Herzliya



Breast Cancer BRCA1/BRCA2 data



False Discovery Rate (FDR)

What fraction of the observed discoveries (e.g DE) is explained by what is expected at random (under a null-model)?

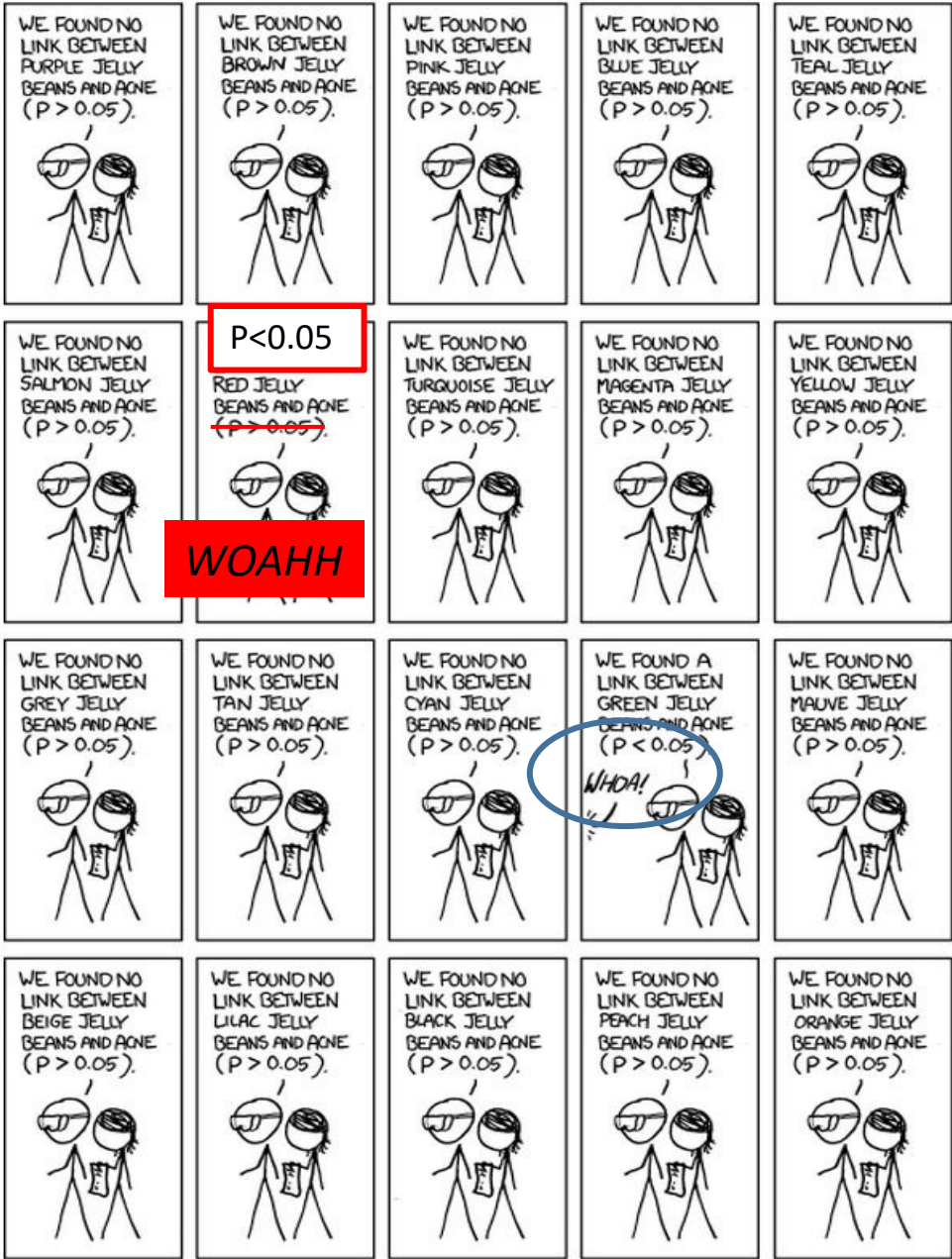
Expected

$$FDR(p) = \frac{pN}{O(p)}$$

Observed: # of tests for which $p\text{-val} \leq p$

More events

What if two colors
were linked at 0.05?
Three?



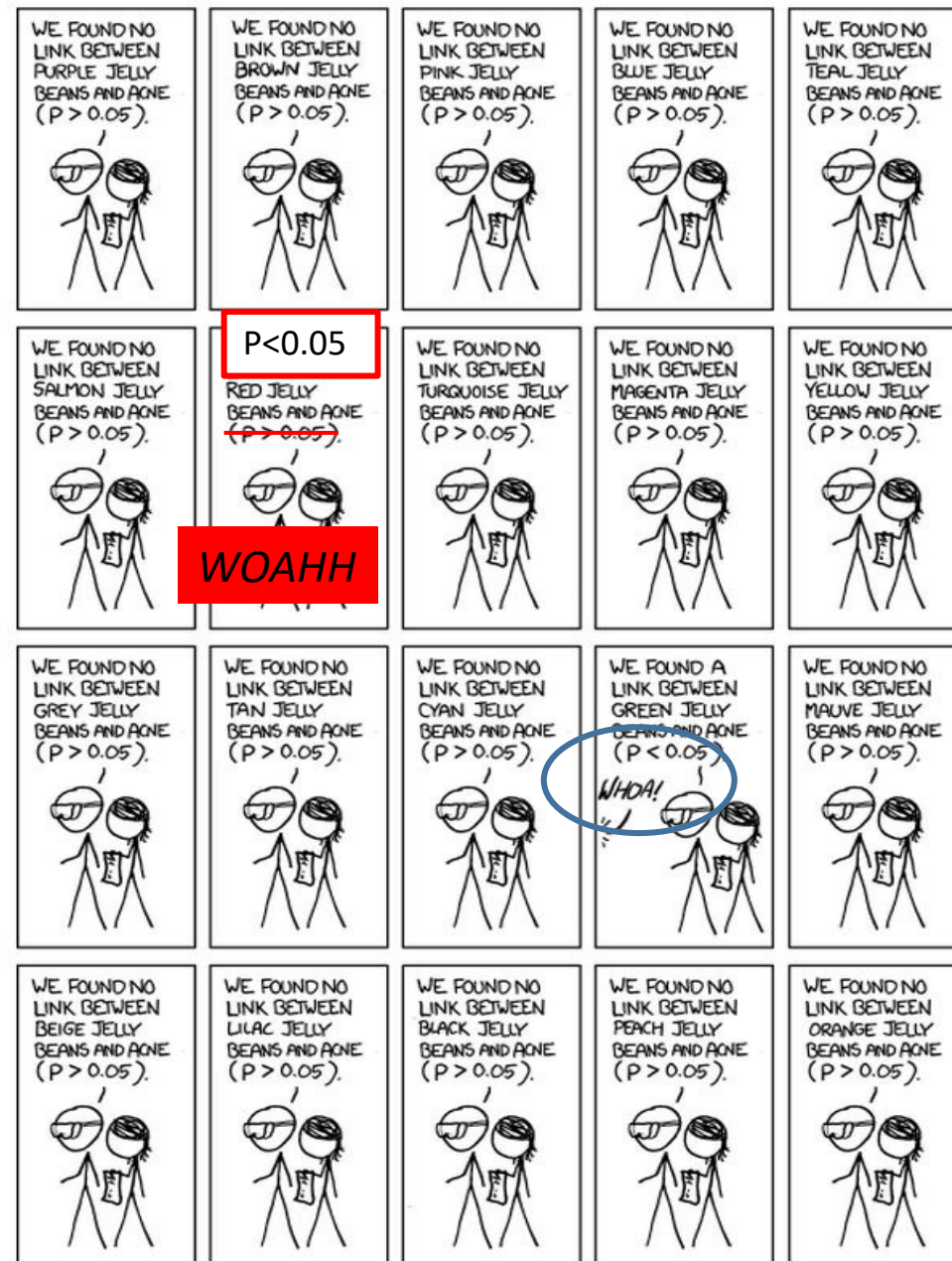
FDR at any i

$$FDR(p) = \frac{pN}{O(p)}$$

$$FDR(i) = \frac{p(i)N}{i}$$

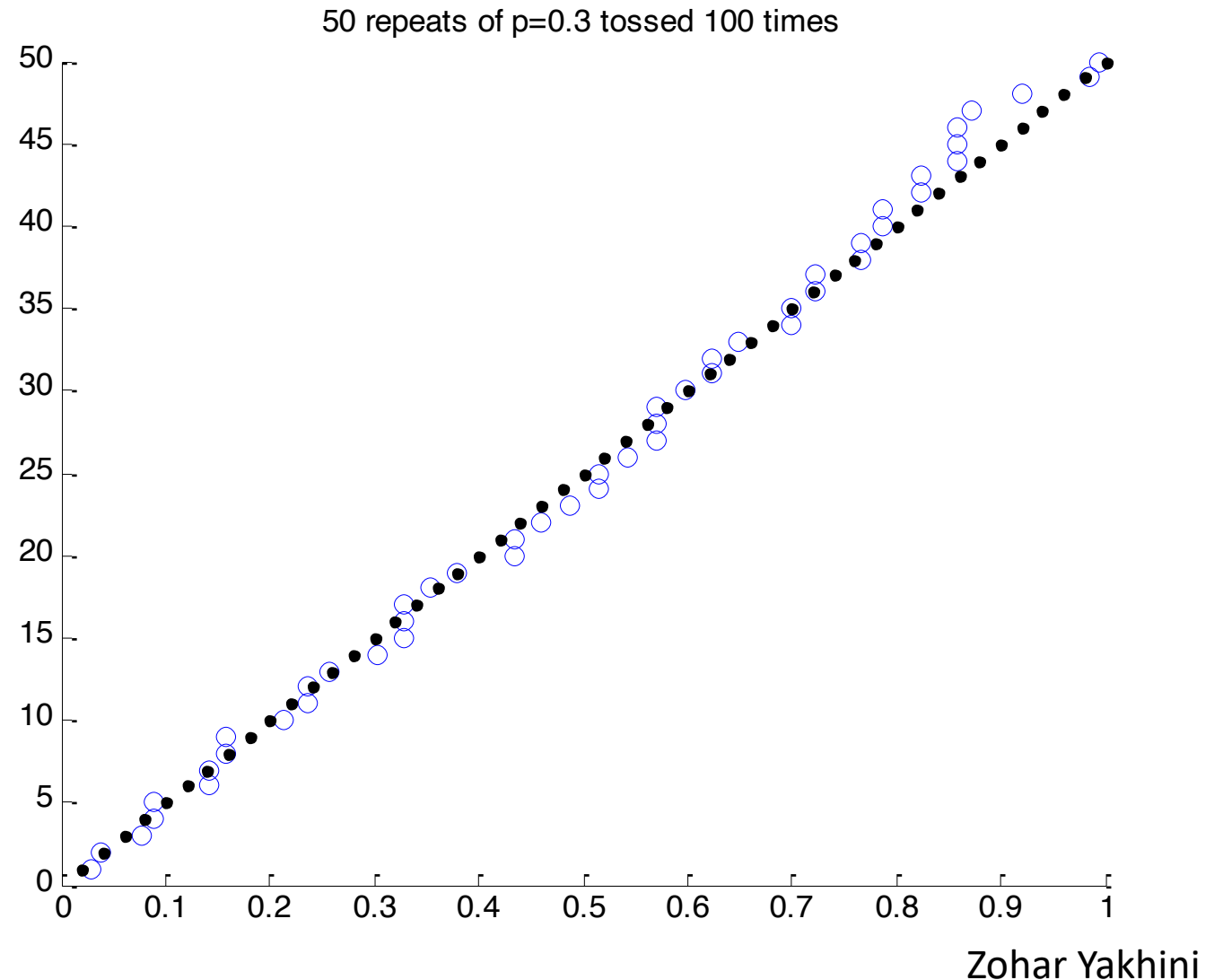
More events

$$FDR(2) = ?$$



Distribution of p-values under the null model

- Generating data using a coin with $p = 0.3$
- under a null model of $p = 0.3$

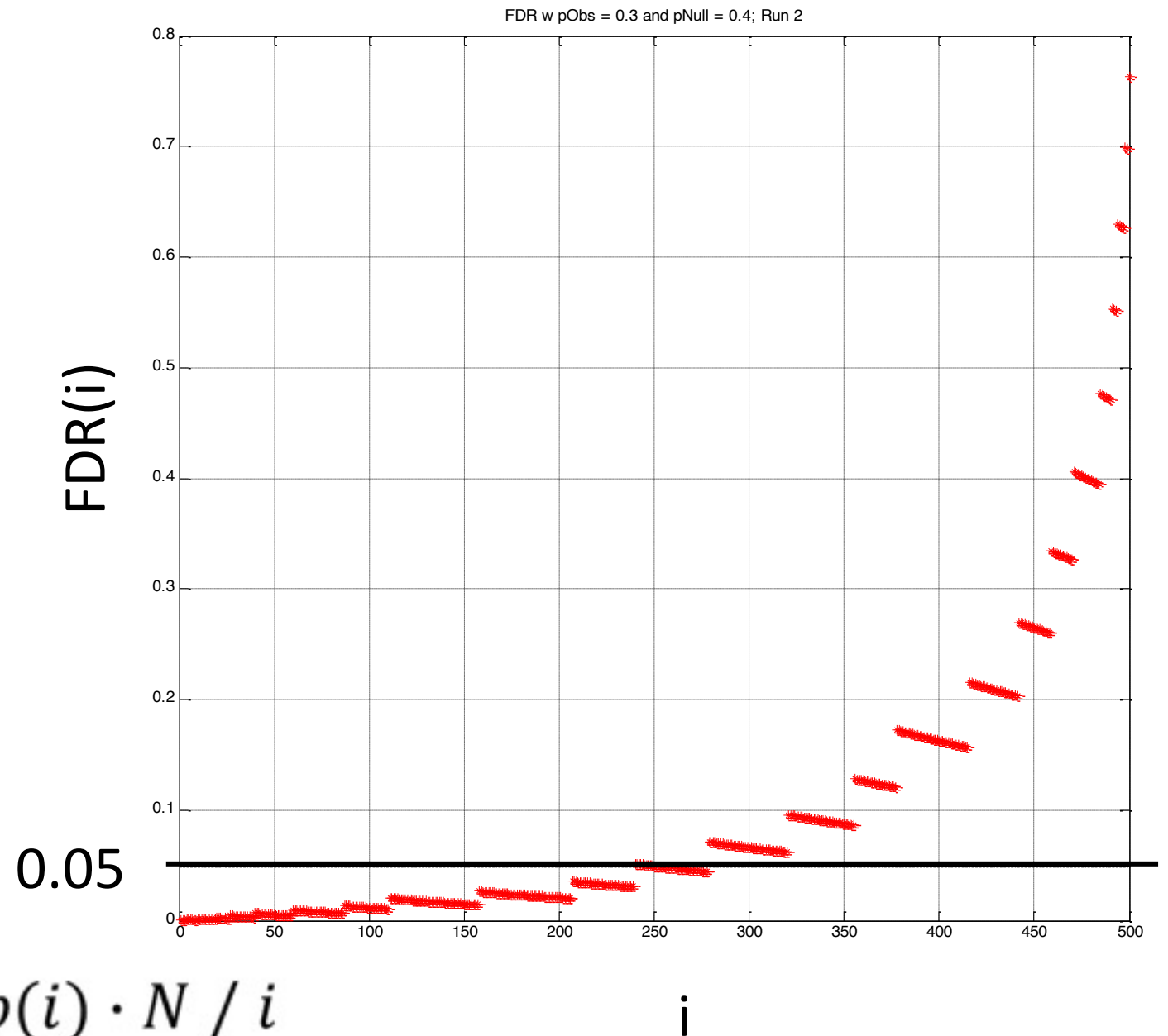


FDR

- Generating data using a coin with $p = 0.3$
- under a null model of $p = 0.4$



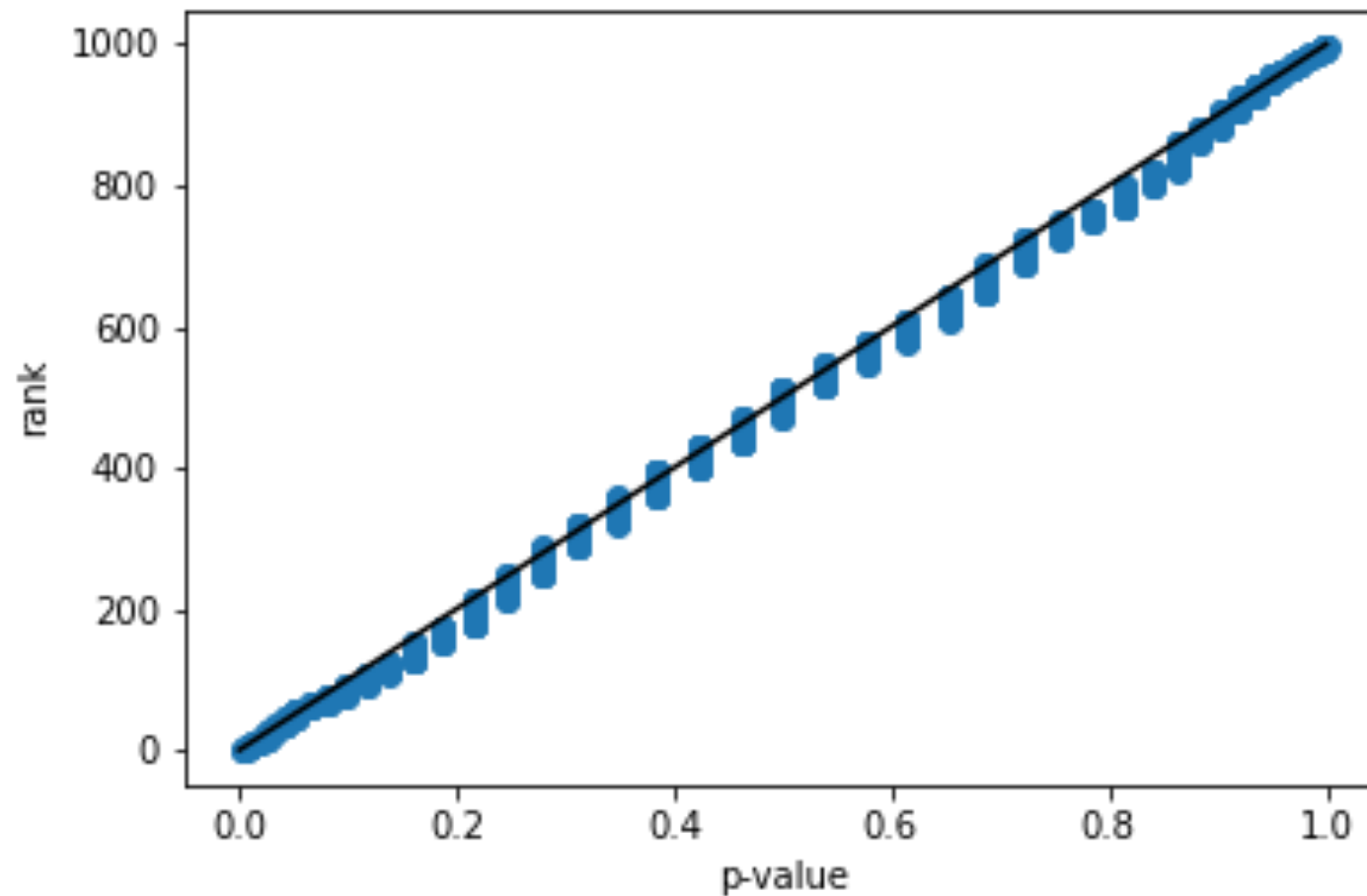
$$FDR(i) = p(i) \cdot N / i$$



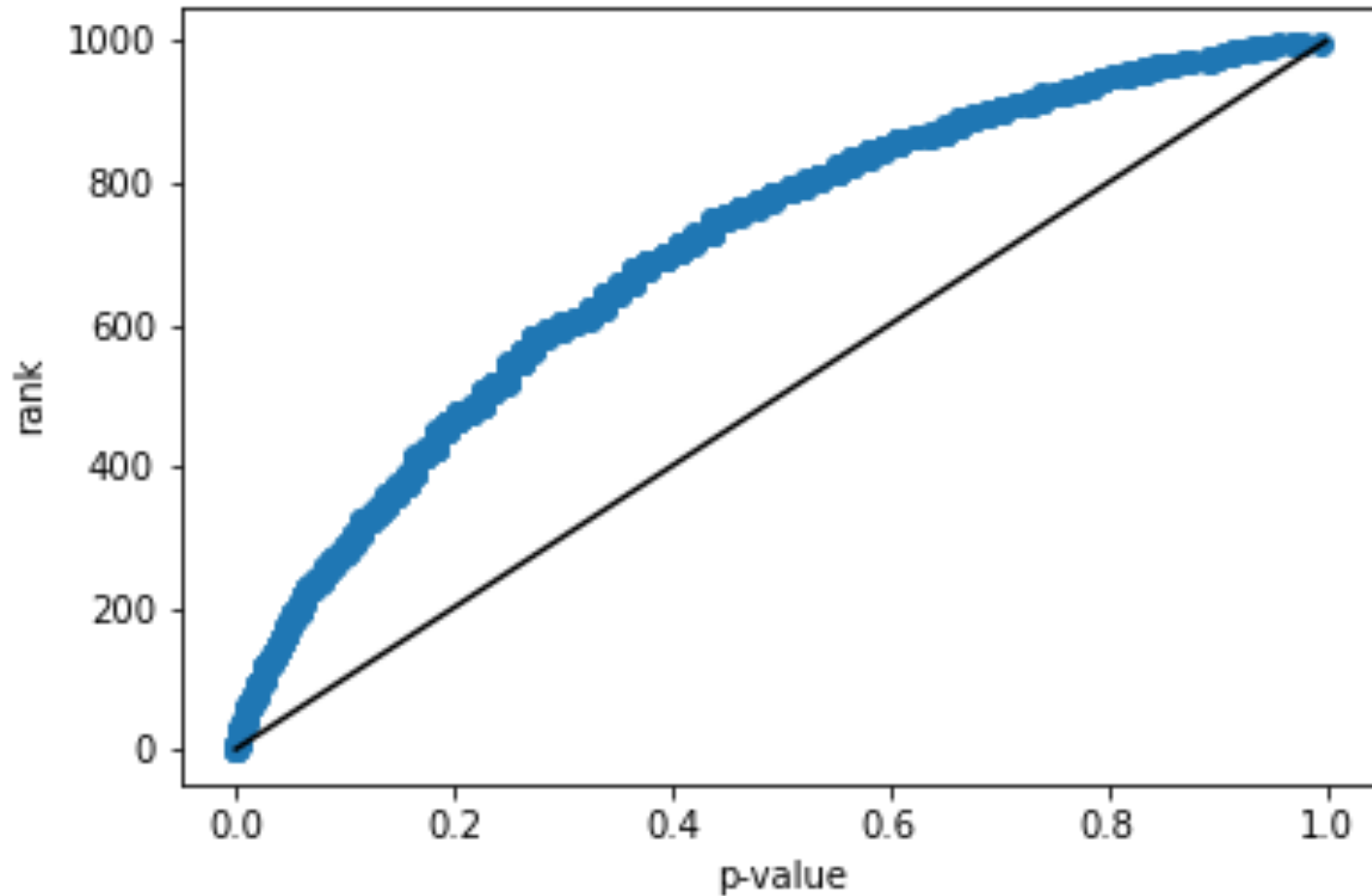
A more adequate correction

$$\text{RobFDR}(i) = \min_{j \geq i} \left(\frac{p(j) \cdot N}{j} \right)$$

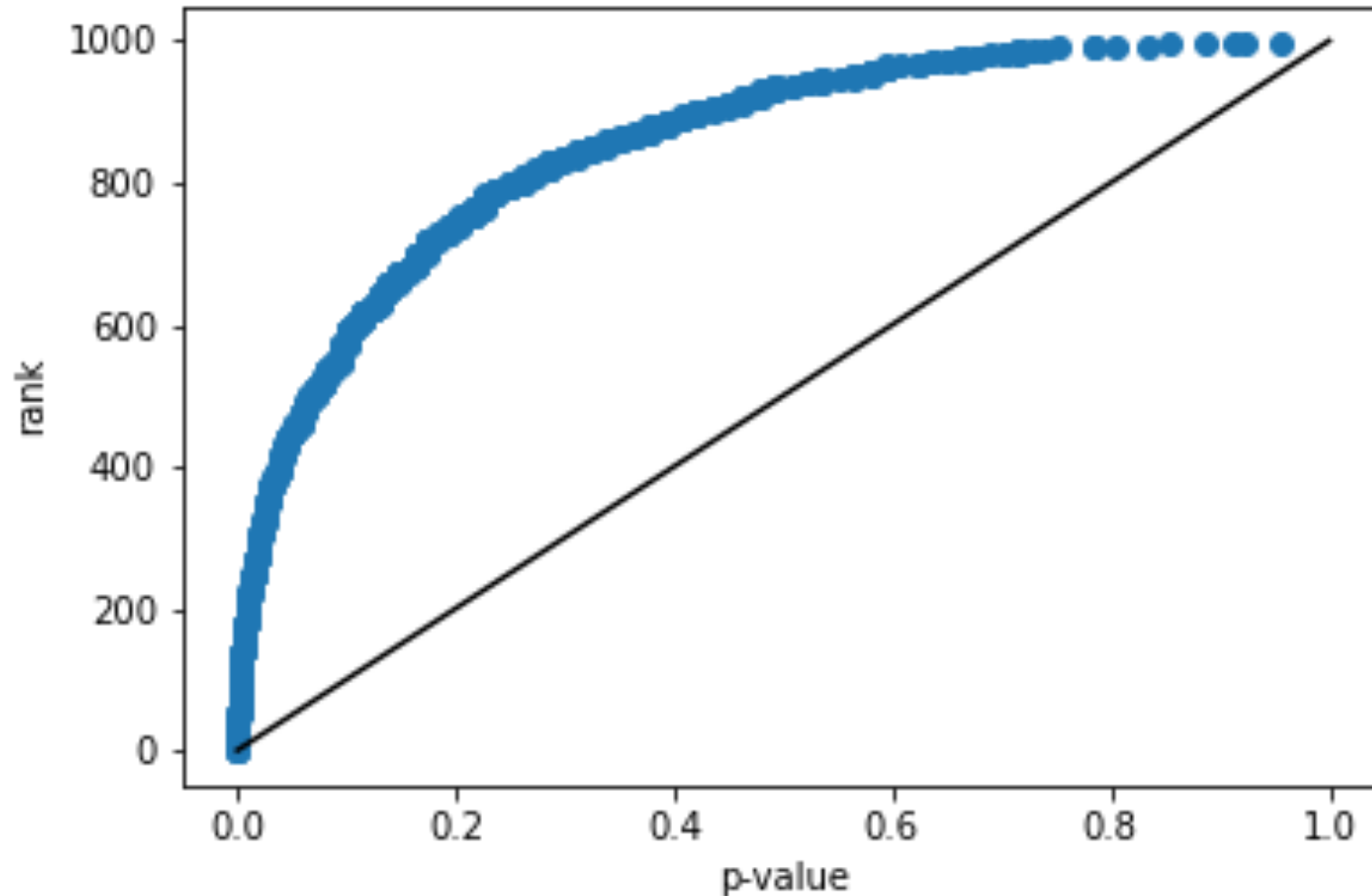
WRS FDR: $A \sim N(0,1)$ and $B \sim N(0,1)$, $n=20,20$



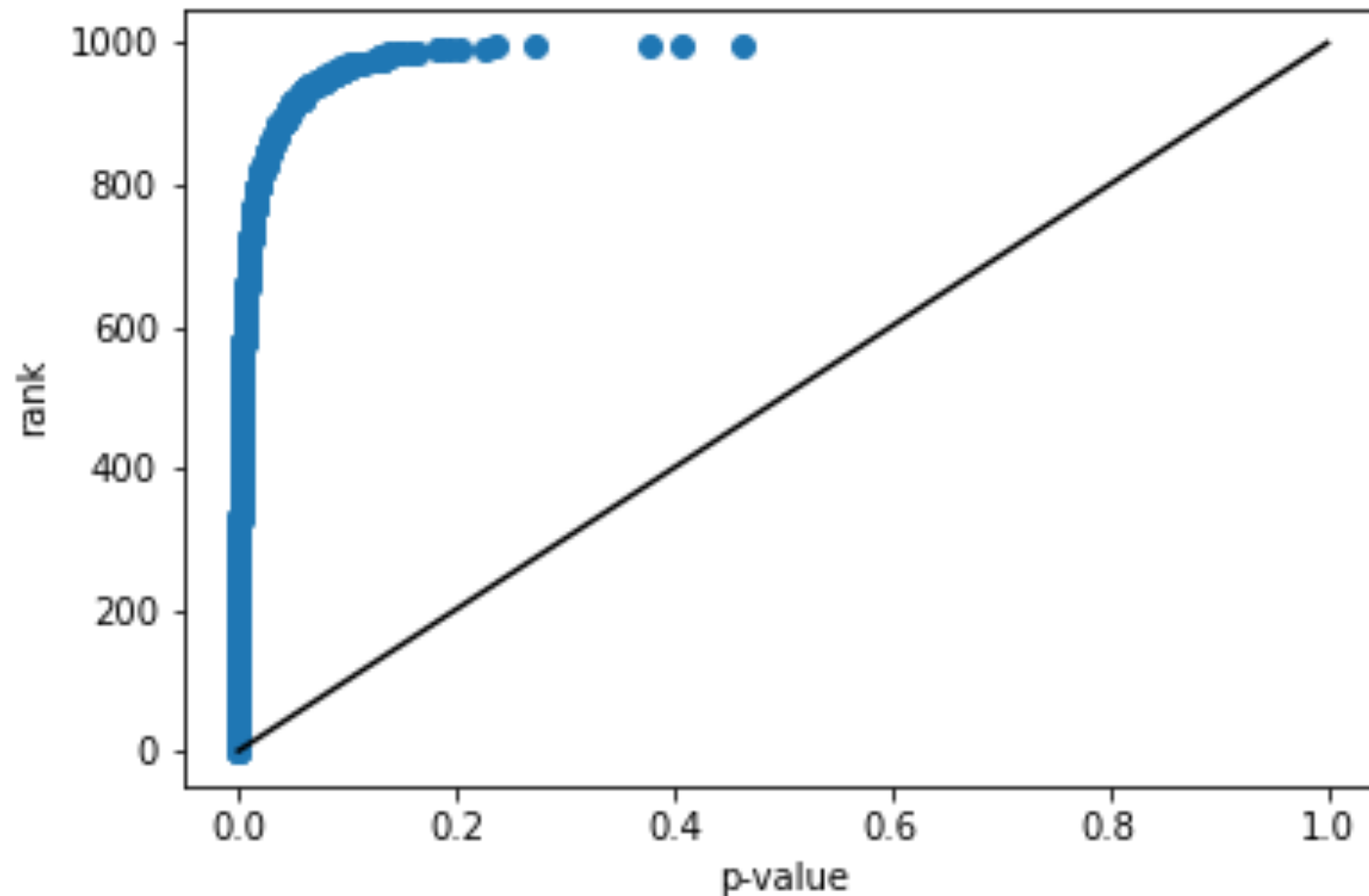
WRS FDR: $A \sim N(0,1)$ and $B \sim N(0.25,1)$, $n=20,20$



WRS FDR: $A \sim N(0,1)$ and $B \sim N(?,1)$, $n=20,20$



WRS FDR: $A \sim N(0,1)$ and $B \sim N(1,1)$, $n=20,20$



FDR – the procedure

Yoav Benjamini and
Yosef Hochberg
1995

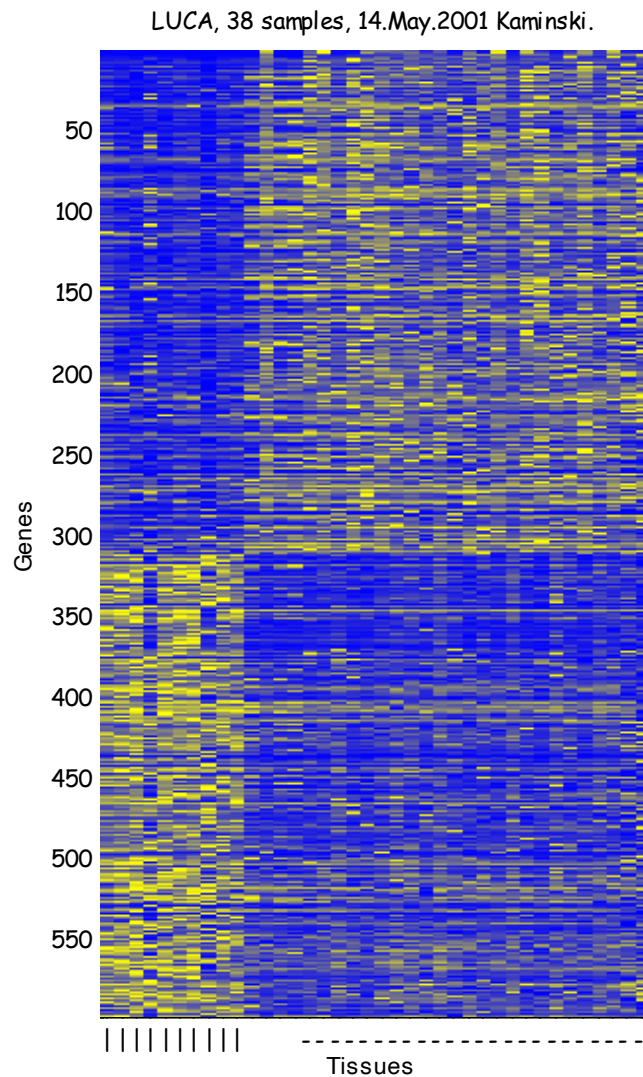


- Assume that we performed N measurements (comparisons, observations etc)
- Rank the computed significance of the findings:
 $p(1) \leq p(2) \leq p(3) \leq p(4) \leq \dots \leq p(N-1) \leq p(N)$
- Under the null model, the expected number of observations with p-value better than $p(i)$ is $p(i) \cdot N$
- The false discovery rate at i is therefore:

$$\text{RobFDR}(i) = \min_{j \geq i} \left(\frac{p(j) \cdot N}{j} \right)$$

- A corrected hypothesis testing in this case would be to find the max i that satisfies $\text{FDR}(i) \leq \tau$, where τ (e.g 0.05) is the required false discovery rate.

In the context of DGE ...



We observed (say) 200 genes
at FDR=0.05, using a WRS test

Multiple testing and FDR

- Greater data volumes require more careful inferential statistics approaches.
- Approaches to addressing multiple testing:
 - Bonferroni correction
 - Report FDR results
 - Simulations under a null

