Wavefront propagation for X-ray beamlines with SRW

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Outline:

- Prologue
- Brief introduction to scalar wave optics
- Free-space propagation with SRW
- Thin optical elements
- Thick optical elements
- Where to learn more?

"Synchrotron Radiation Workshop"
(electrodynamics simulation code for SR emission and propagation)

The project was started in **1997-98** by **Pascal Elleaume** and **Oleg Chubar** after completition of the **Radia** project. **First official version** of SRW was developed at ESRF (written in C++, interfaced to IGOR Pro); Chubar and Elleaume "Accurate and efficient computation of synchrotron radiation in the near field region", Proc. EPAC-98, 1177-1179 (1998).

The **main** open-source **repository (2012)**, containing all C/C++ sources, C API, all interfaces and project development files, is on **GitHub**:

https://github.com/ochubar/SRW

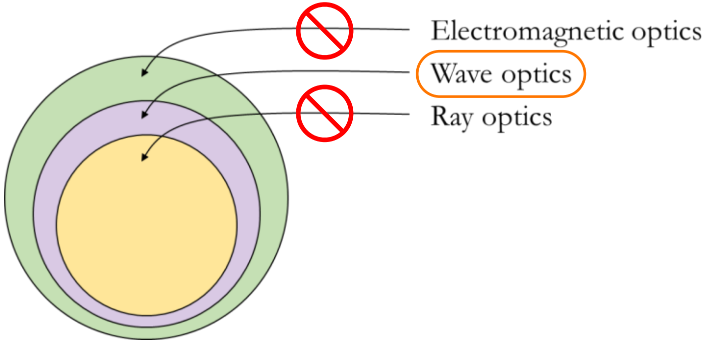
Prologue

Choosing an **adequate optical theory** for simulations:

“As simple as possible, as accurate as necessary.”

We use **physical optics** when **diffraction effects cannot be neglected**:

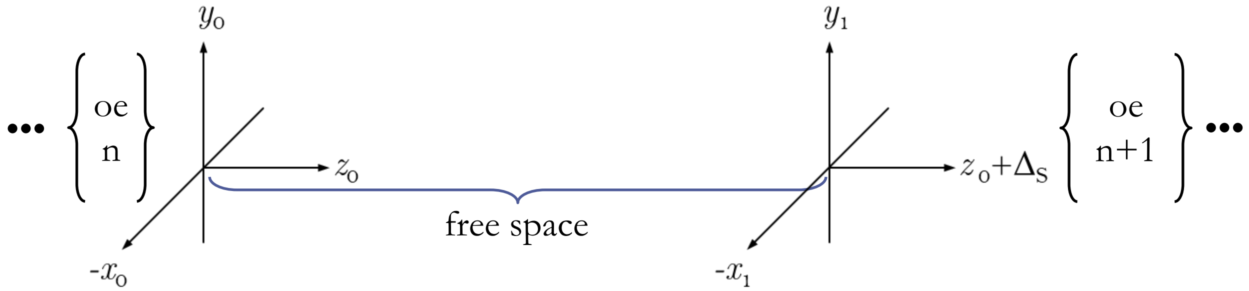
- IR, UV and soft x-ray beamlines in current and low-emittance light sources;
- whenever spatial filtering causes the increase in the coherent fraction (CF):
  - “*increasing the propagation distance*” – cf. van Cittert-Zenike theorem
  - “*closing the aperture to match the coherence length*”
  - “*slitting down the secondary source*”…



(a) hierarchical optical theories

Prologue

Wave-optical simulations deal with two classes of beamline elements:



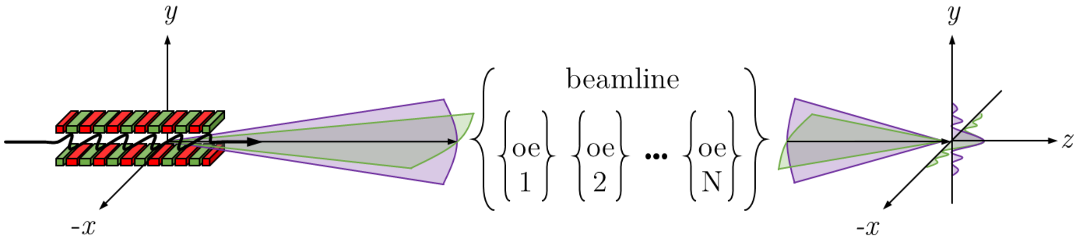
(a) generic beamline

→ (thin or thick) **optical elements** as transmission elements in **projection approximation;**

→ **free spaces** in between optical elements with near- and far-field **diffraction integrals** (e.g. Huygens-Fresnel, Fresnel or Fraunhofer integrals)**;**

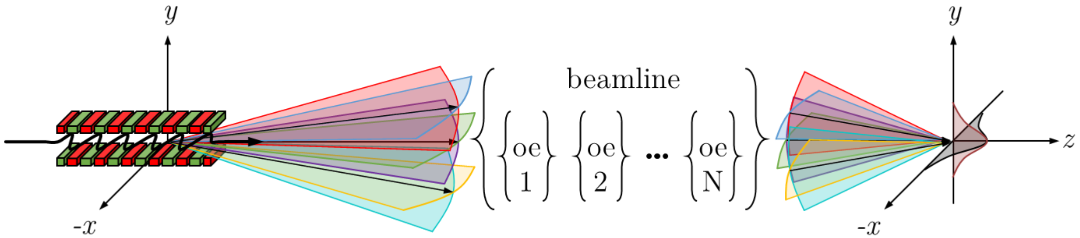
Prologue

Fully-coherent simulations (point source):



(a) fully-coherent simulation (filament beam, zero emittance, single electron...)

Partially-coherent simulations (extended source):



(b) partially-coherent simulation (thick beam, finite emittance, multi-electron...)

Scalar wave theory – how did we get here?

$$\nabla \times \boldsymbol{E}(x,y,z,t) = -\frac{\partial \boldsymbol{B}(x,y,z,t)}{\partial t}$$

$$\left(\varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} - \nabla^2\right) \boldsymbol{E}(x,y,z,t) = \boldsymbol{0}$$

1.) application of $\nabla \times \bullet$ to both sides of **Faraday's law** in an **uncharged** and **non-conducting medium.** Also remembering that $\nabla \times (\nabla \times \bullet) = \nabla(\nabla \cdot \bullet) - \nabla^2 \bullet$ results in the **d'Alembert wave equation.**

$$\left(\varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} - \nabla^2\right) u(x,y,z,t) = 0$$

$$u = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} U(x,y,z) \exp(-\omega t) \mathrm{d}t$$

2.) **each component** of $\boldsymbol{E}(x,y,z,t)$ **individually satisfies** the **wave equation.** The complex scalar **solution** of the **wave equation** can be **decomposed** as **superposition** of **monochromatic fields.**

$$(\nabla^2 + k) U(x,y,z) = 0$$

3.) this is the **Helmholtz equation.** Given a volume in space and boundary conditions, the scalar diffraction theory consists in finding solution to the Helmholtz equation.

$$\iiint_V (U \nabla^2 G - G \nabla^2 U)\, dv = \iint_S \left(U \frac{\partial G}{\partial n} - G \frac{\partial U}{\partial n}\right) ds$$

$$U(x,y,z) = \frac{1}{4\pi} \iint_S \left\{ \frac{\partial U}{\partial n} \left[\frac{\exp(jk|\vec{r}|)}{|\vec{r}|}\right] - U \frac{\partial}{\partial n} \left[\frac{\exp(jk|\vec{r}|)}{|\vec{r}|}\right] \right\} ds$$

4.) **Calculation** of $U(x,y,z)$ at an observation point in space can be done using **Green's theorem.** An **arbitrary** and **careful choice** of the **Green's function** G results in the **integral theorem of Helmholtz and Kirchhoff.** Yet another careful **choice** of **Green's function** and **boundary conditions** yield in the **Rayleigh-Sommerfeld** formulation of **diffraction.**

Paganin, *Coherent X-Ray Optics*, (2006)

Goodman, *Introduction to Fourier Optics*, (2017)

Scalar wave theory – Rayleigh-Sommerfeld formulation

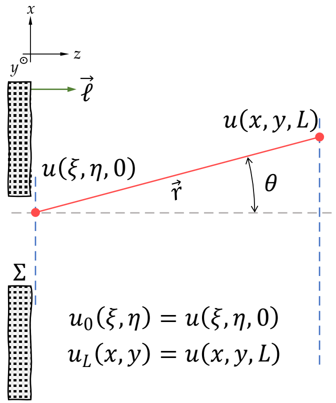
If $u_0(x, y)$ vanishes in the shadow of the screen and is undisturbed in Σ (aperture):

$$u_L(x,y) = -\frac{1}{2\pi}\iint_{\Sigma} u_0(\xi,\eta)\left(jk - \frac{1}{|\vec{r}|}\right)\frac{\exp(jk|\vec{r}|)}{|\vec{r}|}\cos(\vec{\ell},\vec{r})\mathrm{d}s\,,$$

with $|\vec{r}| = \sqrt{L^2 + (x-\xi)^2 + (y-\eta)^2}$, $\cos\left(\vec{\ell},\vec{r}\right) = L/|\vec{r}|$ and $\mathrm{d}s = \mathrm{d}\xi\mathrm{d}\eta$.

The above equation is a **convolution** between the input field and a kernel:

$$\begin{aligned} u_0(x,y) * h(x,y) &= \iint_{-\infty}^{\infty} u_0(\xi,\eta)\cdot h(x-\xi, y-\eta)\mathrm{d}\xi\mathrm{d}\eta \\ &= \mathcal{F}^{-1}\{\mathcal{F}\{u_0(x,y)\}\cdot\mathcal{F}\{h(x,y)\}\} \end{aligned}$$



(a) formulation of diffraction by a plane-screen

The kernel of the RS diffraction integral is given by:

$$h_{\mathrm{RS}}(x,y) = h_1 + h_2 = \frac{L}{j\lambda}\frac{\exp(\,jk|\vec{r}|)}{|\vec{r}|^2} + \frac{L}{2\pi}\frac{\exp(\,jk|\vec{r}|)}{|\vec{r}|^3}$$

For most cases of interest $|\vec{r}| \gg \lambda$ and hence, h can be approximated by h_1.

Goodman, *Introduction to Fourier Optics*, (2017)

Scalar wave theory – Huygens-Fresnel formulation

Applying $r \gg \lambda$ to the first Rayleigh-Sommerfeld solution:

$$u_L(x,y) = \frac{1}{j\lambda}\iint_{\Sigma} u_0(\xi,\eta)\,\frac{\exp(jk|\vec{r}|)}{|\vec{r}|}\,\frac{L}{|\vec{r}|}\,\mathrm{d}s$$

Which is a **convolution** with kernel given by:

$$h_{\mathrm{HF}}(x,y) = \frac{L}{j\lambda}\,\frac{\exp(\,jk|\vec{r}|)}{|\vec{r}|^2}$$

The propagated wave can be calculated as:

$$\boxed{u_L(x,y) = \mathcal{F}^{-1}\left\{\mathcal{F}\{u_0(x,y)\}\cdot\mathcal{F}\{h_{\mathrm{HF}}(x,y)\}\right\}}$$

The kernel $\mathcal{F}\{h_{\mathrm{HF}}(x,y)\}$ has no know closed form, so three FFTs are necessary for numerical evaluation. Using the convolution theorem with FFTs is computationally advantageous for numerical problems.

Goodman, *Introduction to Fourier Optics*, (2017)

Scalar wave theory – Huygens-Fresnel formulation

$$\boxed{u_L(x,y) = \mathcal{F}^{-1}\left\{\mathcal{F}\{u_0(x,y)\} \cdot \mathcal{F}\{h_{\mathrm{HF}}(x,y)\}\right\}}$$

"the split propagator/Huygens-Fresnel diffraction integral"

Short description: the **Huygens-Fresnel diffraction integral** is implemented by using the convolution theorem with **3x FFT**.

General use:

- strong (de)magnification systems (e.g. nano KB, FZP…) where the paraxial approximation is less recommended;

Comments:

- preserves number of pixel and ranges of the input plane;
- works for strongly astigmatic systems;
- recently implemented and not extensive used as the other propagators.

He et al, *Proc. SPIE* 12697(03) (2023)

Scalar wave theory – the Fresnel diffraction

"standard"

The **paraxial approximation** of the Huygens-Fresnel integral can be obtained by using the binomial expansion to the square root in $|\vec{r}|$ provided that $L^2 \gg (x-\xi)^2$, $L^2 \gg (y-\eta)^2$:

$$u_L(x,y) = \frac{\exp(jkL)}{j\lambda L} \iint_{\Sigma} u_0(\xi,\eta) \exp\left\{\frac{jk}{2L}\left[(x-\xi)^2 + (y-\eta)^2\right]\right\} \mathrm{d}s$$

The diffraction integral above is a **convolution** between the input field u_0 and a kernel h_F :

$$h_\mathrm{F}(x,y) = \frac{\exp(jkL)}{j\lambda L} \exp\left[\frac{jk}{2L}(x^2+y^2)\right],$$

which has analytical Fourier transform $H_\mathrm{F}(f_x,f_y) = \exp(jkL)\exp\left[-j\pi\lambda L\left(f_x^2+f_y^2\right)\right]$. Hence, the calculation of the Fresnel diffraction integral is done using two FFT operations

$$\boxed{u_L(x,y) = \mathcal{F}^{-1}\left\{\mathcal{F}\{u_0(x,y)\} \cdot H_\mathrm{F}(f_x,f_y)\right\}}$$

Goodman, *Introduction to Fourier Optics*, (2017)

Scalar wave theory – the Fresnel diffraction

"standard"

$$\boxed{u_L(x,y) = \exp(jkL) \cdot \mathcal{F}^{-1}\{\mathcal{F}\{u_0(x,y)\} \cdot \exp[-j\pi\lambda L(f_x^2 + f_y^2)]\}}$$

"Standard Fresnel"

Short description: the **standard Fresnel** diffraction integral is implemented by using the convolution theorem with **2x FFT**.

General use:

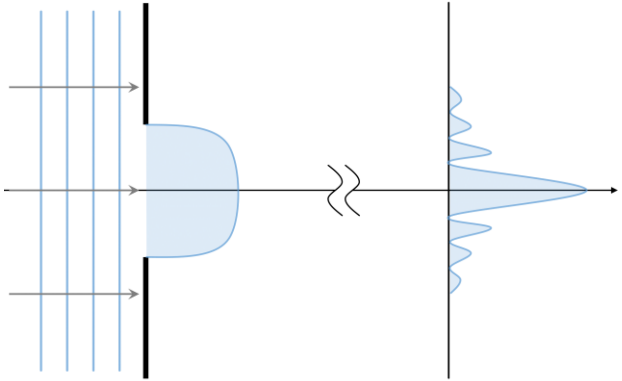
- propagation over free-space with gentle (de)magnification;
- to be used before slits, ideal lenses and smooth phase elements;

Comments:

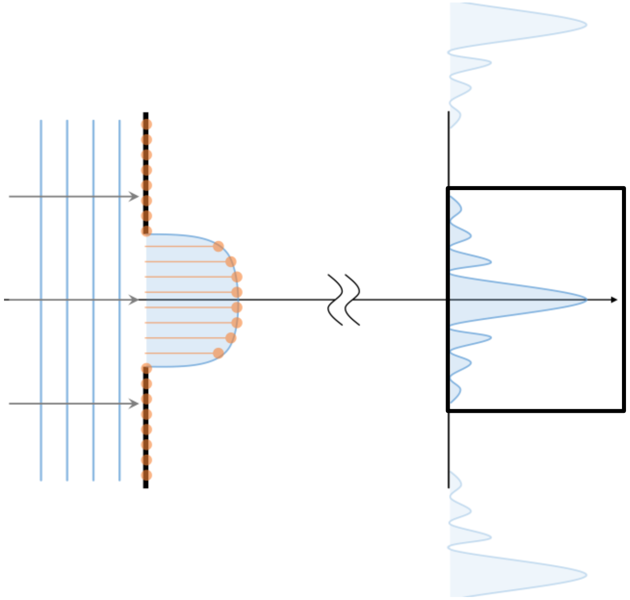
- preserves number of pixel and ranges of the input plane;
- works for strongly astigmatic systems;

Scalar wave theory – the Fresnel diffraction

Issues: when calculating numerically the convolution-type or the Fourier transformation-type integrals replicas and aliasing occur.



(a) analytical solution

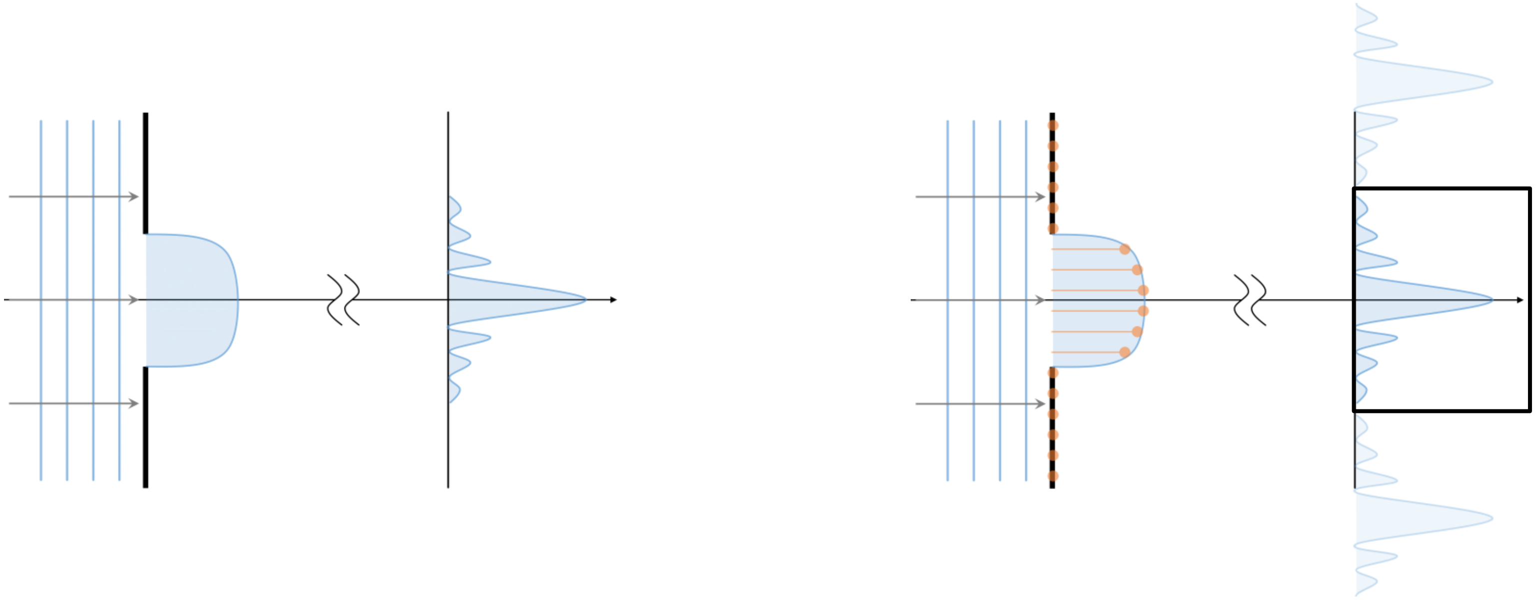


(b) numerical calculation with sampling artifacts (replicas)

Kelly, *J. Opt. Soc. Am. A* 31(4), 755 (2014)

Scalar wave theory – the Fresnel diffraction

Issues: when calculating numerically the convolution-type or the Fourier transformation-type integrals replicas and aliasing occur.



(a) analytical solution

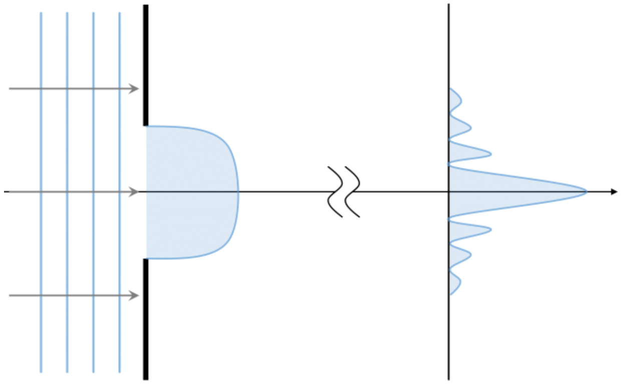
(b) numerical calculation with sampling artifacts (replicas)

Kelly, *J. Opt. Soc. Am. A* 31(4), 755 (2014)

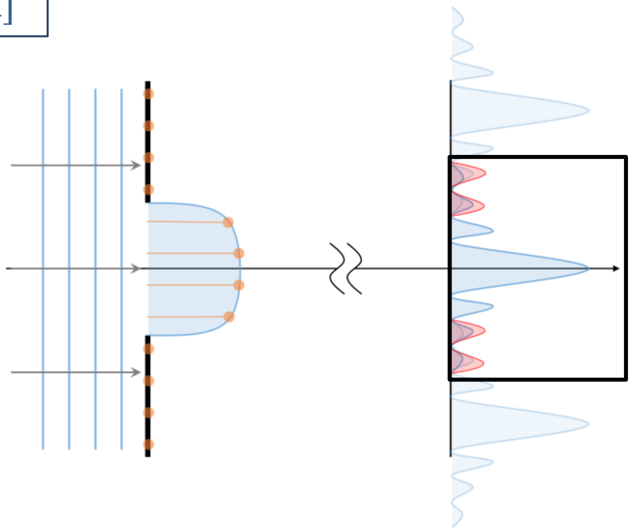
Scalar wave theory – the Fresnel diffraction

Issues: when calculating numerically the convolution-type or the Fourier transformation-type integrals replicas and aliasing occur.

$$\boxed{N = [p \cdot \Delta\Omega^2 \cdot R/\lambda]}$$



(a) analytical solution



(b) numerical calculation with sampling artifacts (replicas)

Kelly, *J. Opt. Soc. Am. A* 31(4), 755 (2014)

Scalar wave theory – the Fresnel diffraction

"quadratic term"

Assuming the input field has a **quadratic phase term** defined by the wavefront curvature (R_x, R_y) centred at (x_0, y_0):

$$u_0(x,y) = g_0(x,y) \cdot \exp\left\{\frac{jk}{2}\left[\frac{(x-x_0)^2}{R_x} + \frac{(y-y_0)^2}{R_y}\right]\right\},$$

The Fresnel diffraction integral can be re-written as:

$$u_L(x,y) = \exp\left\{\frac{ik}{2}\left[\frac{(x-x_0)^2}{(R_x+L)} + \frac{(y-y_0)^2}{(R_y+L)}\right]\right\} \cdot$$
$$\frac{\exp(jkL)}{j\lambda L} \cdot \iint_\Sigma g_0(\xi,\eta)\exp\left\{\frac{ik}{2L}\left[\frac{R_x+L}{R_x}\left(\frac{R_x x + L x_0}{R_x+L} - \xi\right)^2 + \frac{R_y+L}{R_y}\left(\frac{R_y y + L y_0}{R_y+L} - \eta\right)^2\right]\right\} \mathrm{d}s$$

Which is a **convolution** integral with the reduced coordinates:

$$(\hat{x}, \hat{y}) = \left(\frac{R_x x + L x_0}{R_x + L}, \frac{R_y y + L y_0}{R_y + L}\right)$$

Chubar and Celestre, *Opt. Express* 27(20), 28750 (2019)

Scalar wave theory – the Fresnel diffraction

"quadratic term"

The kernel of the transformation as a function of the reduced coordinates is given by:

$$h_{\mathrm{QPT}}(x,y) = \frac{\exp(jkL)}{j\lambda L}\exp\left[\frac{ik}{2L}\left(\frac{R_x+L}{R_x}\hat{x}^2 + \frac{R_y+L}{R_y}\hat{y}^2\right)\right],$$

with analytical Fourier transform $H_{\mathrm{QPT}}\left(f_x, f_y\right)$:

$$H_{\mathrm{QPT}}\left(f_x, f_y\right) = \exp(jkL)\cdot\sqrt{\frac{R_xR_y}{(R_x+L)\left(R_y+L\right)}}\cdot\exp\left[-j\pi L\lambda\left(\frac{R_x}{R_x+L}f_x^2 + \frac{R_y}{R_y+L}f_y^2\right)\right]$$

The calculation of the **Fresnel transform** with the **analytical treatment of the quadratic phase term:**

$$\boxed{u_L(x,y) = \exp\left\{\frac{ik}{2}\left[\frac{(x-x_0)^2}{(R_x+L)} + \frac{(y-y_0)^2}{\left(R_y+L\right)}\right]\right\}\cdot\mathcal{F}^{-1}\left\{\mathcal{F}\{g_0(x,y)\}\cdot H_{\mathrm{QPT}}\left(f_x,f_y\right)\right\}}$$

Chubar and Celestre, *Opt. Express* 27(20), 28750 (2019)

Scalar wave theory – the Fresnel diffraction

"quadratic term"

$$u_L(x,y) = \exp\left\{\frac{ik}{2}\left[\frac{(x-x_0)^2}{(R+L)} + \frac{(y-y_0)^2}{(R+L)}\right]\right\} \cdot \mathcal{F}^{-1}\{\mathcal{F}\{g_0(x,y)\} \cdot H_{\mathrm{QPT}}(f_x, f_y)\}$$

"Fresnel with analytical treatment of the quadratic (leading) phase terms"

Short description: the **quadratic phase term** of the input wave is removed, relaxing sampling requirements. This implementation of the Fresnel diffraction integral uses the convolution theorem with **2x FFT**.

General use:

- most prolific SRW propagator – being used with a wide range of free-space propagation;
- to be used before complex optical elements (e.g. transmission elements, curved mirrors…);
- can be used for propagation *from* and *to* image planes;

Comments:

- preserves number of pixel but the ranges are re-scaled as $\Delta x_L = \Delta x_0 (R+L)/R$ and $\Delta y_L = \Delta y_0 (R+L)/R$;
- works for strongly astigmatic systems;
- has singularities when $R \approx -L$.

Chubar and Celestre, *Opt. Express* 27(20), 28750 (2019)

Scalar wave theory – the Fresnel diffraction

"quadratic term – special"

$$u_L(x,y) = \exp\left\{\frac{ik}{2}\left[\frac{(x-x_0)^2}{(R_x+L)} + \frac{(y-y_0)^2}{\left(R_y+L\right)}\right]\right\} \cdot \mathcal{F}^{-1}\{\mathcal{F}\{g_0(x,y)\} \cdot H_{\mathrm{QPT}}\left(f_x,f_y\right)\}$$

"Fresnel with analytical treatment of the quadratic (leading) phase terms, yet with different processing near a waist"

Short description: the **quadratic phase term** of the input wave is removed, relaxing sampling requirements. This implementation of the Fresnel diffraction integral uses the convolution theorem with **2x FFT** but **with a different estimation of R_x and R_y near the beam waist.**

General use:

- to be used before complex optical elements (e.g. transmission elements, curved mirrors…);
- can be used for propagation *from* and *to* image planes (e.g. very small slits);
- strong diffracting elements

Comments:

- preserves number of pixel but the ranges are re-scaled;
- works for strongly astigmatic systems;
- has singularities when $R_x \approx -L$ and $R_y \approx -L$.

Chubar and Celestre, *Opt. Express* 27(20), 28750 (2019)

Scalar wave theory – the Fraunhofer diffraction

"from waist"

Expanding the quadratic terms in the exponential function and collecting terms in the standard Fresnel propagator:

$$u_L(x,y) = \frac{\exp(jkL)}{j\lambda L} \exp\left[\frac{jk}{2L}(x^2 + y^2)\right] \iint_\Sigma u_0(\xi,\eta) \cdot \exp\left[\frac{jk}{2L}(\xi^2 + \eta^2)\right] \cdot \exp\left\{-\frac{jk}{L}(x\xi + y\eta)\right\} \mathrm{d}s$$

For sufficiently large propagation distances, i.e. $a^2/\lambda L \ll 1$, we arrive at the **Fraunhofer** regime:

$$u_L(x,y) = \frac{\exp(jkL)}{j\lambda L} \exp\left[\frac{jk}{2L}(x^2 + y^2)\right] \iint_\Sigma u_0(\xi,\eta) \cdot \exp\left\{-\frac{jk}{L}(x\xi + y\eta)\right\} \mathrm{d}s$$

which, apart from the multiplicative factors, is the Fourier transform of the input field $u_0(x,y)$ evaluated at frequencies $f_x = x/\lambda L$ and $f_y = y/\lambda L$:

$$\boxed{u_L(x,y) \propto \mathcal{F}\{u_0(x,y)\}}$$

Goodman, *Introduction to Fourier Optics*, (2017)

Scalar wave theory – the Fraunhofer diffraction

“from waist”

$$\boxed{u_L(x,y) \propto \mathcal{F}\{u_0(x,y)\}}$$

“For propagation from a waist over a ~large distance”

Short description: propagator based on the far-field approximation (Fraunhofer) using 1x FFT.

General use:

- propagation of a wavefront emerging from a focal plane in both vertical and horizontal directions – *from a waist to a ~large distance*;
- output plane several times larger than the input plane (e.g. scattering experiments);

Comments:

- preserves number of pixel but the ranges are re-scaled;
- fails for astigmatic systems.

Scalar wave theory – the optical Fourier transform

“to waist”

We consider now a converging wavefront $u_0(x, y) = g_0(x, y) \cdot \exp[-jk(x^2 + y^2)/2q]$, where q is the distance between wave and focusing plane. We then plug it into the Fresnel integral:

$$u_L(x, y) = \frac{\exp(jkL)}{j\lambda L} \exp\left[\frac{jk}{2L}(x^2 + y^2)\right] \cdot$$
$$\iint_\Sigma g_0(\xi, \eta) \cdot \exp\left[-\frac{jk}{2q}(\xi^2 + \eta^2)\right] \cdot \exp\left[\frac{jk}{2L}(\xi^2 + \eta^2)\right] \cdot \exp\left\{-\frac{jk}{L}(x\xi + y\eta)\right\} \mathrm{d}s$$

If the wavefront $u_0(x, y)$ is propagated to the image plane, that is, if $L = q$, the above equation becomes:

$$u_L(x, y) = \frac{\exp(jkL)}{j\lambda L} \exp\left[\frac{jk}{2L}(x^2 + y^2)\right] \cdot \iint_\Sigma g_0(\xi, \eta) \cdot \exp\left\{-\frac{jk}{L}(x\xi + y\eta)\right\} \mathrm{d}s$$

Which is a Fourier transform of the field $g_0(x, y)$:

$$\boxed{u_L(x, y) \propto \mathcal{F}\{g_0(x, y)\}}$$

Goodman, *Introduction to Fourier Optics*, (2017)

Scalar wave theory – the optical Fourier transform

"to waist"

$$\boxed{u_L(x,y) \propto \mathcal{F}\{g_0(x,y)\}}$$

"For propagation over some distance to a waist"

Short description: propagator based on the **Fourier transforming property** of a lens using **1x FFT**.

General use:

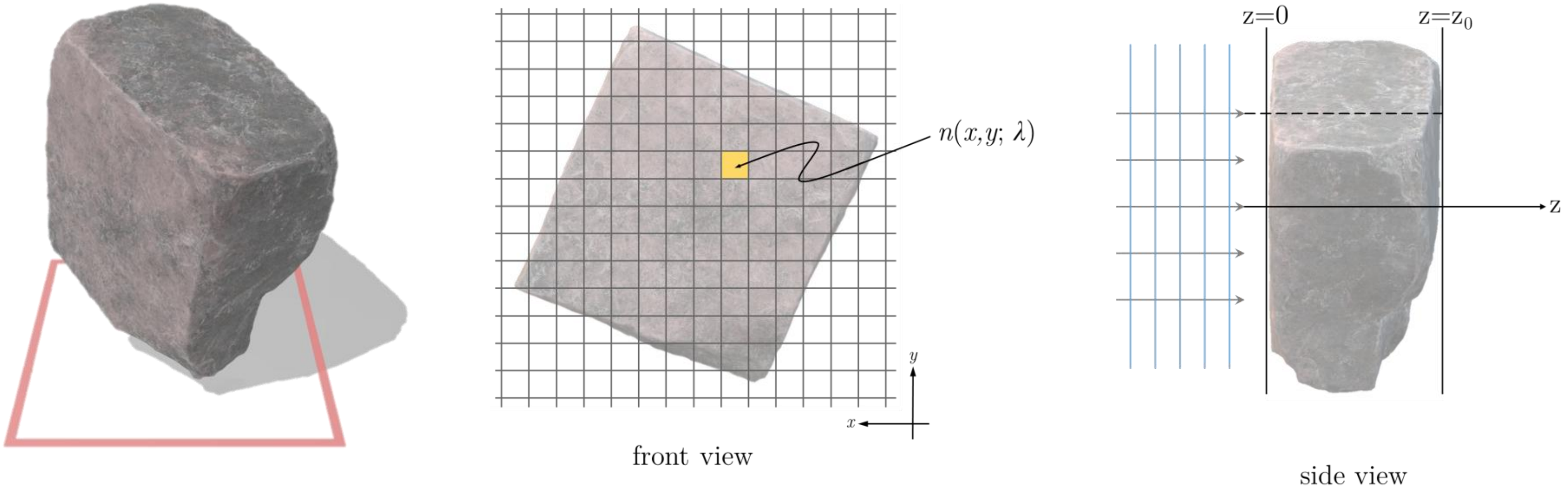
- propagation of a wavefront to a waist (image plane);
- output plane several times smaller than the input plane;

Comments:

- preserves number of pixel but the ranges are re-scaled;
- fails for astigmatic systems.

Thin optical elements

Consider the following 3D scattering element:



(a) representation of a weak 3D scattering volume

Paganin, *Coherent X-Ray Optics*, (2006)

Thin optical elements

If the scatterer is **sufficiently weak** as to **minimally disturb the path** that the wave-field would have taken in its absence:

$$u_{\mathrm{thin}}(x,y) \approx \exp\left\{-\frac{jk}{2}\int_{z=0}^{z=z_0}[1-n^2(x,y)]\,\mathrm{d}z\right\}u_0(x,y)$$

Since $n = 1-\delta + j\cdot\beta$ and because $1-\delta \approx 1$ we can approximate: $1-n^2 \approx 2(\delta + j\cdot\beta\,)$:

$$u_{\mathrm{thin}}(x,y) \approx \exp\{-jk[\delta(x,y)+j\cdot\beta(x,y)]\Delta_z(x,y)\}\,u_0(x,y)$$

The complex transmission operator in projection approximation can be written as:

$$T[\Delta_z(x,y)] = \exp\{-jk[\delta(x,y)+j\cdot\beta(x,y)]\Delta_z(x,y)\} = \sqrt{T_{\mathrm{BL}}(\Delta_z)}\cdot\exp[j\phi(\Delta_z)]$$

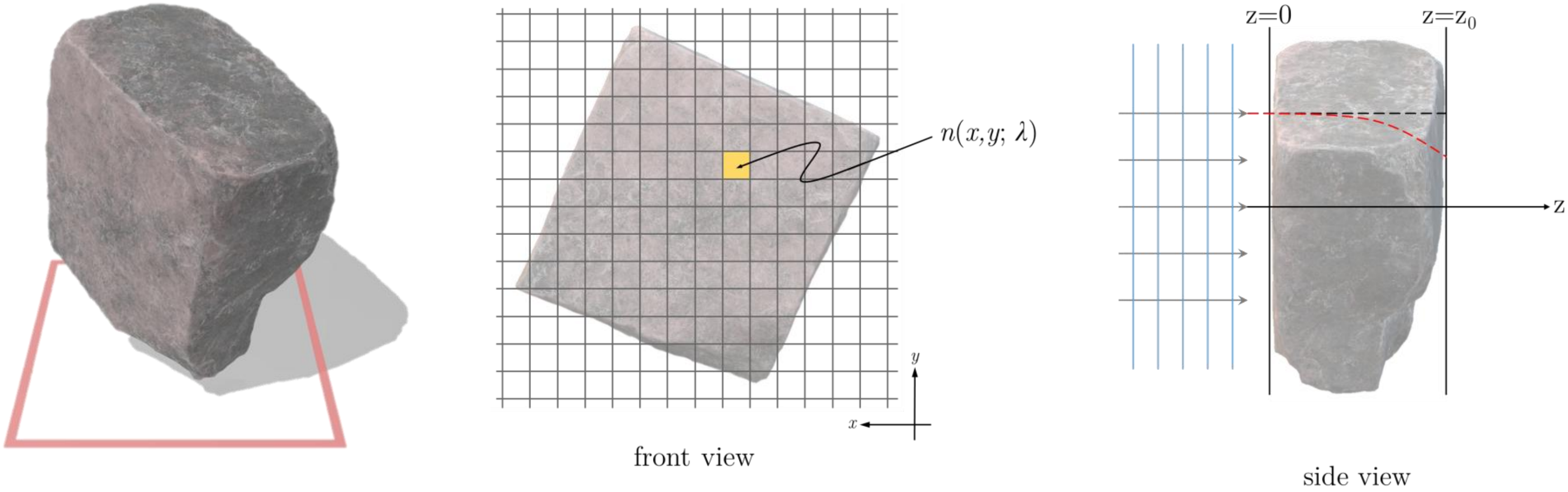
Finally:

$$\boxed{u_{\mathrm{thin}}(x,y) = T[\Delta_z(x,y)]\cdot u_0(x,y)}$$

Paganin, *Coherent X-Ray Optics*, (2006)

Thick optical elements

We consider now the following 3D scattering element:



(a) representation of thick scattering volume

Paganin, *Coherent X-Ray Optics*, (2006)

Thick optical elements

A **thick** optical **element** can be **sliced** into a number N of parallel slabs **until the projection approximation holds** between two adjacent slices **separated by vacuum.**

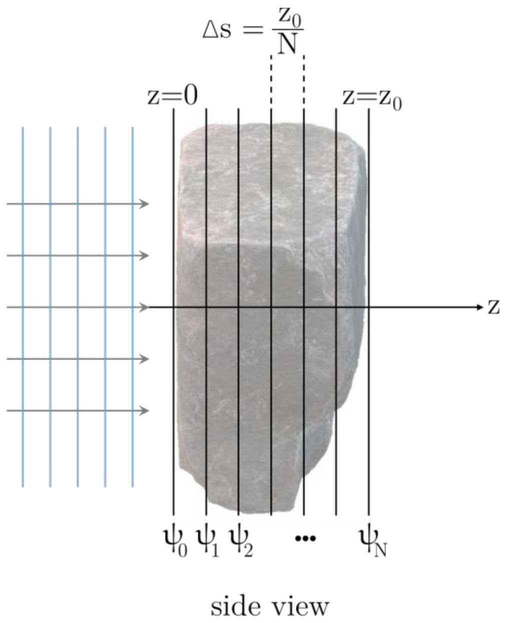
The wave propagation through this thick element is given by:

$$\boxed{u_{MS}(x,y) = \prod_{i=1}^{N} \mathcal{D}(\Delta s) \cdot [T_i(\Delta_z) \cdot u_0\,(x,y)]}$$

Where $\mathcal{D}(\Delta s)$ is an operator form of a free-space propagator over Δs.

See also:

- Li, Wojcik and Jacobsen, *Opt. Express* 25(3), 1831 (2017)
- Munro, *J. Opt. Soc. Am. A* 36(7), 1197 (2019)



(a) representation of a tick scattering volume as a series of thin elements (multi-slicing)

Paganin, *Coherent X-Ray Optics*, (2006)

Thick optical elements

Alternatively, a **thick** optical **element** (e.g. grazing incidence mirrors) can be represented by:

$$u_{\text{thick}}(x, y, x^{-}, y^{-}) \approx \boldsymbol{G}(x, y) \cdot \exp[jk \cdot \Lambda(x, y)]\, \delta_D[x^{-} - u(x, y)]\delta_D[y^{-} - v(x, y)] \cdot u_0(x^{-}, y^{-})$$

where the superscripts '-' indicate 'before interaction' and:

- $\mathbf{G}(x, y)$ is a matrix function defining local transformations of the electric field components;
- $\Lambda(x, y)$ is a scalar function defining the corresponding optical path difference;
- $\delta_D[x^{-} - u(x, y)]$ and $\delta_D[y^{-} - v(x, y)]$ are scalar functions defining the transformation of the coordinates for points in the transverse planes before and after the optical element.

Algorithms for their numerical calculation can be found by using the **stationary phase method** and/or by **applying** (locally) the laws of **geometrical optics and boundary conditions** for the electric field components

See also:

- Zachariasen, *Theory of X-Ray Diffraction in Crystals* (1945)
- Sutter, Chubar and Suvorov, *Proc. SPIE* 9202(3), 1831 (2014)
- Sutter et al, *Proc. SPIE* 11493(0V), 1831 (2020)

Canestrari, Chubar and Reininger, *J Synchrotron Rad* 21(5), 1110–1121 (2014)

Where to learn more?

SRW:

- Chubar, "Precise computation of electron-beam radiation in nonuniform magnetic fields as a tool for beam diagnostics," *Review of Scientific Instruments* 66(2), 1872–1874 (1995)
- Chubar and Elleaume, "Accurate And Efficient Computation Of Synchrotron Radiation In The Near Field Region", *Proc. EPAC98* 1177-1179 (1998)
- Chubar et al, "Development of partially-coherent wavefront propagation simulation methods for 3rd and 4th generation synchrotron radiation *sources*", *Proc. SPIE* 8141, (2011)
- Laundy et al, "Partial coherence and imperfect optics at a synchrotron radiation source modeled by wavefront propagation", *Proc. SPIE* 9209, (2014)
- Chubar et al, ""Main functions, recent updates, and applications of Synchrotron Radiation Workshop code", *Proc. SPIE* 10388, (2017)
- Li and Chubar, "Memory and CPU efficient coherent mode decomposition of partially coherent synchrotron radiation with subtraction of common quadratic phase terms," *Opt. Express* 30(4), 5896 (2022)
- Chubar et al, "Physical optics simulations for synchrotron radiation sources," *J. Opt. Soc. Am. A* 39(12), (2022)

Physical optics:

- Goodman, *Introduction to Fourier Optics*, (2017)
- Paganin, *Coherent X-Ray Optics*, (2006)
- Southwell, "Validity of the Fresnel approximation in the near field," *J. Opt. Soc. Am.* 71(1), 7 (1981)
- Rees, "The validity of the Fresnel approximation," *Eur. J. Phys.* 8(1), 44–48 (1987)
- Kelly, "Numerical calculation of the Fresnel transform," *J. Opt. Soc. Am. A* 31(4), 755 (2014)

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