Least squares method

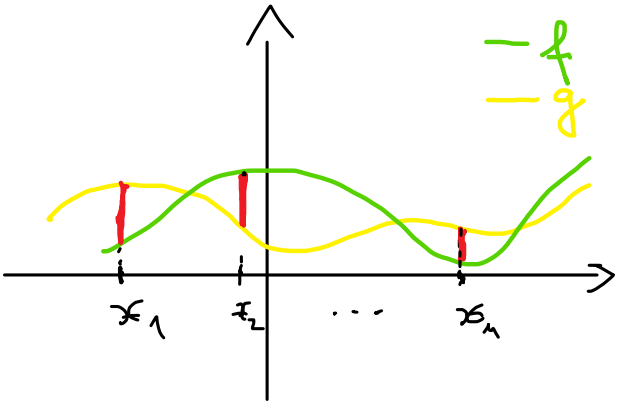
$\langle f, g \rangle$ = inner/dot product of f and g

(l.g.) $= \sum_{k=1}^{m} f(x_k) \cdot g(x_k)$

$\|f\|$ = norm of f

$= \sqrt{\langle f, f \rangle}$

(l.g.) $= \sqrt{\sum_{k=1}^{m} f^2(x_k)}$



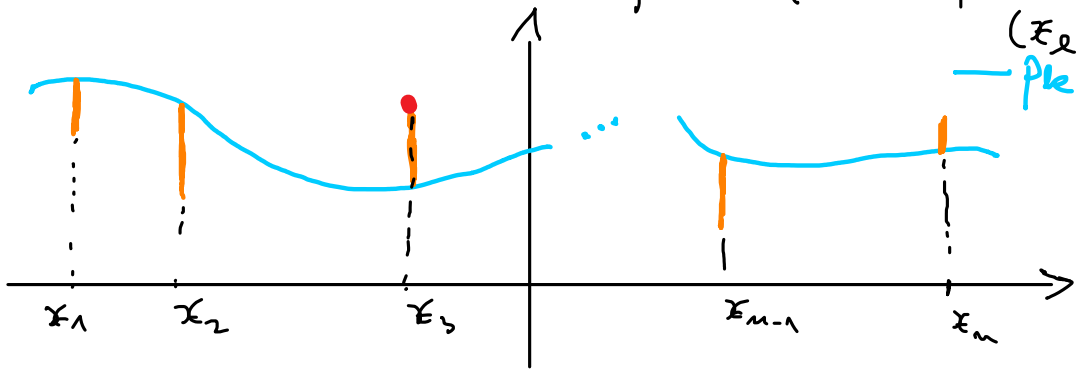
$\|f - g\|$ = distance between f and g

(l g.) $= \sqrt{\sum_{k=1}^{m} (f(x_k) - g(x_k))^2}$

(?) Find pol. p_k of deg. $\leq k$ s.t.

$\|f - p_k\| = \min_{p \in \mathbb{P}_k} \|f - p\|$ is minimal,

where $\mathbb{P}_k$ is the space of all pol. of deg. $\leq k$.



Input: x → nodes
f → values at x
k → degree (us. low)

Output: coeffs of p_k pol. that best fits the points in the sense of least square

$p_n(X) = a_k \cdot X^k + \ldots + a_1 X + a_0$

$\langle f, g \rangle = \sum_{k=1}^{n} f(x_k) \cdot g(x_k)$