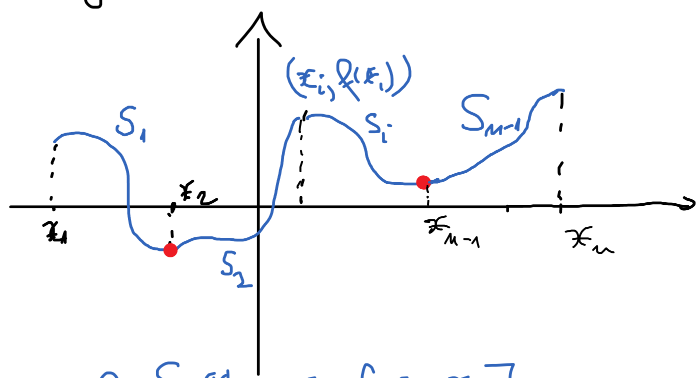
Lab 6

Monday, March 29, 2021 2:43 PM

Cubic splines

(deg. ≤ 3)



cubic spline $\leftarrow S(x) = \begin{cases} S_1(x), & x \in [x_1, x_2] \\ S_2(x), & x \in [x_2, x_3] \\ \vdots \\ S_{n-1}(x), & x \in [x_{n-1}, x_n] \end{cases}$ s.t. $S \in C^2_{[x_1, x_n]}$ (S, S', S'' are cont.)

$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$, $x \in [x_i, x_{i+1}]$.

$S_i(x_i) = f(x_i)$, $S_i(x_{i+1}) = f(x_{i+1})$, $i = \overline{1, n-1}$

$S_i'(x_{i+1}) = S_{i+1}'(x_{i+1})$, $S_i''(x_{i+1}) = S_{i+1}''(x_{i+1})$, $i = \overline{1, n-2}$

Number of conditions: $2(n-1) + (n-2) + (n-2) = 4n - 6$

Number of unknown coeffs: $4(n-1) = 4n - 4$

We need two more conditions:

- natural spline: 2nd derivatives at first and last nodes are 0 ($S''(x_1) = S''(x_n) = 0$)

(plug load splines)

pp = csape(nodes, values, 'variational')

- clamped: pp = csape(nodes, values, 'complete', der_vals)
- deBoor: pp = csape(nodes, values, 'not-a-knot')