## PFB Upchannelisation Question

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From my understanding, given the time stream vector x, the telescope gives us  $B^Tx$ , which is a coarser PFB, and we want Ax, which is the result of a fine PFB. The minimisation procedure gives us:

$$C = (B^T B + \tilde{Q})^{-1} B^T A, \tag{1}$$

which is the coefficient that we apply to  $B^Tx$  that gives us the fine PFB. So in the end,  $CB^Tx$  is the closest approximation we have to Ax.

If we run some numbers, however, it seems that the dimension won't match. Say that  $B^T$  is a 4-tap,  $2n_c = 512$  PFB, and we want A as a 4-tap,  $2n_c = 2048$  PFB. Then, A should be of dimension  $1024 \times \dim x$ , and  $B^T$  should be of dimension  $256 \times \dim x$ . Obviously, they couldn't multiply together, so Eq. (1) doesn't really compute.

I'm not quite sure about what you mean by using several blocks of the short PFB; do you mean that we cut the length of  $B^Tx$  to what we need?

## Giving myself a hypothetical scenario

The below scenario also gives something wrong, and I hope that I didn't make any big mistake somewhere, but here's the scenario:

We have  $B^T$  that does the coarse PFB, which is 4-tap and  $n_{CB}$  channels. We also have A that does the fine PFB, which is also 4-tap and  $n_{CA}$  channels. We also have the relationship  $n_{CA} = 4n_{CB}$  (we upchannelise by a factor of 4). Let the input time stream be denoted by x, which is of length  $\dim(x) = 128n_{CB}$ .

The first operation is  $B^Tx$ . As a result, it is clear that  $B^T$  takes input of length  $128n_{CB}$ . Since it's a 4-tap PFB, the output would be of length  $61n_{CB}$ , if we move down  $2n_{CB}$  indices every instance of the PFB. In other words,  $B^T$  is a matrix of size  $61n_{CB} \times 128n_{CB}$ 

Now, we take  $AB^Tx$ . Since  $n_{CA} = 4n_{CB}$ , after cutting off the end to make the input divisible, we have that the input that A takes is of dimension  $60n_{CB} = 15n_{CA}$ . Since the PFB is 4-tap, the first instance of PFB uses the first  $8n_{CA}$  data, leaving  $7n_{CA}$  for the further instances. Making it divisible by 2, we have  $6n_{CA}$  left for the next few small PFBs, which in the end gives us an output dimension of  $4n_{CA} = 16n_{CB}$ . As a result, A is a matrix of size  $16n_{CB} \times 60n_{CB}$ .

This doesn't make sense, since when we feed the output back to  $B^T$  in the next step, the dimensions would be all wrong.