The foundations of the rigorous study of *analysis* were laid in the nineteenth century, notably by the mathematicians Cauchy and Weierstrass. Central to the study of this subject are the formal definitions of *limits* and *continuity*.

Let D be a subset of $\mathbf{R}$ and let $f\colon D \to \mathbf{R}$ be a real-valued function on D. The function f is said to be *continuous* on D if, for all $\epsilon > 0$ and for all $x \in D$ there exists some $\delta > 0$ (which may depend on x) such that if $y \in D$ satisfies

$$|y - x| < \delta$$

then

$$|f(y) - f(x)| < \epsilon.$$

One may readily verify that if f and g are continuous functions on D then the functions $f + g$, $f - g$ and $f.g$ are continuous. If in addition g is everywhere non-zero then f/g is continuous.