Variational Multiscale Method in Julia

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Abstract

High-Altitude Pseudo-Satellites (HAPS) are Unmanned Aerial Vehicles (UAV), which fly at an altitude ranging from 15 to 20 km, at a low Reynolds number regimes, offering long-lasting local aerial coverage. Conventional airfoils are not suitable for these vehicles. Different physical phenomena can occur, such as Laminar Separation Bubbles (LSB) [1; 2; 3], flow separation and transition which are extremely sensitive to variations in Reynolds number, pressure gradient and environmental disturbances. The PhD project aims to develop, test and calibrate, suitable numerical methods for this specific use. For achieving this purpose, the main focus is to implement a two–scale Variational MultiScale (VMS) method in the Julia programming language which follows the procedure explained by Bazilevs et al. [4]. It is an evolution of the classic well know Streamline Upwind Petrov–Galerkin (SUPG). In practice, the incompressible and time-dependent Navier-Stokes is solved using the Finite Element Method as an implicit Large Eddy Method (LES) for turbulence modelling.

The VMS has been implemented in Julia programming language using the Gridap library [5; 6]. It has been tested on simple 2D cases with satisfactory results also at high Reynolds. In specific, it solves accurately the lid driven cavity problem up to Reynolds 10 000 and capture the laminar boundary layer over a flat plate. The code implementing VMS is capable of capturing correctly also unsteady phenomena as it has been proven by the vortex shedding on a cylinder at Reynolds number 1000. As ODEs solvers are used both the $\theta - method$ and $\alpha - method$. It has been shown that for the $\theta - method$ θ for the pressure field has to be set to 1 for avoiding instabilities. The 2D Taylor-Green vortex case has been used as a further validation but also for evaluating the grid convergence. As expected, the error decreases with the increase of mesh resolution, both for velocity and pressure. The code runs also fully in parallel, using MPI and PETSc as a solver [7]. The parallelization efficiency for the 2D Taylor-Green vortex has been tested with positive results both for the weak and strong scalability.

Keywords: VMS, Variational Multiscale Method, Julia, Finite Element Mehthod, Incompressible Navier Stokes

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