Force Set Selection for Vibration Environment Expansion

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Theory of Predictive Modeling

CS 580/PHSCS 513R

Problem Statement

- Aerospace components often experience significant levels of vibration which can cause failure and / or degrade their intended functionality.
- The flight vibration environment can be estimated by using a model of the system of interest and the forces that it experiences in operation. The distributed loads applied to the system can often be reasonably modeled as a set of point forces, though it is challenging to know where and how many forces to place such that they accurately capture the environment. Additionally, flight environment measurements are often sparse.
- The goal of this project is to develop a robust method for accurately estimating these loads and to better understand the conditions for accurate environment estimation generally.

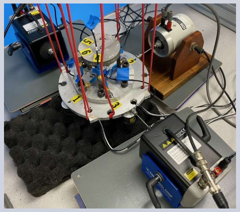
Select the number, location, and orientation of forces that will most accurately represent the flight environment given only accelerometer measurements.

Background

- The "optimal" set of forces satisfies the following equation, and they can be calculated using the pseudo-inverse if the 2-norm is selected. A set of measured accelerations is the "training data" used to estimate the forces.

$$\hat{F} = \arg min_F \, ||HX_{Train} - F||$$

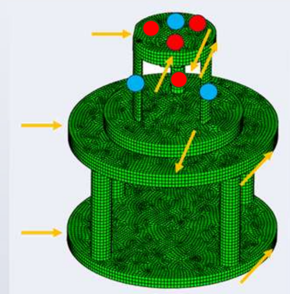
- Ideally, these forces accurately recreate the responses at all locations on the vehicle, which correspond to the "test data."
- When there are more measured responses than forces, they are calculated with least squares error, though often only a few responses are known.
- In machine learning problems, there are often many more parameters than training data points, which begs the question: would using more forces than measurements work well in environment estimation?



Forces are selected to minimize error at points with known responses (**training data**). An accurate force set would recreate the response globally (**test data**).

Methodology

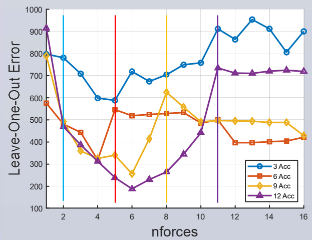
- Fourteen forces are applied to a finite element model at various locations to create a flight environment, representing the actual global response of the vehicle in "flight".
- The responses are divided into training and test accelerometers which are used to estimate forces and to quantify the model's accuracy, respectively.
- A different set of forces is created on the same finite element model, and these are used to attempt to capture the true flight environment given the set of training accelerometers.
- The number of forces and accelerometers is varied together to better understand which combinations are the most and least accurate.
- Leave-one-out cross validation error is also calculated and compared to the test accelerometer error to understand when cross validation can be used to predict model success in practice.



The number of forces and training accelerometers is varied, and error is calculated on the test accelerometers to assess model accuracy.

Results – Double Descent

- Initially, adding additional forces reduces leave-one-out error as it reduces the model's **bias** significantly.
- Past a point – around 6 forces in the yellow and purple lines – the **variance** starts to increase significantly as the transfer function matrix becomes increasingly **ill-conditioned**.
- At the **interpolation points** (vertical lines), the error (generally) peaks, followed by a decrease in error when more shakers are added. Doing so reduces the error due to **aliasing**, hence the overall decrease.



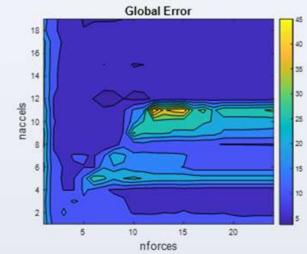
Error initially decreases as forces are added, then increases as overfitting occurs, and decreases again after the interpolation point.

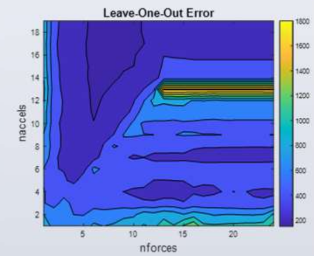
Results – Number of Forces

- The model is most accurate when there are 3 or more forces and 9 or more training accelerometers (see contour plots below).
- Results are generally poorer when there are more forces than training accelerometers.
- Surprisingly, the model is relatively accurate when there are >10 forces and only ~5 training accelerometers. This suggests that reasonable models can be selected in some cases where data is sparse, which I did not previously think was possible.

Results – Leave-One-Out Cross Validation

- The leave-one-out error is closely correlated with the global error at most combinations of forces and accelerometers.
- There is significant disagreement when there are only a few training accelerometers, which is reasonable as leave-one-out cross validation is most accurate when it averages over many errors.
- Overall, leave-one-out cross validation is a reasonable approach to predicting model accuracy with sparse measurements.





Conclusions

- Double descent is observed when calibrating models of flight vehicle dynamics.
- The most accurate models are obtained when there are more accelerometers than shakers, and when there are many of both.
- Though generally less accurate, an accurate model can be obtained when there are many shakers and few accelerometers in some cases.
- The theoretical conditions under which this occurs are not yet well understood, though leave-one-out cross-validation is a reasonable way to evaluate model accuracy, provided that there are at least a few accelerometers. The exact number differs from system to system, but the approach was most accurate with 4+ accelerometers for this system.
- The flight environment at other locations can be estimated using this approach, which is very useful when predicting system performance and performing vibration qualification tests.

Link to Code / Repository

The data and code used in this study can be accessed at: https://github.com/barcusmehling/Behling_580_Project