



# Unifying Evolution, Explanation, and Discernment: A Generative Approach for Dynamic Graph Counterfactuals

Oral paper @ KDD'24



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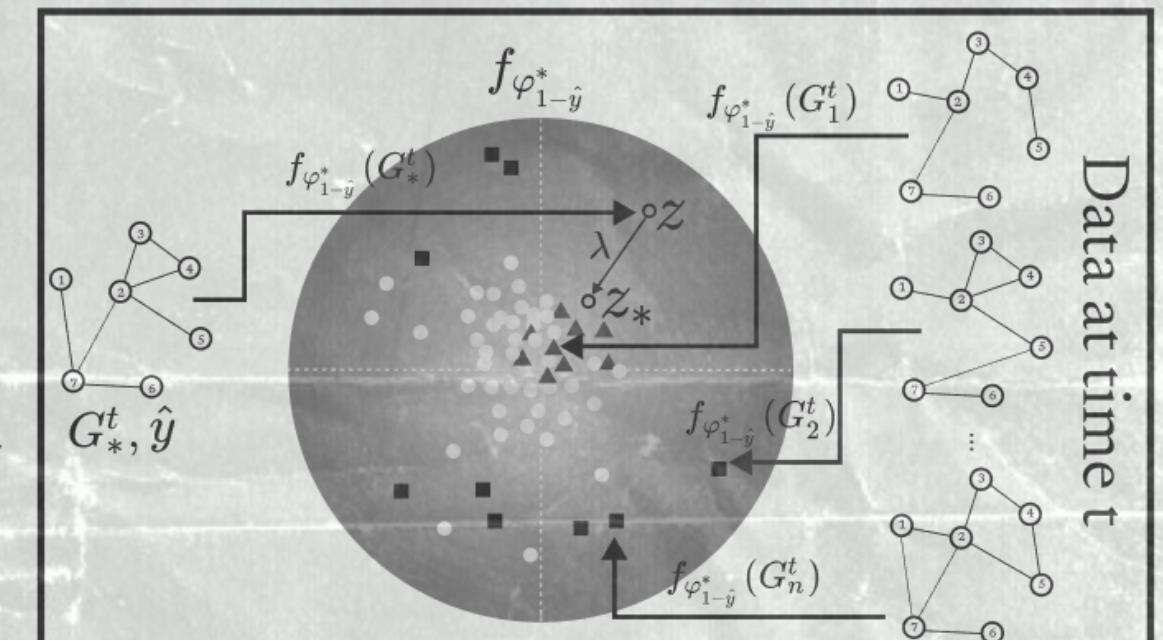
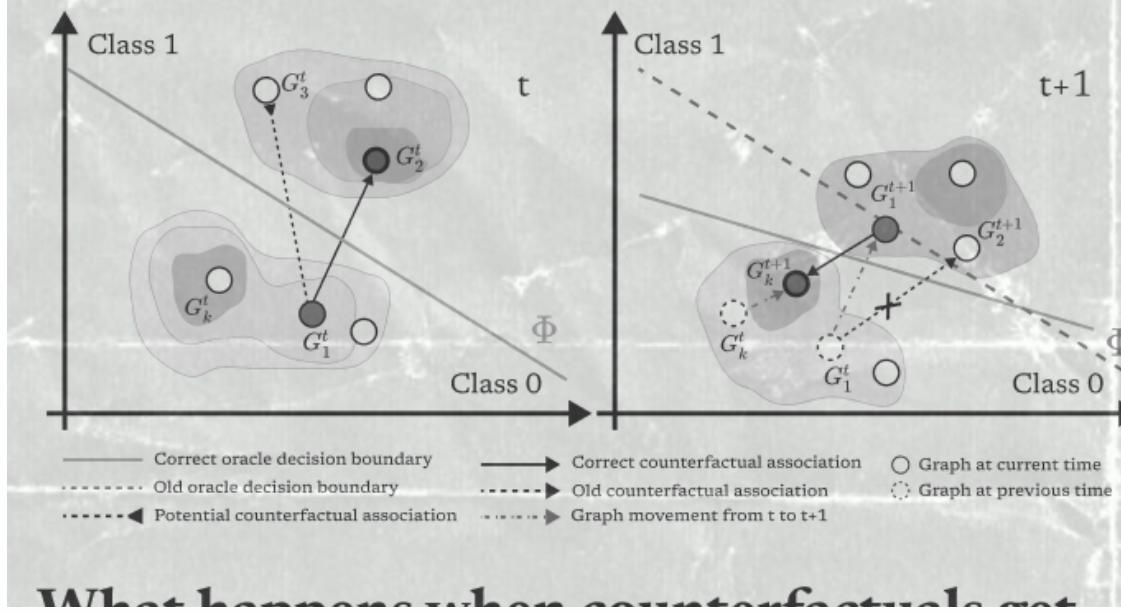
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# UNIFYING EVOLUTION, EXPLANATION, AND DISCERNMENT

## A GENERATIVE APPROACH FOR DYNAMIC GRAPH COUNTERFACTUALS



What happens when counterfactuals get

# Today's Roadmap

- Introduction
  - Good ol' graphs
  - What are counterfactuals?
  - “*The right to be forgotten*” - Pawelczyk et al.
- Pictorial Problem Statement
- Problem Formulation
- Generative Classification (GC) Perspective
  - Bridging Reconstruction and GC

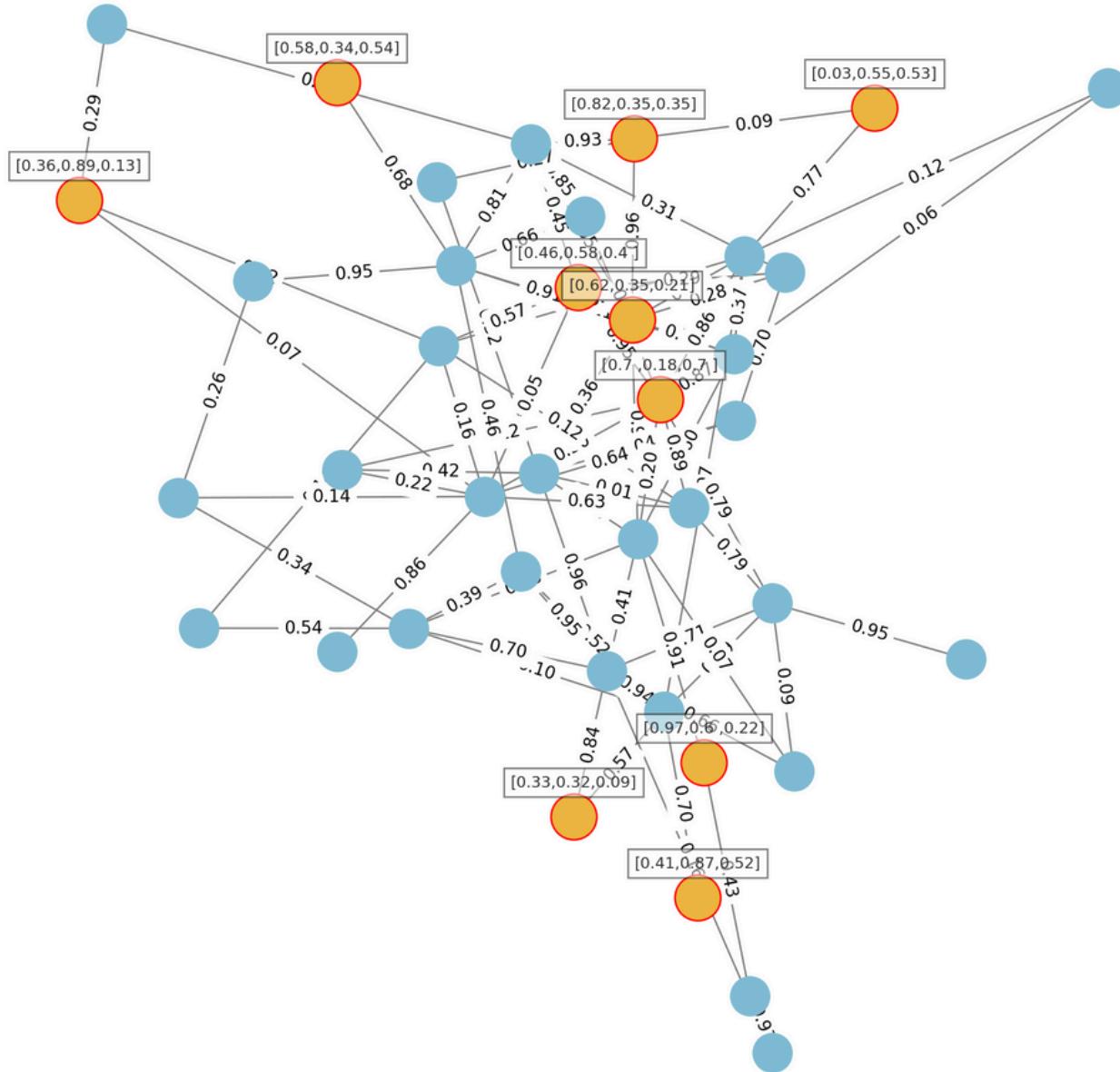
# Today's Roadmap (cont.)

- Fighting out of the **blue corner**: GRACIE!
  - Training
  - Inference and Finding Latent Counterfactuals
  - Dynamic Update
  
- Experiments
  - Synthetic vs. Real-world Datasets
  - Pulling Factor Trade-Off
  - Qualitative Inspection



Oh yeah... almost forgot  
about the **Conclusions** 😊

# Good ol' graphs

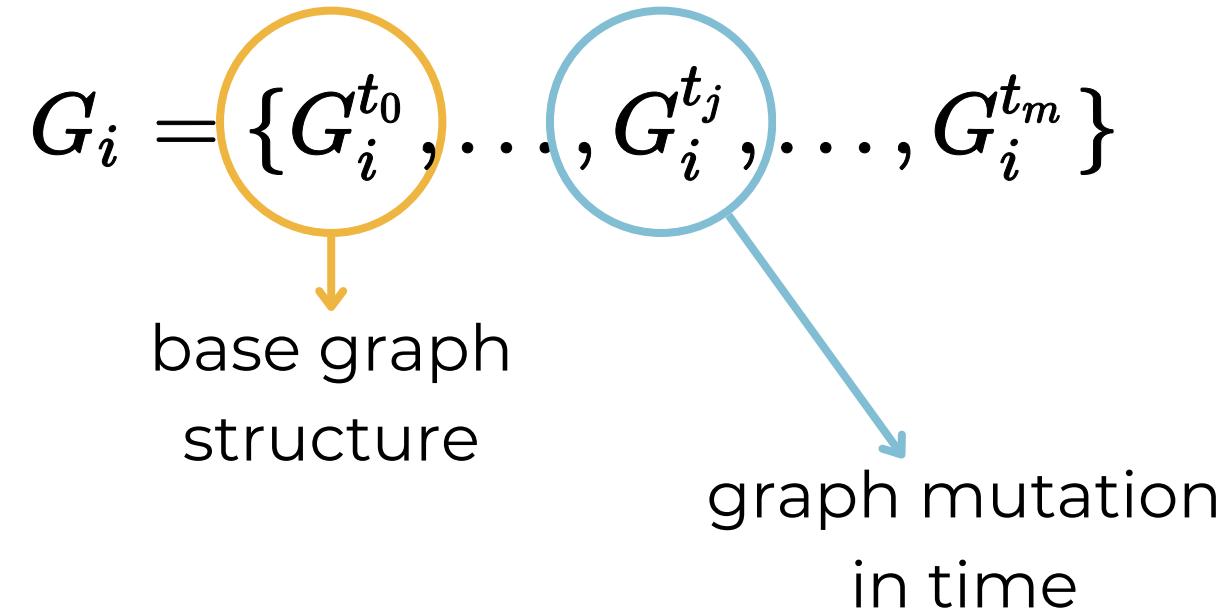
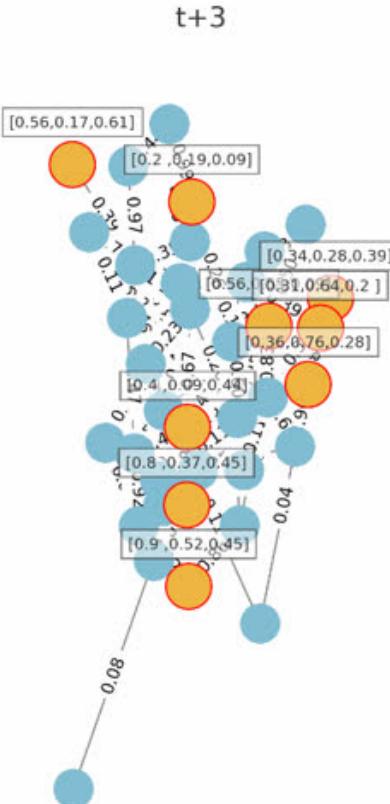


$$G_i = (\mathbf{X}, \mathbf{A}) \in \mathcal{G}$$

$$\mathbf{X} \in \mathbb{R}^{n \times d}$$

$$\mathbf{A} \in \mathbb{R}^{n \times n}$$

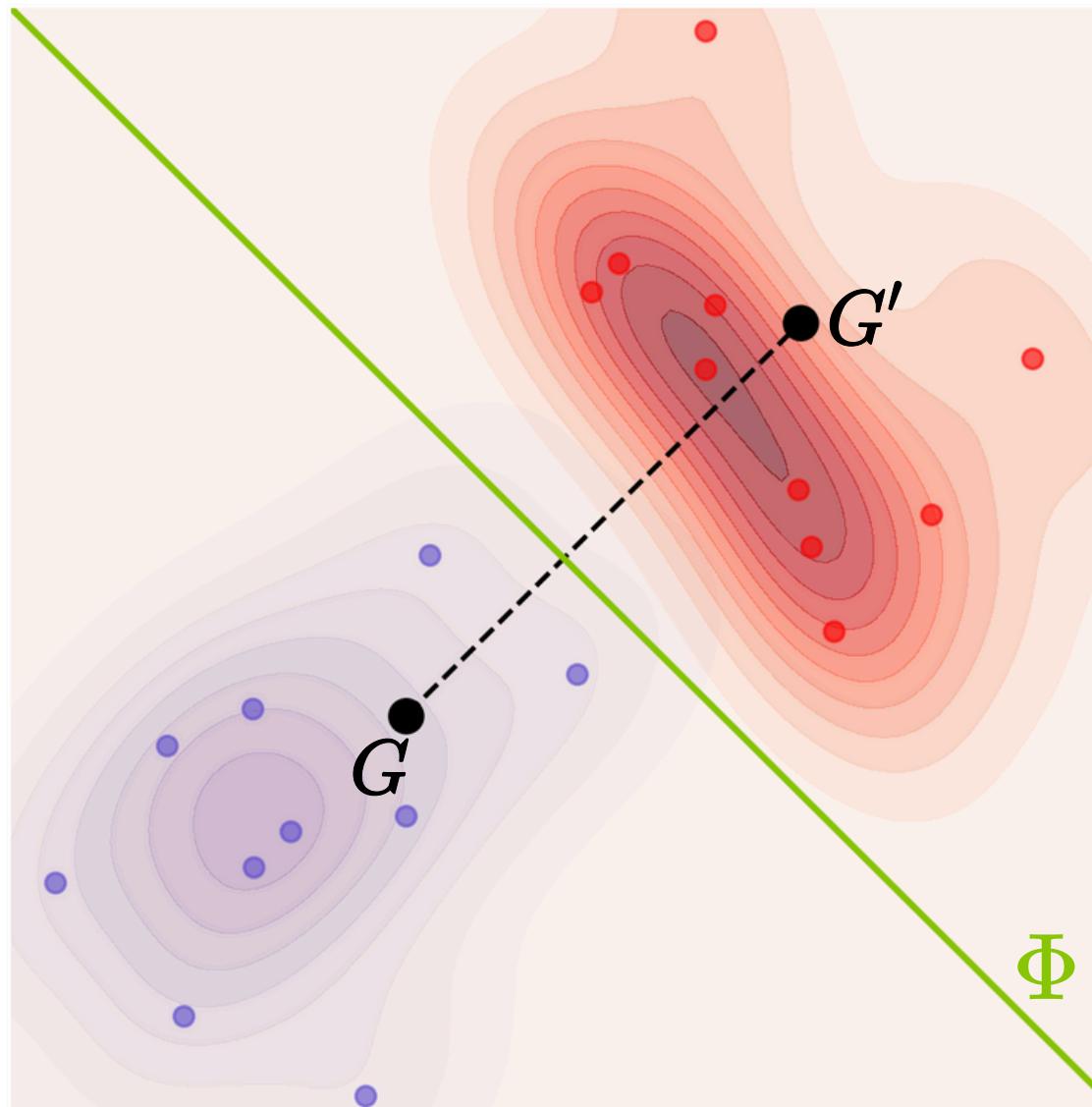
# Good ol' graphs (contd.)



## Possible modifications in time:

- Node additions/removal
- Edge additions/removal

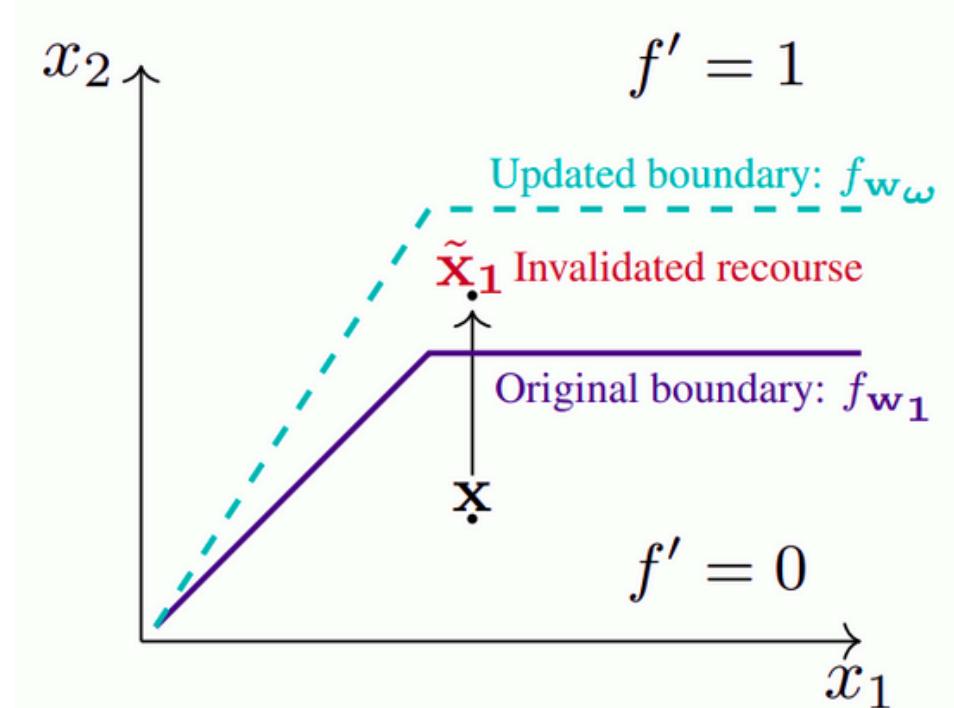
# What are Counterfactuals?



$$\Phi(G) \neq \Phi(G')$$

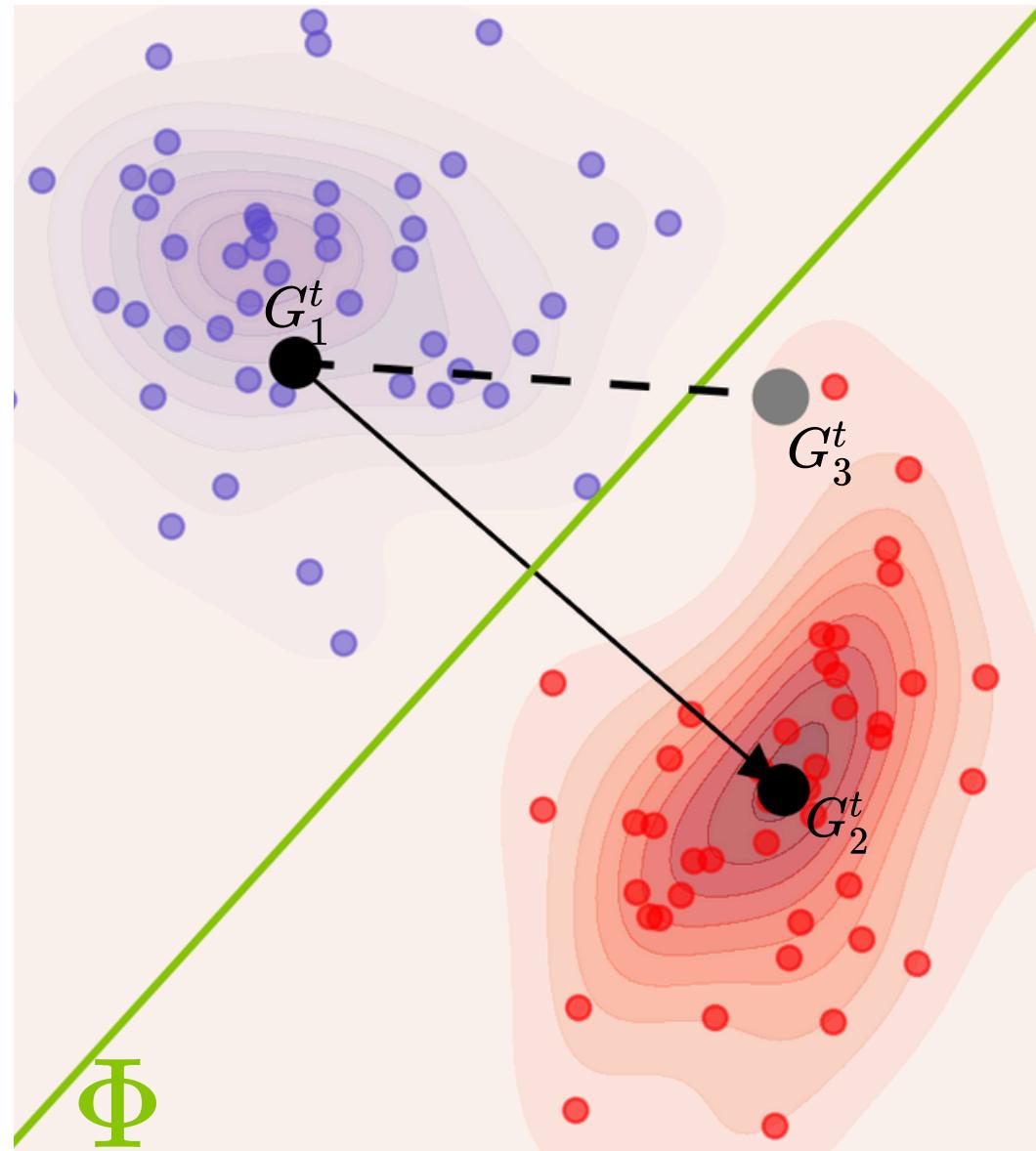
# “The right to be forgotten”

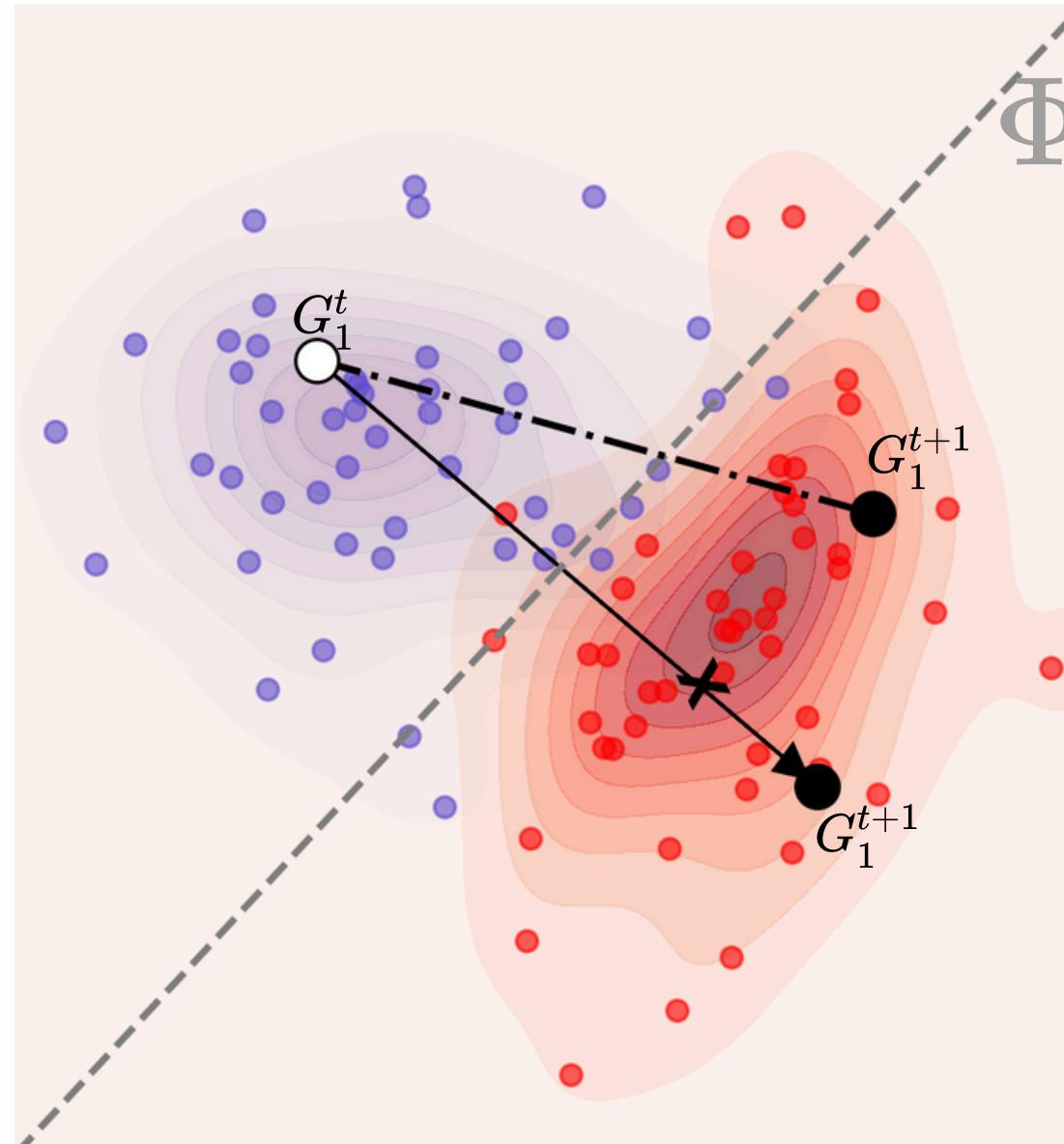
- Counterfactuals can become invalidated when data is deleted
- Paweleczyk et al. identify data points that, when deleted at  $\mathbf{t} + \boldsymbol{\delta}$ , invalidate the counterfactuals at time  $\mathbf{t}$



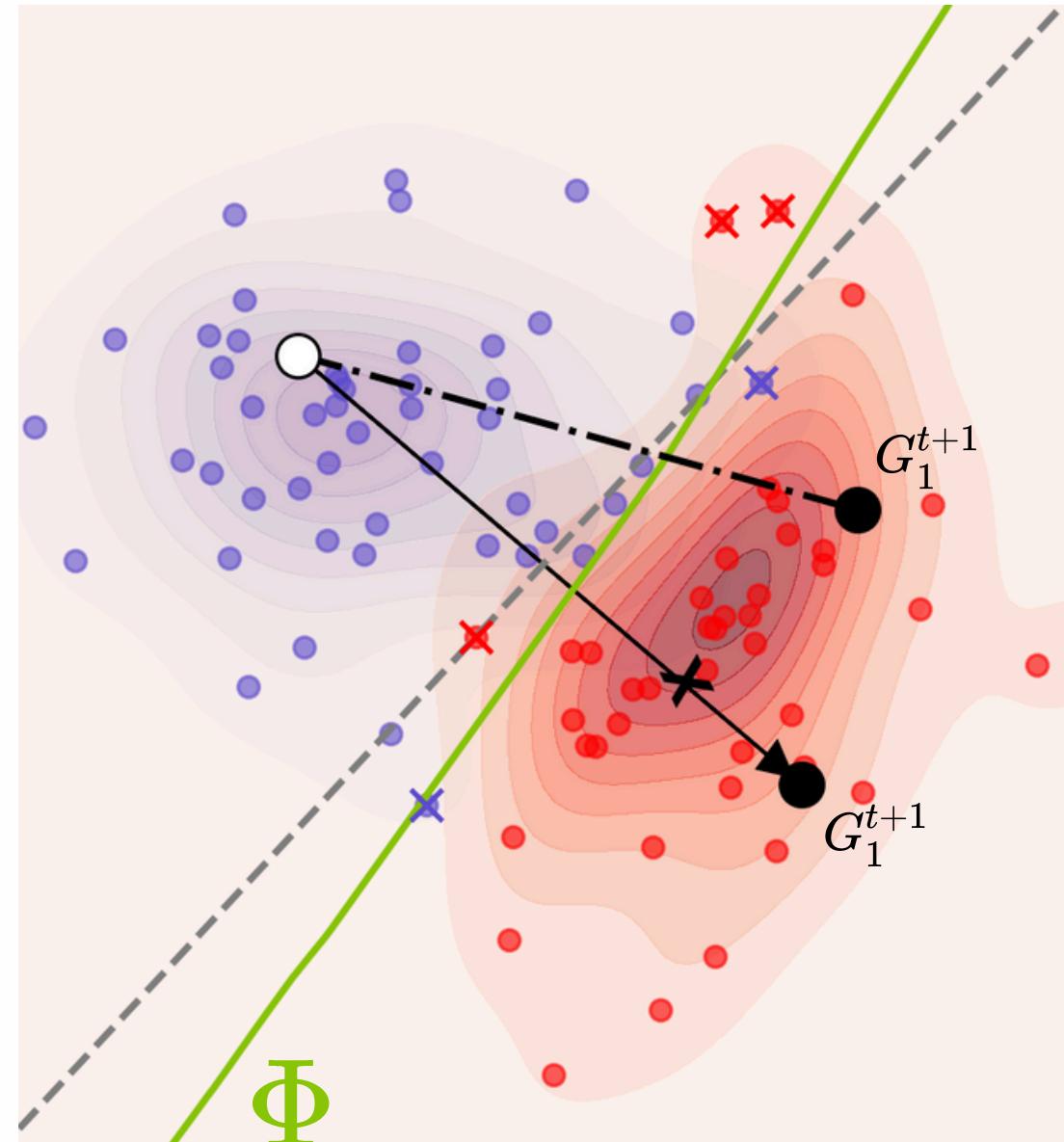
Martin Paweleczyk, Tobias Leemann, Asia Biega, and Gjergji Kasneci. 2023. On the Trade-Off between Actionable Explanations and the Right to be Forgotten. In Proc. of the 11th International Conference on Learning Representations

# Pictorial Problem Statement

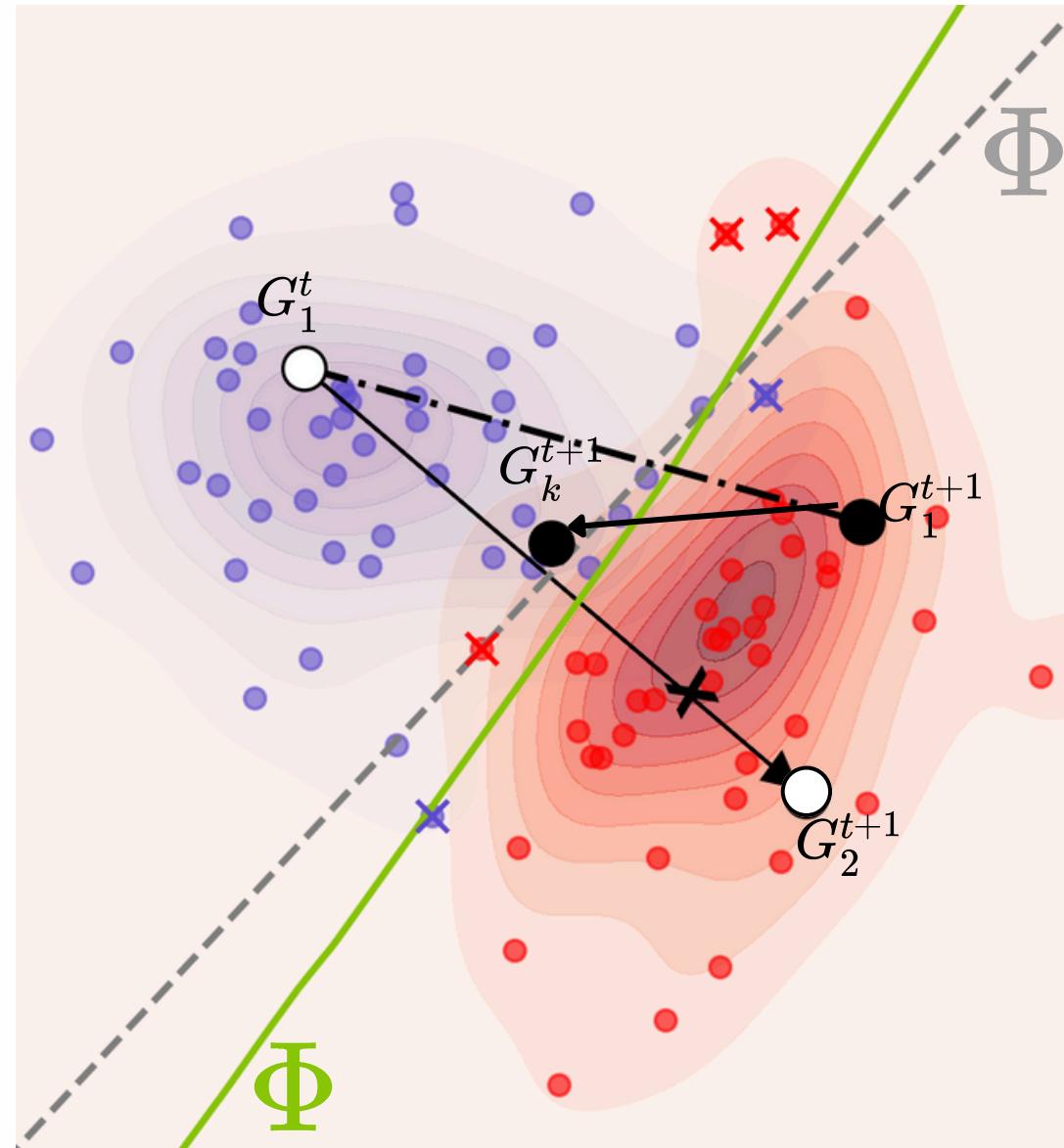




$t+1$



$t+1$



$t+1$

# Problem Formulation

$$\mathcal{E}_\Phi(G_i^t) = \arg \max_{G_j^t \in \mathcal{G}} P_{cf}^t(G_j^t \mid G_i^t, \Phi(G_i^t), \neg\Phi(G_i^t))$$

probability of  $G_j^t$   
being in-  
distribution  
counterfactual of  $G_i^t$

Any other class  
that isn't  $\Phi(G_i^t)$

**Differently from previous work, we shift towards a generative classification (GC) perspective**

# Generative Classification (GC) Perspective

- Generative Classifiers (GCs) perform classification by modeling the full joint distribution of features  $x$  and class labels  $y$

$$\begin{aligned}\hat{y} &= \arg \max_{y \in \mathcal{Y}} p(x, y) = \arg \max_{y \in \mathcal{Y}} p(x|y) p(y) = \\ &= \arg \max_{y \in \mathcal{Y}} \log p(x|y) + \log p(y).\end{aligned}$$

Andrew Ng and Michael Jordan. 2001. On discriminative vs. generative classifiers: A comparison of logistic regression and naive bayes. *Advances in neural information processing systems 14* (2001)

# Generative Classification (GC) Perspective

- Superior generalization capabilities over discriminative classifiers
- Accurately calibrated posteriors
- Increased adversarial robustness

*Ilkay Ulusoy and Christopher M Bishop. 2006. Comparison of generative and discriminative techniques for object detection and classification. In Toward Category-Level Object Recognition. Springer, 173–195*

*Lynton Ardizzone, Radek Mackowiak, Carsten Rother, and Ullrich Köthe. 2020. Training normalizing flows with the information bottleneck for competitive generative classification. Advances in Neural Information Processing Systems 33 (2020), 7828–7840*

*Yingzhen Li, John Bradshaw, and Yash Sharma. 2019. Are generative classifiers more robust to adversarial attacks?. In International Conference on Machine Learning. PMLR, 3804–3814.*

# Variational Graph Autoencoders (VGAEs)

- We consider the following generative model where the graphs  $G$  are generated from factored latent representation  $\mathbf{z}$  and the true class label  $y$

$$p(G|y) = \int_{\mathbf{z} \in \mathcal{Z}} p(G|\mathbf{z}, y) p(\mathbf{z}|y) dz$$

# VGAEs (Decoder)

- To represent  $p(G|y)$ , we use a single VGAE for each class , which is dependent on the class where each node has a latent vector and then define

$$\begin{aligned} p_{\theta_y}(G|\mathbf{z}, y) &= p_{\theta_y}(\mathbf{A}, \mathbf{X}|\mathbf{z}, y) \\ &= p_{\theta_y}(\mathbf{X}|\mathbf{A}, \mathbf{z}, y) p_{\theta_y}(\mathbf{A}|\mathbf{z}, y) \end{aligned}$$

# VGAEs (Encoder)

$$q_{\varphi_y}(\mathbf{z}|G, y) = \prod_{v_i} q_{\varphi_y}(\mathbf{z}_{v_i}|G, y)$$

$$q(z_{v_i}|G, y) = \mathcal{N}\left(z_{v_i} | \mu_{v_i}, \gamma^2 \mathbf{I}\right),$$

> 0 and fixed  
hyperparameter

$$\mu = [\mu_{v_1}, \dots, \mu_{v_n}] = \text{GCN}_{\varphi_y}(G)$$

# Bridging Reconstruction and GC

- We train the VGAEs for each of the classes by optimizing the parameters  $\theta$  and  $\varphi$

$$\text{ELBO}_y (\theta_y, \varphi_y) = \mathbb{E}_{q_{\varphi_y}(z|G,y)} \left[ \log p_{\theta_y}(G|z,y) \right] - \text{KL} \left[ q_{\varphi_y}(z|G,y) \parallel p(z) \right]$$

$$(\theta_y^*, \varphi_y^*) = \arg \max_{\theta_y, \varphi_y} \text{ELBO}_y (\theta_y, \varphi_y) \quad \forall y \in \mathcal{Y}$$

# Bridging Reconstruction and GC

- Having obtained a generative latent variable model of a specific class, we can now exploit its power to perform generative classification
- If the variational family is expressive enough, the ELBO converges to the logarithm of the true class-conditional probability
- Use the generative models to compare different class probabilities and perform generative classification

# Bridging Reconstruction and GC

**Proposition 1:** Comparing Distance-Augmented Reconstruction Losses performs Implicit GC

*If the density model is sufficiently expressive, i.e., it covers the true data generating process, computing*

$$\hat{y} = \arg \min_{y \in \mathcal{Y}} \frac{1}{2} \left( \mathbb{E}_{q_{\varphi_y^*}(z|G,y)} \left[ \frac{\|g_{\theta_y^*}(z) - G\|_2^2}{\sigma^2} \right] + \|f_{\varphi_y^*}(G)\|_2^2 \right] - \log p(y),$$

*is equivalent to performing generative classification for an input graph.*

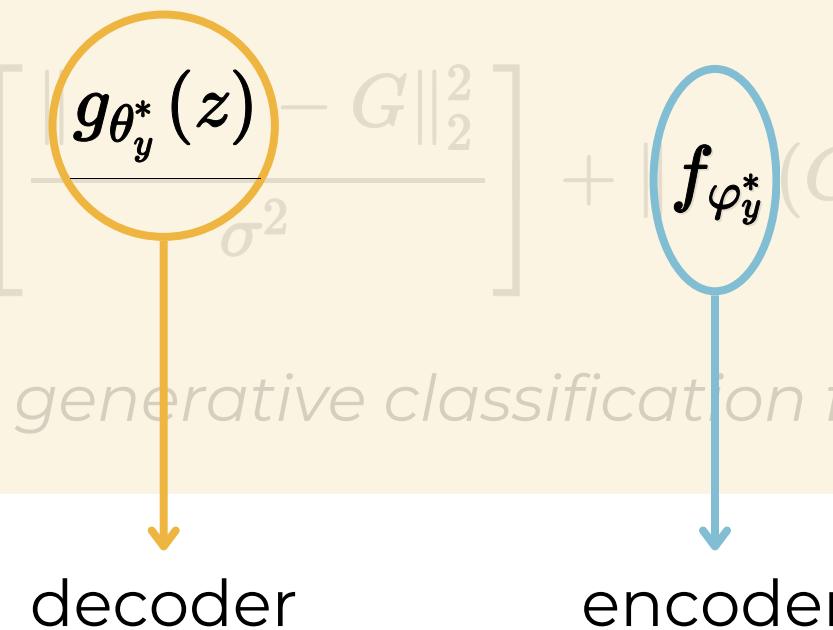
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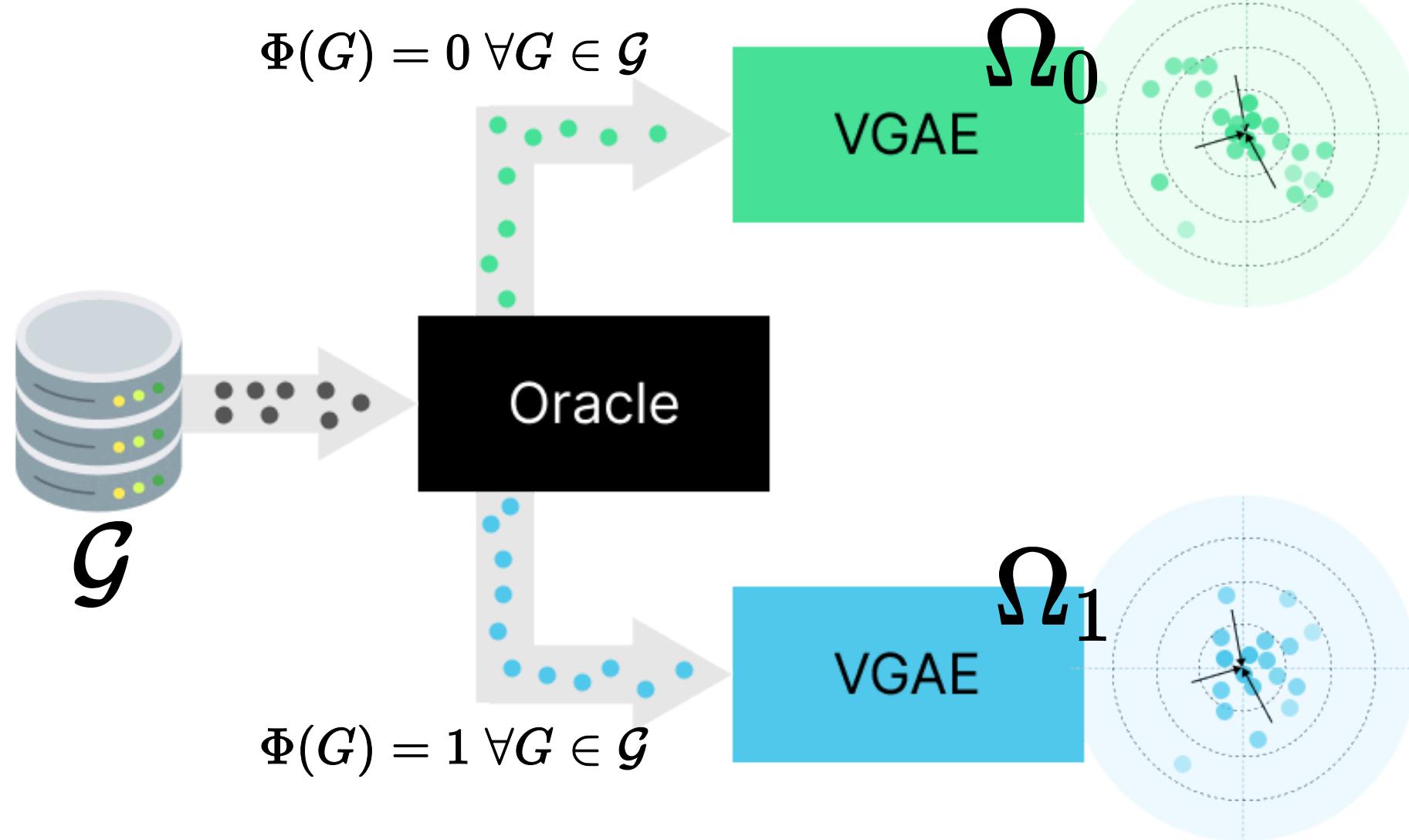
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# GRACIE

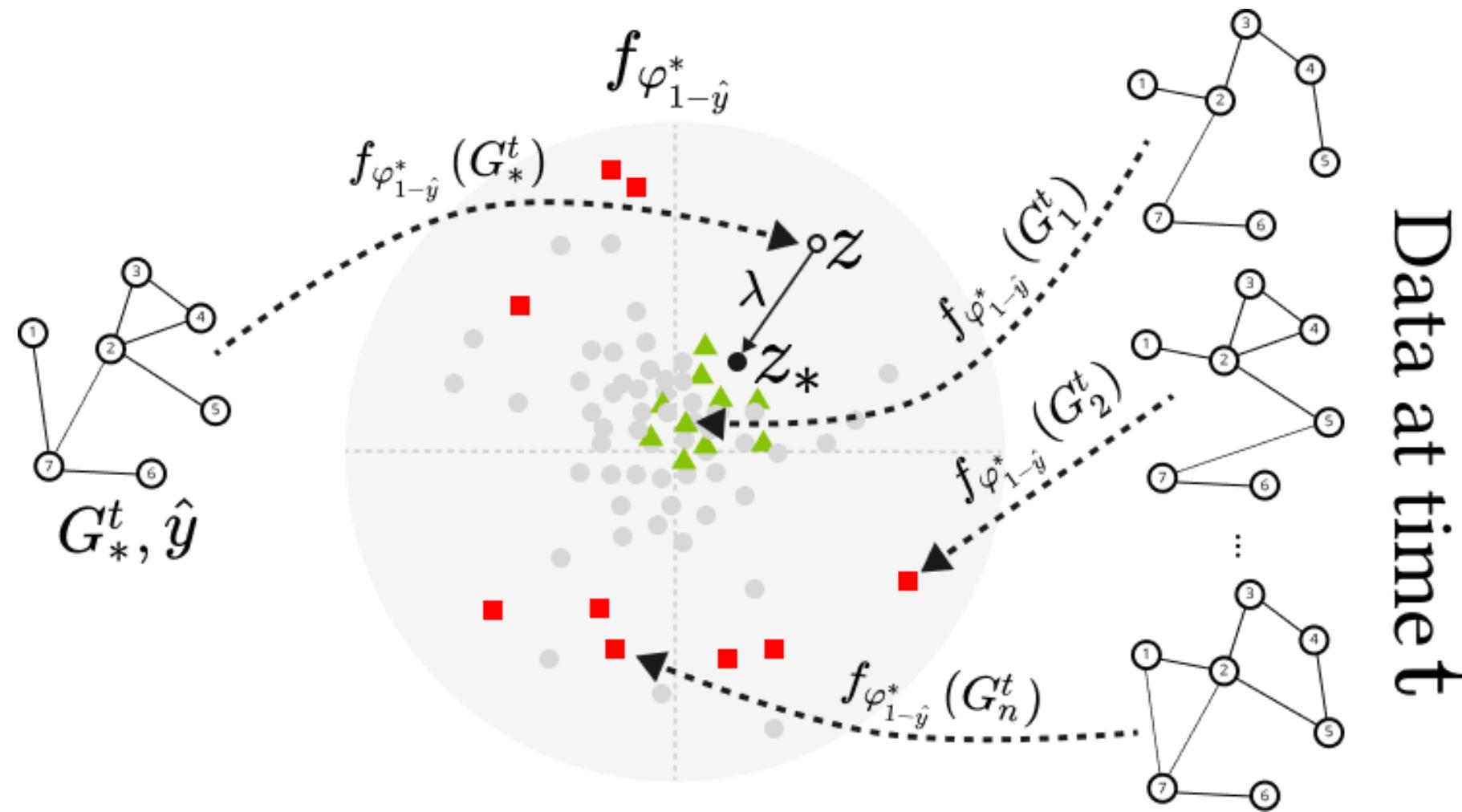
# Class Representation Experts



# Training

$$\begin{aligned} -\text{ELBO}_y(\theta_y, \varphi_y) &= \mathcal{L}_{rec} + \mathcal{L}_{dist} \\ &= \frac{1}{2} \left( \mathbb{E}_{q_{\varphi_y}(\mathbf{z}|G)} \left[ \frac{\|g_{\theta_y}(\mathbf{z}) - G\|_2^2}{\sigma^2} \right] + \|f_{\varphi_y}(G)\|_2^2 \right) \end{aligned}$$

# Inference and Finding Latent Counterfactuals



# Dynamic Update

- Use the learned representation of the VGAEs
- For each graph, find  $k$  candidate counterfactuals close to the center of the VGAE responsible to learn the counterfactual class
- We can use these counterfactuals to update the counterfactual VGAE and the graph itself to update the factual VGAE
- **GRACIE is semi-supervised in the first snapshot, and completely unsupervised in the next snapshots**

# Experiments

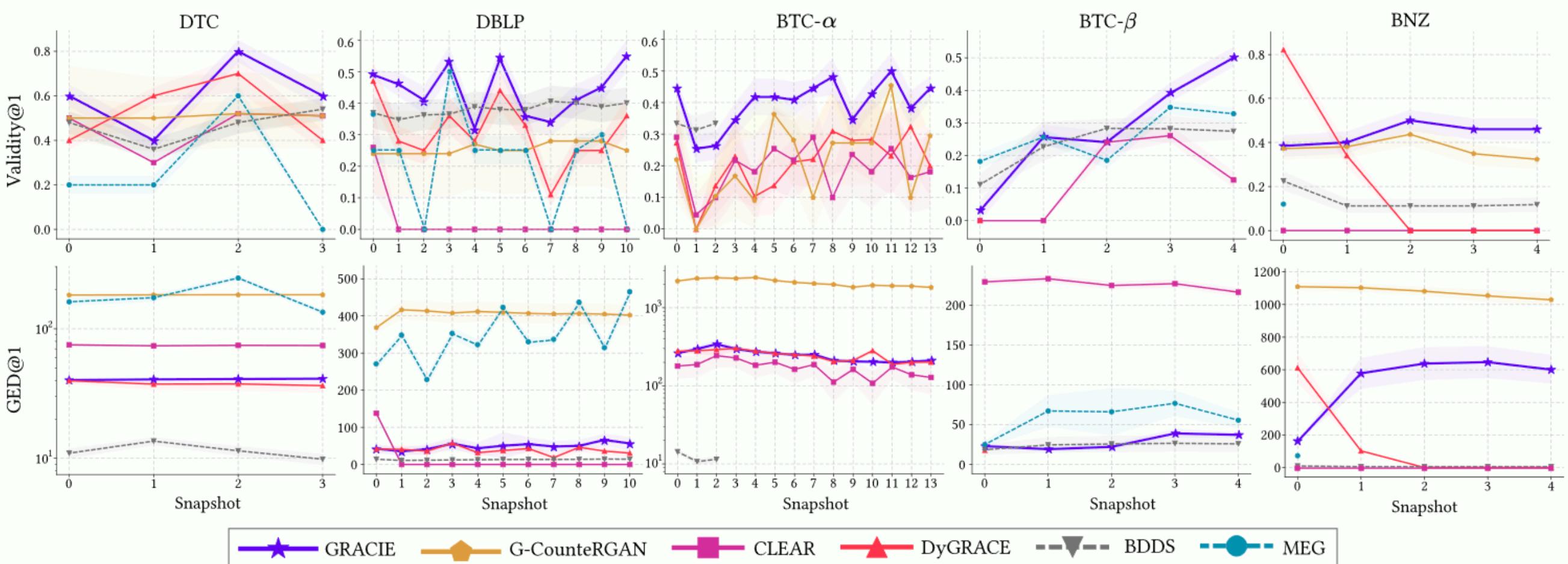
# Synthetic vs. Real-world Datasets

	DTC	DBLP	BTC- $\alpha$	BTC- $\beta$	BNZ
BDDS	0.465	<u>0.381</u>	<u>0.360</u> <sup>†</sup>	0.235	0.136
MEG	0.250	0.209	$\times$	<u>0.260</u>	0.120 <sup>†</sup>
CLEAR	0.458	0.024	0.214	0.125	0.000
G-CounteRGAN	0.507	0.256	0.236	$\times$	<u>0.404</u>
DyGRACE	<u>0.525</u>	0.307	0.232	0.000 <sup>†</sup>	0.232
GRACIE	<b>0.600</b>	<b>0.442</b>	<b>0.440</b>	<b>0.284</b>	<b>0.441</b>

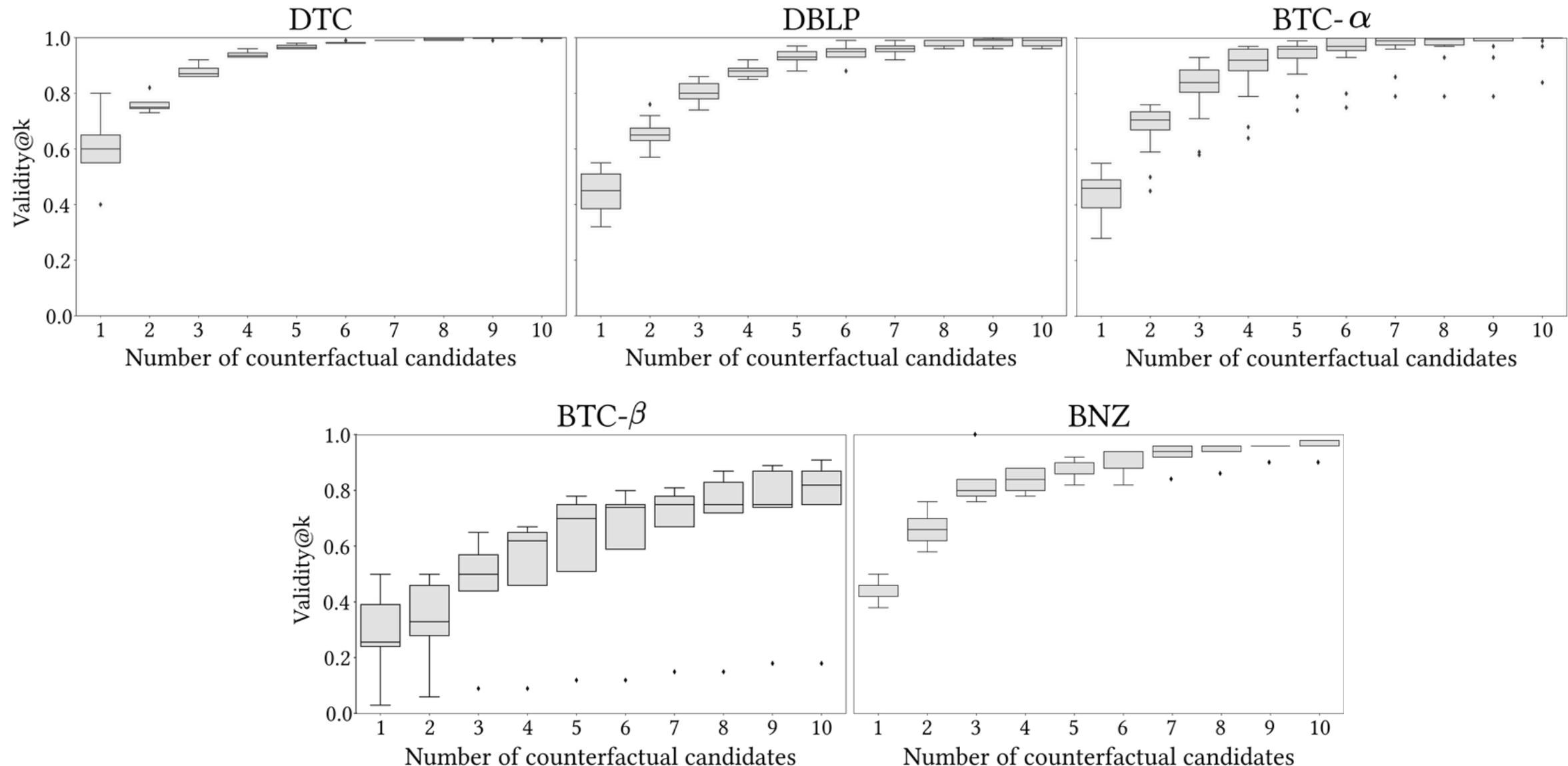
<sup>a</sup>The criterion of non-convergence is to fail to produce at least one counterfactual within 14 days of execution on an HPC SGE Cluster of 6 nodes with 360 cumulative cores, 1.2Tb of RAM, and two GPUs (i.e., one Nvidia A30 and one A100).

	GRACIE	
	w/o Bonferroni ( <i>p</i> -value .05)	w/ Bonferroni ( <i>p</i> -value .01)
BDDS	$2.472 \times 10^{-6}$	$3.708 \times 10^{-5}$
MEG	$1.784 \times 10^{-15}$	$2.676 \times 10^{-14}$
G-CounteRGAN	$1.090 \times 10^{-5}$	$1.635 \times 10^{-4}$
CLEAR	$9.354 \times 10^{-13}$	$1.403 \times 10^{-11}$
DyGRACE	$2.014 \times 10^{-6}$	$3.021 \times 10^{-5}$

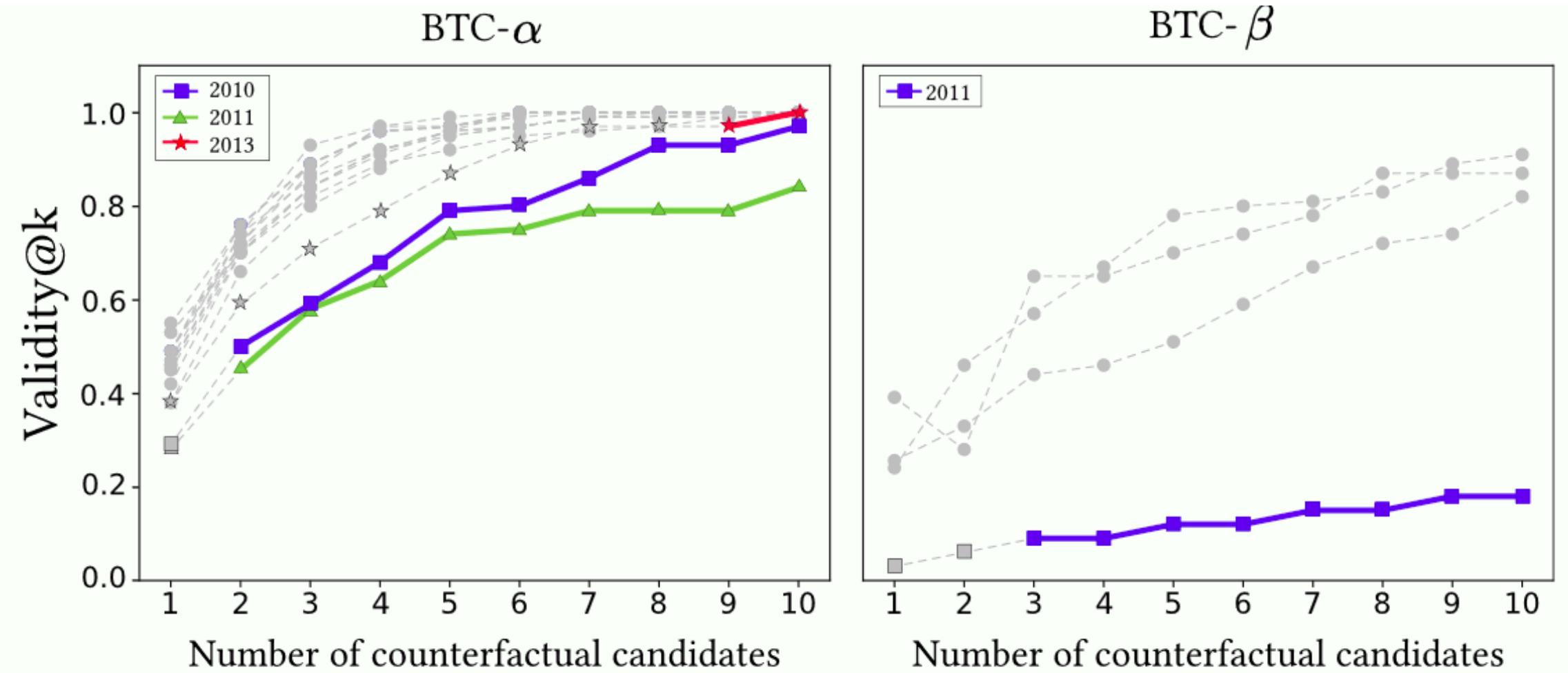
# Synthetic vs. Real-world Datasets



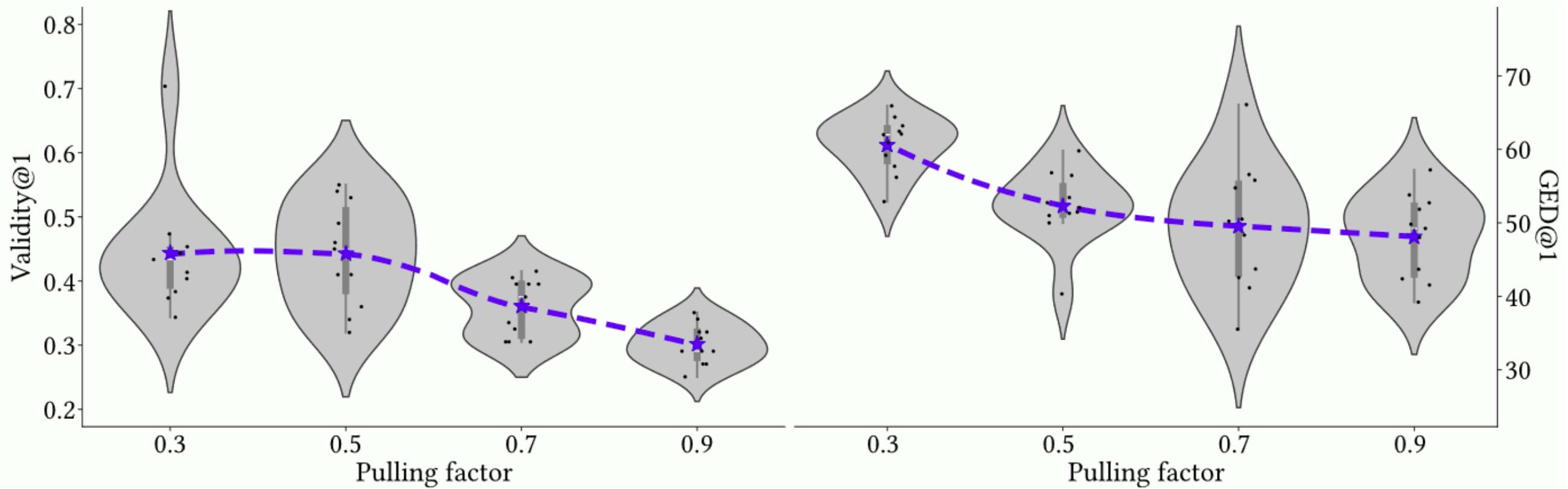
# More sampling = more validity



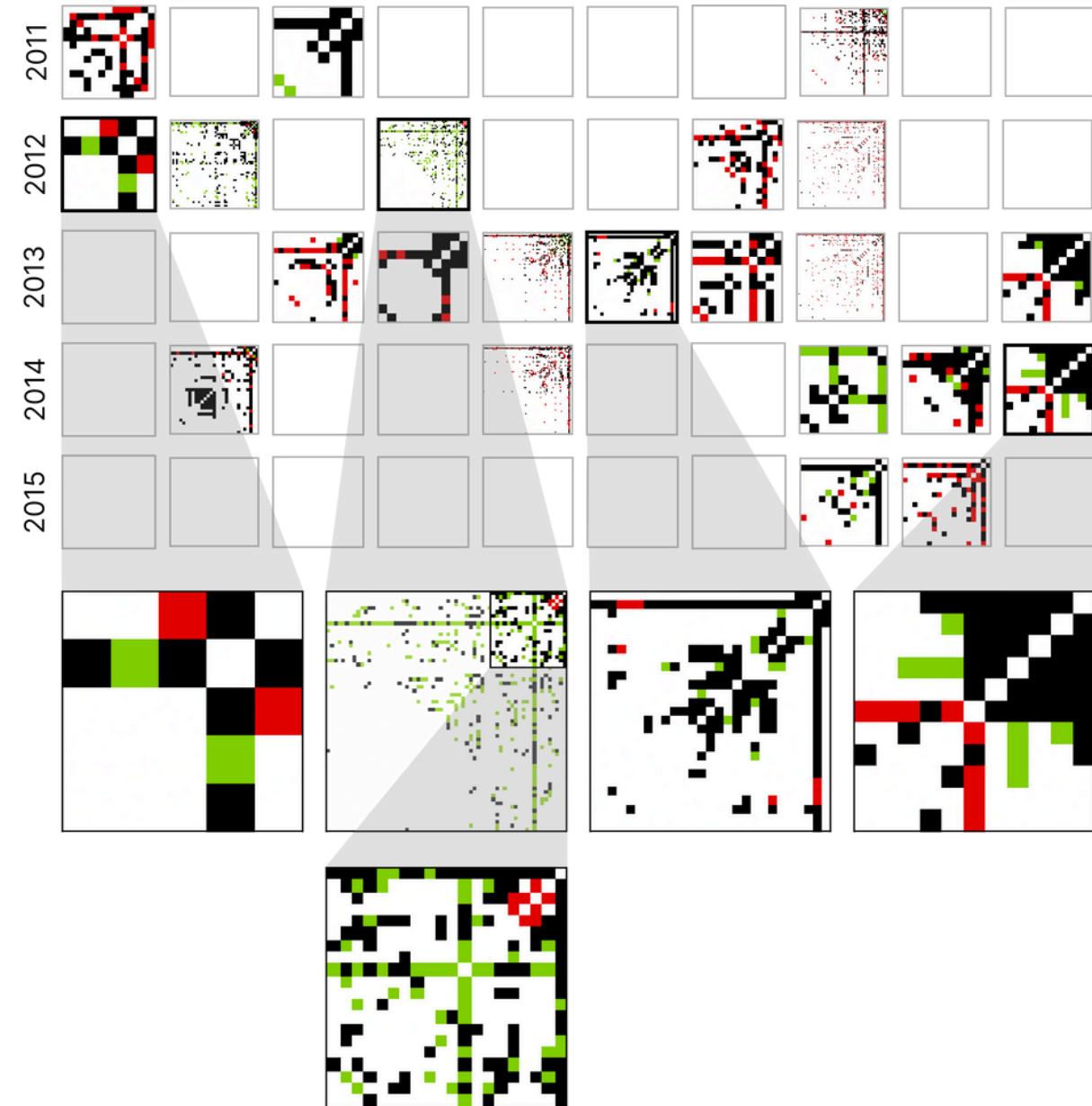
# A closer look on sampling



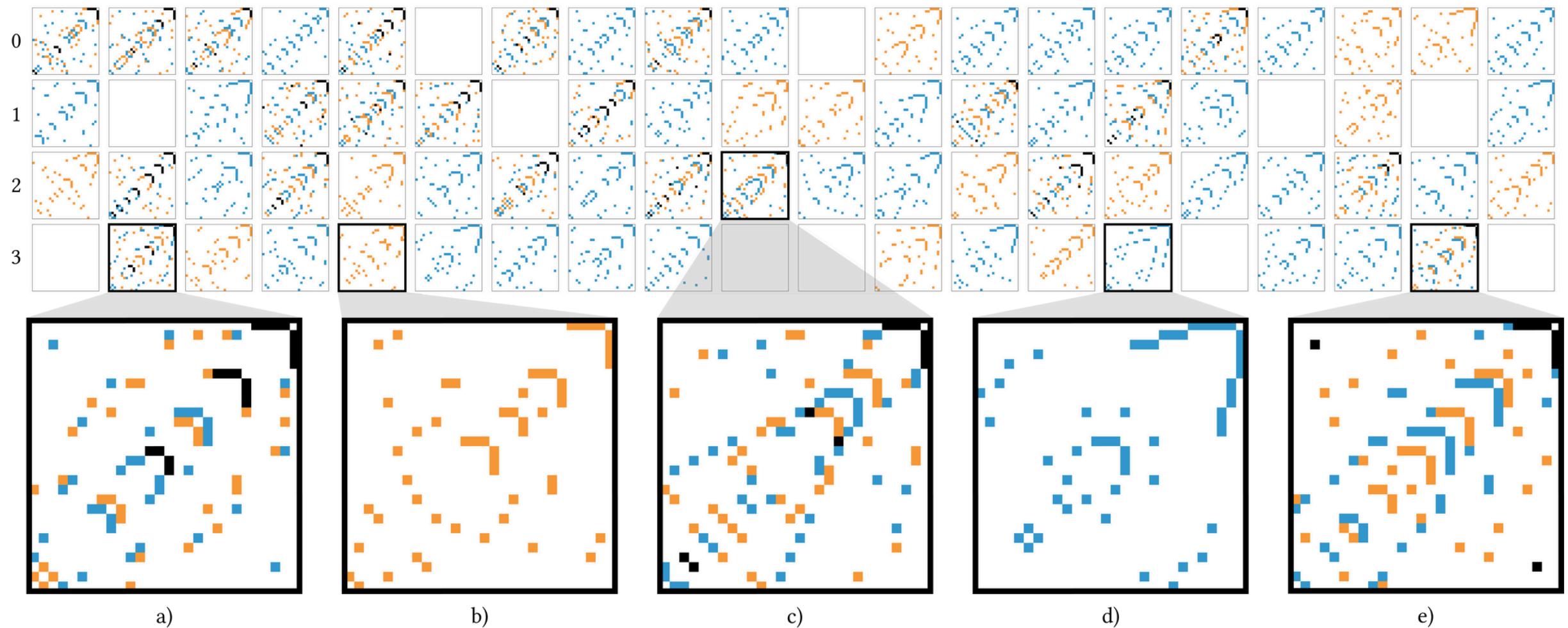
# Effect of pulling factor



# Qualitative on BTC- $\beta$



GRACIE vs BDDS



# Conclusions

- GRACIE is one of the first generative approaches to address dynamic counterfactual explainability in the context of temporal graphs
- We leverage VGAEs, self-supervisedly, to learn class representations and adapt to data distribution shifts
- Improvement of 13.1% in producing valid counterfactuals than SoTA
- **The center of the latent space of the VGAEs should be used to find valid counterfactual**

# Thank you!

