(1)

(a) $x[n]=\delta[n] \rightarrow X(e^{jw})=\sum_{-\infty}^{+\infty} x[n]e^{-jwn}$

$$X(e^{jw})=\sum_{-\infty}^{+\infty}\delta[n]e^{-jwn} \xrightarrow{n=0} 1\times e^{-jw0}=\boxed{1}$$

(b) $x[n]=a^n u[n] \quad |a|<1 \qquad X(e^{jw})=\sum_{n=0}^{\infty} a^n e^{-jwn}=\sum_{n=0}^{\infty}(ae^{-jw})^n=\boxed{\frac{1}{1-ae^{-jw}}}$

دنباله هندسی با $|q|<1$

(c) $x[n]=a^{|n|} \quad |a|<1 \qquad X(e^{jw})=\sum_{n=-\infty}^{+\infty} a^{|n|}e^{-jwn}=\sum_{n=0}^{+\infty}a^n e^{-jwn}+\sum_{n=-\infty}^{-1}a^{-n}e^{-jwn}$

$$=\frac{1}{1-ae^{-jw}}+\sum_{n=-\infty}^{0}(a^{-1}e^{-jw})^n=\frac{1}{1-ae^{-jw}}+\sum_{n=0}^{+\infty}(ae^{jw})^n=\boxed{\frac{1}{1-ae^{-jw}}+\frac{1}{1-ae^{jw}}}$$

(d) $x[n]=\begin{cases}1 & |n|\leq N_1\\ 0 & ow\end{cases} \rightarrow X(e^{jw})=\sum_{n=-N_1}^{N_1}e^{-jwn}=e^{+jwN_1}+e^{-jw(-N_1+1)}+\dots+e^{-jwN_1}$

$a_1=e^{jwN_1}$

$r=e^{-jw} \rightarrow \qquad S_n=\frac{a_1(1-r^{n+1})}{1-r}=\boxed{\frac{e^{jwN_1}\left(1-e^{-2jwN_1}\right)}{1-e^{-jw}}}$

$a_{2N_1+1}=e^{-jwN_1}$

(e) $x[n]=\cos(\frac{2\pi}{5}n) \qquad X(e^{jw})=\sum_{n=-\infty}^{+\infty}\cos(\frac{2\pi}{5}n)e^{-jwn}=\sum_{n=-\infty}^{+\infty}\left(\frac{e^{\frac{2\pi nj}{5}}+e^{\frac{-2\pi nj}{5}}}{2}\right)e^{-jwn}$

$$=\frac{1}{2}\sum_{n=-\infty}^{+\infty}\left(e^{\frac{2\pi j}{5}}\times e^{-jw}\right)^n+\left(e^{\frac{-2\pi j}{5}}\times e^{-jw}\right)^n=\frac{1}{2}\sum_{n=-\infty}^{+\infty}e^{j(\frac{2\pi}{5}-jw)n}+e^{j(\frac{-2\pi}{5}-jw)n}$$

$$=\frac{1}{2}\sum_{n=0}^{+\infty}e^{j(\frac{2\pi}{5}-jw)n}+e^{-j(\frac{2\pi}{5}-jw)n}+e^{j(\frac{-2\pi}{5}-jw)n}+e^{-j(\frac{-2\pi}{5}-jw)n}$$

$$=\boxed{\sum_{n=0}^{+\infty}\cos\left((\tfrac{2\pi}{5}-jw)n\right)+\cos\left((\tfrac{-2\pi}{5}-jw)n\right)}$$

۴) $x[n] = u[n+1] - u[n-2] \rightarrow X(e^{j\omega}) = \sum_{n=-1}^{1} e^{-j\omega n} = \dfrac{\sin(\frac{3}{2}\omega)}{\sin(\frac{1}{2}\omega)}$ ← طبق فرمول

۹) $x[n] = u[n] \rightarrow X(e^{j\omega}) = \sum_{n=0}^{\infty} e^{-j\omega n} = \boxed{\dfrac{1}{1-e^{-j\omega}}}$

۸) $x[n] \longrightarrow X(e^{j\omega}) = \sum_{n=0}^{9} x[n] e^{-j\omega n} = 1\times 1 + 2e^{-j\omega} + \dots + 2e^{-j\omega 9}$

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۲) $X(e^{j\omega}) = \dfrac{1}{(1-ae^{-j\omega})^2} \qquad x[n] = \dfrac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} \frac{1}{(1-ae^{-j\omega})^2} e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{2\pi} \frac{e^{j\omega n} e^{2j\omega}}{(e^{j\omega}-a)^2} d\omega$$

$$= \frac{1}{2\pi}\int_{2\pi} \frac{e^{j\omega}(e^{n+2})\, d\omega}{(e^{j\omega}-a)^2} = \frac{e^{n+2}}{2\pi}\int_{2\pi} \frac{e^{j\omega}}{(e^{j\omega}-a)^2} d\omega$$

حل $\rightarrow \left((e^{j\omega}-a)^{-1}\right)' = \dfrac{je^{j\omega}}{(e^{j\omega}-a)^2} \longrightarrow \dfrac{e^{n+2}}{2\pi} \times \dfrac{(e^{j\omega}-a)^{-1}}{j}\Bigg|_0^{2\pi}$

$$\rightarrow \boxed{\frac{e^{n+2}}{2\pi j}\left(\frac{1}{e^{2\pi j}-a} - \frac{1}{1-a}\right)} = x[n]$$

(3) $X(e^{j\omega}) = re^{j\theta}$ $\qquad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$

$\boxed{\theta = \omega}$

$$\to x[n] = \frac{1}{2\pi}\int_{-\pi}^{-\pi/2}\left(\frac{2\omega}{\pi}+2\right)e^{j\omega n}d\omega + \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{\pi/2}^{\pi}\left(\frac{2\omega}{\pi}+2\right)e$$

$$\Rightarrow x[n] = \frac{1}{2}\left(\int_{-\pi}^{-\pi/2}\frac{2}{\pi}\omega e^{j\omega n} + 2e^{j\omega n}d\omega + \int_{-\pi/2}^{\pi/2} 2e^{j\omega n}d\omega + \int_{\pi/2}^{\pi}\left(\frac{-2\omega}{\pi}e^{j\omega n}\right) + 2e^{j\omega n}d\omega\right)$$

$$x[n] = \frac{1}{2}\left(\frac{2}{jn}e^{j\omega n}\Big|_{-\pi}^{-\pi/2} + \frac{1}{jn}e^{j\omega n}\Big|_{-\pi/2}^{\pi/2} + \frac{2}{jn}e^{j\omega n}\Big|_{\pi/2}^{\pi}\right) + \int_{\pi/2}^{\pi} -\frac{2\omega}{\pi}e^{j\omega n} - \frac{2\omega}{\pi}e^{-j\omega n}$$

$$x[n] = \frac{1}{2}\left(\frac{2}{jn}e^{\frac{-\pi jn}{2}} - \frac{2}{jn}e^{-\pi jn} + \frac{1}{jn}e^{jn\pi/2} - \frac{1}{jn}e^{-jn\pi/2} + \frac{-2}{\pi}\int_{\pi/2}^{\pi}\omega\left(2\cos(\omega n)\right)d\omega\right)$$

$$\to x[n] = \frac{1}{jn}\left(\cancel{e^{-\pi jn/2}} - e^{-\pi jn} + \cancel{e^{jn\pi/2}} - e^{-jn\pi/2} + e^{jn\pi} - e^{jn\pi/2}\right) + \frac{-1}{\pi}\left(\int\right)$$

$$x[n] = \frac{1}{jn}\left(2j\sin(n\pi) - 2\cos(n\pi/2)\right) - \frac{2}{\pi}\left(\int_{\pi/2}^{\pi}\omega\cos(\omega n)\,d\omega\right)$$

$$\Rightarrow x[n] = \frac{2\sin(n\pi)}{n} + \frac{2j\cos(n\pi/2)}{n} - \frac{2}{\pi}\left(\frac{n\omega\sin(n\omega)+\cos(n\omega)}{n^2}\right)$$

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منطقه ۴

۴

$$x[n] = b^n u[n]$$

$$h[n] = a^n u[n] \rightarrow y[n] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{+\infty} b^k u[k]\, a^{n-k} u[n-k] = \sum_{k=0}^{+\infty} b^k a^{n-k} u[n-k]$$

$$= \sum_{k=n}^{+\infty} b^k a^{n-k} = a^n \sum_{k=n}^{+\infty} b^k a^{-k} = a^n \times \sum_{k=n}^{+\infty} \left(b/a\right)^k$$

$$\boxed{= a^n \times \frac{\left(1 - (b/a)^{n+1}\right)}{1 - b/a}}$$