① $x[n] = \cos(\pi/۳\, n) + 1 = \dfrac{e^{j\pi/۳\, n}}{۲} + \dfrac{e^{-j\pi/۳\, n}}{۲} + e^{j\pi/۲\, \cdot 0}$

$$\boxed{\begin{aligned} a_0 &= 1 \\ a_1 = a_{-1} &= 1/۲ \end{aligned}}$$

$y[0] = ۲$

$y[1] = -1/۲$

$y[-1] = -1/۲$

دوباره تکرار می‌شود

ادامه ② !

$x_۲(t) = \cos(\omega t) = \dfrac{e^{j\omega t} + e^{-j\omega t}}{۲}$

$\rightarrow \boxed{a_1 = a_{-1} = 1/۲}$

$y(t) = a_1 H(j\omega \times 1) e^{j\omega t} + a_{-1} H(j\omega \times -1) e^{-j\omega t}$

$= \frac{1}{۲}\left(\frac{-1}{j\omega}\left(e^{-۲j\omega} - 1\right) e^{j\omega t} + \frac{1}{j\omega}\left(e^{۲j\omega} - 1\right) e^{-j\omega t} \right)$

$= \frac{1}{۲j\omega}\left(\left(1 - e^{-۲j\omega}\right) e^{j\omega t} + \left(e^{۲j\omega} - 1\right) e^{-j\omega t} \right)$

$= \frac{1}{۲j\omega}\left(\left(e^{j\omega t} - e^{-j\omega t}\right) + \left(-e^{j\omega(t-۲)} + e^{-j\omega(t-۲)}\right) \right)$

$= \dfrac{e^{j\omega t} - e^{-j\omega t}}{۲j\omega} + \dfrac{e^{j\omega(t-۲)} - e^{-j\omega(t-۲)}}{۲j\omega}$

الف) ۲

$$h(t) = \begin{cases} 1 & 0 \le t \le 2 \\ 0 & o.w. \end{cases}$$

$$y(t) = \sum_{k=-\infty}^{+\infty} a_k H(jw_k) e^{jw_k t}$$

$$h(jw) = \int_{-\infty}^{+\infty} h(t) e^{-jwt} dt = \int_0^2 e^{-jwt} dt = \frac{-1}{jw} e^{-jwt} \Big|_0^2$$

$$\boxed{= \frac{-1}{jw} \left(e^{-2jw} - 1 \right)}$$

$$x_1(t) = e^{jwt} = e^{j \times \frac{2\pi}{T} t \times k} \longrightarrow \boxed{a_1 = 1}$$

$$y(t) = \sum_{k=-\infty}^{+\infty} a_k H(jw_k) e^{jw_k t} = a_1 \times H(jw) e^{jwt}$$

$$= 1 \times \frac{-1}{jw} \left(e^{-2jw} - 1 \right) e^{jwt} = \frac{-1}{jw} \left(e^{-2jw} - 1 \right) \left(\cos(wt) + j \sin(wt) \right)$$

$$\frac{-1}{j} = j \Rightarrow y(t) = \frac{j}{w} \left(e^{-2jw} - 1 \right) \left(\cos(wt) + j \sin(wt) \right)$$

$$= \frac{\left(\cos(2w) + j \sin(2w) - 1 \right)}{w} \left(-\sin(wt) + j \cos(wt) \right)$$

$$\boxed{= -F(w) \sin(wt) + j F(w) \cos(wt)}$$

$$= \boxed{\frac{\sin(\omega t)}{\omega} - \frac{\sin(\omega(t-۲))}{\omega}}$$

---

(۳)

$$h(t) = ۳e^{-۱۰t}u(t)$$

$$h(j\omega_k) = \int_{-\infty}^{+\infty} h(t)\, e^{-j\omega_k t}\, dt = \int_{0}^{+\infty} ۳e^{-۱۰t} \times e^{-j\omega_k t}\, dt$$

$$= ۳\int_{0}^{+\infty} e^{t(-۱۰-j\omega_k)}\, dt = \frac{۳}{-۱۰-j\omega_k}\left(e^{t(-۱۰-j\omega_k)}\Big|_{0}^{\infty} \right)$$

$$\Rightarrow \boxed{h(j\omega_k) = \frac{۳}{-۱۰-j\omega_k}}$$

$$x(t) = ۳\cos(۵t) = \frac{۳}{۲}e^{j۵t} + \frac{۳}{۲}e^{-j۵t}$$

$$\boxed{a_1 = a_{-1} = ۳/۲}$$

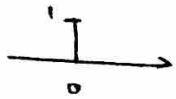
$$y(t) = ۳/۲ \times \frac{۳}{-۱۰-j\omega} e^{j\omega t} + ۳/۲ \times \frac{۳}{-۱۰+j\omega} e^{-j\omega t}$$

$$\boxed{y(t) = ۹/۴\left(\frac{e^{j\omega t}}{-۱۰-j\omega} + \frac{e^{-j\omega t}}{-۱۰+j\omega} \right)}$$

٤ .a

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j\omega t}\, dt$$



$\Leftarrow$ $\boxed{t=0}$ در همه‌جا صفر است به جز

می‌دانیم : $\int_{-\infty}^{+\infty} \delta(t)\, A\, dt = \boxed{A}$

$\boxed{\text{جواب} = 1}$

.b

$$x(t) = e^{-at} u(t)$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt = \int_{0}^{+\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= \int_{0}^{+\infty} e^{t(-a-j\omega)}\, dt = \frac{-1}{a+j\omega} e^{t(-a-j\omega)}\Big|_{0}^{+\infty} = \boxed{\frac{1}{a+j\omega}}$$

$$\boxed{X(j\omega) = \frac{1}{a+j\omega}}$$

.c

$$x(t) = e^{-\alpha|t|}$$

$$X(j\omega) = \int_{-\infty}^{+\infty} e^{-\alpha|t|} e^{-j\omega t}\, dt$$

$$= \int_{0}^{+\infty} e^{-\alpha t} e^{-j\omega t}\, dt + \int_{-\infty}^{0} e^{\alpha t} e^{-j\omega t}\, dt$$

$$= \int_{0}^{+\infty} e^{t(-\alpha - j\omega)} + e^{t(-\alpha + j\omega)} \, dt$$

$$= \frac{-1}{\alpha + j\omega} e^{t(-\alpha - j\omega)} + \frac{1}{-\alpha + j\omega} e^{t(-\alpha + j\omega)} \Bigg|_{0}^{+\infty}$$

$$\boxed{= \frac{1}{\alpha + j\omega} + \frac{1}{\alpha - j\omega}}$$

د

$$x(t) = \begin{cases} x & -1 \leq x \leq 1 \\ 0 & \text{ow} \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} \, dt = \int_{-1}^{+1} t e^{-j\omega t} \, dt$$

$$= t \times \frac{-e^{-j\omega t}}{j\omega} + \frac{e^{-j\omega t}}{\omega^2} \Bigg|_{-1}^{+1}$$

$$= \frac{-e^{-j\omega}}{j\omega} + \frac{e^{-j\omega}}{\omega^2} - \left(+ \frac{e^{+j\omega}}{j\omega} + \frac{e^{j\omega}}{\omega^2} \right)$$

$$= \frac{-(e^{-j\omega} + e^{j\omega})}{j\omega} - \frac{e^{j\omega} - e^{-j\omega}}{\omega^2}$$

$$\boxed{= \frac{-2\cos(\omega)}{j\omega} - \frac{2j \sin(\omega)}{\omega^2}}$$

۵. ⓐ

$$x(t)=\frac{1}{۲\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega t}$$

$$x(t-t_0)=\frac{1}{۲\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega(t-t_0)} = \frac{1}{۲\pi}\int_{-\infty}^{+\infty} \underbrace{\left(X(j\omega)e^{-j\omega t_0}\right)}_{X'(j\omega)} e^{j\omega t}$$

ⓑ خاصیت جابجایی

$$x(t)=\frac{1}{۲\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega t}$$

$$x^{*}(t)=\frac{1}{۲\pi}\int_{-\infty}^{+\infty}\left(X(j\omega)e^{j\omega t}\right)^{*}=\frac{1}{۲\pi}\int_{-\infty}^{+\infty} X^{*}(j\omega)\, e^{-j\omega t}$$

$$=\frac{1}{۲\pi}\int_{-\infty}^{+\infty} X^{*}(-(-j\omega))\, e^{-j\omega t} \quad \boxed{=\frac{1}{۲\pi}\int_{-\infty}^{+\infty} X^{*}(-j\omega)\, e^{j\omega t}}$$

٥.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega \qquad X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(at) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+aj\omega t}\, d\omega$$

$$\omega = \frac{\omega}{a} \rightarrow d\omega = \frac{d\omega}{a} \Rightarrow x(at) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X\left(\frac{j\omega}{a}\right) e^{j\omega t}\, \frac{d\omega}{a}$$

$$\boxed{\rightarrow X'(j\omega) = X\left(\frac{j\omega}{a}\right) \times \frac{1}{a}}$$

به صورت قدر مطلق در نظر می‌گیریم

تا ضرب‌ها قرینه نشوند