$$x[1-n] = x[-(n+1)]$$

(1)

$$x[n] \to x(n-1) \Longrightarrow a_1 = -e^{-j\omega \times 1}$$

$$\Rightarrow a_{-2} = e^{-2j\omega}$$

$$\Rightarrow a_3 = -je^{-5j\omega}$$

$$x[n] \to x[-n] \to \begin{cases} a_{-1} = e^{-1j\omega} \\ a_{+2} = e^{-2j\omega} \\ a_{-5} = -je^{-5j\omega} \end{cases}$$

(٩)

$$x(t) = \sin^{2}(t) = \left(\frac{e^{jt} - e^{-jt}}{2j}\right)^{2}$$

$$= \frac{e^{2jt} + e^{-2jt} - 2}{-4}$$

$$a_{2} = a_{-2} = -1/4$$

$$a_{0} = 1/2$$

$$\frac{2\pi}{T} = \omega_0$$

$$x(t)\cos\left(\frac{2\pi t}{T}\right) = x(t)\cos(\omega_0 t) = \sum a_k e^{jk\omega_0 t} \times \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right)$$

$$= \sum_{k=-\infty}^{k=2+\infty} \frac{a_k}{2}\left(e^{jk\omega_0 t(k+1)} + e^{jk\omega_0 t(k-1)}\right)$$

برای مثال $\rightarrow \frac{a_k}{2} e^{jk\omega_0 t(k+1)} + \frac{a_{k+2}}{2} e^{jk\omega_0 t(k+1)}$

در نتیجه $\rightarrow$ $$a_k = \frac{a_{k-1} + a_{k+1}}{2}$$

(۴)

$$x_1(t) = 2x(t - 0.5)$$

۱- اثر $x(t - 0.5) \longrightarrow a_k \times e^{\frac{-j\omega k}{2}}$

$$2x(t) \longrightarrow 2a_k \times e^{\frac{-j\omega k}{2}}$$

حدید $a_k = \begin{cases} e^{\frac{-j\omega k}{2}} = 1 & k = 0 \\ \dfrac{\sin(\frac{k\pi}{2}) \times 2 \times e^{\frac{-j\omega k}{2}}}{k\pi} & \end{cases}$ $\omega = \frac{2\pi}{2} = \pi$

$$\frac{2}{k\pi} \times e^{\frac{-j\pi k}{2}} \left(\frac{e^{\frac{k\pi j}{2}} - e^{\frac{-k\pi j}{2}}}{2j} \right)$$

$$= \frac{1}{k\pi j} \times \left(1 - e^{-k\pi j}\right) = a_k$$

$a_k \in \mathbb{R}$ ← (a) (b)

$N = 5$ ← (b)

$$x[n] = \sum_{k=\langle 5 \rangle} a_k e^{j\omega_0 k n}$$

$$a_k = \frac{1}{5} \sum_{n=\langle 5 \rangle} x[n] \times e^{-j\omega_0 k n}$$

$$-1 = e^{j\pi} \rightarrow \sum_{n=\langle 5 \rangle} e^{\frac{j2\pi}{5} n} \times x[n] = 5$$

$$\rightarrow \boxed{a_{-1} = 1} \Rightarrow \boxed{a_1 = 1}$$

از اینجا

$$\rightarrow (-1)^{\frac{2\pi}{5}} = 1^{\frac{2\pi}{5}} \rightarrow e^{0}$$

$$\sum_{n=\langle 5 \rangle} e^{j0\frac{2\pi}{5}n} x[n] = 0 \rightarrow \boxed{a_0 = 1}$$

$$\frac{1}{N} \sum |x[n]|^2 = \sum |a_k|^2 \rightarrow \sum |a_k|^2 = 5$$

$$a_0^2 + a_1^2 + 2a_2^2 = 5 \rightarrow |a_2| = 1 \xrightarrow{\text{طبق}} a_2 = a_{-2} = 1$$