$$X(e^{jw}) = \sum_{k=-\infty}^{+\infty} x[n] e^{-jw_k n}$$ ①

(a)

$$\rightarrow X(e^{jw}) = \sum_{n=0}^{+\infty} (1/2)^n e^{-jw_k n} = \sum_{n=0}^{+\infty} \left(\frac{e^{-jw_k}}{2}\right)^n$$

$$= \boxed{\frac{1}{1 - \frac{e^{-jw_k}}{2}}}$$

(b)

$$X(e^{jw}) = \sum_{n=0}^{+\infty} a^n \sin(w_0 n) e^{-jw_k n}$$

$$= \frac{1}{2j} \sum_{n=0}^{+\infty} \left(a^n e^{jw_0 n} e^{-jw_k n}\right) - \left(a^n e^{+jw_0 n} e^{-jw_k n}\right)$$

$$= \frac{1}{2j} \sum_{n=0}^{+\infty} \left(a e^{jw_0} e^{-jw_k}\right)^n - \left(a e^{+jw_0} e^{-jw_k}\right)^n$$

$$= \boxed{\frac{1}{2j} \times \left(\frac{1}{1 - a e^{j(w_0 - w_k)}} - \frac{1}{1 - a^{-j(w_0 + w_k)}}\right)}$$

ج. ۴-۱

$$X(e^{jw}) = \sum_{n=-2}^{\infty} (1/4)^n e^{-jw_K n} = (1/4)^{-2} e^{2jw_K} + (1/4)^{-1} e^{jw_K} + \frac{1}{1 - \frac{e^{-jw_K}}{4}}$$

طبق ۵

د.

$$X(e^{jw}) = \sum_{n=0}^{3} e^{-jw_K n} = e^{0} + e^{-jw_K} + e^{-2jw_K} + e^{-3jw_K}$$

۲

$$H(e^{jw}) = \sum_{n=-\infty}^{+\infty} \left[(1/2)^n \cos(n\pi/2)\right] u[n] \times e^{-jwn}$$

$$= \sum_{n=0}^{+\infty} (1/2)^n \cos(n\pi/2) e^{-jw_K n}$$

$$= \sum_{n=0}^{+\infty} \frac{(1/2)^n e^{jn\pi/2} e^{-jw_K n}}{2} + \frac{(1/2)^n e^{-jn\pi/2} e^{-jw_K n}}{2}$$

$$= 1/2 \times \sum_{n=0}^{\infty} \left(\frac{e^{j\pi/2} e^{-jw_K}}{2}\right)^n + \left(\frac{e^{-j\pi/2} e^{-jw_K}}{2}\right)^n$$

$$= 1/2 \left(\frac{1}{1 - \frac{e^{j\pi/2 - jw_K}}{2}} + \frac{1}{1 - \frac{e^{-j\pi/2 - jw_K}}{2}} \right) = \frac{1}{1 + 1/4\, e^{-2jw_K}}$$

(۳ ادامه)

$$\cos(n\pi/2) \rightarrow a_1 = a_{-1} = 1/2$$

$$y[n] = \sum a_k H(e^{j\omega}) e^{j\omega_k n}$$

$$\rightarrow y[n] = 1/2 \times \Sigma/e \, e^{-j\pi/2 n} + 1/2 \times \Sigma/e \, e^{j\pi/2 n}$$

$$= \Sigma/e \cos(\pi/2 n)$$

(a) Σ

$$w = 0 \rightarrow X(e^{jw}) = \sum x(n) = \Sigma$$

(b)

$$w = \pi \rightarrow e^{-j\pi n} \rightarrow \sum (-1) x[n] = 0$$

(c)

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw}) e^{jwn} dw \rightarrow x[0] = -1 \rightarrow$$

$$\int_{-\pi}^{\pi} X(e^{jw}) dw = -2\pi$$

(d)

$$\int_{2\pi} |X(e^{jw})|^2 dw = 2\pi \sum |x[n]|^2 = 34\pi$$ پارسوال

(e)

$$\int_{2\pi} \frac{|dX(e^{jw})|^2}{dw} dw = 2\pi \sum |1/j\, n\, x[n]|^2 = 2\pi \times 1C2$$

(f)

$$\boxed{X(e^{jw})_R = 1/2 \left(X(e^{jw}) + X^*(e^{jw}) \right) = 1/2 \left(x[n] + x[-n] \right)}$$

۵

$$\boxed{x_1[n] = \left(\tfrac{1}{2}\right)^n u[n] \rightarrow X_1(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega_k}}}$$

$$y_1[n] = n\,x[n] + \sum_{k=0}^{n} x[n] \rightarrow Y_1(e^{j\omega}) = \frac{1}{\left(1-\frac{1}{2}e^{-j\omega_k}\right)^2}$$

$$Y_1(e^{j\omega}) = X_1(e^{j\omega}) \times H(e^{j\omega}) \rightarrow \boxed{H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega_k}}}$$

$$\left.\begin{aligned} X_2(e^{j\omega}) &= \frac{Y_2(e^{j\omega})}{H(e^{j\omega})} \\ Y_2(e^{j\omega}) &= \frac{1}{1-\frac{1}{2}e^{j\omega_k}} \end{aligned}\right\} \rightarrow \boxed{X_2(e^{j\omega}) = 1 \rightarrow x[n] = \delta[n]}$$

۶

$$\boxed{\text{سمت چپ} = h_1[n] * h_2[n]}$$

ضرب

$$\text{سمت راست} = \frac{1}{2\pi}\int H_1(e^{j\omega}) H_2(e^{j\omega})\, d\omega \longrightarrow h_2[n]$$

کانولوشن

$$\Rightarrow \boxed{h_3[n] = h_1[n] \hat{*} h_2[n]}$$