University of South-Eastern Norway

Solution to exam in course FM1220 Automatic Control

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Solution to Problem 1 (5 %) Mathematical block diagram

We can start by writing the model is written as:

$$y'' = \left(B\sqrt{y'} + Cu + Dd\right)/A \tag{1}$$

The block diagram can be drawn as shown in Figure 10.

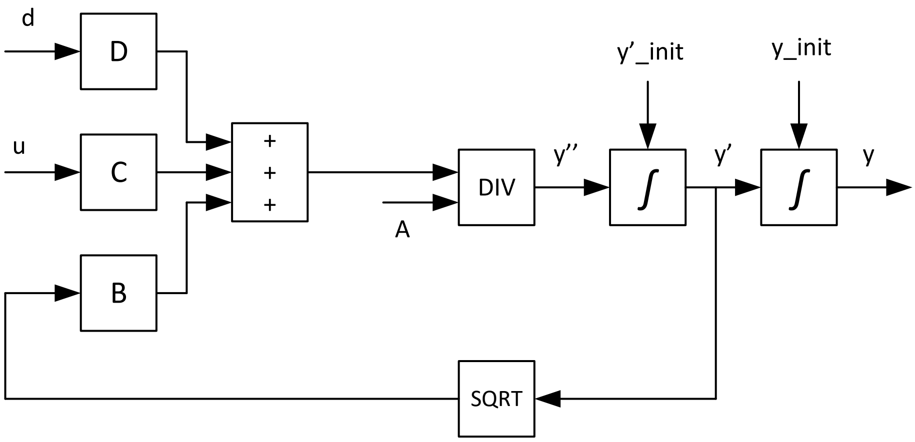

Figure 1: Problem 1: Block diagram.

Solution to Problem 2 (8 %) Simulation algorithm

We start by writing the model as a state space model. Firstly, we write the model as:

$$y'' = \left(B\sqrt{y'} + Cu + Dd\right)/A \tag{2}$$

Then we define state variables:

$$x_1 = y$$

$$x_2 = y'$$

The state space model becomes:

$$x_1' = x_2$$

$$x_2' = (B\sqrt{x_2} + Cu + Dd)/A$$

$$y = x_1$$

Simulation algorithm based on Euler forward integration of x_1' and x_2':

- Initialization (before the simulation loop):
  - x1_k = x1_init
  - x2_k = x2_init
- Inside the simulation loop:
  - Limitation of x1_k and x2_k between respective min and max values
  - Euler forward integration:
    - dx1_dt_k = x2_k
    - dx2_dt_k = (B*sqrt(x2_k) + C*u_k + D*d_k)/A
    - x1_kp1 = x1_k + dt*dx1_dt_k
    - x2_kp1 = x2_k + dt*dx2_dt_k
    - y_k = x1_k
  - Any use of x1_k, x_2, and y_k, e.g. storing in an array for later plotting
  - Time index shift:
    - x1_k = x1_kp1
    - x2_k = x2_kp1
  - After the simulation loop: Plotting, saving simulation data to file, etc.

Solution to Problem 3 (5 %) Adaptation of a dynamic model to data

Defining the following optimization criterion to be minimized: Sum of squared prediction errors over the pertinent time interval of the time series, where the prediction error is the difference between the observed output and the simulated output. The model parameters are the optimization variables to be found using a proper optimization method, e.g. the grid method or an search (iterative) method.

Solution to Problem 4 (6 %) Transfer function of PID controller

Given the continuous-time PID controller:

$$u(t) = u_{\text{man}} + K_c e(t) + \frac{K_c}{T_i}\int_0^t e\,d\tau + K_c T_d e'(t) \tag{3}$$

Laplace transformation:

$$u(s) = K_c e(s) + \frac{K_c}{T_i}\cdot\frac{1}{s}\cdot e(s) + K_c T_d \cdot s \cdot e(s) \tag{4}$$

Transfer function:

$$C(s) = \frac{u(s)}{e(s)} = K_c + \frac{K_c}{T_i s} + K_c T_d s \tag{5}$$

Solution to Problem 5 (8 %) Process dynamics

Time-constant with time-delay:

- Transfer function:

$$H_1(s) = \frac{K}{Ts+1}e^{-\tau s}$$

- The unit-step response is shown in Figure 2.

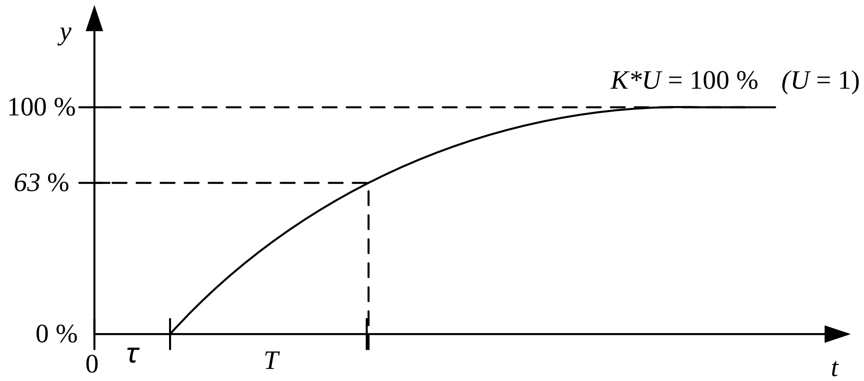

Figure 2: Problem 5: Unit-step response of time-constant with time-delay.

Integrator with time-delay:

- Transfer function:

$$H_2(s) = \frac{K_i}{s}e^{-\tau s}$$

- The unit-step response is shown in Figure 3.

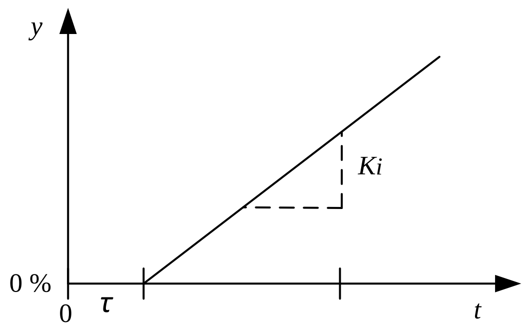

Figure 3: Problem 5: Unit-step response of integrator with time-delay.

Solution to Problem 6 (10 %) PID tuning with Ziegler-Nichols

In the program, the integral time, Ti, of the PI controller is set to a very large number, here: 1e9. The critical gain, Kcu, is found by trial and error as Kcu = 8. The corresponding sustained oscillations are shown in Figure 4. The period is read off as Pu = 7.8 s. The PI

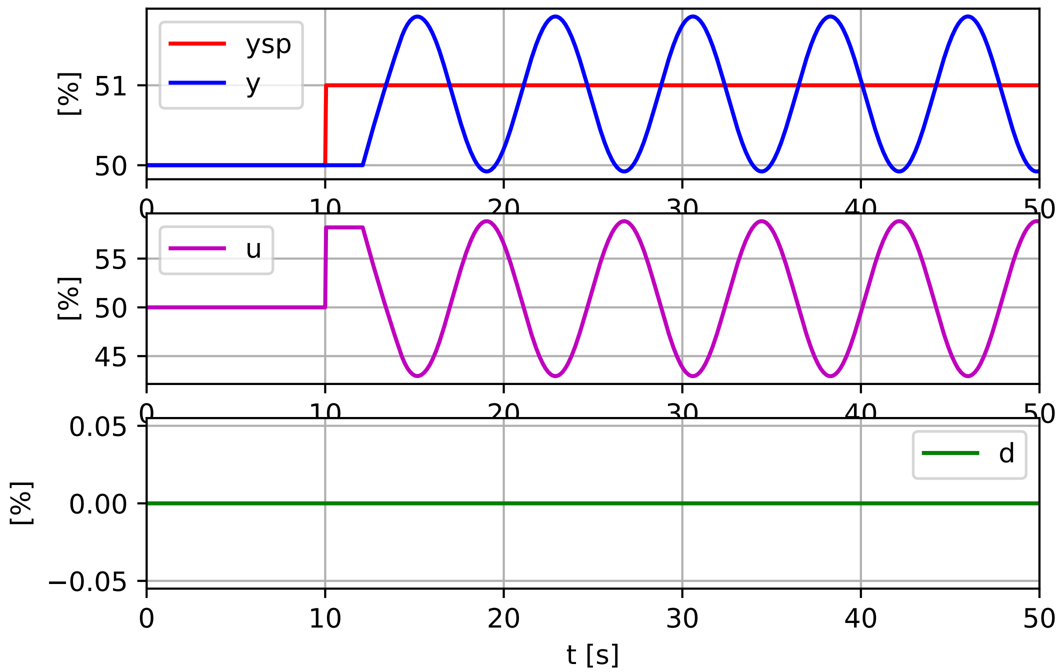

Figure 4: Problem 6: Sustained oscillations.

settings become:

$$K_c = 0.45 K_{cu} = 0.45 \cdot 8.2 = 3.7$$

$$T_i = \frac{P_u}{1.2} = \frac{7.8}{1.2} = 6.5 \text{ s}$$

Figure 5 shows the response due to a unit step in the setpoint. The amplitude decay ratio is clearly less than one quarter. Therefore, the stability is better than the Ziegler-Nichols "one quarter decay ratio" stability.

Solution to Problem 7 (5 %) Transfer function of a control system

$$\begin{aligned} e(s) &= y_{\text{sp}}(s) - y(s) \\ &= y_{\text{sp}}(s) - P(s) \cdot u(s) \\ &= y_{\text{sp}}(s) - P(s) \cdot C(s) \cdot e(s) \end{aligned}$$

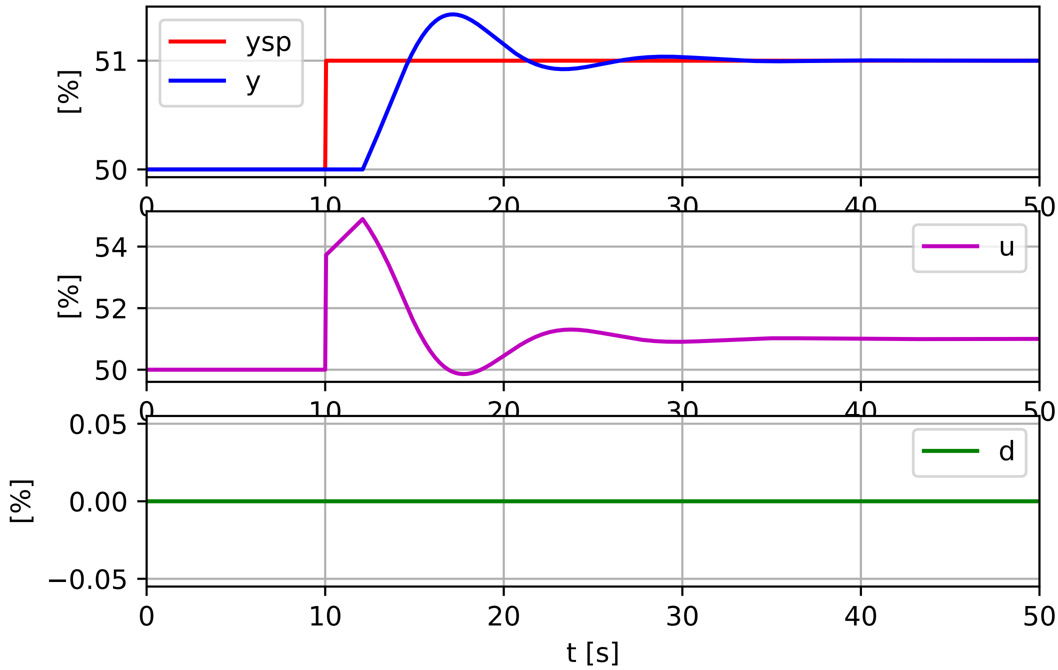

Figure 5: Problem 6: Step response with tuned PI controller.

Solving for $e(s)$ gives:

$$e(s) = \frac{1}{1 + P(s) \cdot C(s)} y_{\text{sp}}(s)$$

So, the transfer function from $y_{\text{sp}}(s)$ to $e(s)$ is:

$$\frac{e(s)}{y_{\text{sp}}(s)} = \frac{1}{1 + P(s) \cdot C(s)} = \frac{1}{1 + L(s)} = S(s)$$

where $L(s)$ is the loop transfer function. $S(s)$ is called the sensitivity transfer function.

Solution to Problem 8 (5 %) Stability margins

The gain margin becomes:

$$\text{GM} = \frac{5}{2} = 2.5$$

which is in the range [1.7, 4.0] of acceptable GM values.

The phase margin becomes:

$$\text{PM} = 360° \cdot \frac{10}{90} = 40°$$

which is in the range [30°, 45°] of acceptable PM values.

The control system has acceptable stability since both GM and PM are in respectively acceptable ranges.

Solution to Problem 9 (12 %) Averaging level control

1. (6 %) Figure 6 shows a plot of the simulated level. The level margin is approximately 0.18 m.

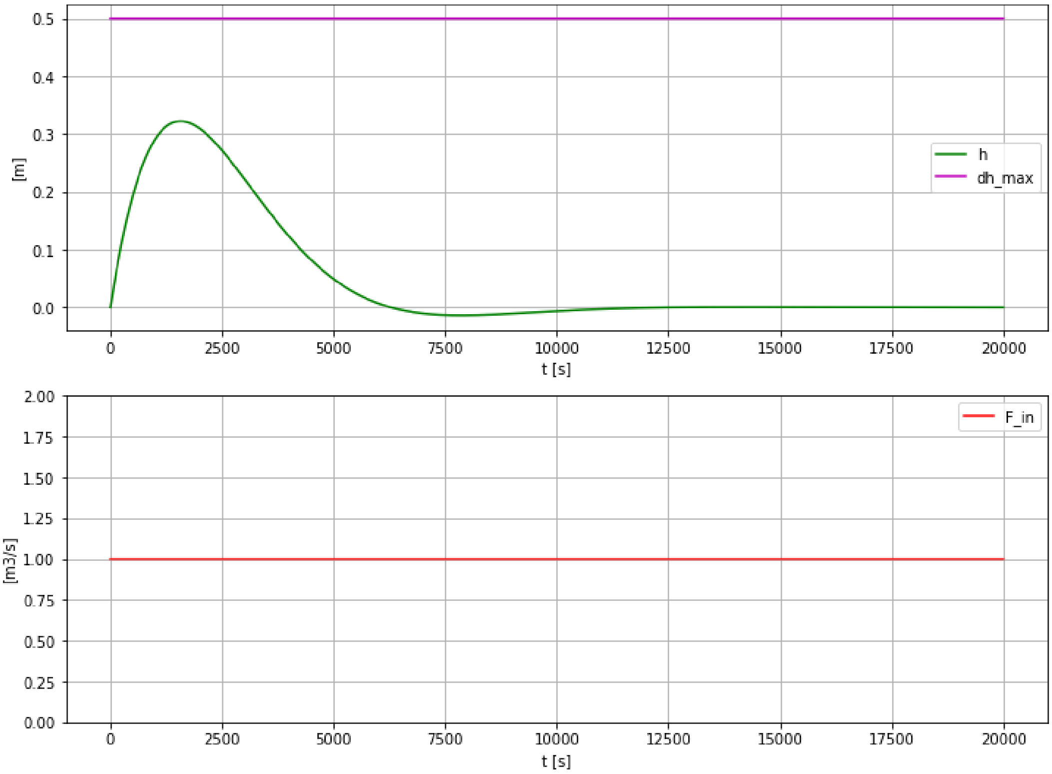

Figure 6: Problem 9: Plot of simulated level.

2. (6 %) Figur 7 shows F_in with period t_p_F_in = 1250 s and the resulting response in F_out. From Figure 7 we see that the ratio between the amplitude of F_out and the amplitude of F_in is approximately 0.2. The Bode plot shown in Figure 8 also indicates an amplitude gain of approximately 0.2 at frequency 0.0008 = 1/1250 Hz of the sinusoidal F_in. So, the simulation and the Bode plot are in accordance.

Solution to Problem 10 (8 %) Cascade control system

One example: Figure 9 shows a P&I diagram of a temperature controlled heat exchanger in which a cold process water flow is heated using a heating medium. The control structure is cascade control where the primary loop is a temperature control loop for the temperature of

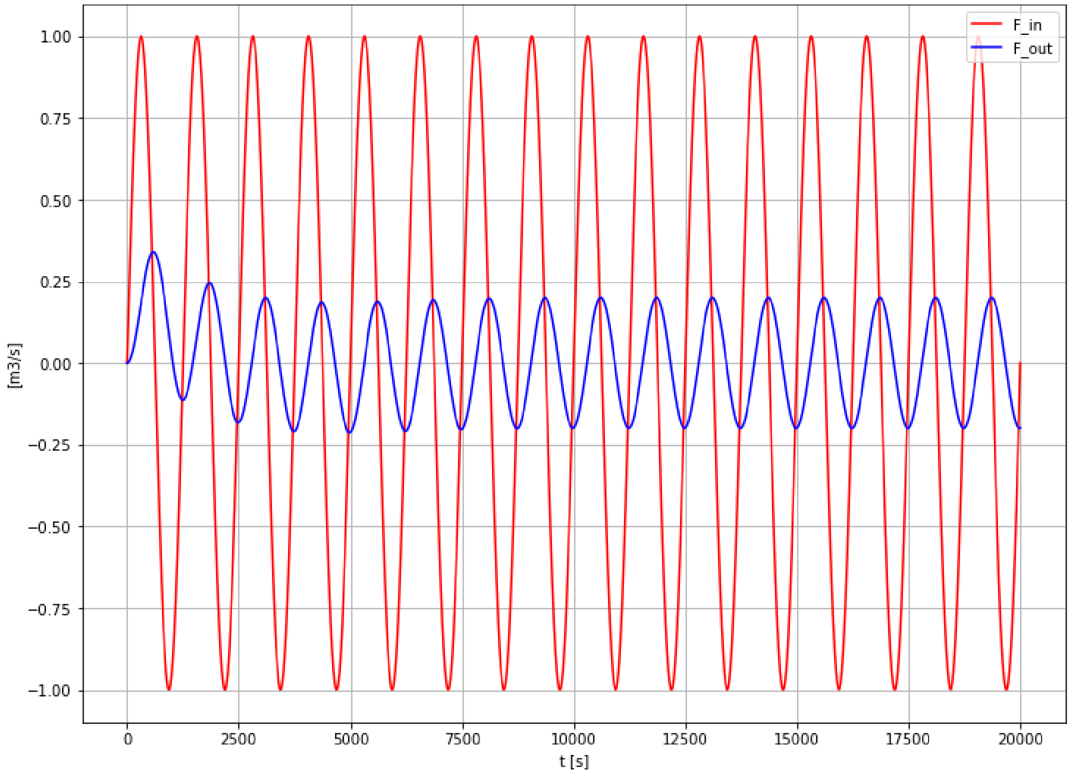

Figure 7: Problem 9: F_out with t_p_F_in = 1250 s.

the outflow of the heat exchanger. The secondary loop is a flow control loop of the hot water stream that compensates for variations in the heating medium flow caused by variations in the pressure supply (disturbance), so that the flow variations are minimized, which again minimizes variations in the temperature of the heat exchanger outflow.

Solution to Problem 11 (6 %) Feedforward control

The process model is repeated here:

$$Ay'' = B\sqrt{y'} + Cu + Dd$$

Substituting y with the setpoint y_{sp} and then solving for the control variable gives the feedforward controller:

$$u_{\mathrm{ff}} = \left(Ay''_{\mathrm{sp}} - B\sqrt{y'_{\mathrm{sp}}} - Dd\right)/C \tag{6}$$

All quantities on the right side of (6) must be known for the feedforward controller to be realizable.

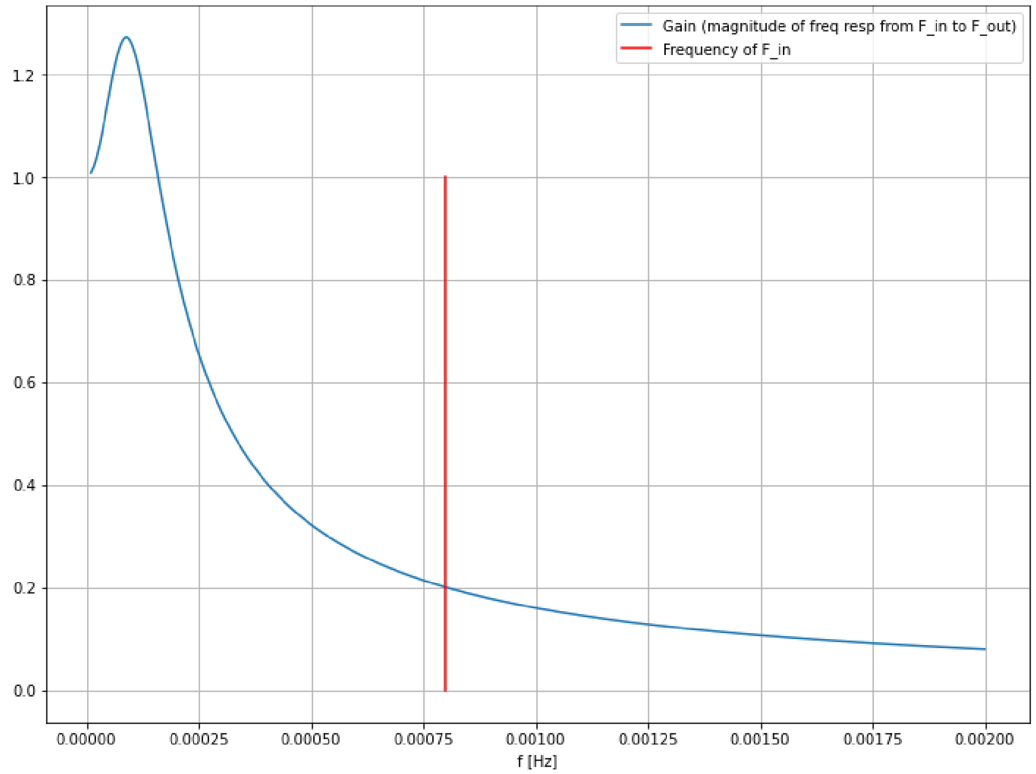

Figure 8: Problem 9: Bode plot of transfer function from F_in to F_out.

Solution to Problem 12 (10 %) Frequency response

The frequency response is found by substituting s by $j\omega$ in $H(s)$, and then writing $H(j\omega)$ on polar form (note that $j = e^{j\pi/2}$):

$$\begin{aligned} H(j\omega) &= \frac{K_i}{j\omega} e^{-\tau j\omega} \\ &= \frac{K_i}{e^{j\cdot\frac{\pi}{2}}\omega} e^{-\tau j\omega} \\ &= \frac{K_i}{\omega} e^{j\left(-\tau\omega - \frac{\pi}{2}\right)} \end{aligned}$$

The amplitude (magnitude):

$$|H(j\omega)| = \frac{K_i}{\omega}$$

The phase:

$$\arg H(j\omega) = \tau\omega - \frac{\pi}{2} \text{ [rad]}$$

Solution to Problem 13 (12 %) Control structure of a process line

Figure 10 shows the P&I diagram of the control system.

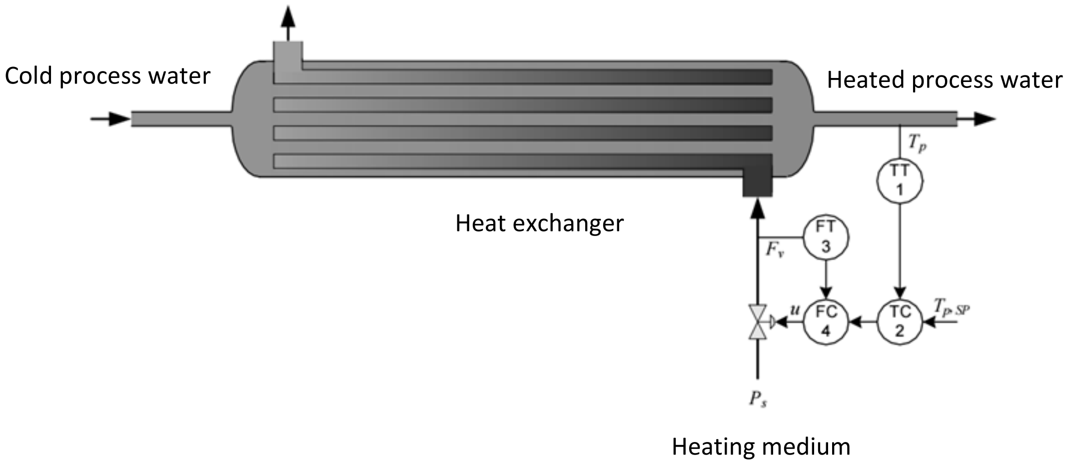

Figure 9: Problem 10: Heat exchanger.

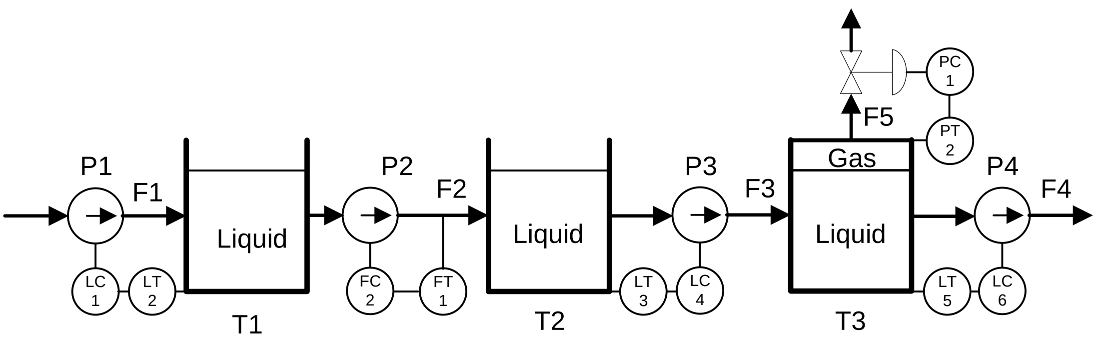

Figure 10: Process line.