University of South-Eastern Norway

Solution to resit exam in FM1220 Automatic Control on 22nd February 2021

Exam duration: 4 hours 30 minutes. Weight in final grade of the course: 100%. Exam format: Home exam with all aids allowed, but online cooperation is prohibited.

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Solution to Problem 1 (5 %) Piping & Instrumentation Diagram

See Figure 1.

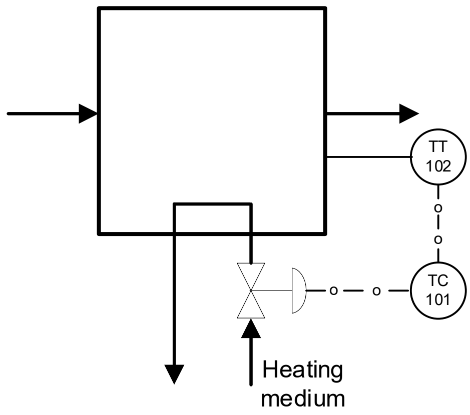

Figure 1

Solution to Problem 2 (5 %) Simulation algorithm

We can start by writing the model,

a*y’’ = b*y’ + c*y + d*u

as a state space model. Firstly,

y’’ = (b*y’ + c*y + d*u)/a

Then defining state variables x1 = y and x2 = y’ giving the state space model:

x1’ = x2

x2’ = (b*x2 + c*x1 + d*u)/a

y = x1

Simulation algorithm:

Initialization (before the simulation loop):

x1_k = x1_init

x2_k = x2_init

Inside the simulation loop:

Limitation of x1_k and x2_k between respective min and max values

Euler integration:

dx1_dt_k = x2_k

dx2_dt_k = (b*x2_k + c*x1_k + d*u_k)/a

x1_kp1 = x1_k + dt*dx1_dt_k

x2_kp1 = x2_k + dt*dx2_dt_k

y_k = x1_k

Any use of x1_k, x_2, and y_k, e.g. storing in an array for later plotting

Time index shift:

x1_k = x1_kp1

x2_k = x2_kp1

After the simulation loop:

Plotting, saving simulation data to file, etc.

Solution to Problem 3 (5 %) Dynamics

See Figure 2.

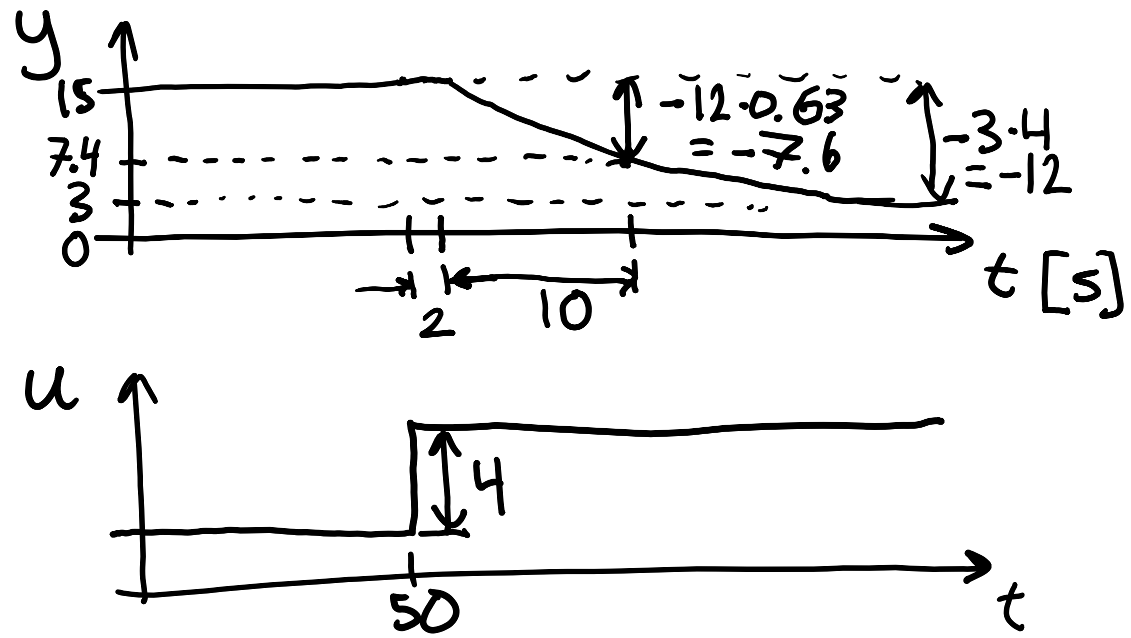

Figure 2

Solution to Problem 4 (10%) PID tuning for averaging level control

Process integrator gain is Ki = 1/A. Skogestad PI settings for an integrator process:

Kc = 1/(Ki*Tc) = A/Tc = 5/100 = 0.05 (m3/s)/m

Ti = 2*Tc = 2*100 = 200 s

Solution to Problem 5 PID tuning with the Good Gain method

a. (10 %) A "good gain" seems to be KcGG = 5.0. Figure 3 shows the step response with Kc = KcGG and Ti = inf. From the response the Tou is read off as Tou = 7.5 s, giving the following PI settings:
Kc = 0.8*KcGG = 0.8*5.0 = 4.0
Ti = 1.5*Tou = 11.25 s

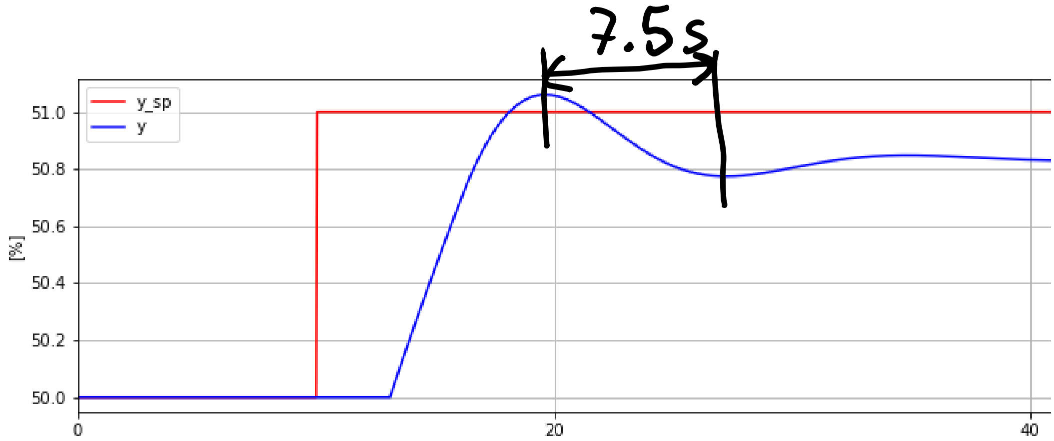

Figure 3

Figure 4 shows the response with these PI settings. The stability of the control system is acceptable.

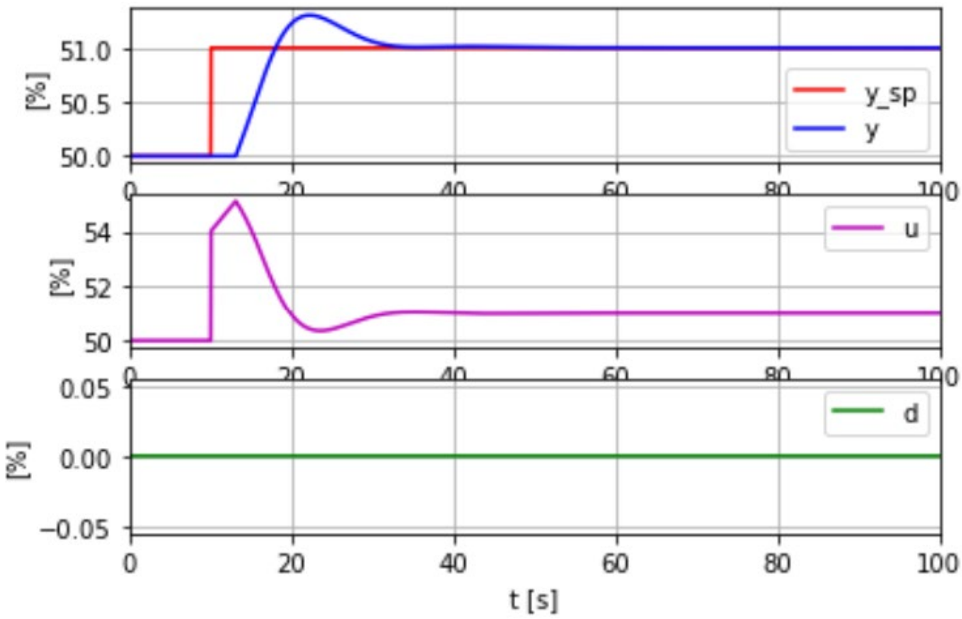

Figure 4

b (5%) The response shown in Figure 4 indicates that the steady state control error with the PI controller is zero. Figure 5 shows the responses with the I-term deactivated, i.e. with Ti = infinity. The steady state control error is read off from the response as y_sp – y in Figure 5, or with the following Python code:

```
y_sp_array[-1] - y_array[-1]
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The steady state control error is 0.2 (assuming y_sp = 51).

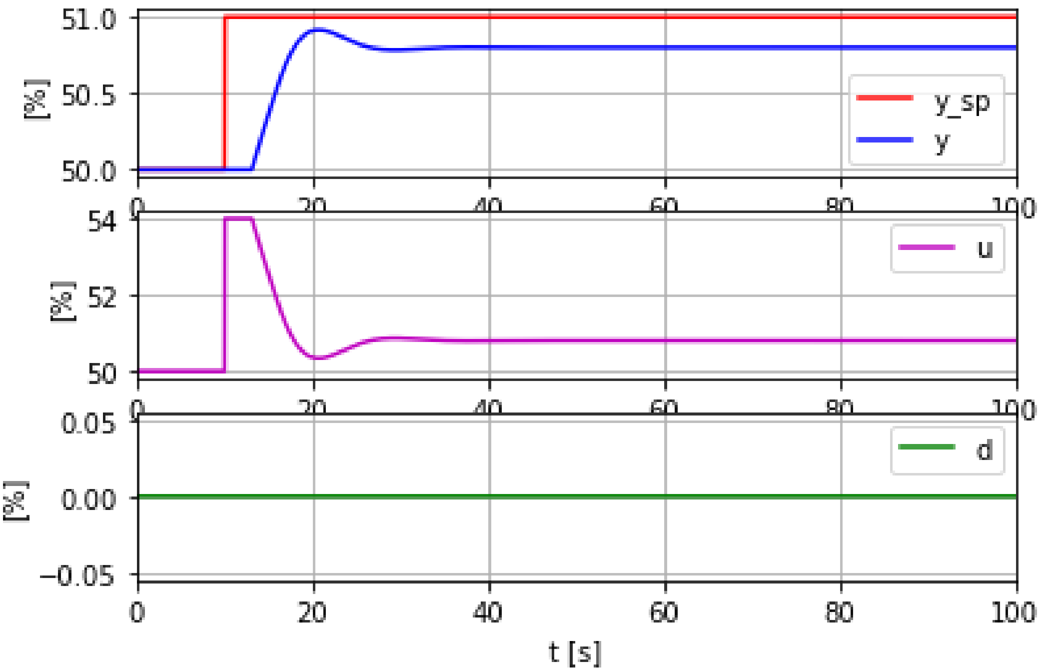

Figure 5

Solution to Problem 6 (5 %) Cascade control

See Figure 6.

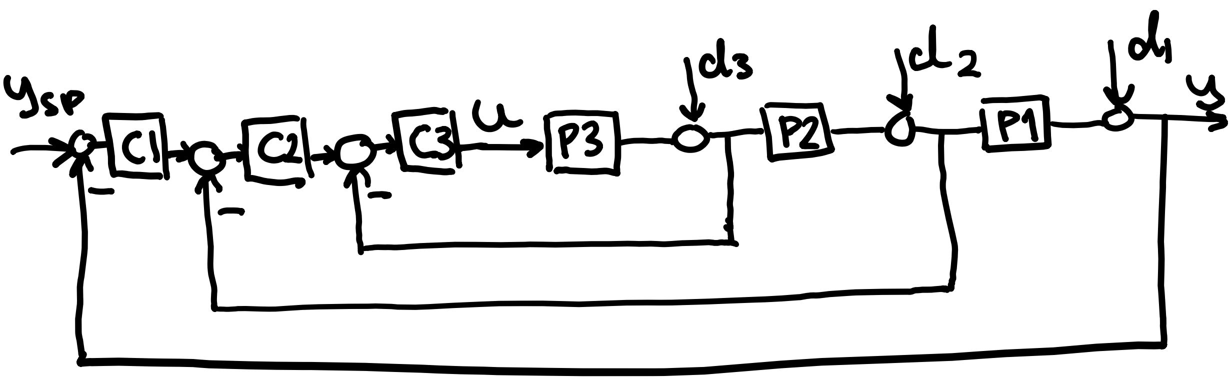

Figure 6

Solution to Problem 7 (10 %) Feedforward control

Two possible applications:

1. Dynamic positioning (ship position control): Feedforward from measured wind speed and angle, estimated water current, and position setpoint. See Figure 7.

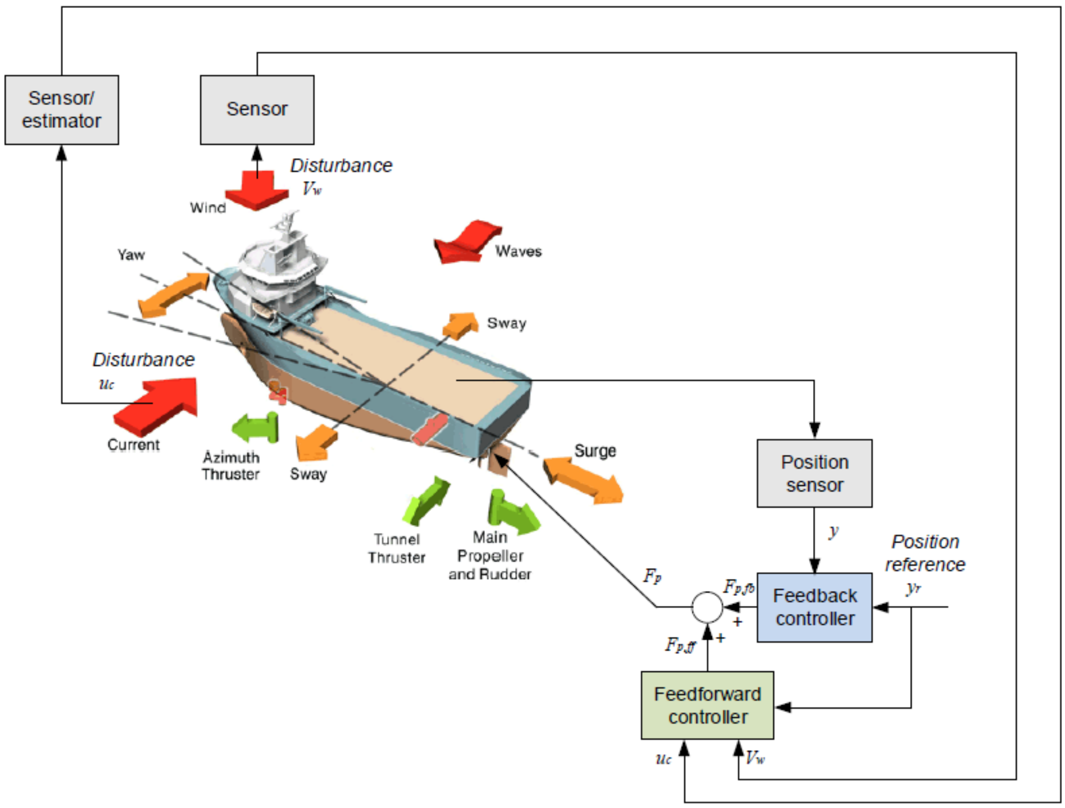

Figure 7: Feedforward (and feedback) control of ship position.

2. Level control: Feedforward from measured outflow (disturbance) and level setpoint. See Figure 8.

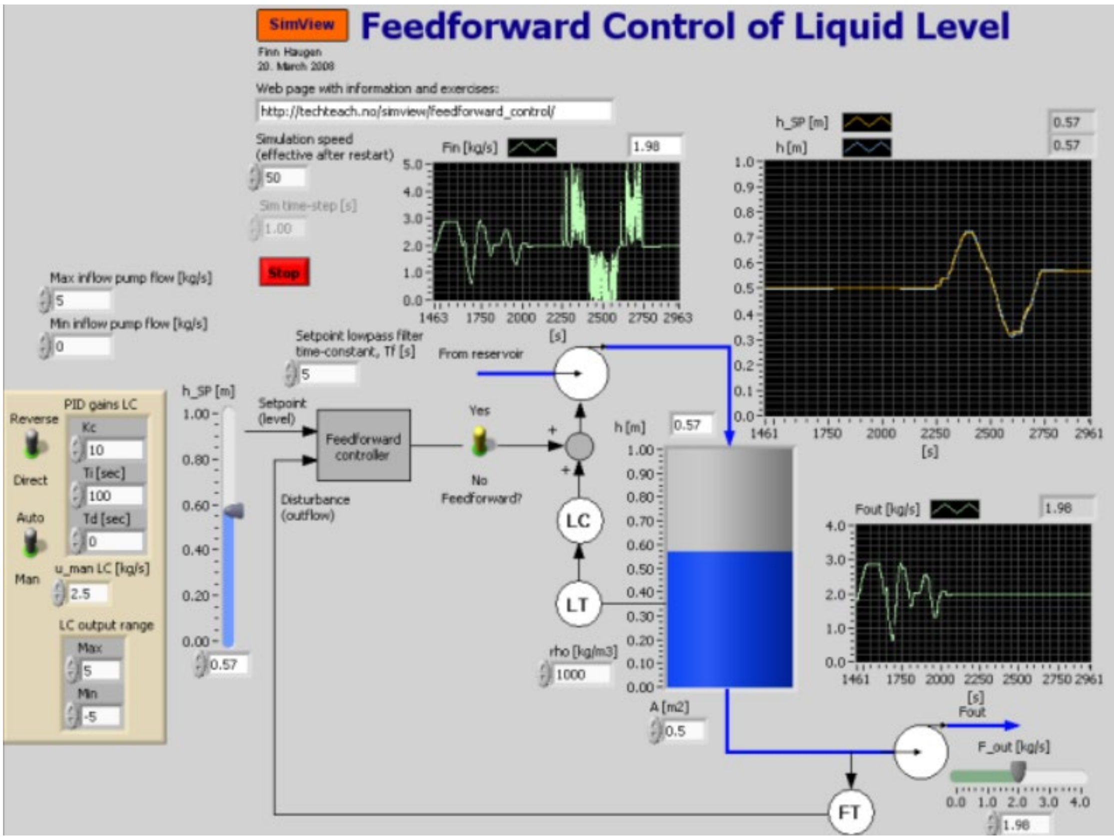

Figure 8: Feedforward (and feedback) control of liquid level.

Solution to Problem 8 (5 %) Transfer function

Taking the Laplace transformation of the differential equation gives (here, any non-zero initial values of y and y' are neglected):

A*s*y(s) + B*y(s) + C*u(s) + D*d(s) = 0

Solving for y(s) gives:

y(s) = [-C/(A*s + B)]*u(s) + [-D/(A*s + B)] *d(s)

Hence, the transfer function from u to y is:

Hu(s) = [-C/(A*s + B)]

And the transfer function from d to y is:

Hd(s) = [-D/(A*s + B)]

Solution to Problem 9 (15 %) Transfer function of a control system

Process transfer function:

$$Hp(s) = K/(T*s + 1)$$

PI controller transfer function:

$$Hc(s) = Kc + Kc/(Ti*s) = Kc*(Ti*s + 1)/(Ti*s)$$

Tracking transfer function of the control system:

$$T(s) = Hc*Hp/(Hc*Hp + 1)$$

which is:

$$T(s) = N(s)/D(s) = \text{Numerator}/\text{Denominator}$$

where:

$$D(s) = Ti*T*s^2 + (Kc*K + 1)*Ti*s + Kc*K$$

which is the characteristic polynomial.

The order of the control system the order of D(s): 2.

The poles are the roots of D(s), i.e. the solutions of:

$$D(s) = 0$$

The requirement of the poles of the control system for the control system to be asymptotically stable is that all the poles are in the left half plane (the imaginary axis excluded).

Solution to Problem 10 (15 %) Analysis of a control system

a) (7.5 %) Figure 9 shows the Bode diagram as shown in Spyder.

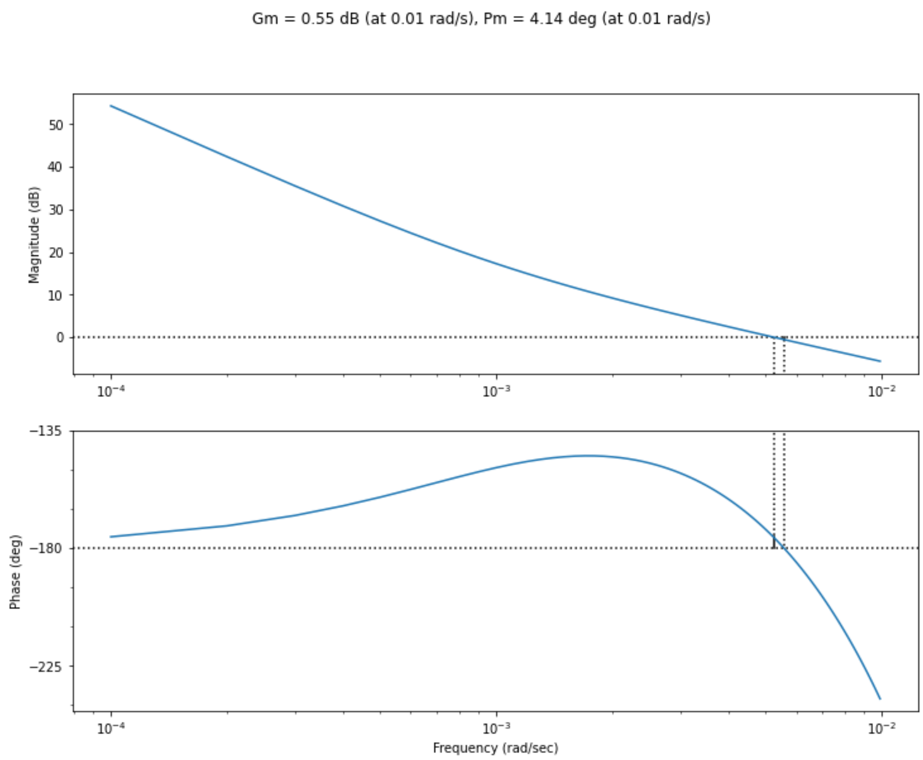

Figure 9

We see that GM = 0.55 dB, which corresponds to 10^(0.55/20) = 1.065.
PM = 4.14 degrees.
Acceptable stability margins are:

1.7 <= GM <= 4.0
30 degrees <= PM <= 45 degrees

Both GM and PM are outside these limits. The stability of the control system is therefore not satisfactory (it is necessary that both are within their acceptable ranges).

b) (7.5 %) By trial and error, Kcu is found as 6.1. The corresponding Pu is read of from the simulated response as Pu = 1000 s, see Figure 10.

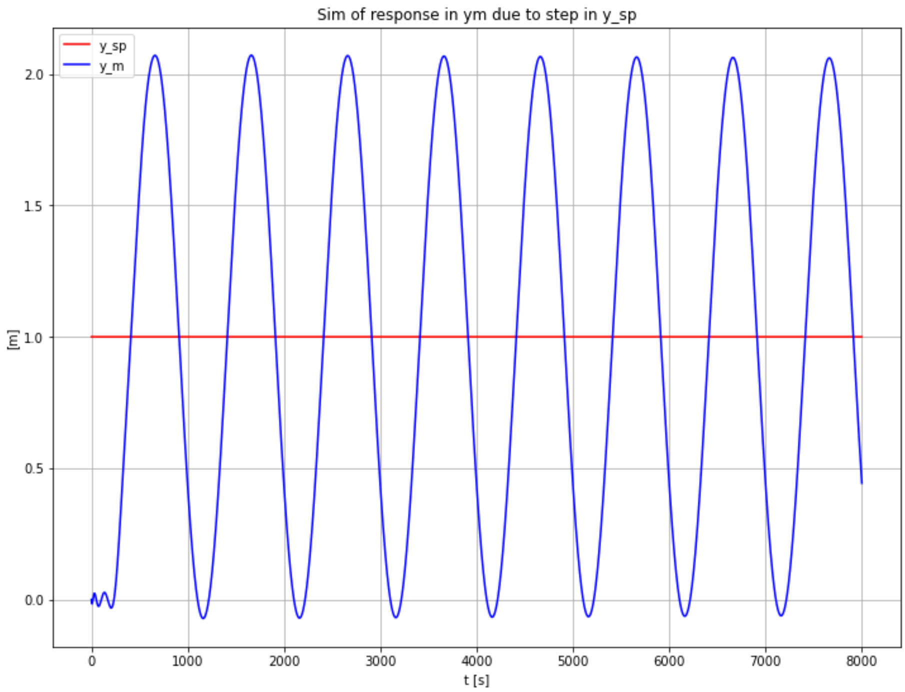

Figure 10

The Ziegler-Nichols' method then gives:
Kp = 0.45 * 6.1 = 2.75
Ti = 1000/1.2 = 833 s
Figure 11 shows the Bode plots.

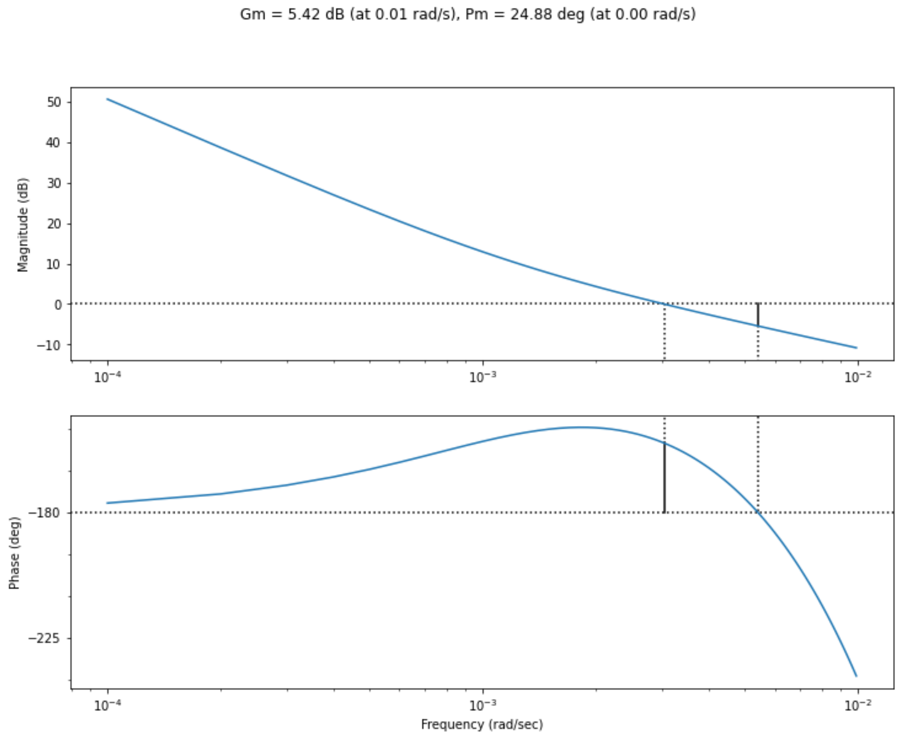

Figure 11

From Figure 11:

GM is 5.42 dB = 10^(5.42/20) = 1.87, which is acceptable.

PM is 24.88 degrees, which is not accetable (too small value).

Figure 12 shows the simulated response (however, the exam does not ask for this plot).

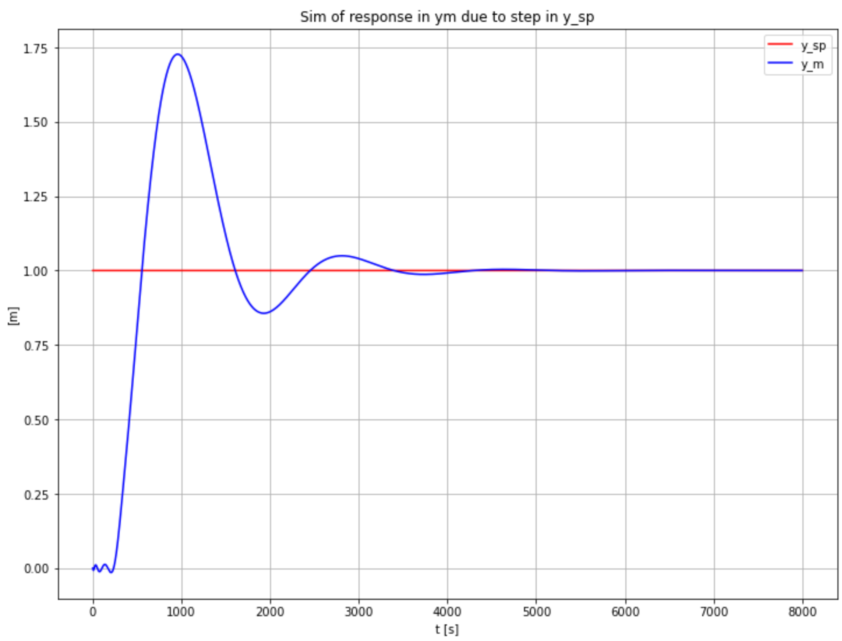

Figure 12

Solution to Problem 11 (10 %) Control structure of a process line

See Figure 13.

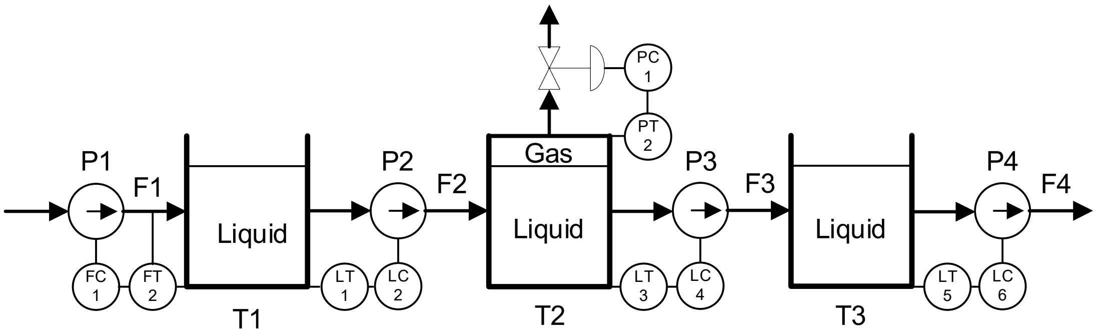

Figure 13