Solution to exam in FM1220 Automatic Control on 16th December 2020

Exam duration: 4 hours 30 minutes. Weight in final grade of the course: 100%. Exam format: Home exam with all aids allowed, but online cooperation is prohibited.

Teacher: Finn Aakre Haugen (finn.haugen@usn.no).

Solution to Problem 1 (5 %) Piping & Instrumentation Diagram

See Figure 1.

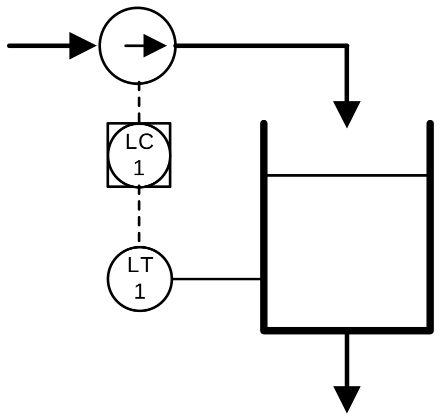


Figure 1

Solution to Problem 2 (5 %) Simulation algorithm

Before the simulation loop:

Initialization: y_k = y_init

Inside the simulation loop:

Limitation of y_k between y_min and y_max (with e.g. the clip function of numpy)

dy_dt_k = [b*sqrt(y_k) + c*u_k]/a

y_kp1 = y_k + dt*dy_dt_k

Any use of y_k, e.g. storing in an array for later plotting

Time index shift: y_k = y_kp1

After the simulation loop:

Plotting, saving simulation data to file, etc.

Solution to Problem 3 (5 %) Dynamics

See Figure 2.

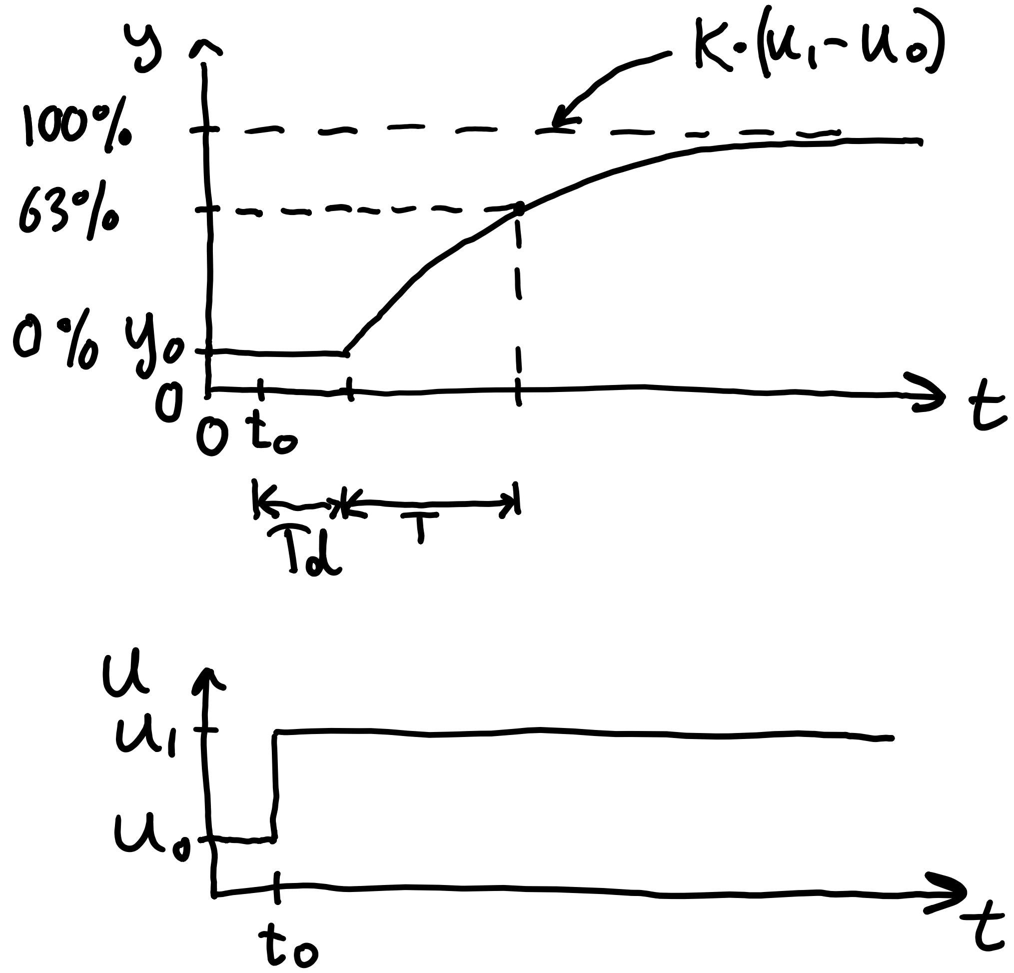


Figure 2

Solution to Problem 4 (10%) PID tuning for averaging level control

Given:

A = 50 m2

dF_in = 100 L/s = 0.1 m3/s

dh = 20 cm = 0.2 m

This gives:

Tc = A * dh/dF_in = 50 * 0.2/0.1 = 100 s

Kp = - A/Tc = - dF_in/dh = - 0.1/0.2 = - 0.5 (m3/s)/m

Ti = 2 * Tc = 200 s

Solution to Problem 5 PID tuning with Relaxed Ziegler-Nichols' method

a. (10 %) Kcu = 10.8 (and Ti = 1e9, which is already given in the program) gives the steady state oscillatory response shown in Figure 3. This response is the Ziegler-Nichols' oscillations. The period is read off from the plot as approx. Pu = 12 s.

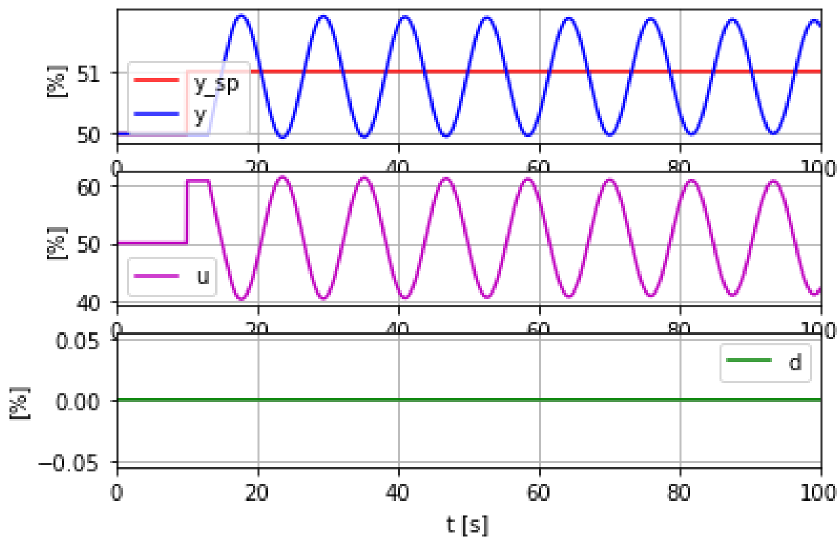


Figure 3

The PI settings become: Kc = 0.25 * Kcu = 0.25 * 10.8 = 2.7, and Ti = 1.25 * Pu = 1.25 * 12 = 15 s.

Figure 4 shows the response with these PI settings. The stability of the control system is acceptable.

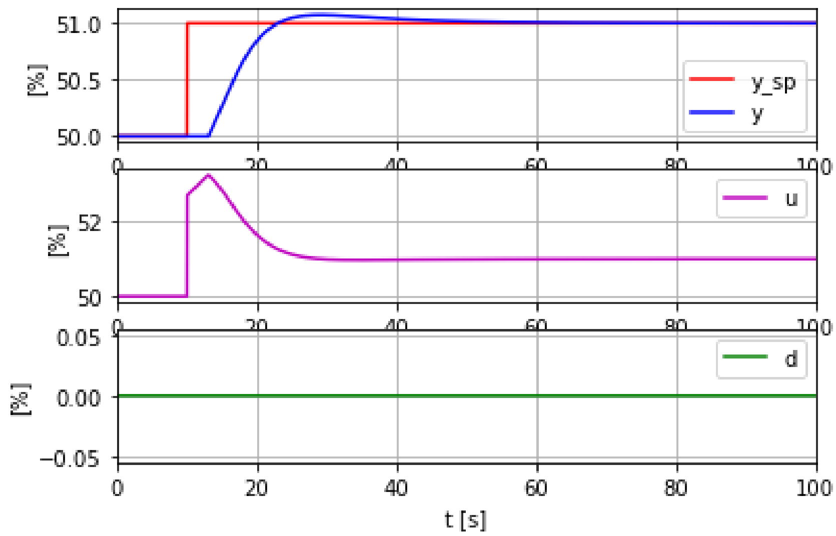


Figure 4

b. (5%) The setpoint is fixed to 50, while the disturbance is changed as a step, here from 0 to 1 (or to some other value different from 0), see Figure 5.

```
# Setting setpoint and disturbance:
if t_k <= 10:
    y_sp_k = 50  # [%]
    d_k = 0  # [%]
else:
    y_sp_k = 50  # [%]
    d_k = 1  # [%]
```

Figure 5

Figure 6 shows the response due to this step change. The steady state control error is zero, so the compensation is perfect in steady state.

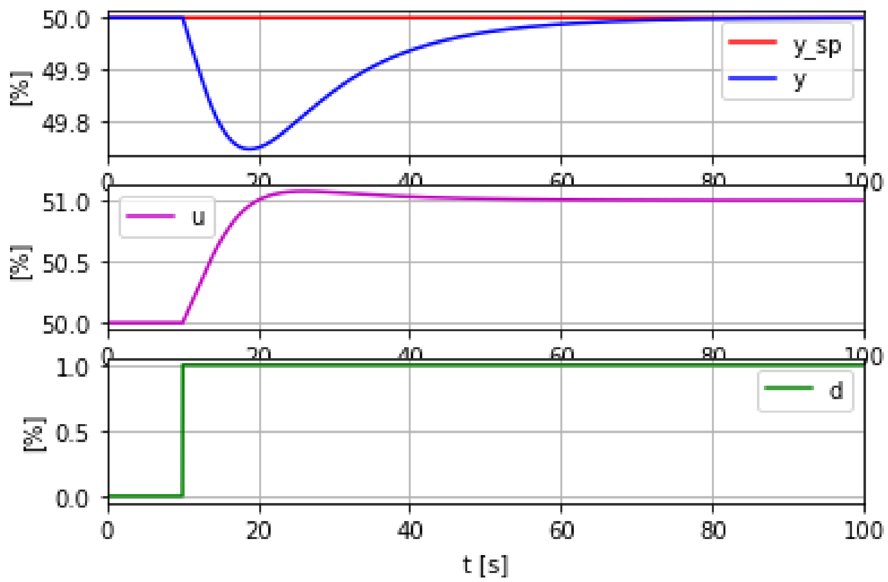


Figure 6

Solution to Problem 6 (5 %) Cascade control

The purpose of the flow control is to compensate for disturbances, e.g. pressure variations, which affect the flow, and for nonlinear relation between the pump control signal and the actual flow, so that the actual flow value becomes equal to the flow value requested by the level controller (flow setpoint).

Primary control loop: Level control loop. Secondary loop: Flow control loop. Tertiary control loop: Rotational speed of the pump motor.

Solution to Problem 7 (10%) Feedforward control

Solving the model for the control signal, u, and substituting y with y_sp gives the feedforward controller:

u_{ff} = [T*y_sp' + y_sp – K2*d]/K1

All quantities on the right side must be known.

The feedforward controller is based on a mathematical model of the process, and on the measurements (or otherwise known values) of the disturbances which appears in the model. In practice, there will always be errors in the model (the model is not perfect) and inaccuracies in the measurements. Therefore, the practical feedforward controller will not calculate the perfect control signal, causing the control error to become different from zero. The feedback controller implements error-driven control, and will therefore reduce this error. In particular, assuming that the feedback controller has integral action, the feedback controller will ensure that the steady state control error is zero (despite model and measurement errors).

Solution to Problem 8 (5%) Transfer function

Taking the Laplace transformation of the differential equation gives (here, any non-zero initial values of y and y' are neglected):

s*s*y(s) = - a1*s*y(s) - a0*y(s) + b*u(s) + c*d(s)

Solving for y(s) gives:

y(s) = [b/(s*s + a1*s + a0)]u(s) + [c/(s*s + a1*s + a0)]d(s)

Hence, the transfer function from u to y is [b/(s*s + a1*s + a0)].

Solution to Problem 9 (15 %) Tuning and analysis of a control system

Process dynamics: Integrator.

Tuning the P controller: Controller transfer function is Hc(s) = Kc. Loop transfer function is L(s) = Hc*Hp = Kc*K/s. Tracking transfer function is T(s) = L/(1 + L) = Kc*K/(s + Kc*K) = 1/(T*s + 1) where closed loop time constant is T = 1/(Kc*K). Solving for Kc gives Kc = 1/(T*K).

Pole of control system is root of characteristic equation s + Kc*K = 0. Hence, pole is p = - Kc*K = -1/T.

Assuming K is negative, Kc becomes negative (since T must be positive to have a stable control system). A negative Kc means that the controller has direct action.

Solution to Problem 10 (15 %) Analysis of a control system

a) (7.5 %) From Bode diagram:
   GM = 3.65 dB, which corresponds to 10^(3,65/20) = 1.52.
   PM = 21.56 degrees.
   Acceptable stability margins are 1.7 <= GM <= 4.0 and 30 degrees <= PM <= 45 degrees. Both GM and PM are outside these limits. The stability of the control system is therefore not satisfactory (it is necessary that both are within their acceptable ranges).

   Figure 7 shows the simulated level response.

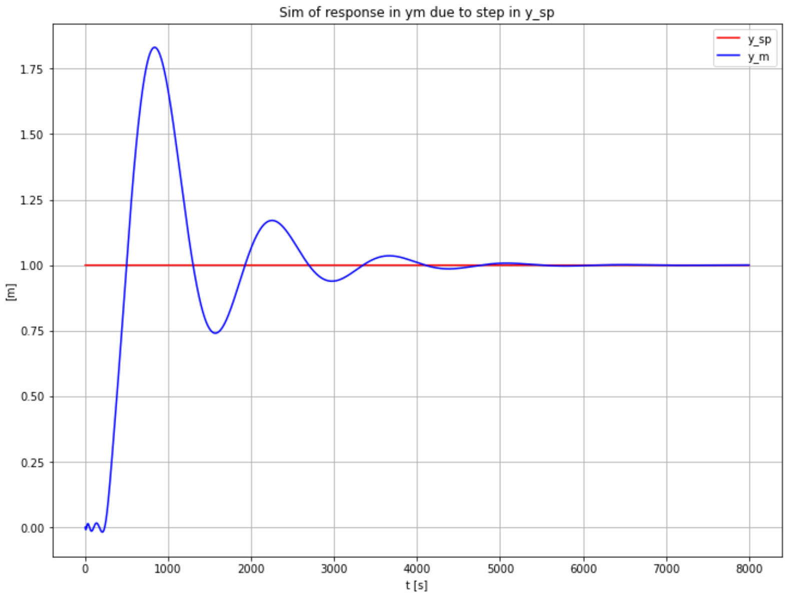


Figure 7

b) (7.5 %) In the program, Kc = 3.5, which is used as Kcu in Repeated Ziegler-Nichols' method. In Figure 6, we read off a period of about 1400 s, which is used as Pu in the Repeated Ziegler-Nichols' method. We then get:
Kp = 0.45 * 3.5 = 1.58
Ti = 1400/1.2 = 1167 s
GM is now 10.8 dB = 10^(10.8/20) = 3.5.
PM is 38.8 degrees.
Both GM and PM have accetable values.
Figure 8 shows the simulated response (stability seems ok).

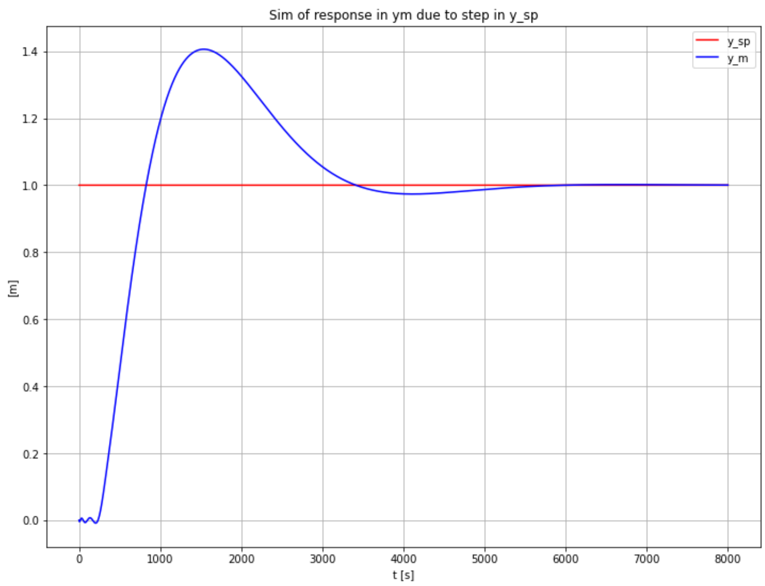
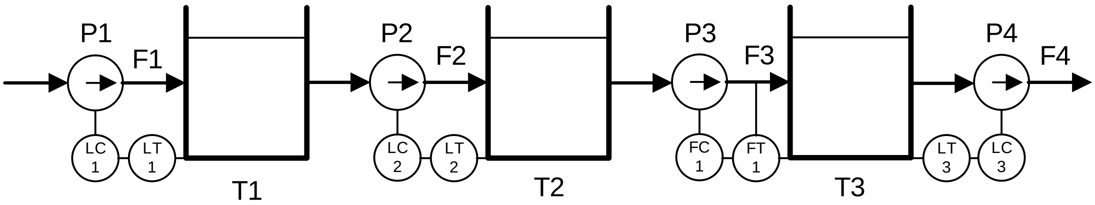


Figure 8

Solution to Problem 11 (10 %) Control structure of a process line

See Figure 9.



F = Flow
T = Tank
P = Pump

Figure 9