

Large Scale Structure: comparison of theory with observation (II)



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The Sloan Digital Sky Survey 2.5m telescope
Apache Point, New Mexico

Outline

- Measuring density fluctuations and two-point clustering in spectroscopic surveys
- Estimating errors
- Choosing a data vector
- Geometric Distortions
- Baryon Acoustic Oscillations: theory and measurement

Galaxy Redshift Surveys in 3 easy steps

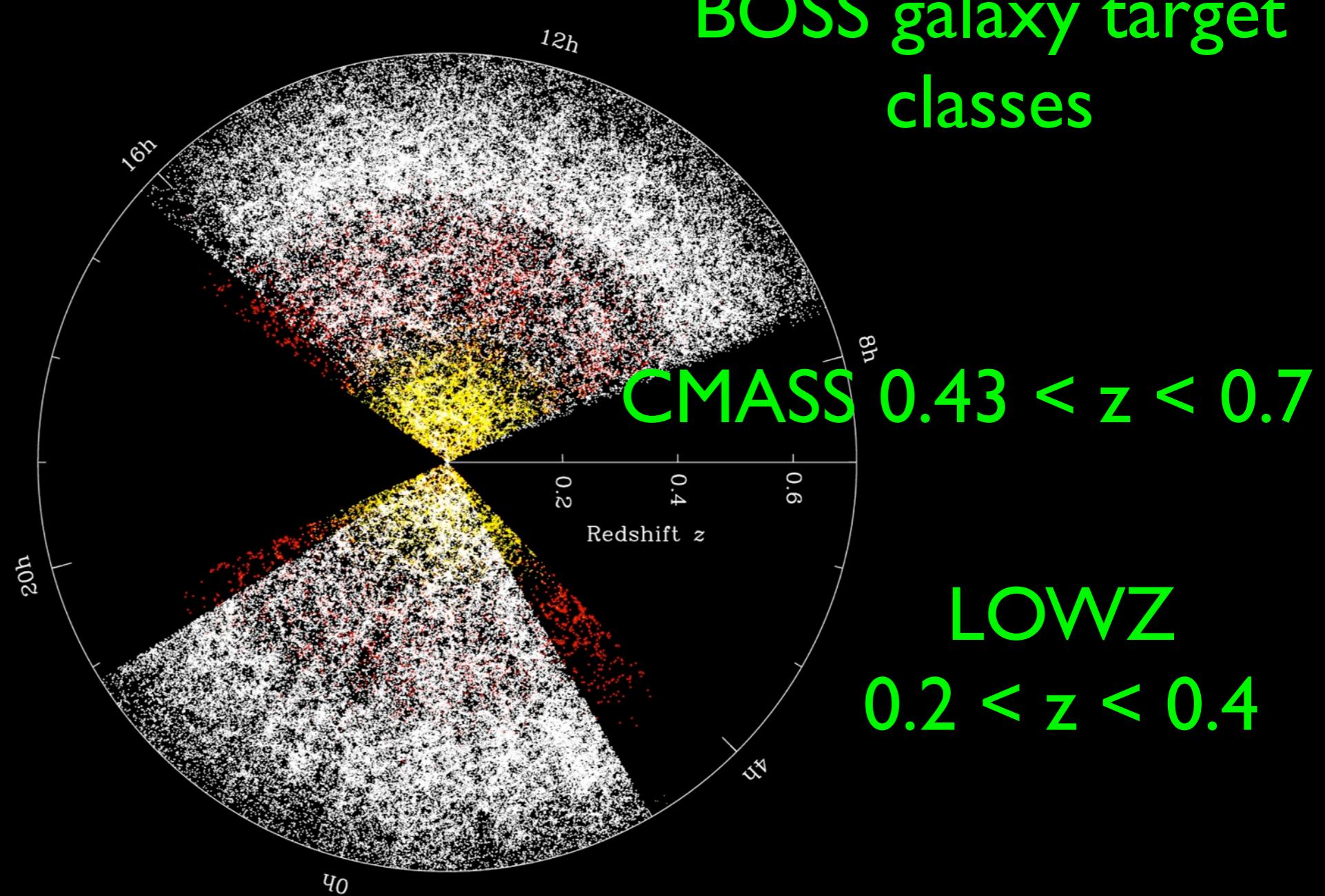
- Assemble a homogeneous list of targets from imaging data [magnitude and/or color-cuts]
- Measure redshifts [take spectra]
- Make a three-dimensional galaxy density map

Galaxy Redshift Surveys in 3 easy steps

SDSS-III BOSS

SDSS Main

SDSS LRGs

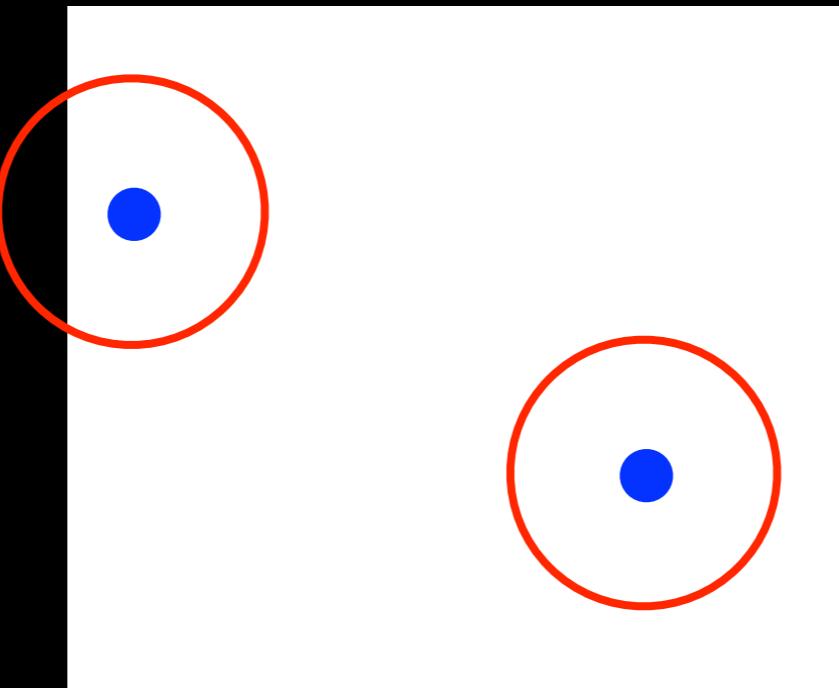


Estimating the 3d galaxy density

- Before we can estimate the galaxy density field from the measured list of 3d galaxy positions, we need to know the probability that we could have observed a galaxy at every point in the sky.
- This is the survey selection function.

Estimating the 3d galaxy density

- As a simple example, consider a square survey
- Many possible pairs at the separation of the red circle cannot be observed for the left-most point



Estimating the 3d galaxy density

- If our survey was instead a very long narrow strip, we would be able to observe even fewer pairs at this separation!



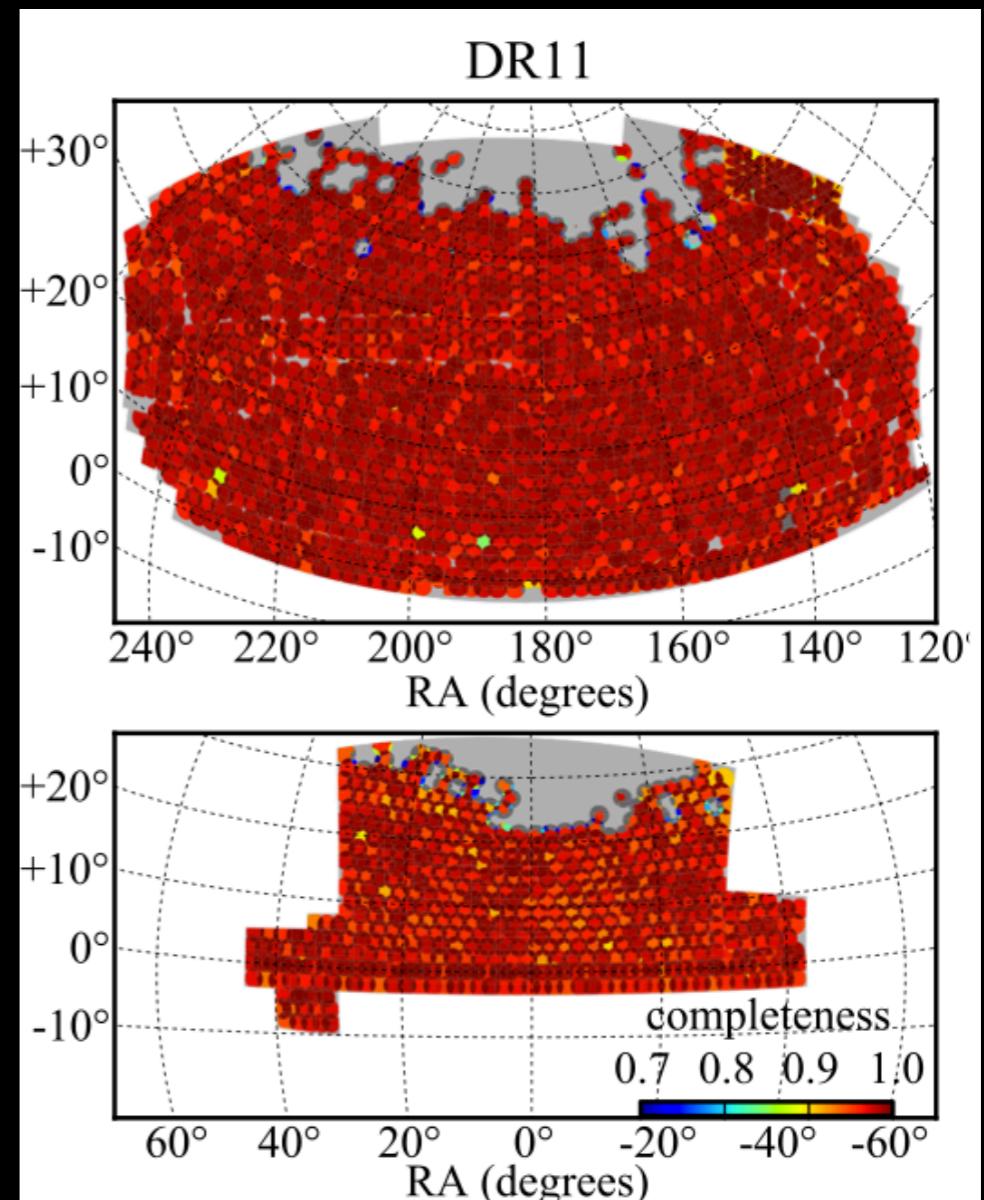
Estimating the 3d galaxy density: BOSS

- For BOSS, the selection function can be simplified into an angular component and a radial (line-of-sight) component

The Angular Selection Function

The geometry of the survey is defined the intersection of the imaging data (grey swath) with the spectroscopic plates.

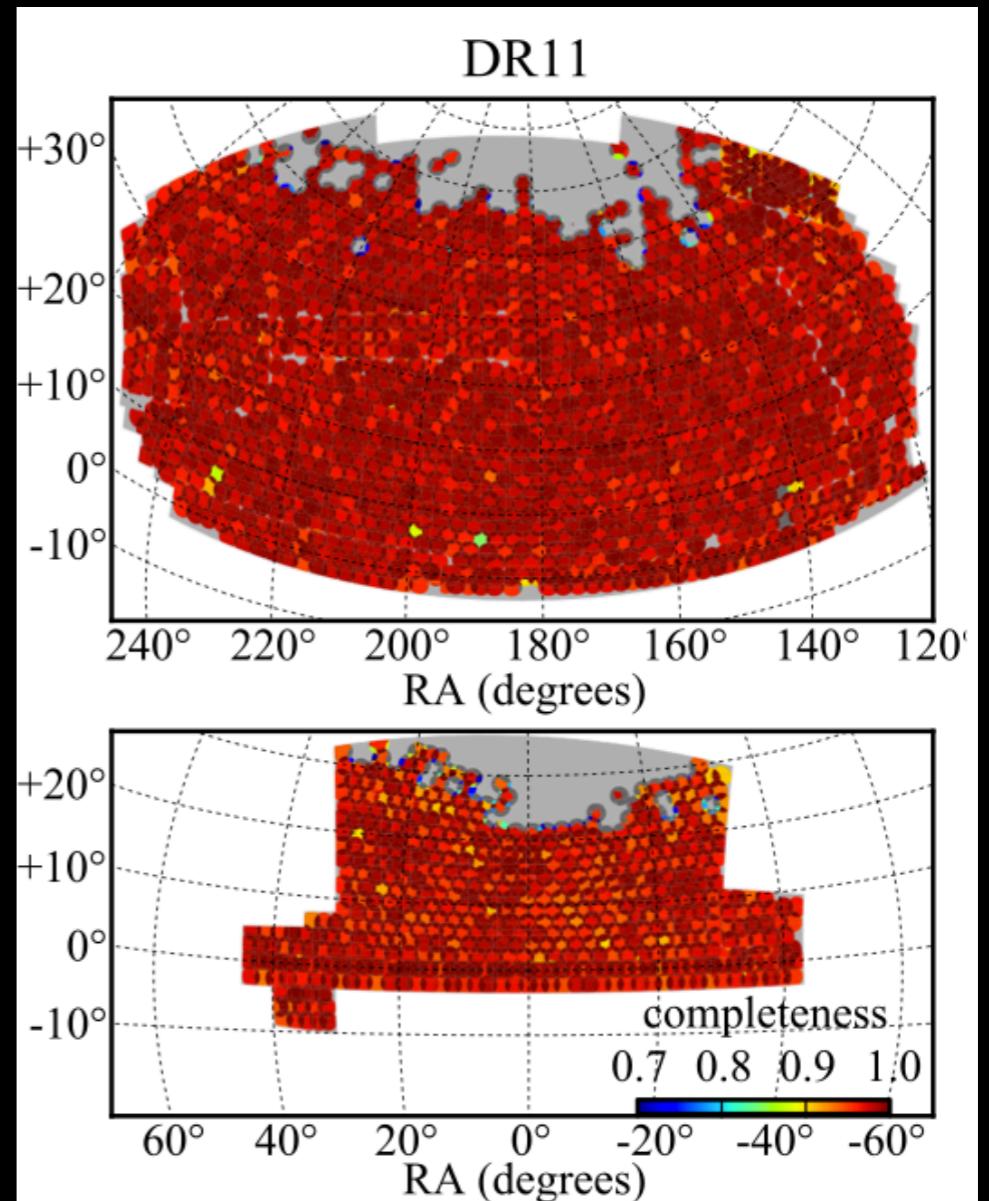
In each unique intersection of plates, we define the completeness as the probability that a spectrum was measured for all the targets in that region (color bar).



Anderson et al. 2014, 1312.4877

The Angular Selection Function

If you observed 5 galaxies in a region with completeness = 0.5, how many galaxies did you miss in that region?

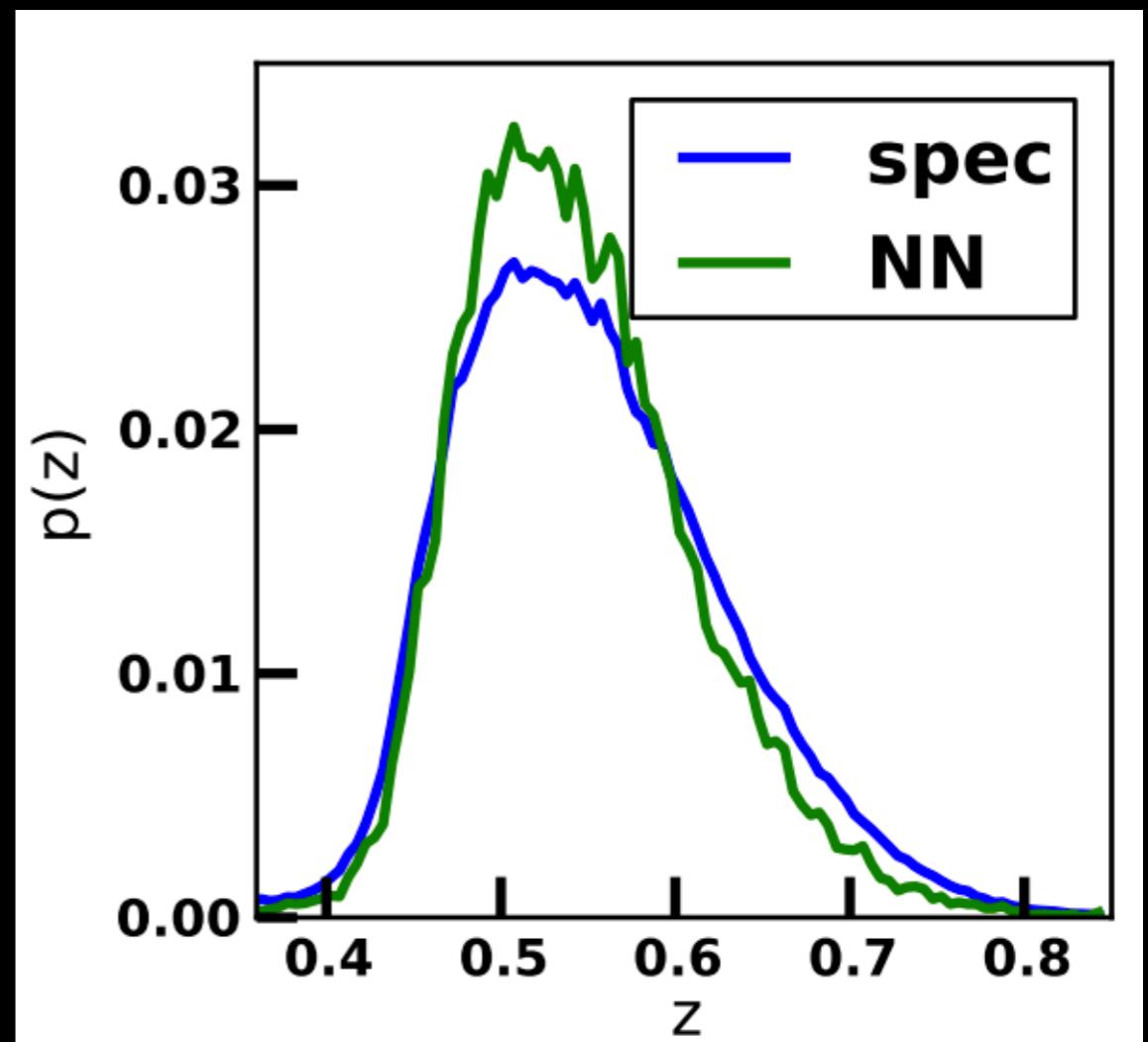


Anderson et al. 2014, 1312.4877

The radial selection function

What is the probability distribution for the observed redshift z of the target galaxy sample?

We estimate this directly from the observed redshifts.



Reid et al. 2014, 1404.3742

Describing the total selection function

- For BOSS, the final selection function of the survey is just the product of the angular and radial selection functions.
- For all practical purposes, we describe the selection function by creating a random set of points that have exactly the same selection function as the galaxies. By definition they have $\xi = 0$.

Describing the total selection function

- To create the random set of galaxies --
- First we draw generate random positions on the sky (ra, dec) with probability proportional to the angular completeness map.
- Then we randomly assign* a measured galaxy redshift to each random point. This ensures that the galaxy sample and random sample have the same radial selection function.

* with probability proportional to the weight of each galaxy

The 3d galaxy density

- Within any angular patch on the sky $d\Omega$ and redshift interval dz (covering volume δV_i), our best estimate of the **galaxy overdensity** in that volume is

$$\delta_{\text{gal},i} = N_{D,i}/N_{R,i} * (N_{R,\text{tot}}/N_{D,\text{tot}})$$

Number of Data/Random
galaxies in volume δV_i

Total number of Data/Random
galaxies in entire survey volume

Describing the total selection function

- We use this set of random points to compute any clustering statistics of interest, for example --
- the galaxy two-point correlation function $\xi(r)$ using the Landy-Szalay estimator [ApJ 412, 64 (1993)]
- the galaxy power spectrum [Feldman, Kaiser, Peacock ApJ 426, 23 (1994); Yamamoto, Sato, Huetsi Prog. Theor. Phys. 120, 609 (2008)]

Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- There are n_d real galaxies (“data”) in your survey, and you generate n_r random galaxies (“randoms”). You can make n_r as big as you want so it does not contribute to your error in measuring ξ . I typically use $n_r = 50n_d$.
- We estimate ξ by counting up the number of unique data-data (DD), data-random (DR) and random-random (RR) pairs.

Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- The total number of unique DD, DR, and RR pairs in the survey is $n_d(n_d - 1)/2$, $n_d(n_d - 1)/2$, $n_d n_r$, and $n_r(n_r - 1)/2$ respectively.
- The Landy-Szalay estimator is

$$\xi = [DD f_{dd} - 2*DR f_{dr} + RR]/RR$$

where f_{dd} , f_{dr} match the counts of DD and DR to RR:

$$f_{dd} = n_r(n_r - 1)/n_d(n_d - 1); f_{dr} = n_r(n_r - 1)/2n_d n_r$$

Computing ξ : Landy-Szalay estimator Integral Constraint

- Since we estimated the expected number density of galaxies from the observed redshift distribution (after accounting for the observational selection function), our naive estimate of ξ (or P) is biased:

$$\langle 1 + \bar{\xi}(r) \rangle = \frac{1 + \xi(r)}{1 + \xi_\Omega(r)}$$

↑

Our LS estimator

← true ξ

← true ξ averaged over survey volume;
approaches 0 for large survey volumes.

Computing ξ : Landy-Szalay estimator Integral Constraint

- We can account for this bias when computing the theoretical expectation for our observations

$$\langle 1 + \bar{\xi}(r) \rangle = \frac{1 + \xi(r)}{1 + \xi_\Omega(r)}$$

↑

Our LS estimator

true ξ

true ξ averaged over survey volume;
approaches 0 for large survey volumes.

Computing $P(k)$: FKP method

[ApJ 426, 23 (1994)]

- Since we have a direct estimate of the galaxy overdensity at each point, we can compute the Fourier transform of the observed galaxy density field:

weighting function [usually just depends on radial (redshift) coordinate]

Normalized galaxy overdensity field

observed galaxy density field

normalization factor

random galaxy density field

$$F(\mathbf{r}) \equiv \frac{w(r)[n_g(r) - \alpha n_s(r)]}{[\int d^3 r \bar{n}^2(r) w^2(r)]^{1/2}}$$

Computing $P(k)$: FKP method [ApJ 426, 23 (1994)]

- Since we have a direct estimate of the galaxy overdensity at each point, we can compute the Fourier transform of the observed galaxy density field:

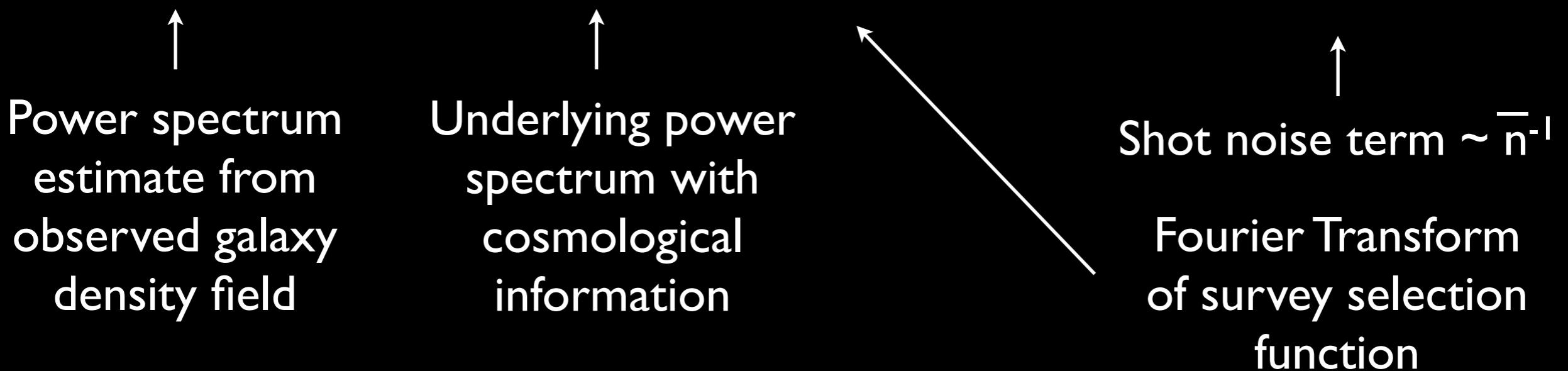
$$\langle |F(\mathbf{k})|^2 \rangle = \frac{\int d^3r \int d^3r' w(\mathbf{r})w(\mathbf{r}') \langle [n_g(\mathbf{r}) - \alpha n_s(\mathbf{r})][n_g(\mathbf{r}') - \alpha n_s(\mathbf{r}')] \rangle e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')} }{\int d^3r \bar{n}^2(\mathbf{r})w^2(\mathbf{r})}$$

Computing $P(k)$: FKP method

[ApJ 426, 23 (1994)]

- Since we have a direct estimate of the galaxy overdensity at each point, we can compute the Fourier transform of the observed galaxy density field:

$$\langle |F(\mathbf{k})|^2 \rangle = \int \frac{d^3k'}{(2\pi)^3} P(k') |G(\mathbf{k} - \mathbf{k}')|^2 + (1 + \alpha) \frac{\int d^3r \bar{n}(\mathbf{r}) w^2(\mathbf{r})}{\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})}$$



Computing $P(k)$: FKP method

[ApJ 426, 23 (1994)]

- FKP showed that to minimize errors on measuring the underlying $P(k)$, the optimal galaxy weights are

$$w_0(r) = \frac{1}{1 + \bar{n}(r)P(k)}$$

↑
Rare galaxies (small \bar{n}) are more valuable [higher weight]

↑
Typically approximated as constant at k of interest

Computing $P(k)$: FKP method

Key Concepts

- Shot noise -- sampling the underlying continuous density field with discrete points (galaxies) produces a \sim constant term ($\sim \bar{n}^{-1}$) in the estimated $P(k)$
- Window Function: Measured $P(k)$ is convolved with the survey window function. If we want to compare observations with theory, we must convolve the theoretical model with the survey window function.
- Optimal weights: Can weight galaxies to minimize observational errors on $P(k)$ or $\xi(r)$

Downloading the data

- Now you can compute ξ and P for yourself!
- You can download SDSS DR10 large scale structure catalogs here: <http://data.sdss3.org/sas/dr10/boss/lss/>
- And make your own subsamples following these instructions: https://www.sdss3.org/dr10/tutorials/lss_galaxy.php

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- Estimating errors
- Choosing a data vector
- Geometric Distortions
- Baryon Acoustic Oscillations: theory and measurement

LSS measurement uncertainties

- For most practical purposes, there are no real *measurement* uncertainties in LSS; that is, the error in the angular position of the galaxy (ra,dec) or redshift is much smaller than the scales on which you are interested in measuring correlations

LSS measurement uncertainties

- Using the simple model from Lecture I (δ_m is Gaussian, Poisson sampled by discrete tracers)

$$\delta_t(\mathbf{k}) = b_t \delta_m(\mathbf{k}) + \epsilon$$

- the error on a bandpower measurement $P(k_i)$ has “cosmic variance” and “sampling variance” terms:

$$\sigma_{P(k_i)}/P(k_i) \propto N_k \frac{b_t^2 P(k_i) + n_t^{-1}}{b_t^2 P(k_i)}$$

LSS measurement uncertainties

- But to estimate LSS uncertainties on $P(k_i)$ or $\xi(s_i)$ precisely, we need to go beyond this simple model and account for several important effects

LSS measurement uncertainties

- Survey Geometry -- the finite volume and complicated geometry of real surveys induce correlations between neighboring modes and change the effective number of independent modes contributing to bandpower $P(k_i)$

LSS measurement uncertainties

- Non-Gaussianity -- we know that non-linear gravitational evolution drives δ_m away from Gaussianity
- A simple example: $\delta_m \geq -1$ always (matter density can't be negative), but dark matter halos reach $\delta_m \geq 200$. This is proof that the one-point distribution of δ_m develops skewness under gravitational evolution.

LSS measurement uncertainties

- Solution -- brute force!
- Generate hundreds or thousands of synthetic surveys (as realistic as possible), and measure the covariance matrix of your observable from this set.

LSS measurement uncertainties

- Approximations to real (expensive to compute) gravitational dynamics + galaxy prescription include Poisson sampling of lognormal matter density fields [easiest to generate], “PTHalos”, “Addgals”, “COLA”, “QPM”, “PATCHY”, real N-body sims + halo model [currently infeasible], hydrodynamic simulations including galaxy formation [REALLY infeasible]
- Then apply survey selection function to include effects of geometry, sampling variance (shot noise), veto masks, etc.

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What to measure? Your choice of data vector matters.

- Several recent papers [e.g. Percival et al., 1302.4841] pointed out that errors in the data covariance matrix estimate propagate into errors on cosmological parameters; this can be mitigated by information compression [i.e., choosing a shorter data vector]
- Percival et al. derives an optimal bin size for $P(k_i)$ or $\xi(s_i)$ for BAO and RSD measurements.

What to measure? Your choice of data vector matters.

- From a theoretical point of view, your data vector should be restricted to scales where you have good reason to believe your model is sufficiently accurate
- Parameter constraints can be quite sensitive to this choice, since the number of available modes is $N_k \propto k^2 \Delta k!$

Summary so far.

- WHAT to measure [$\xi(s_i)$ or $P(k_i)$ in carefully chosen bins]
- HOW to measure it [Landy-Szalay or FKP/Yamamoto]
- HOW to get cosmological parameter constraints [assume Gaussian likelihoods, estimate data covariances from mock surveys]

$$\chi_{\xi}^2 = \sum_{i=1}^n (\xi(s_i) - \xi_{\text{model}}(s_i, \mathbf{p}_i)) C_{\xi,ij}^{-1} (\xi(s_j) - \xi_{\text{model}}(s_j, \mathbf{p}_j))$$

$$\chi_P^2 = \sum_{i=1}^n (P(k_i) - P_{\text{model}}(k_i, \mathbf{p}_i)) C_{P,ij}^{-1} (P(k_j) - P_{\text{model}}(s_j, \mathbf{p}_j))$$

Summary so far.

- Now we'll switch gears and discuss how cosmological parameters enter the models for our LSS observables

$$\chi_{\xi}^2 = \sum_{i=1}^n (\xi(s_i) - \xi_{\text{model}}(s_i, \mathbf{p}_i)) C_{\xi,ij}^{-1} (\xi(s_j) - \xi_{\text{model}}(s_j, \mathbf{p}_j))$$

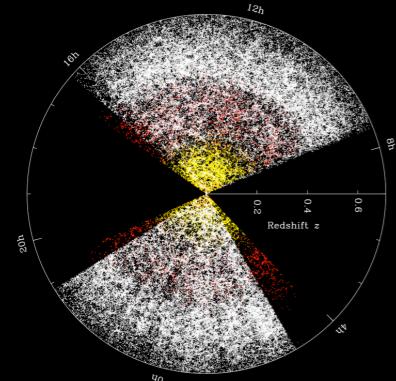
$$\chi_P^2 = \sum_{i=1}^n (P(k_i) - P_{\text{model}}(k_i, \mathbf{p}_i)) C_{P,ij}^{-1} (P(k_j) - P_{\text{model}}(s_j, \mathbf{p}_j))$$

Outline

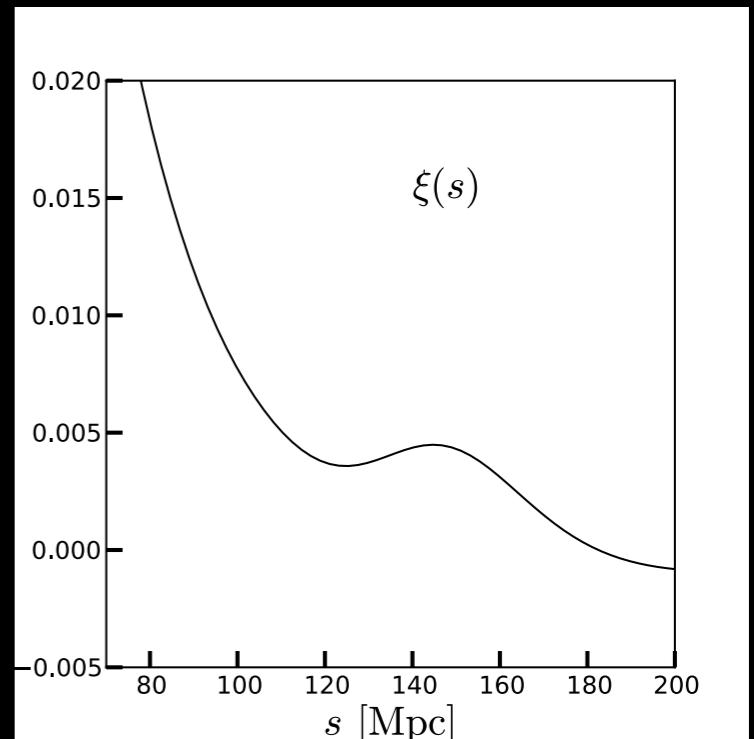
- Measuring density fluctuations and two-point clustering in spectroscopic surveys
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Geometric constraints from galaxy surveys

Observer space:
ra, dec, z



Theory space:
(physical) Mpc



mapping depends on the expansion history $H(z)$ (and therefore cosmological parameters) for z in $[0, z_{\max}]$:

$$X(z) = (1+z)D_A(z) =_0 \int^z c dz'/H(z') \quad [\text{flat universe}]$$

Geometric constraints from galaxy surveys

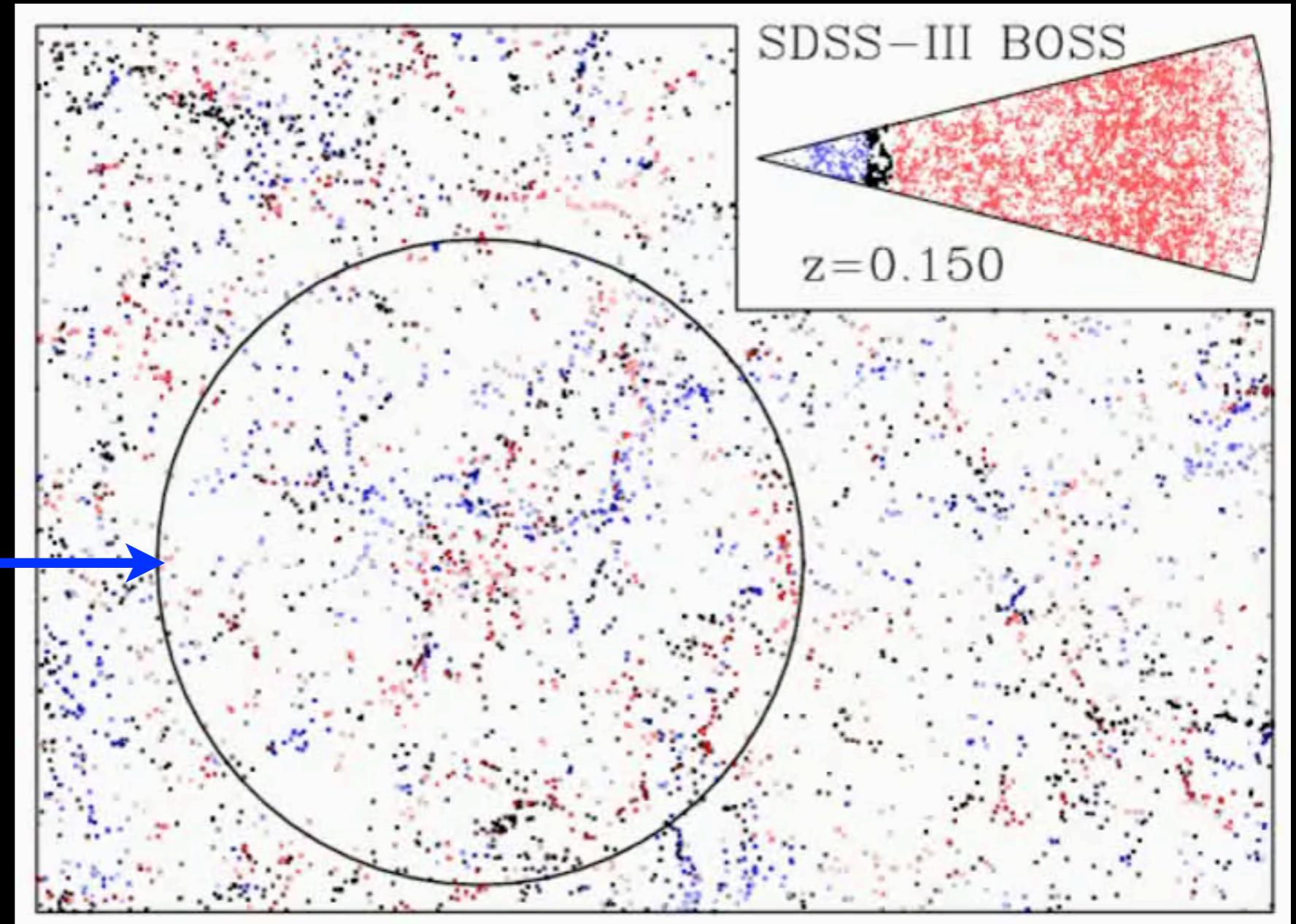
- We have two options:
 - Stick to statistics in observer coordinates -- e.g., compute the angular correlation function/power spectra in MANY redshift bins
 - Assume a fiducial cosmology to convert ra, dec, z to comoving coordinates; account for this choice in theory calculation

Geometric constraints from galaxy surveys

- We have two options:
 - Stick to statistics in observer coordinates -- e.g., compute the angular correlation function/power spectra in MANY redshift bins
 - This is a poor choice because you must have a very long data vector to retain all relevant information

SDSS-III Baryon Oscillation Spectroscopic Survey

Apparent
size of the
BAO ruler



53x38 degree slice; ~20% of DR11

credit: Daniel Eisenstein

Geometric constraints from galaxy surveys

- We have two options:
 - Assume a fiducial cosmology to convert ra, dec, z to comoving coordinates; account for this choice in theory calculation
 - While this seems to be model-dependent, all sensible $X(z)$ are smooth [they are integrals!], and so the resulting geometric distortions can be modeled very accurately.

Geometric constraints from galaxy surveys

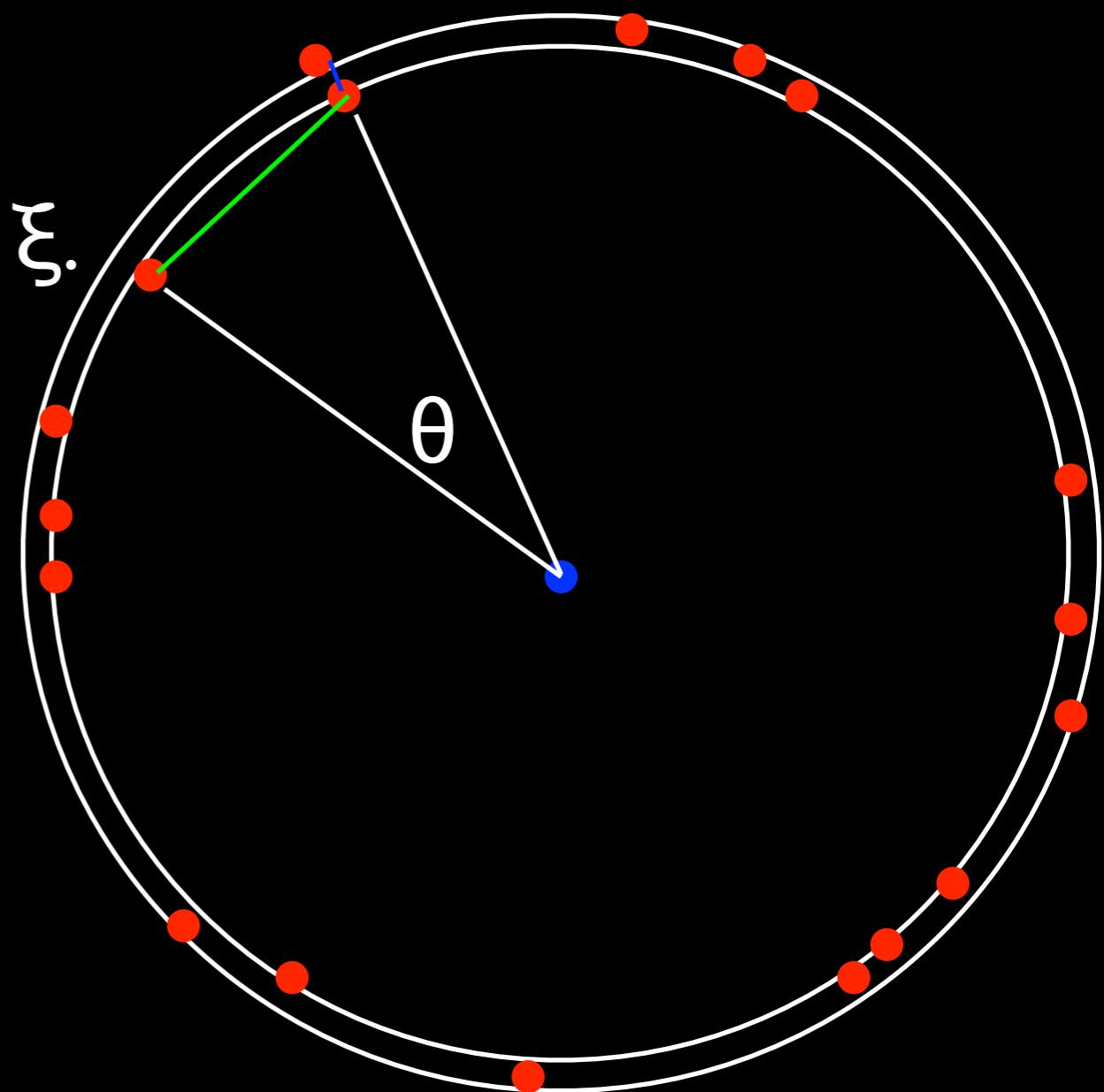
First, imagine a survey restricted to a very narrow redshift range $[z_s \pm dz/2]$. Select a “fiducial” cosmology with which to measure ξ .

The conversion between angular and transverse comoving separation x_{\perp} is

$$x_{\perp} = (1+z_s) D_{A,\text{fid}}(z_s) \theta$$

The conversion between Δz and line-of-sight (LOS) comoving separation x_{\parallel} is

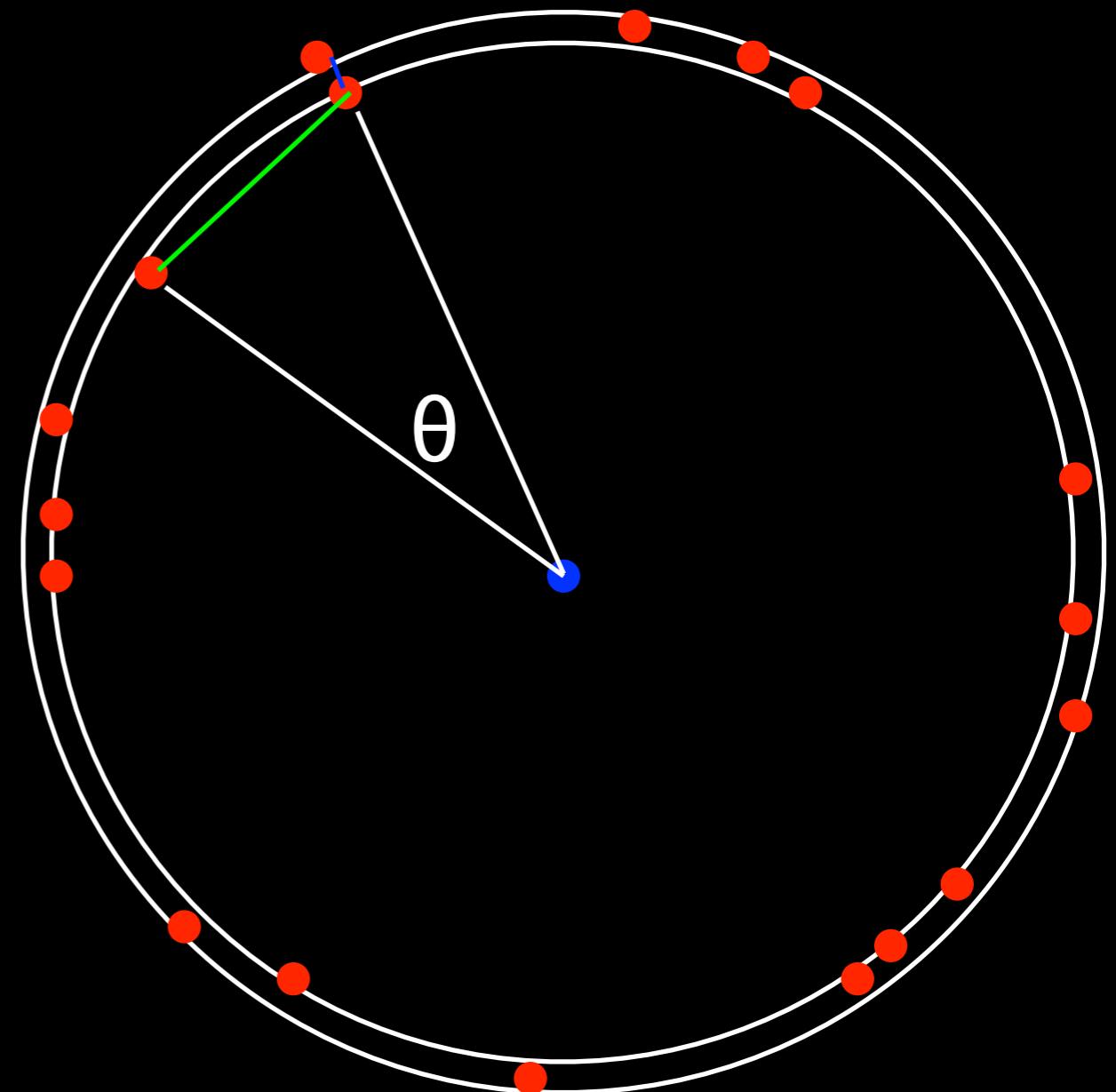
$$x_{\parallel} = c \Delta z / H_{\text{fid}}(z_s)$$



Geometric constraints from galaxy surveys

The theoretical prediction based on a model with a different $D_A(z_s)$ and $H(z_s)$ for the “observed” ξ^{obs} [measured with $D_A(z_s)$ and $H(z_s)$] is

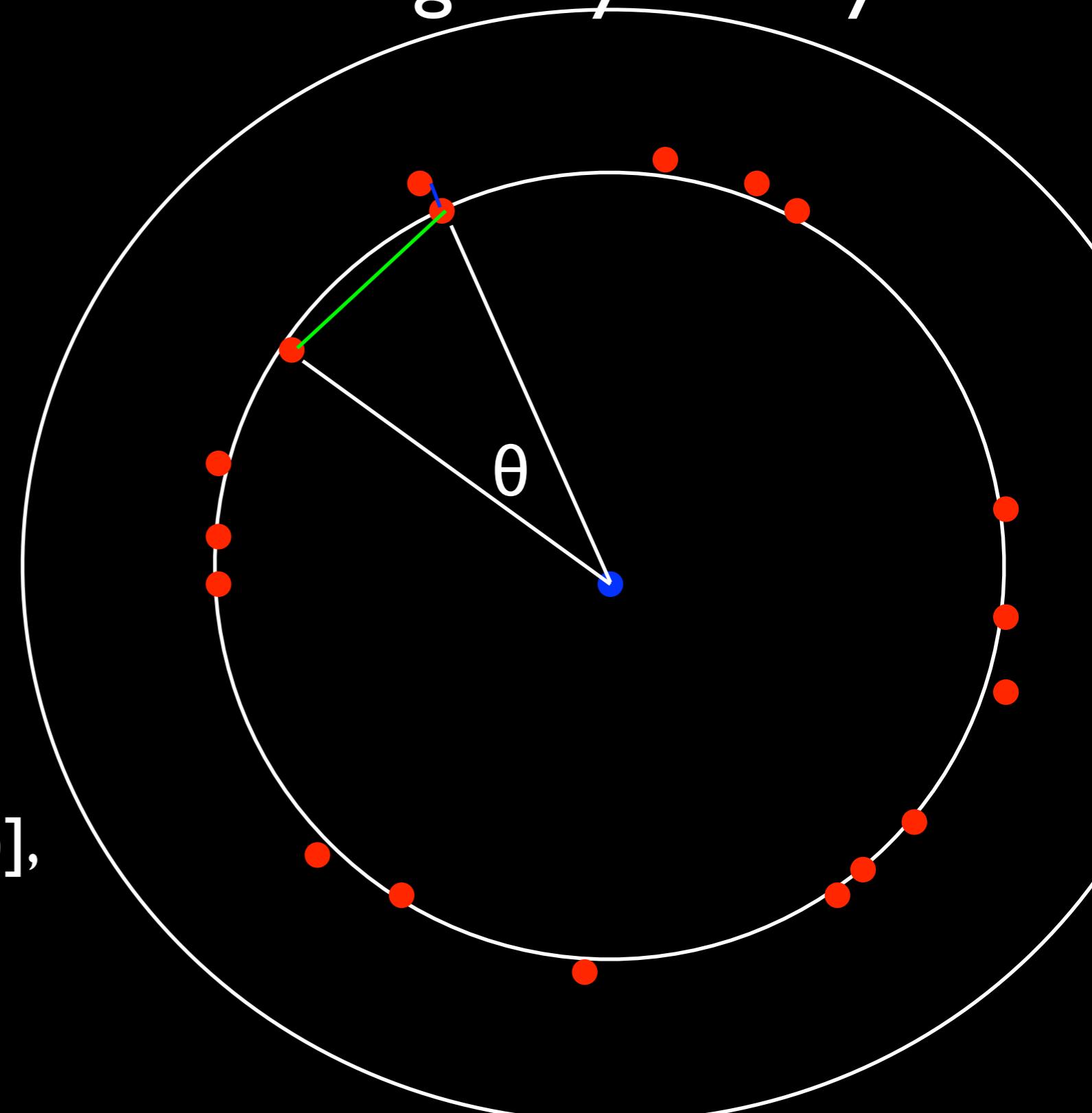
$$\begin{aligned}\xi^{\text{obs}}(s_{\perp}, s_{\parallel}) = \\ \xi^{\text{true}}(s_{\perp} * [D_A(z_s)/D_{A,\text{fid}}(z_s)], \\ s_{\parallel} * [H_{\text{fid}}(z_s)/H(z_s)])\end{aligned}$$



Geometric constraints from galaxy surveys

If we generalize to a thicker redshift range, it is still a very good approximation to assume

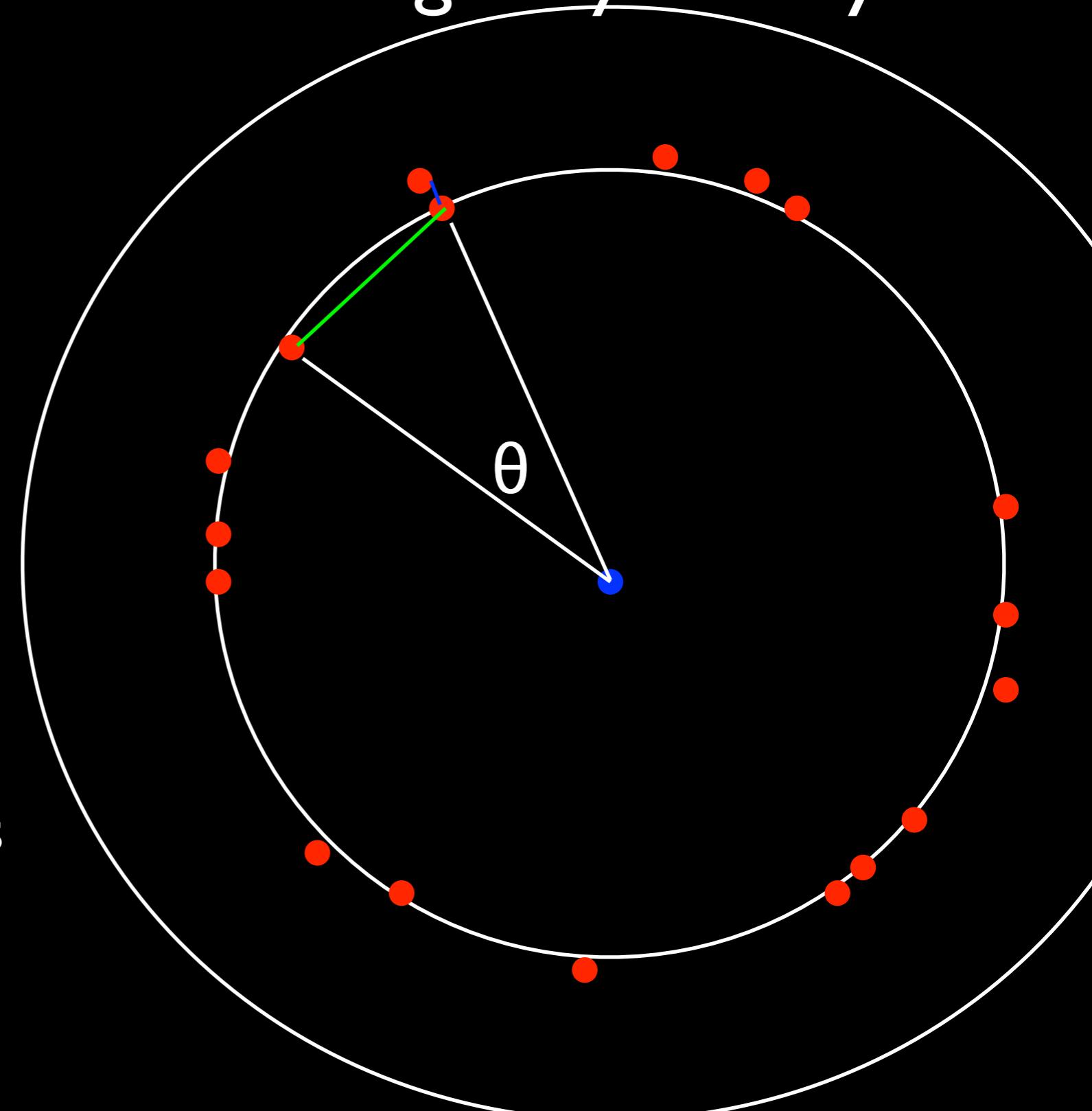
$$\begin{aligned}\xi^{\text{obs}}(s_{\perp}, s_{\parallel}) = \\ \xi^{\text{true}}(s_{\perp} * [D_A(z_{\text{eff}})/D_{A,\text{fid}}(z_{\text{eff}})], \\ s_{\parallel} * [H_{\text{fid}}(z_{\text{eff}})/H(z_{\text{eff}})])\end{aligned}$$



Geometric constraints from galaxy surveys

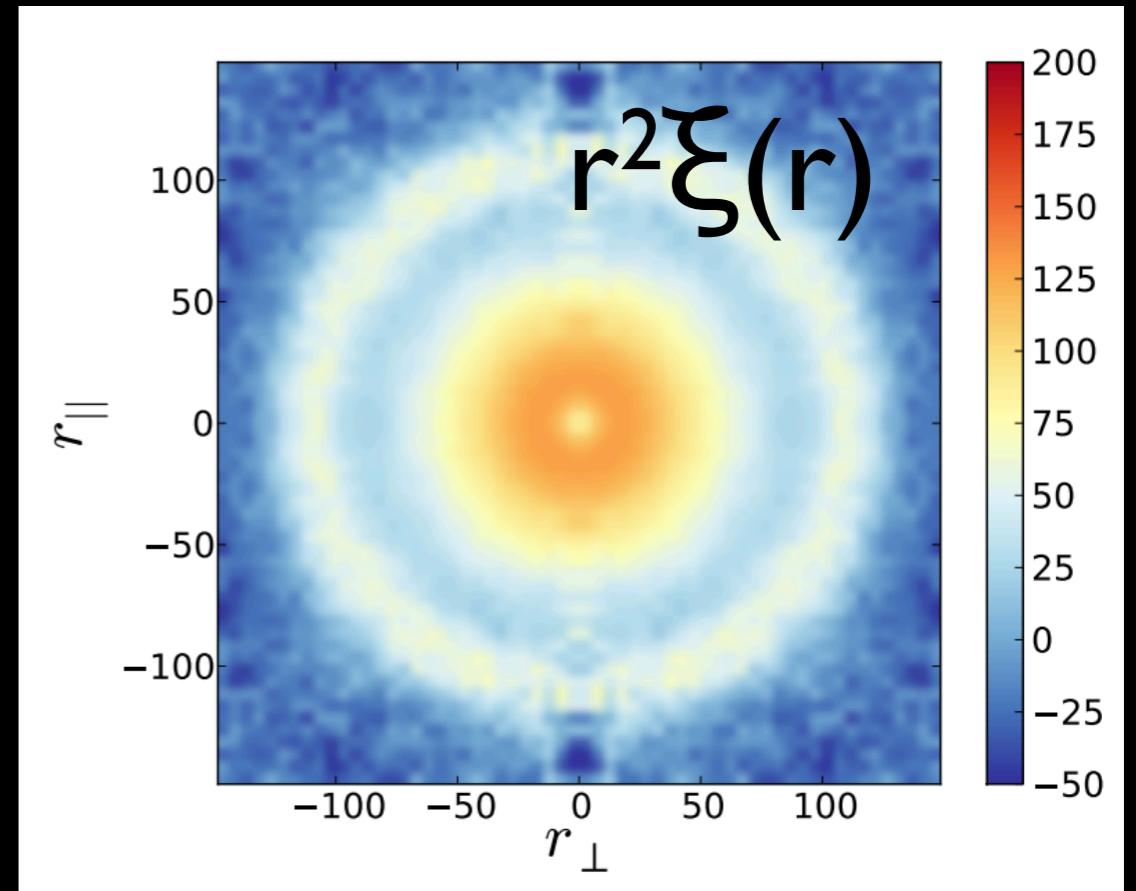
If we generalize to a thicker redshift range, it is still a very good approximation to assume

[though one could easily integrate ξ^{true} over the redshift distribution of pairs to be more exact]



The BAO standard ruler

- ξ^{model} has a feature at a characteristic scale $s_{\text{BAO}} \approx 150 \text{ Mpc}$



[real space; no RSD until Lecture 3]
Padmanabhan et al. 1202.0090

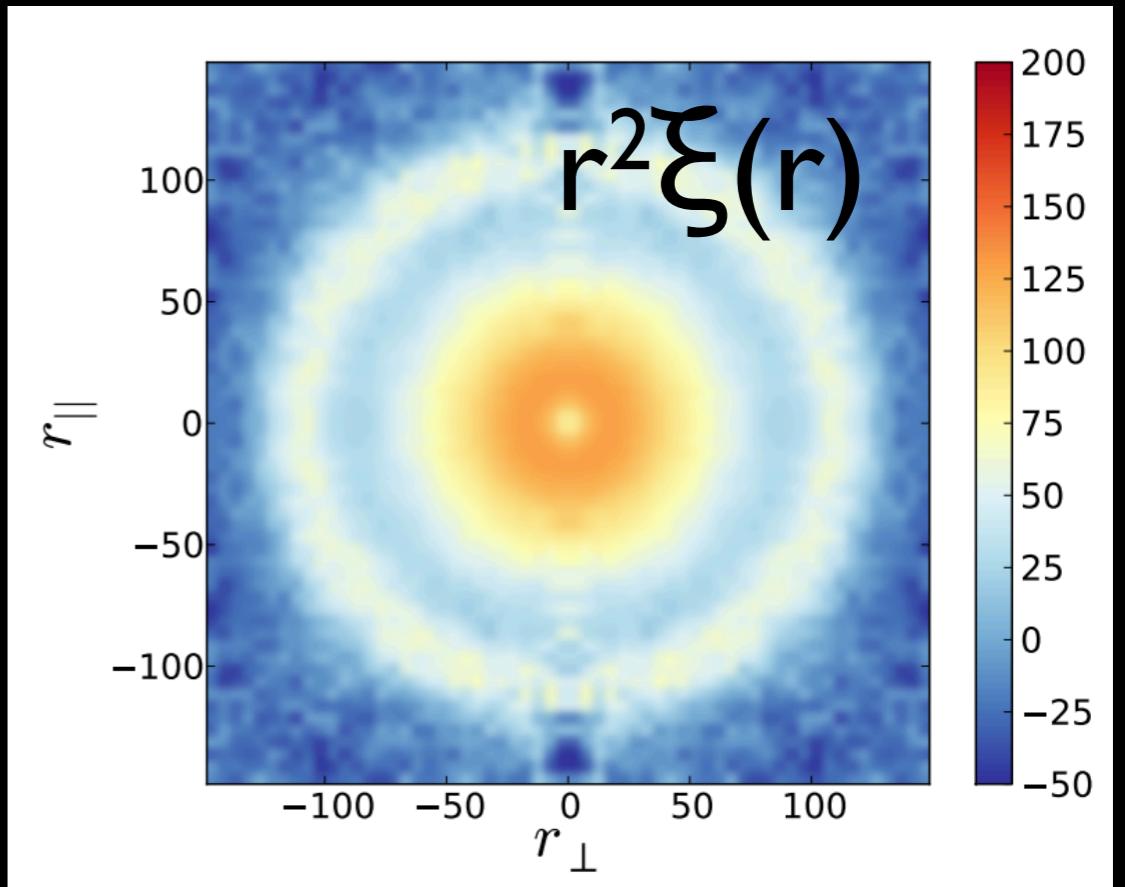
The BAO standard ruler

- Suppose we observe this feature at s_{\perp}' and s_{\parallel}' :

$$\xi^{\text{obs}}(s_{\perp}', s_{\parallel}') =$$

$$\xi^{\text{true}}(s_{\text{BAO}} = s_{\perp}' * [D_A(z_{\text{eff}})/D_{A,\text{fid}}(z_{\text{eff}})],$$

$$s_{\text{BAO}} = s_{\parallel}' * [H_{\text{fid}}(z_{\text{eff}})/H(z_{\text{eff}})])$$

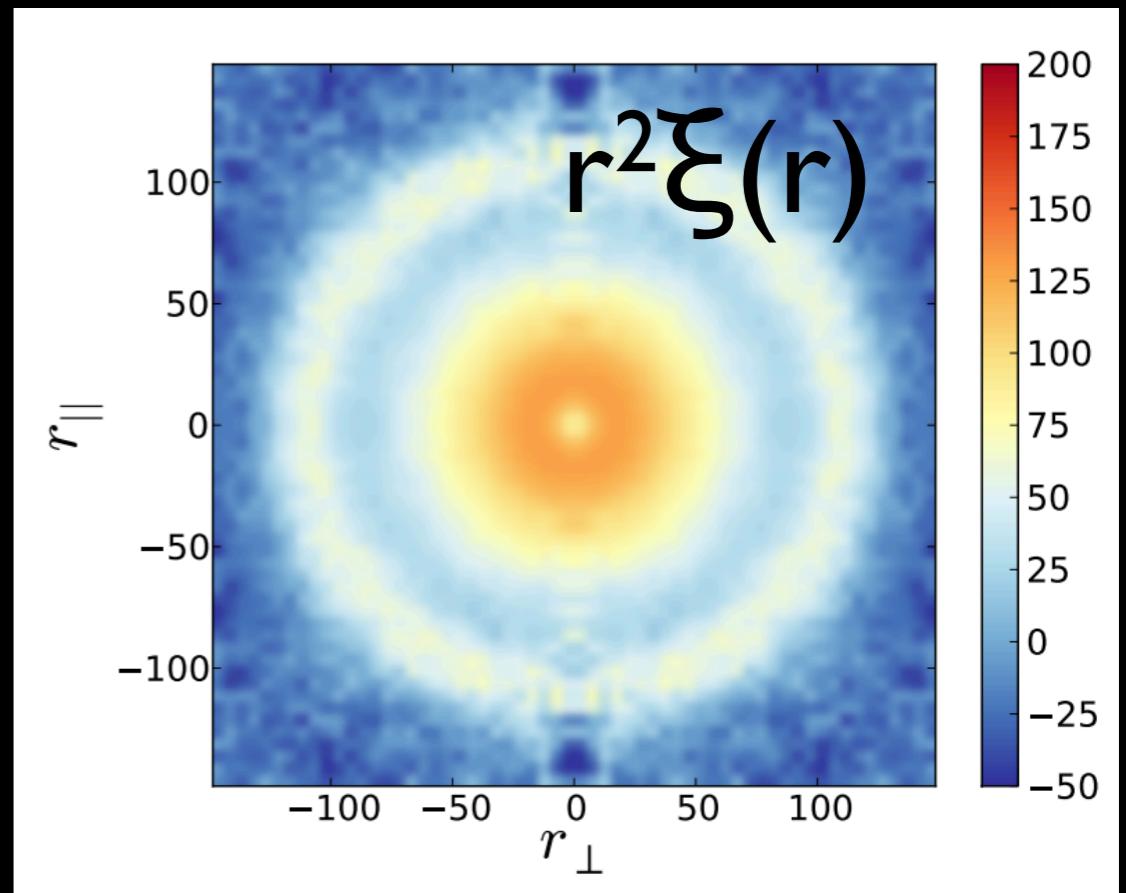


[real space; no RSD until Lecture 3]

Padmanabhan et al. | 202.0090

The BAO standard ruler

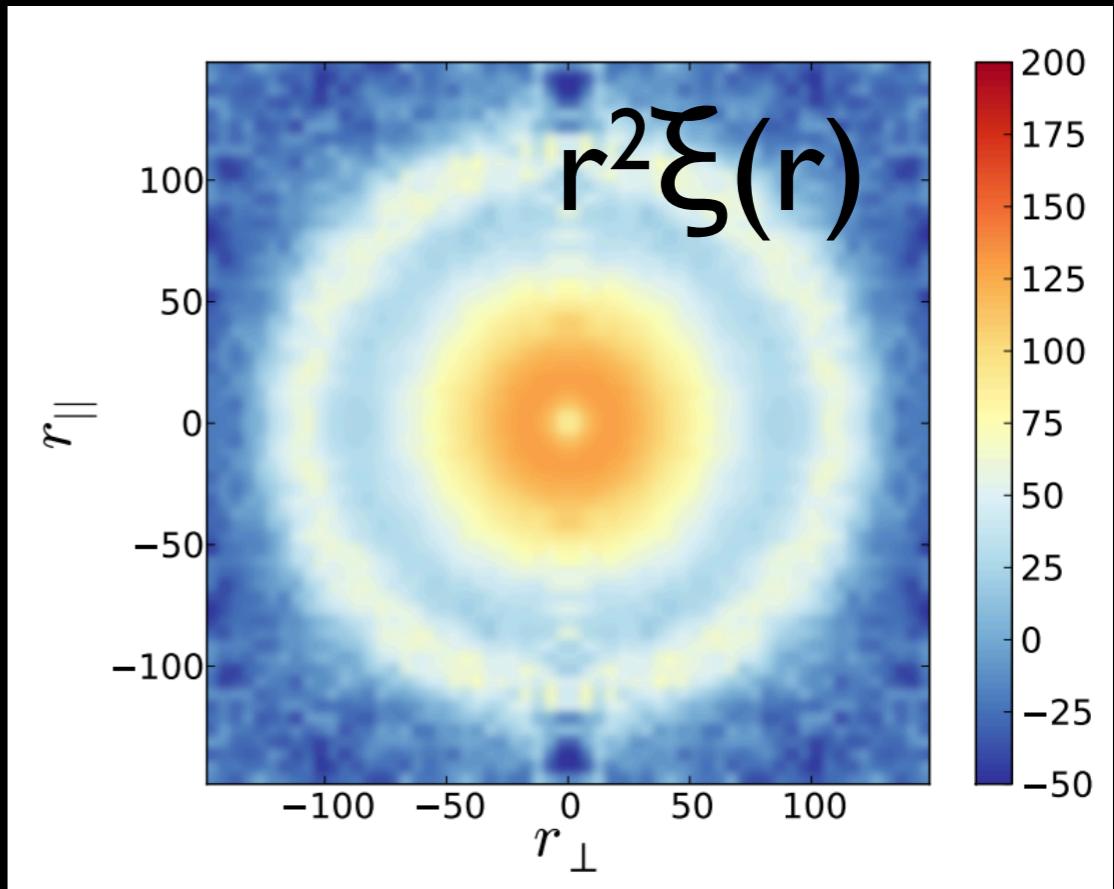
- Then we have measured
 $D_A(z_{\text{eff}})/[s_{\text{BAO}} * D_{A,\text{fid}}(z_{\text{eff}})]$
and
 $H_{\text{fid}}(z_{\text{eff}})/[s_{\text{BAO}} * H(z_{\text{eff}})]!$



[real space; no RSD until Lecture 3]
Padmanabhan et al. | 202.0090

The BAO standard ruler

- BUT! The BAO signal is small, and so most BAO analyses average over all orientations and constrain $D_V(z_{\text{eff}})/s_{\text{BAO}}$.



$$D_V \equiv [(1+z_{\text{eff}})^2 D_A(z_{\text{eff}})^2 * c z_{\text{eff}} / H(z_{\text{eff}})]^{1/3}$$

(geometric mean)

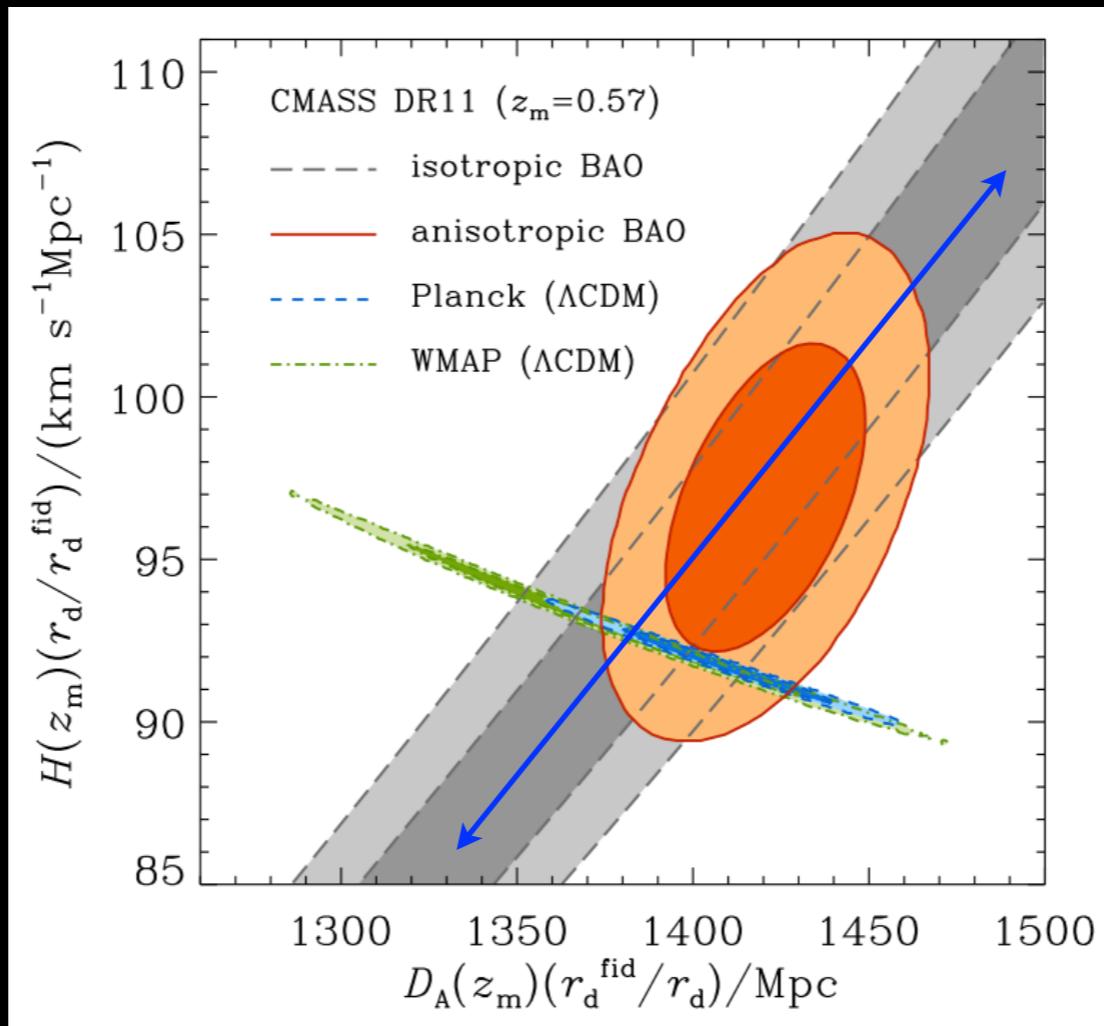
[real space; no RSD until Lecture 3]

Padmanabhan et al. I202.0090

The BAO standard ruler

- Constraints from BOSS DR11:

BOSS, arXiv:1312.4877



D_V approximation

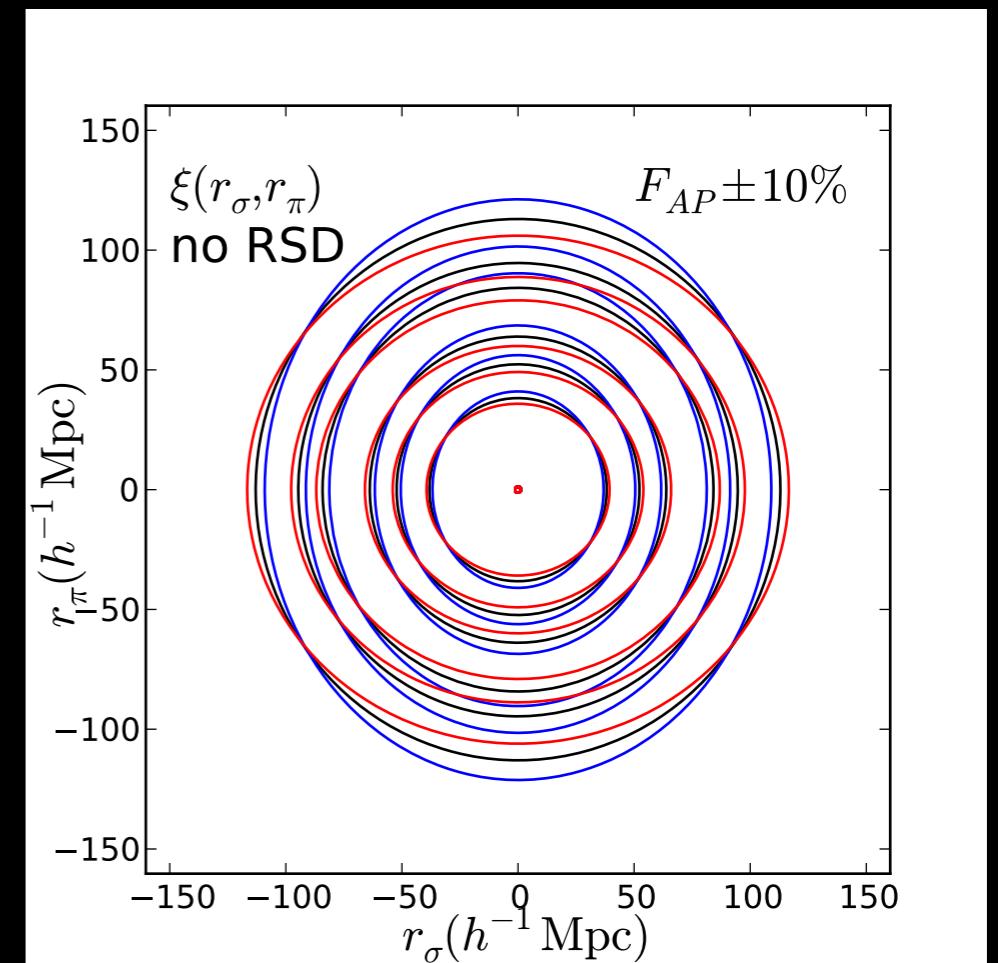
fit to D_A and H

$$D_A(z=0.57) = (1421 \pm 20 \text{ Mpc}) (s_{\text{BAO}} / s_{\text{BAO,fid}})$$

$$H(z=0.57) = (96.8 \pm 3.4 \text{ km s}^{-1} \text{Mpc}^{-1}) (s_{\text{BAO,fid}} / s_{\text{BAO}})$$

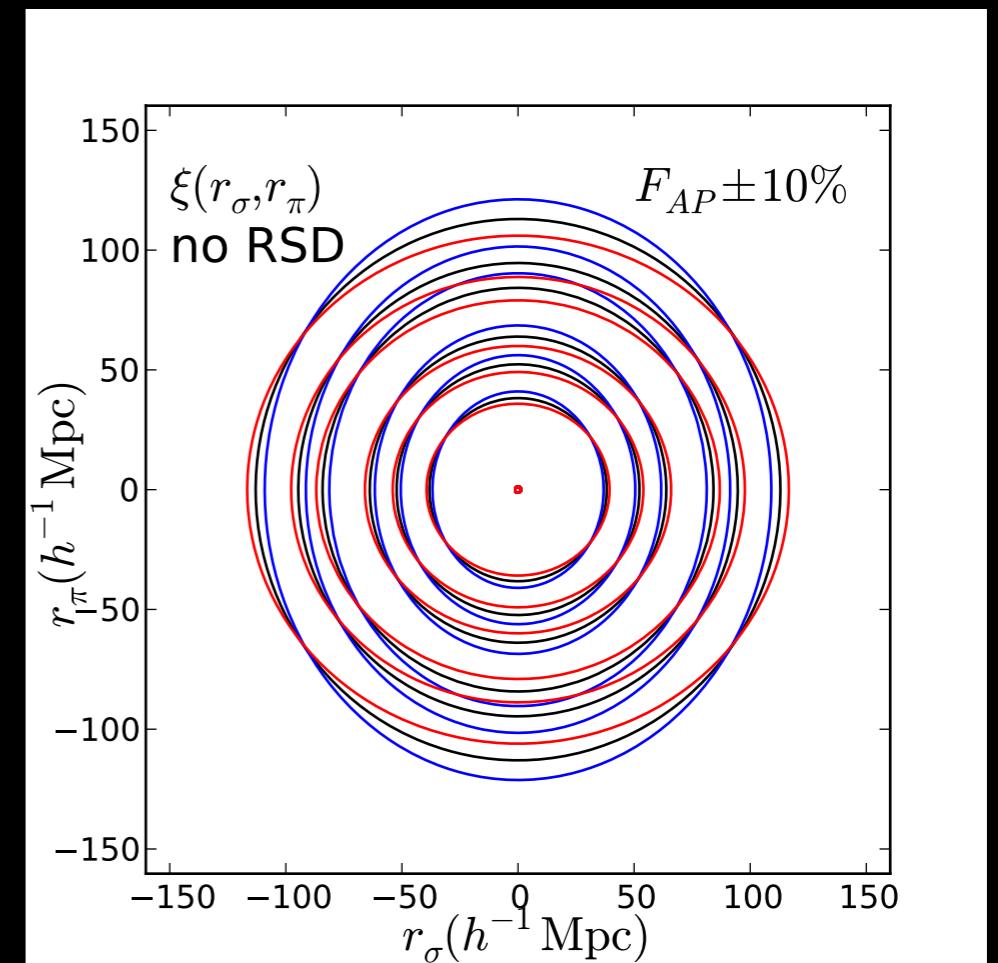
Geometric constraints from galaxy surveys: Alcock-Paczynski (1979) effect

- Assume $\xi(r)$ is isotropic (black circular contours). The observed ξ will appear anisotropic (elliptical contours) if LOS separations are distorted relative to transverse separations; this constrains $F_{AP}(z) = (1+z) D_A(z) H(z)/c$



Geometric constraints from galaxy surveys: Alcock-Paczynski (1979) effect

- The AP effect requires no standard ruler (just isotropy), so information from all scales is useful.



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2014 Shaw Prize awarded to Daniel Eisenstein for BAO

The screenshot shows a computer browser displaying the Shaw Prize website at www.shawprize.org/en/shaw.php?tmp=5&twoid=79&threeid=231&fourid=411. The page title is "THE SHAW PRIZE 邵逸夫獎". The navigation bar includes links for Organization, The Shaw Prize, Shaw Laureates, Shaw Lectures, News & Event, Photo & Video, and Contact Us. The main content area shows the "Announcement of The Shaw Laureates 2014" dated Tuesday, 27 May 2014. It details the awarding of the Shaw Prize in Astronomy to Daniel Eisenstein, Shaun Cole, and John A Peacock for their contributions to the measurements of features in the large-scale structure of galaxies used to constrain the cosmological model including baryon acoustic oscillations and redshift-space distortions.

THE SHAW PRIZE 邵逸夫獎

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News

2014

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Events

Press Release

Announcement of The Shaw Laureates 2014

Tuesday, 27 May 2014. At today's press conference in Hong Kong, The Shaw Prize Foundation announced the Shaw Laureates for 2014. Information was posted on the website www.shawprize.org at Hong Kong time 15:30 (GMT 07:30).

The Shaw Prize consists of three annual prizes: Astronomy, Life Science and Medicine, and Mathematical Sciences, each bearing a monetary award of one million US dollars. This will be the Eleventh year that the Prize has been awarded and the presentation ceremony is scheduled for Wednesday, 24 September 2014.

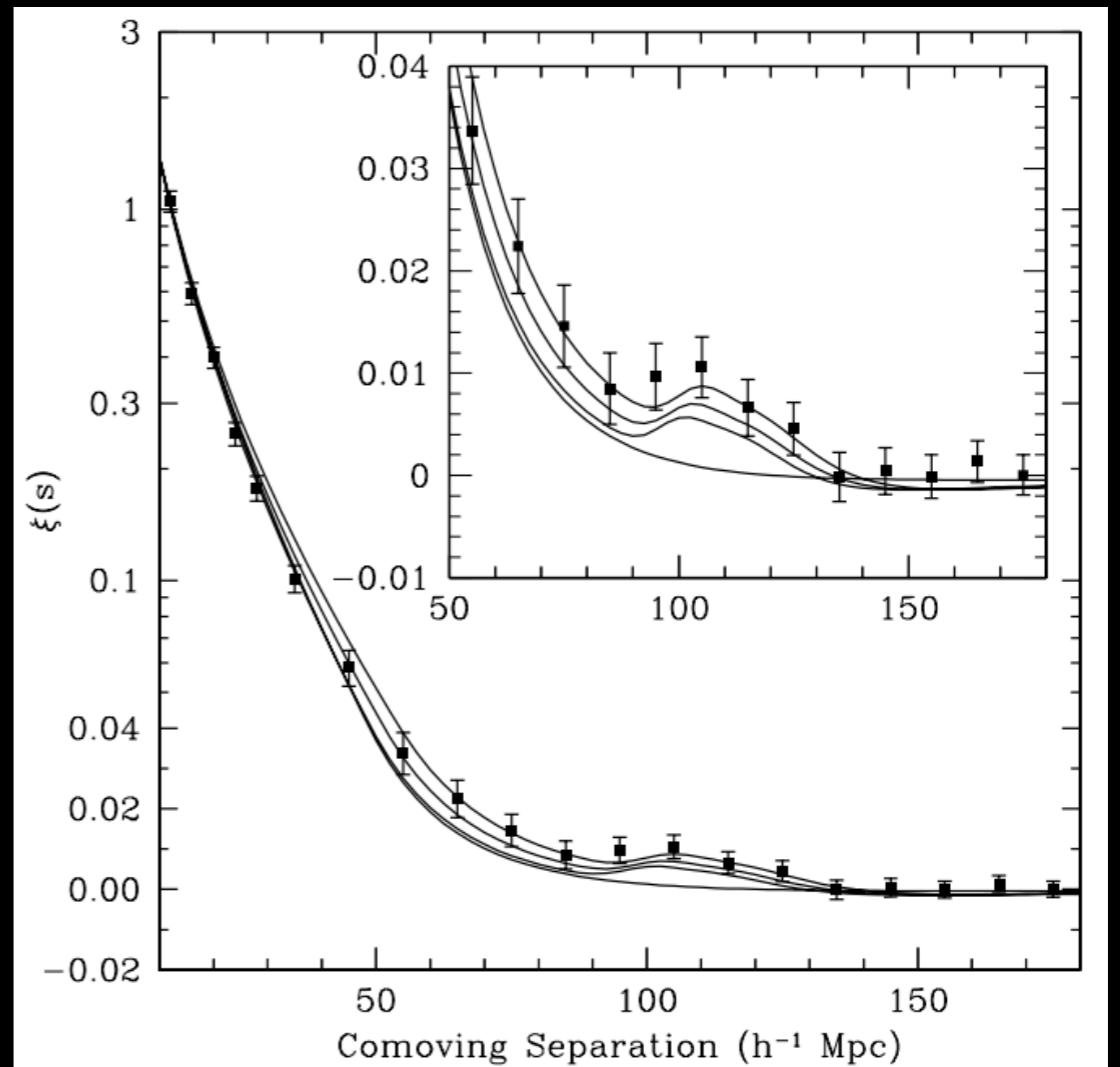
The Shaw Laureates

The Shaw Prize in Astronomy is awarded in one-half to **Daniel Eisenstein**, Professor of Astronomy, Harvard University, USA and the other half in equal shares to **Shaun Cole**, Professor of Physics, Durham University, UK and **John A Peacock**, Professor of Cosmology in the Institute for Astronomy, University of Edinburgh, UK

for their contributions to the measurements of features in the large-scale structure of galaxies used to constrain the cosmological model including baryon acoustic oscillations and redshift-space distortions.

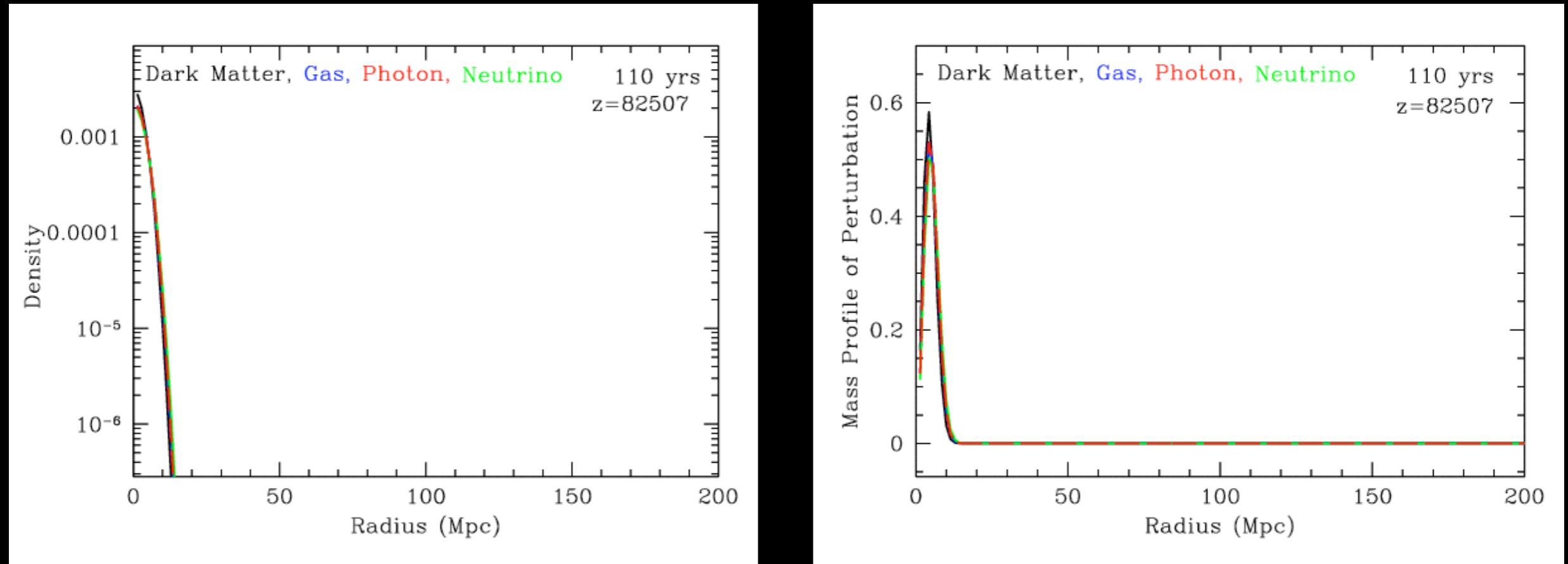
2014 Shaw Prize awarded to Daniel Eisenstein for BAO

Get rich (500k!) on LSS



Eisenstein et al., 2005, ApJ, 633, 560

Features in the initial conditions: Baryon Acoustic Oscillations



$$s_{\text{BAO}} = \int_0^{t_{\text{drag}}} c_s(1+z)dt = \int_{z_{\text{drag}}}^{\infty} \frac{c_s dz}{H(z)} \quad \text{SBAO} = 147.53 \pm 0.64 \text{ Mpc (0.4%!)}$$

(Planck XVI)

http://cmb.as.arizona.edu/~eisenste/acousticpeak/acoustic_physics.html

BAO standard ruler calibration -- caveats?

- Relative calibration requires n_b/n_y and z_{eq} ; absolute also requires $\Omega_m h^2$ [Eisenstein and White, PRD, 70, 103523 (2004)]
- Isocurvature initial conditions shift BAO scale as well; marginalize! [Mangilli, Verde, Beltran, JCAP, 20, 9 (2010)]
- ...?
- The absolute calibration of the BAO standard ruler for “standard” cosmological models depends only on $\Omega_b h^2$ and $\Omega_c h^2$; most of 0.4% uncertainty comes from the latter.

The BAO standard ruler

- If linear perturbation theory were completely accurate, this lecture would be over... BUT!
- Non-linear gravitational evolution degrades the BAO feature in the matter (or galaxy) correlation function
- Some of the BAO information degradation in the evolved (non-Gaussian) field can be undone by the process of reconstruction

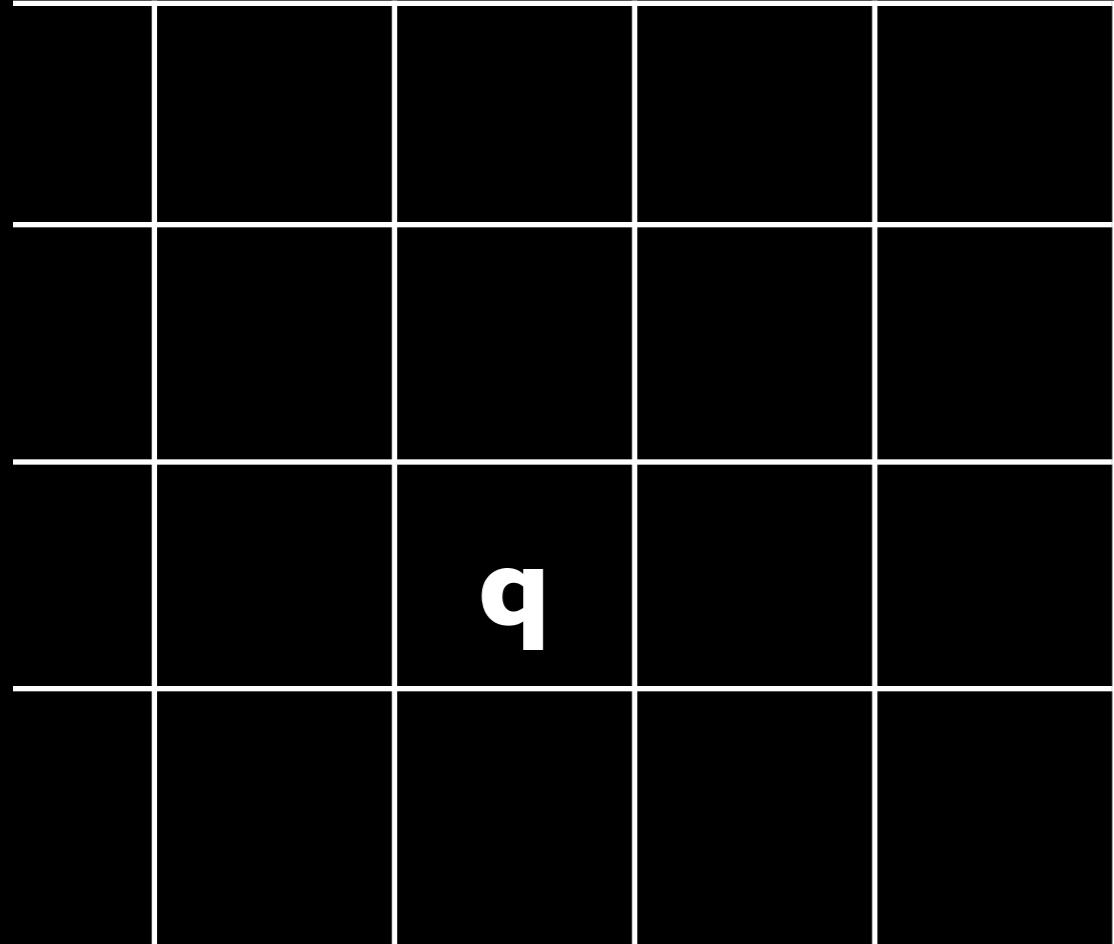
The BAO standard ruler

- The relevant physics can be understood with a very simple model -- the Zel'dovich approximation

Cosmological Perturbation Theory

- The universe is initially completely homogeneous with negligible perturbations
- “Standard” perturbation theory tracks gravitational evolution of density contrast at comoving coordinate \mathbf{x} , $\delta(\mathbf{x})$, and the divergence of the peculiar velocity field $\theta(\mathbf{x}) \equiv -\nabla \cdot \mathbf{v}_P(\mathbf{x})$.
- In SPT (and LPT), $\nabla \times \mathbf{v}_P$ decays and is neglected.

BAO evolution in Zel'dovich approximation



- Consider a **very early time** when the mass distribution is homogeneous, and follow the trajectory of equal mass/volume elements labelled by initial coordinate **q** to position **x** at time t:
$$\mathbf{x} = \mathbf{q} + \Psi(\mathbf{q}, t)$$

BAO evolution in Zel'dovich approximation

$$\mathbf{x} = \mathbf{q} + \Psi(\mathbf{q}, t)$$

- Then $\Psi(\mathbf{q}, t)$ is irrotational and obeys

$$\frac{d^2\Psi}{dt^2} + 2H\frac{d\Psi}{dt} = -\nabla_x \phi[\mathbf{q} + \Psi(\mathbf{q})]$$

$$\nabla_x^2 \phi(\mathbf{x}) = 4\pi G \bar{\rho} a^2 \delta(\mathbf{x})$$

Poisson Eqn

$$\delta(\mathbf{x}) = \int d^3 q \delta^3 [\mathbf{x} - \mathbf{q} - \Psi(\mathbf{q})]$$

Eulerian density contrast in terms of Ψ

[See Matsubara 0711.2521 for details]

BAO evolution in Zel'dovich approximation

$$\mathbf{x} = \mathbf{q} + \Psi(\mathbf{q}, t)$$

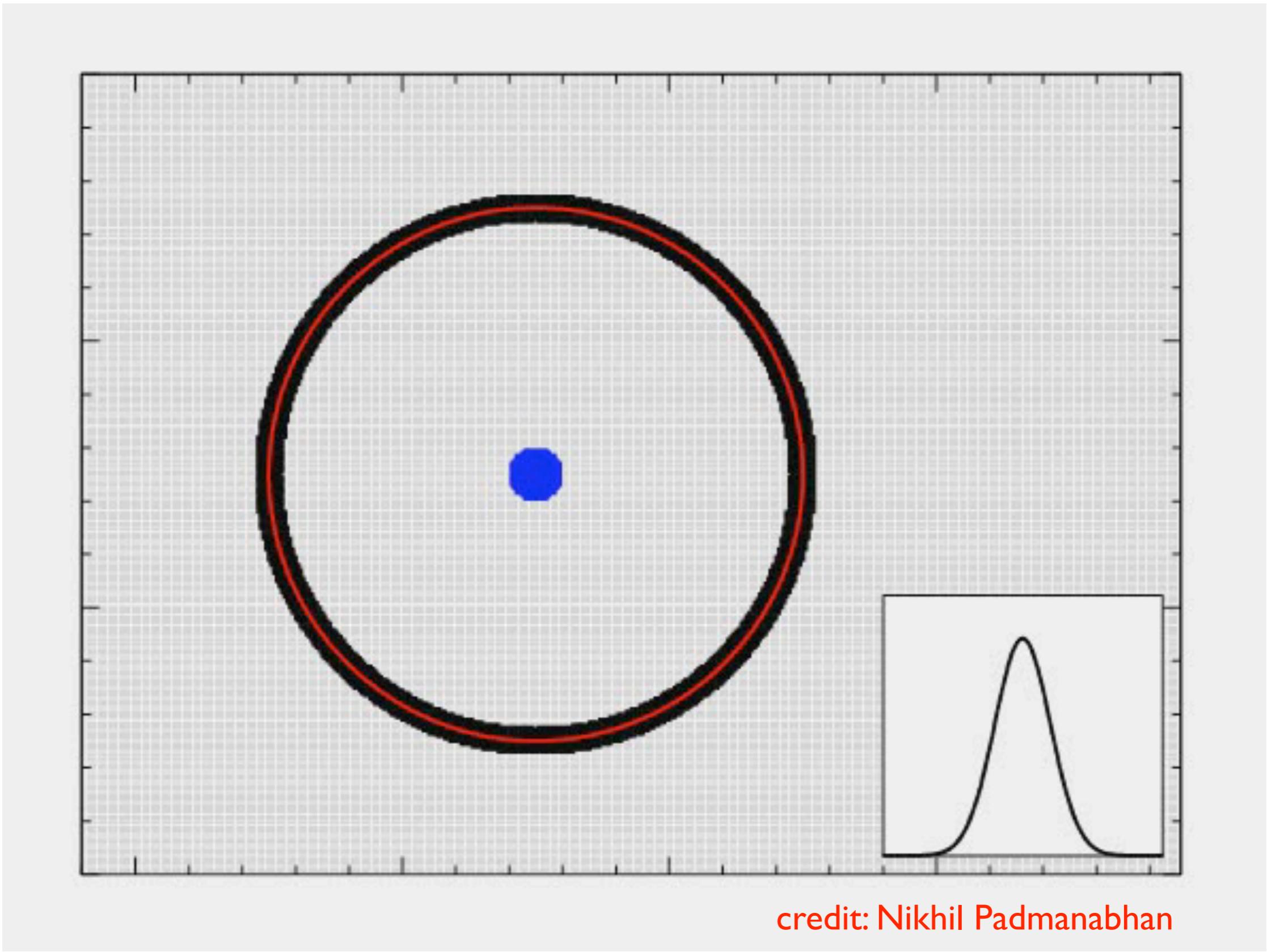
- The Zel'dovich approximation keeps only the first order term for Ψ :

$$\Psi(\mathbf{q}) = \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{q}} \frac{i\mathbf{k}}{k^2} \delta_L(\mathbf{k})$$

↑
Linearly evolved density
contrast at time t

- That is, motions of mass elements follow a straight line; this captures a surprising amount of the features of the LSS/cosmic web!

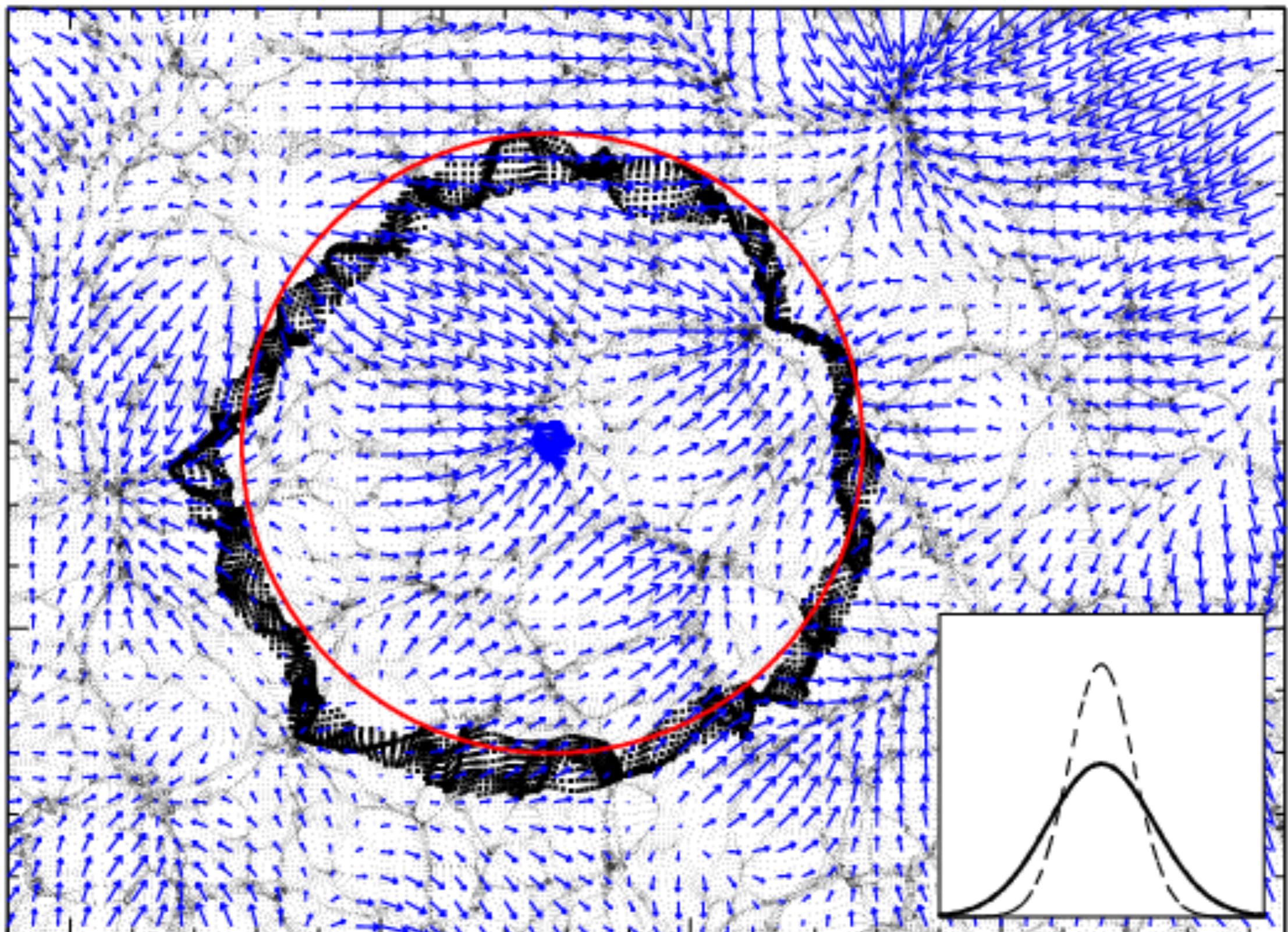
Nonlinear Evolution



BAO evolution in Zel'dovich approximation

- Non-linear evolution smears out the BAO feature.
- This makes it harder measure the BAO location precisely, so your precision degrades.
- However! The relative displacement of particles separated by ~ 150 Mpc is sourced by relatively large scale (linear) density fluctuations

Reconstruction : II



credit: Nikhil Padmanabhan

BAO Reconstruction

- The key insight of Eisenstein et al. 2007 (ApJ, 664, 675) was to use the evolved (non-linear) observed density field to estimate the displacement field

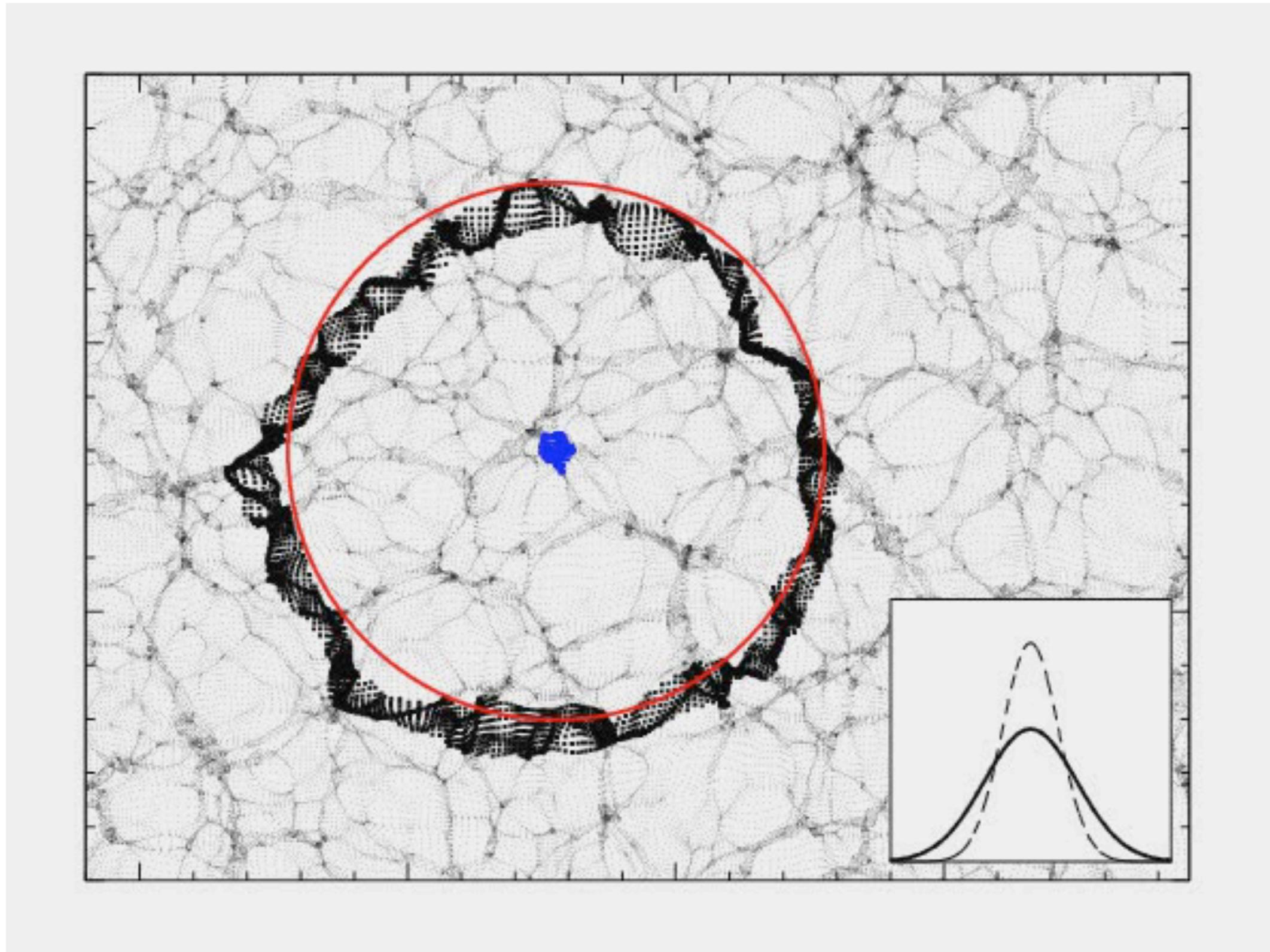
$$\Psi(\mathbf{q}) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{q}} \frac{i\mathbf{k}}{k^2} \delta_L(\mathbf{k})$$



Replace with non-linear
(observed) density field

- Using the estimated displacement field, you can move particles back to their “initial” positions

Reconstruction : III



credit: Nikhil Padmanabhan

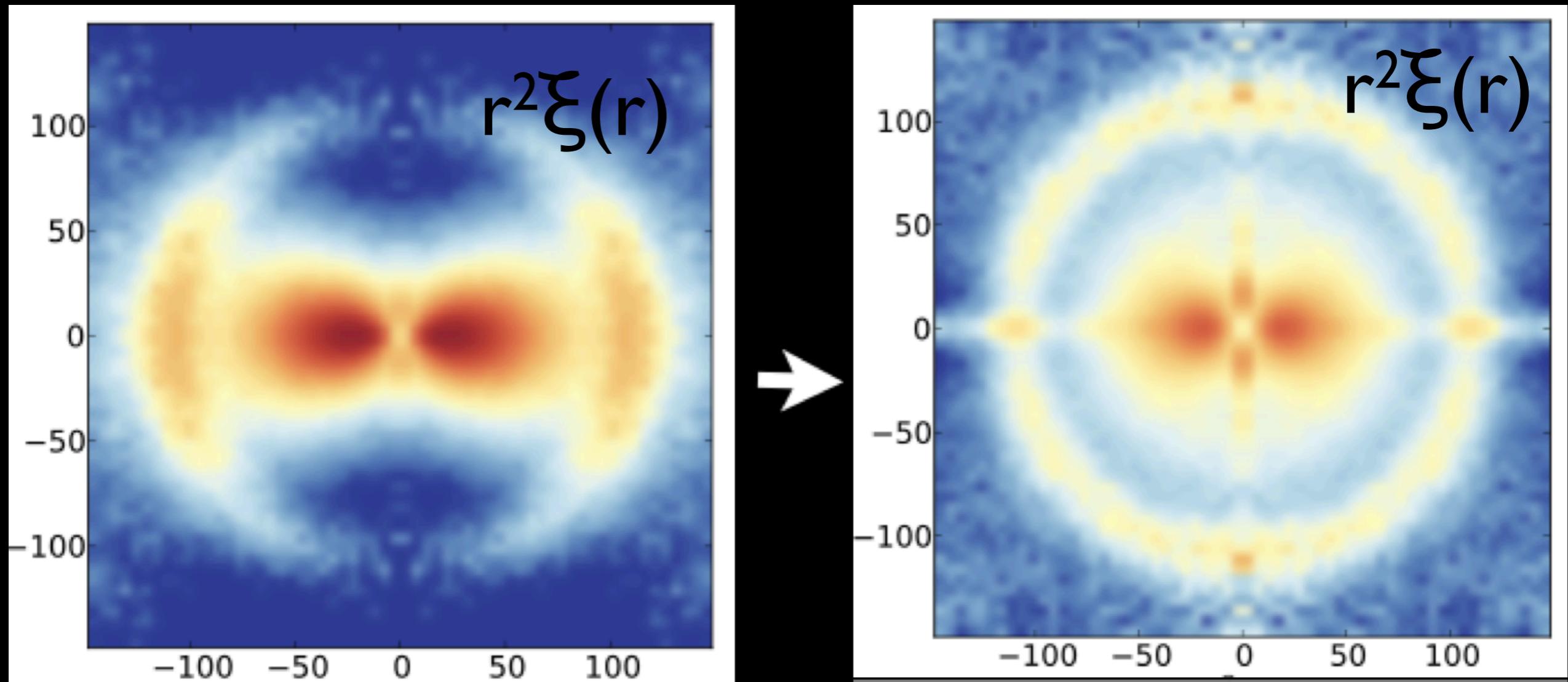
BAO Reconstruction Summary I

- A simple “reconstruction” algorithm partially restores the initial separations of pairs separated at 150 Mpc; it does not undo non-linearity on smaller scales.
- This re-sharpens the BAO feature in the reconstructed density field and increases the signal-to-noise of the BAO distance measurement towards its linear theory value
- Small non-linear shifts ($\sim 0.5\%$; calculable in LPT) are removed by reconstruction.

BAO Reconstruction Summary II

- Note that this is NOT simply a deconvolution! We are restoring information by using phase/higher order correlation information in the observed density field
- It works!

Reconstruction works: mocks

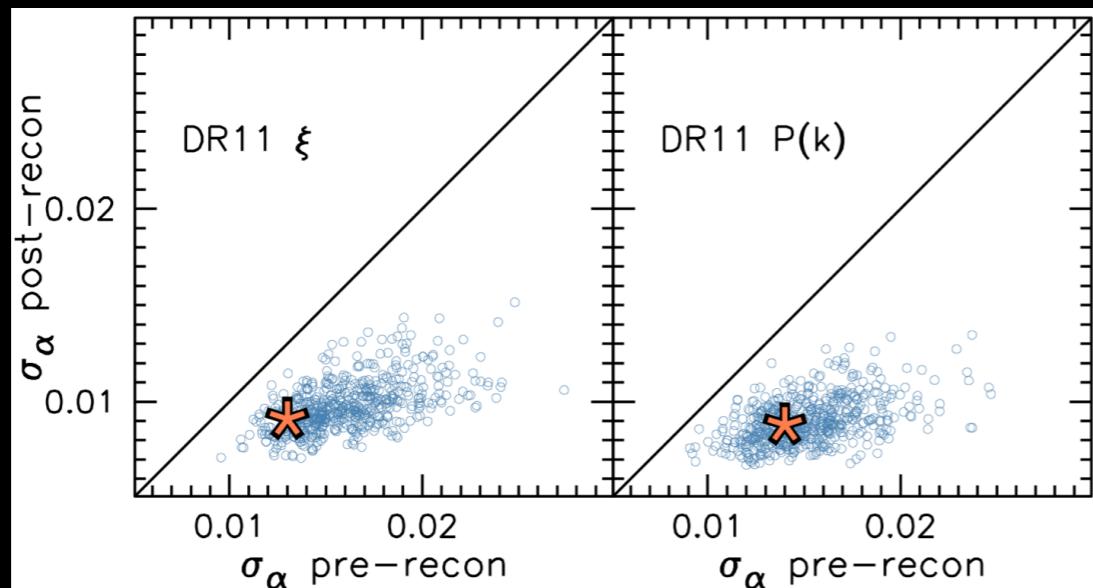


Reconstruction on 160 Las Damas mock galaxy catalogs

Padmanabhan et al., 2012, MNRAS, 427, 2132

Reconstruction works: real surveys!

- SDSS-II LRGs ($z=0.35$): $3.5 \rightarrow 1.9\%$
[Padmanabhan et al., 2012, MNRAS, 427, 2132]
- SDSS-III CMASS ($z=0.57$) : $1.4\% \rightarrow 0.9\%$
[SDSS-III BOSS; arXiv:1312.4877]
- WiggleZ ($z=0.4-0.7$) $\sim 1.5\times$ improvement despite high shot noise/disjoint survey [Kazin et al., arXiv:1401.0358]



BOSS DR11 CMASS
mocks + data
arXiv:1312.4877

Fitting the BAO -- “Philosophy”

- Lots of complicated physical effects alter the galaxy ξ/P away from its linear theory behavior:
 - non-linear gravitational evolution
 - non-linear biasing between tracers and matter field
 - non-linear redshift space distortions
- At the same time, recombination physics + Zel'dovich approximation provides a very accurate model for the exact shape of the BAO feature [i.e., compared with wavelet fitting]

Fitting the BAO -- “Philosophy”

- We will exploit the fact that the messy non-linearities should generically be smooth, and marginalize over “broad-band” (smooth) contributions to the power spectrum/correlation function
- We will use a smoothed template for the BAO feature

Fitting the BAO -- practice

- Start with linear $P(k)/\xi(r)$; damp the BAO feature

$$P^{\text{fit}}(k) = P^{\text{sm}}(k) \left[1 + (O^{\text{lin}}(k/\alpha) - 1)e^{-\frac{1}{2}k^2\Sigma_{nl}^2} \right]$$

- add broadband nuisance terms

$$A^\xi(s) = \frac{a_1}{s^2} + \frac{a_2}{s} + a_3$$

- Marginalize to get $p(\alpha)$

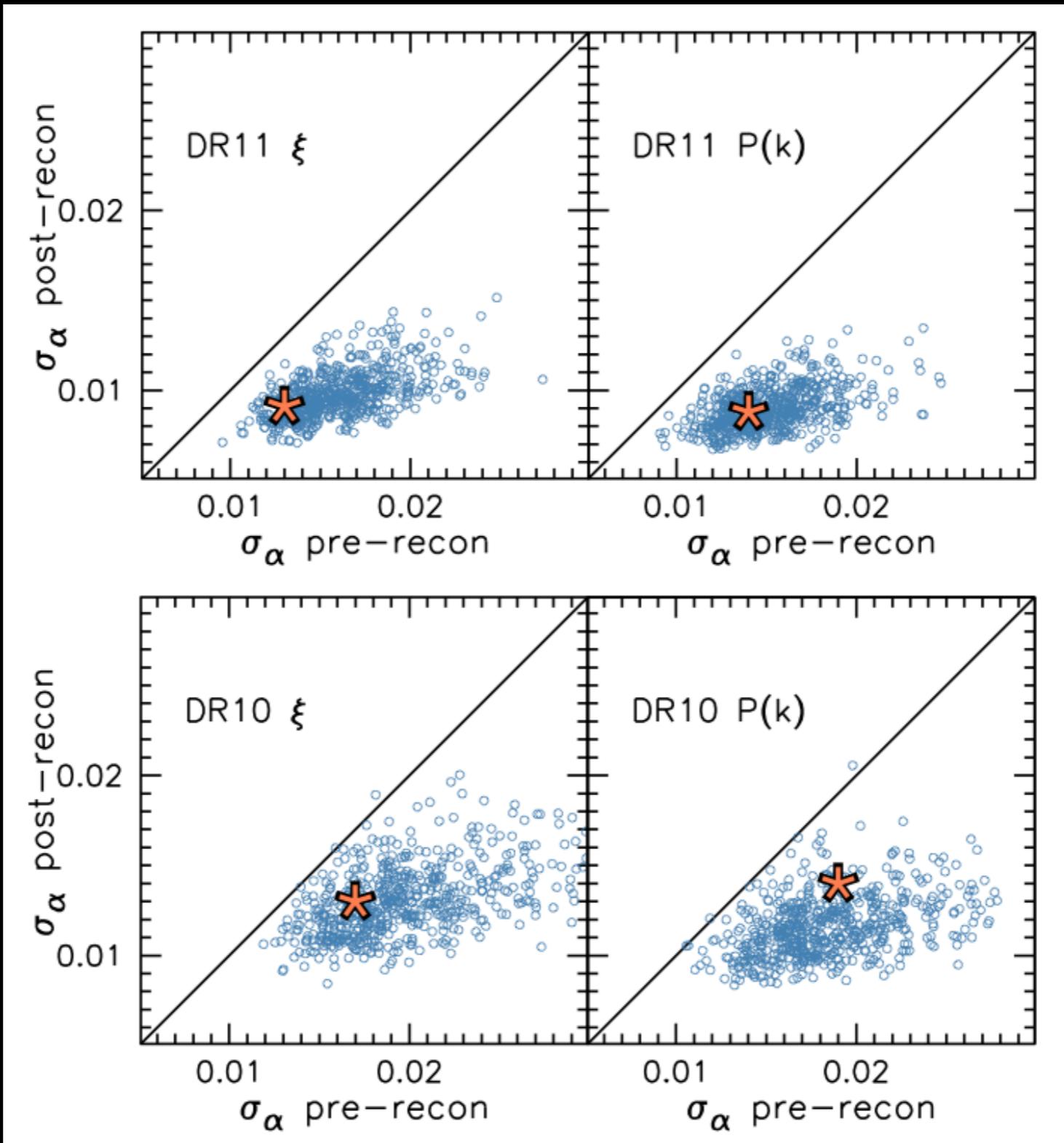
$$\xi^{\text{fit}}(s) = B_\xi^2 \xi^{\text{mod}}(\alpha s) + A^\xi(s)$$

Test with mocks: Fits unbiased

| Estimator | $\langle \alpha \rangle$ | S_α | $\langle \sigma \rangle$ | $\langle \chi^2 \rangle/\text{dof}$ |
|----------------------------------|--------------------------|---------------|--------------------------|-------------------------------------|
| DR11 | | | | |
| Consensus $P(k) + \xi(s)$ | 1.0000 | 0.0090 | 0.0088 | |
| combined $P(k)$ | 1.0001 | 0.0092 | 0.0089 | |
| combined $\xi(s)$ | 0.9999 | 0.0091 | 0.0090 | |
| post-recon $P(k)$ | 1.0001 | 0.0093 | 0.0090 | 28.6/27 |
| post-recon $\xi_0(s)$ | 0.9997 | 0.0095 | 0.0097 | 17.6/17 |
| pre-recon $P(k)$ | 1.0037 | 0.0163 | 0.0151 | 27.7/27 |
| pre-recon $\xi_0(s)$ | 1.0041 | 0.0157 | 0.0159 | 15.7/17 |
| DR10 | | | | |
| post-recon $P(k)$ | 1.0006 | 0.0117 | 0.0116 | 28.4/27 |
| post-recon $\xi_0(s)$ | 1.0014 | 0.0122 | 0.0126 | 17.2/17 |
| pre-recon $P(k)$ | 1.0026 | 0.0187 | 0.0184 | 27.7/27 |
| pre-recon $\xi_0(s)$ | 1.0038 | 0.0188 | 0.0194 | 15.8/17 |

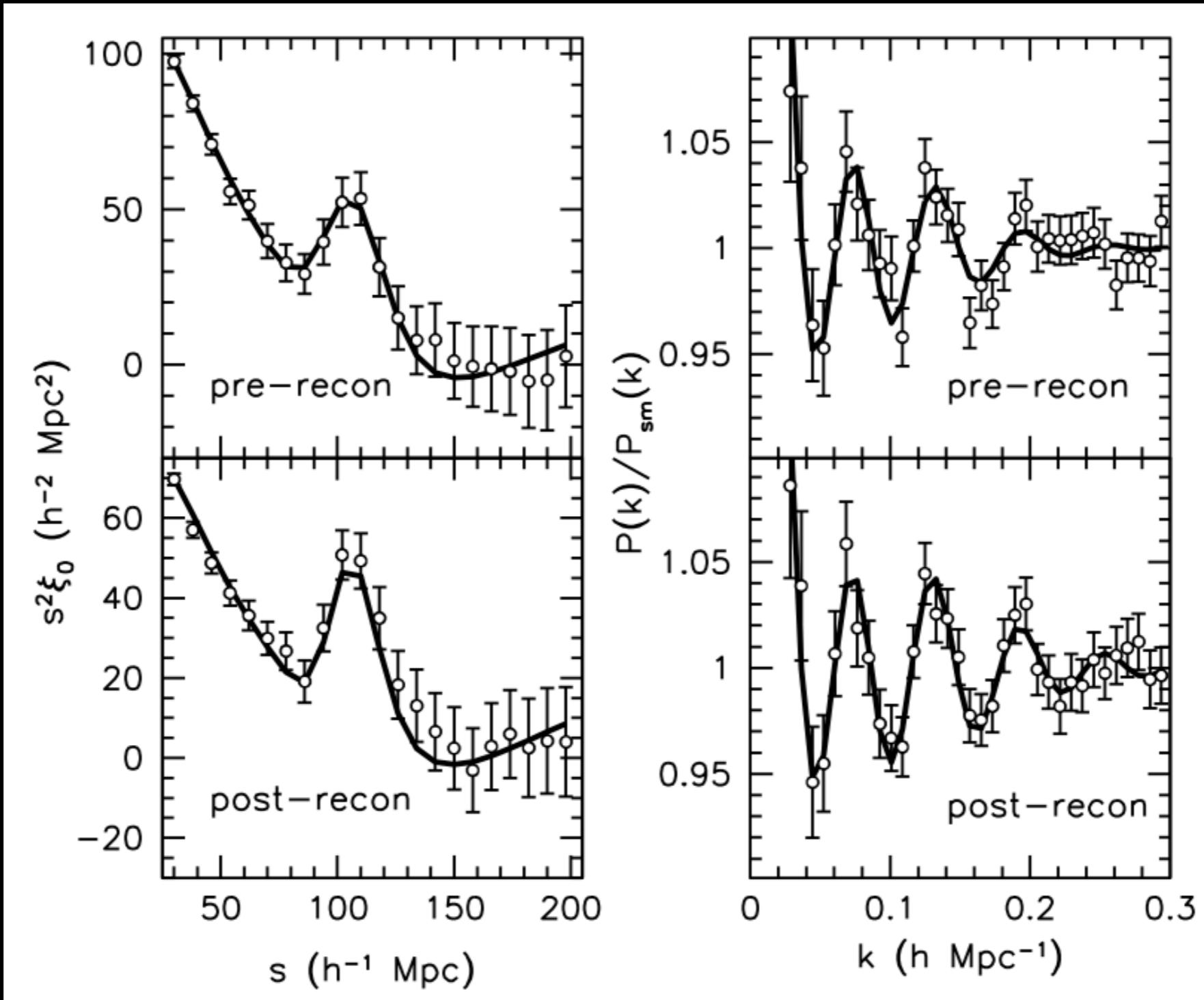
BOSS DR11 results
arXiv:1312.4877

Test with mocks: DR10/11 “typical”



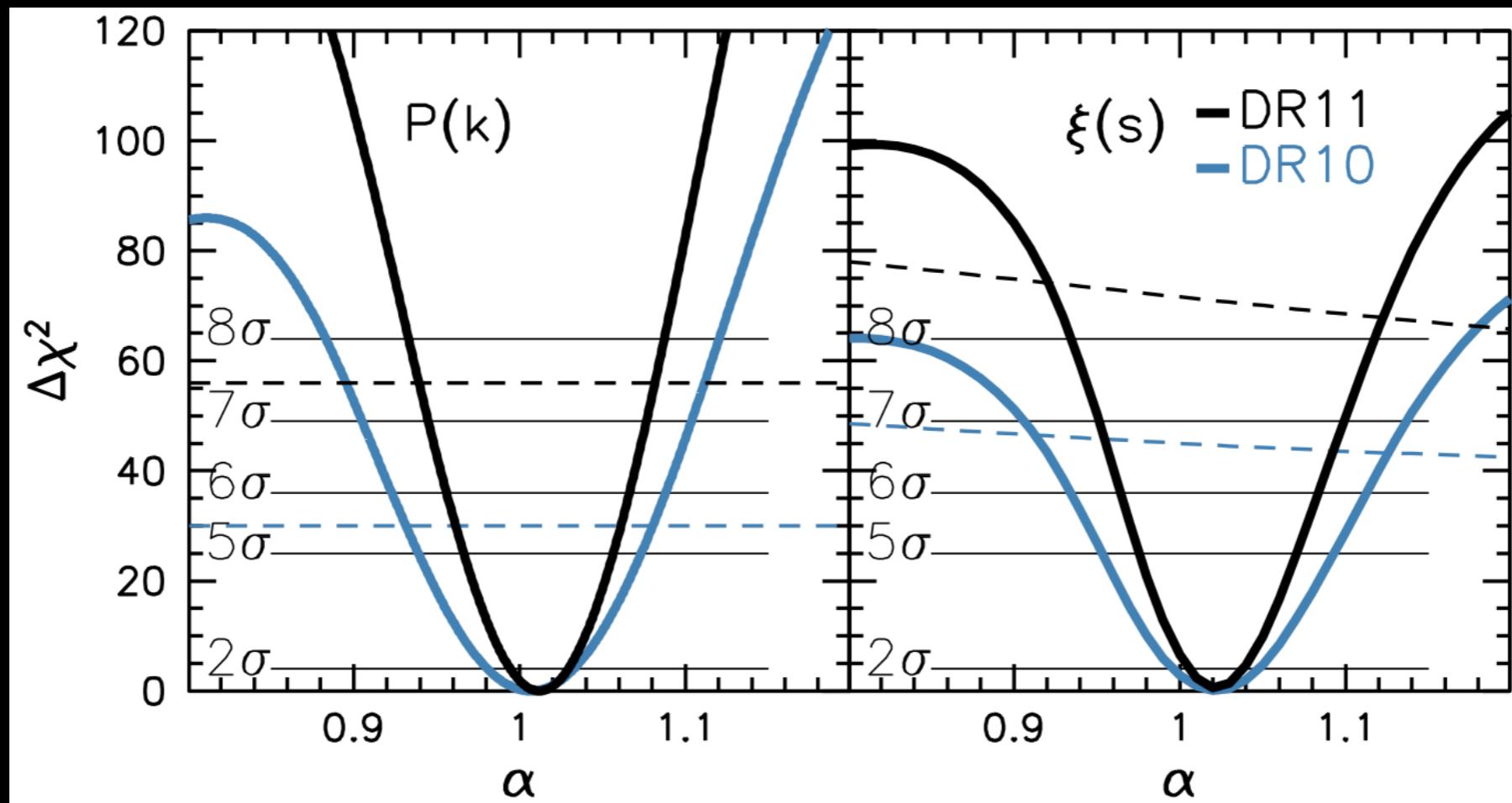
BOSS DR11 results
arXiv:1312.4877

BAO fits: ξ and P



BOSS DR II results
arXiv:1312.4877

BAO in SDSS-III BOSS CMASS sample: 1% distance constraint at $z=0.57$!

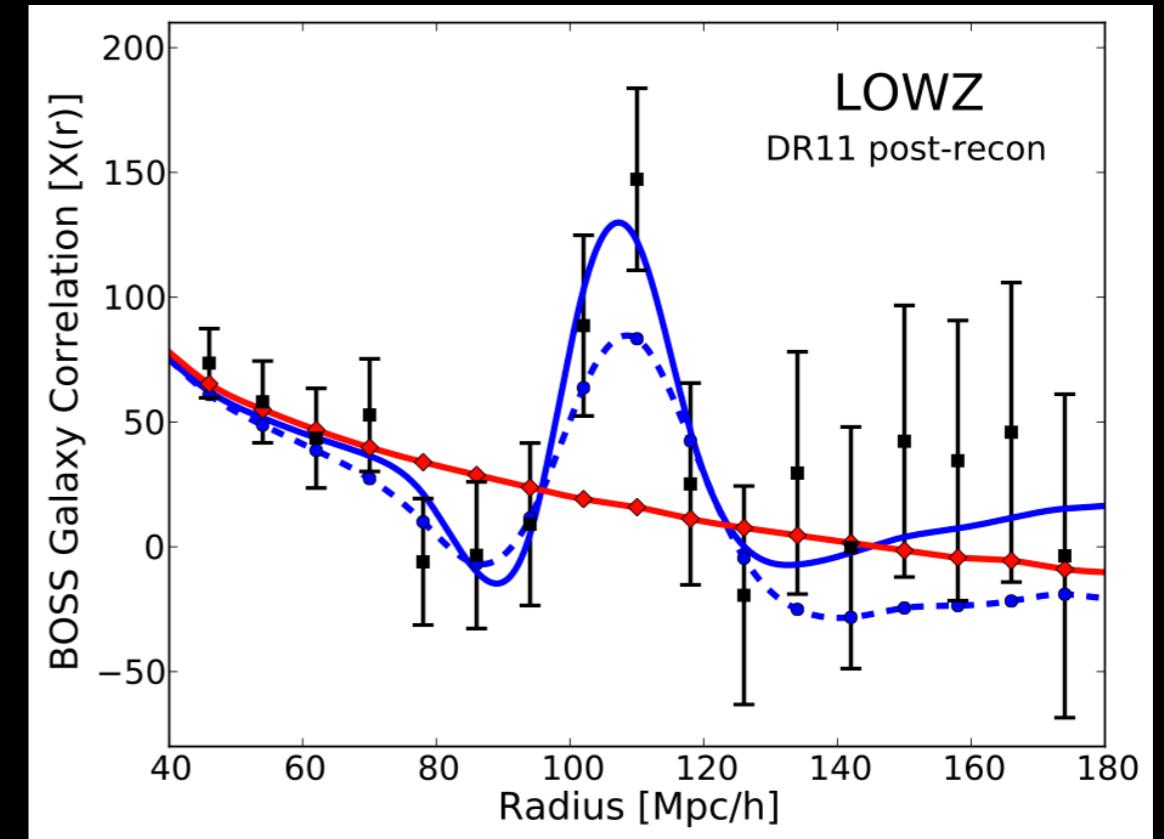
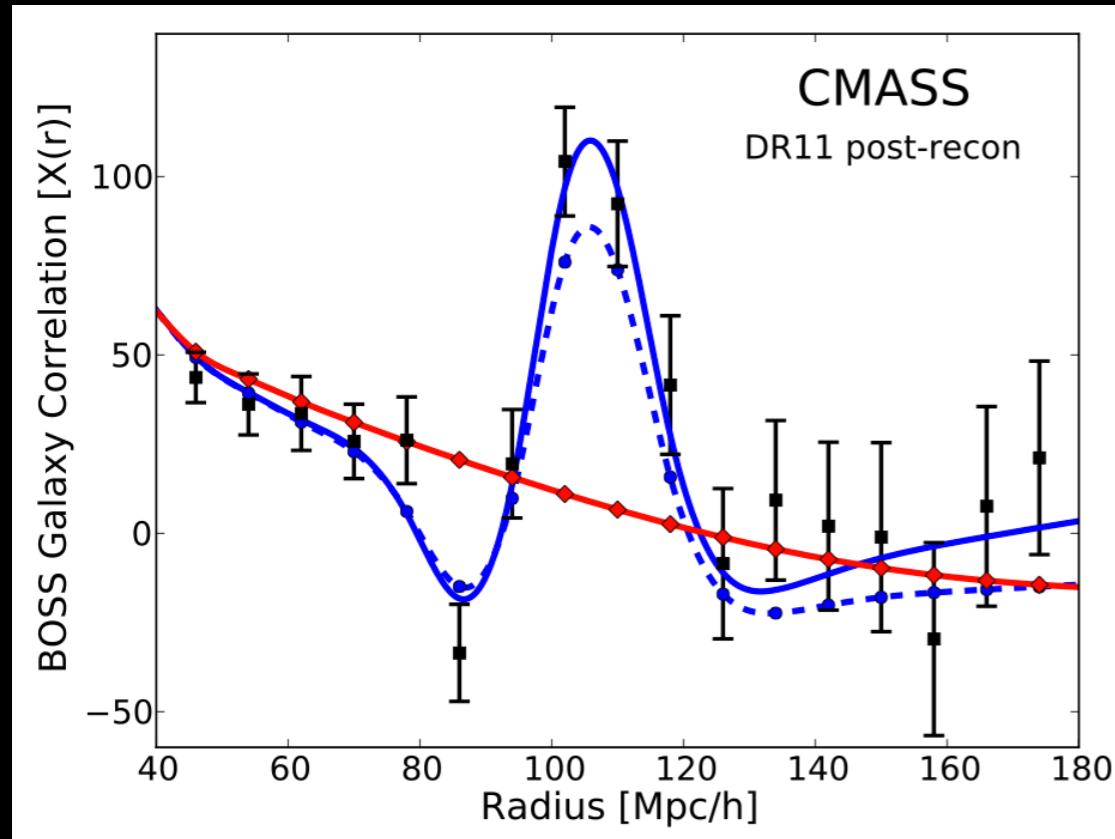


$$D_V(z=0.57) = (2056 \pm 20 \text{ Mpc}) (s_{\text{BAO}} / s_{\text{BAO,fid}})$$

BOSS, arXiv:1312.4877

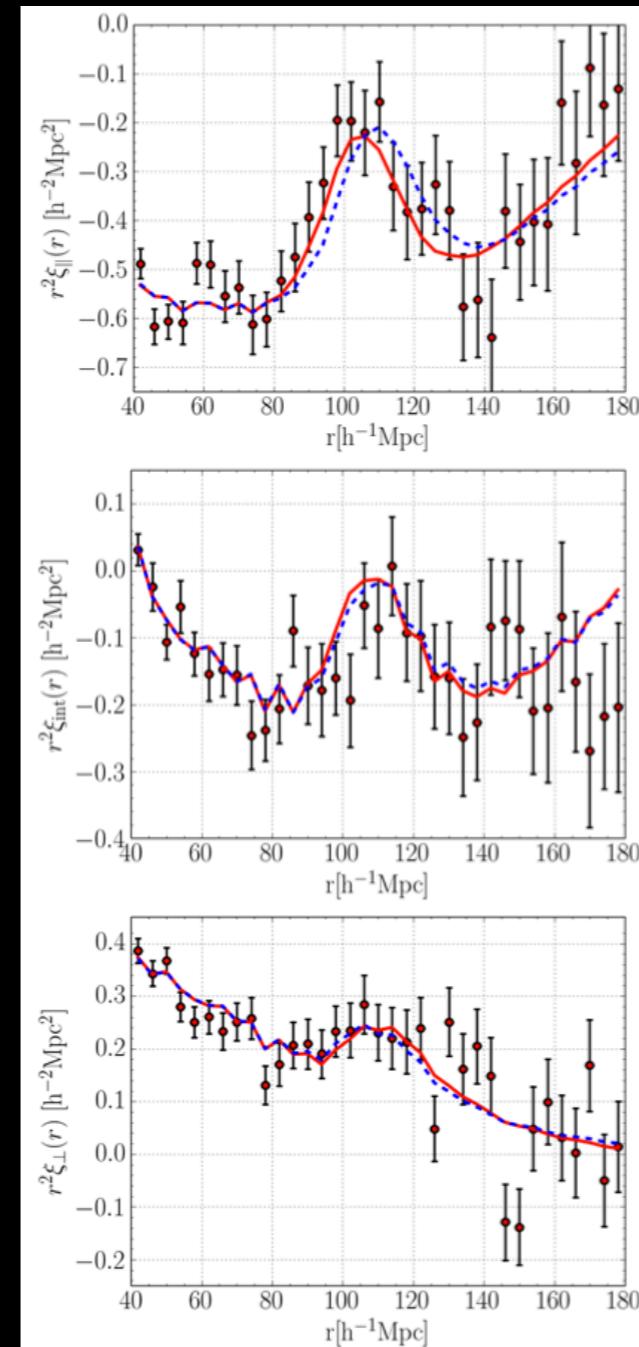
BAO fits: χ^2 by eye allowed (points nearly uncorrelated)

$$X(s_i) = \frac{x_i - a(x_{i-1} + x_{i+1}) - b(x_{i-2} + x_{i+2})}{1 - 2a - 2b}$$



Recent developments in BAO: Ly- α forest

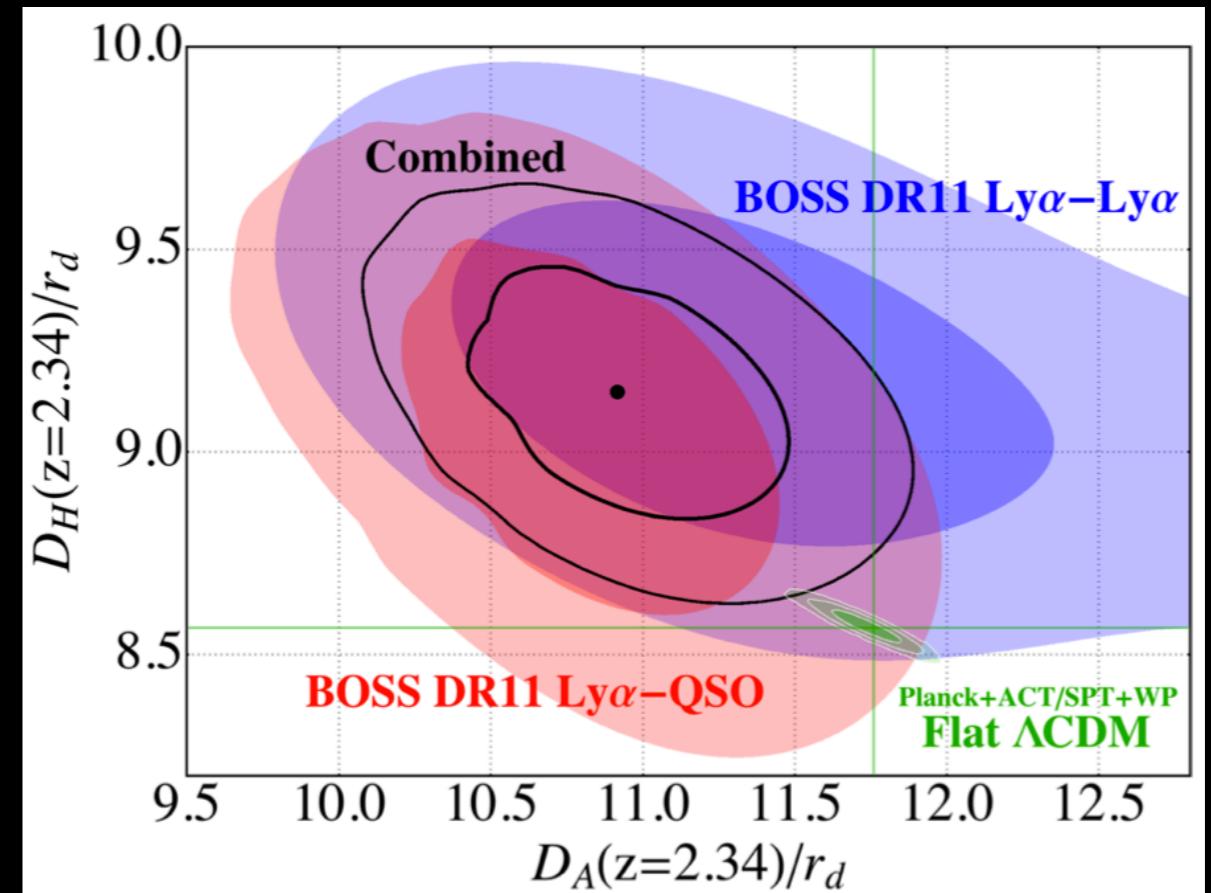
- First detected in BOSS DR9 [Busca et al., Slosar et al.]
- Detected in DR11 in Ly- α auto-correlation [Delubac et al., arXiv:1401.1801] and quasar/Ly- α cross-correlation [Font-Ribera et al., 1311.1767]



Delubac et al., arXiv:1401.1801

Recent developments in BAO: Ly- α forest

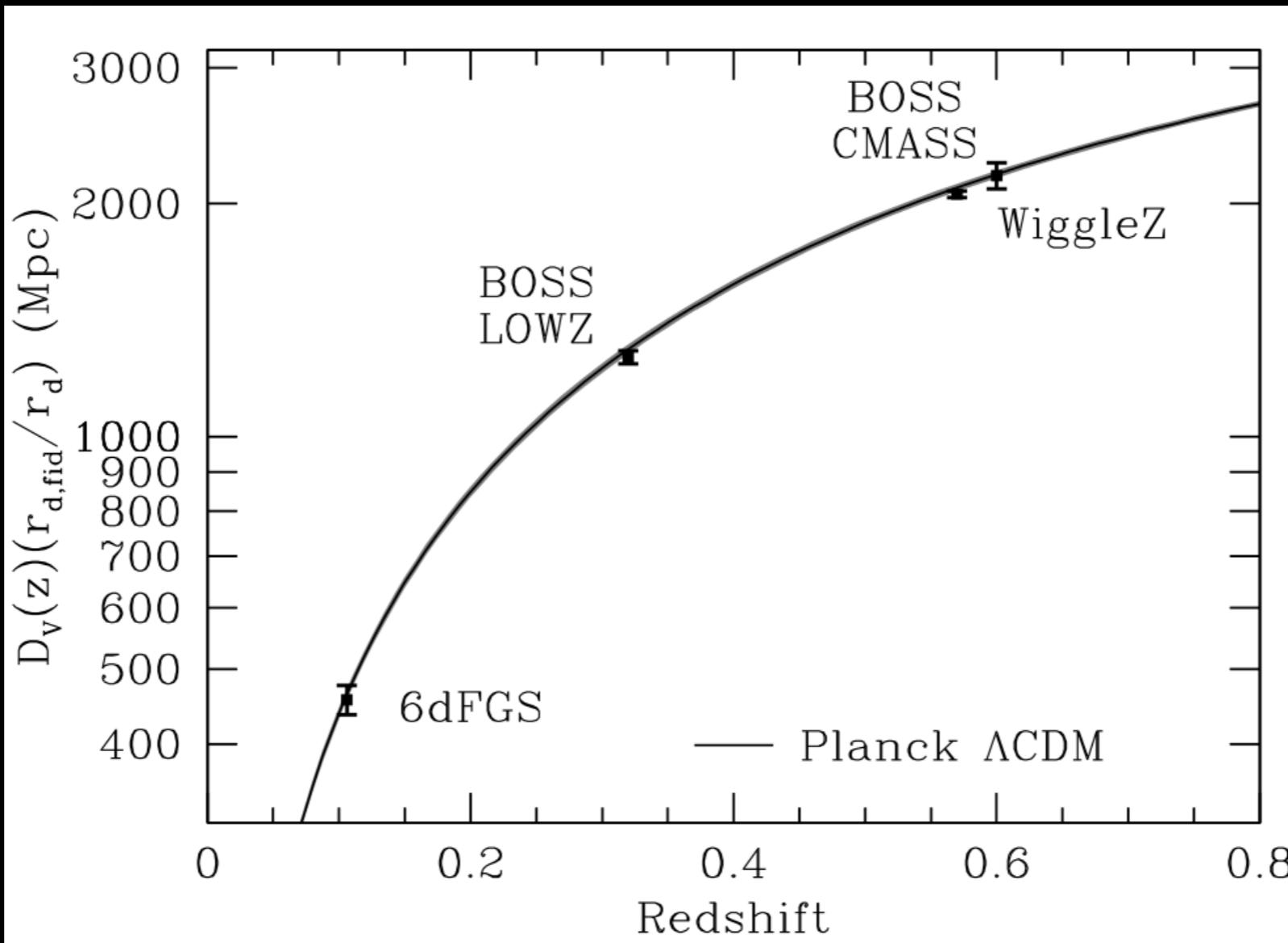
- Another $\sim 2.5\sigma$ “tension”
- Not simply solved by a normal Λ CDM extension



Delubac et al., arXiv:1401.1801

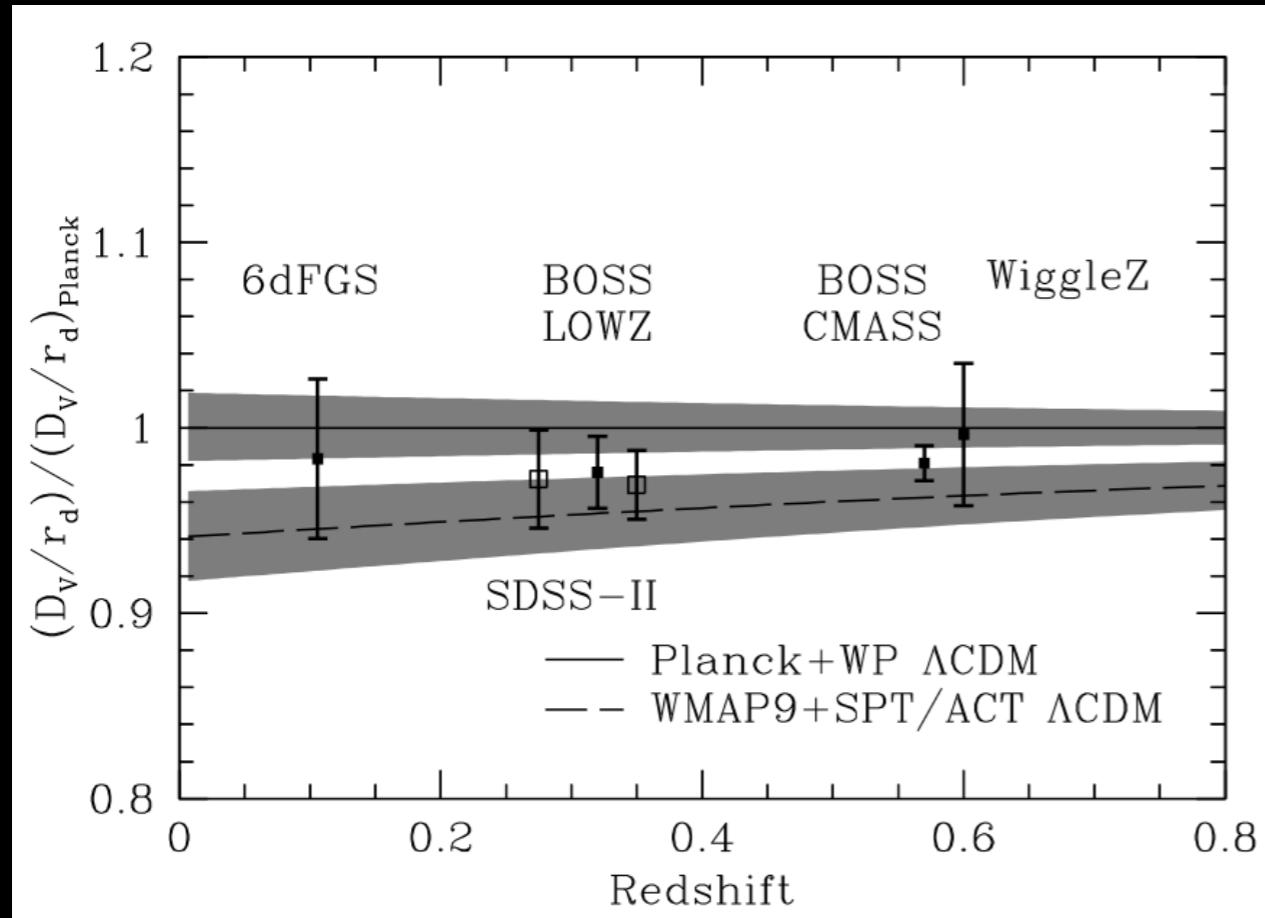
Cosmological Implications of current galaxy BAO measurements

Galaxy BAO distance ladder vs CMB Λ CDM predictions



BOSS, arXiv:1312.4877

Galaxy BAO distance ladder vs CMB Λ CDM predictions



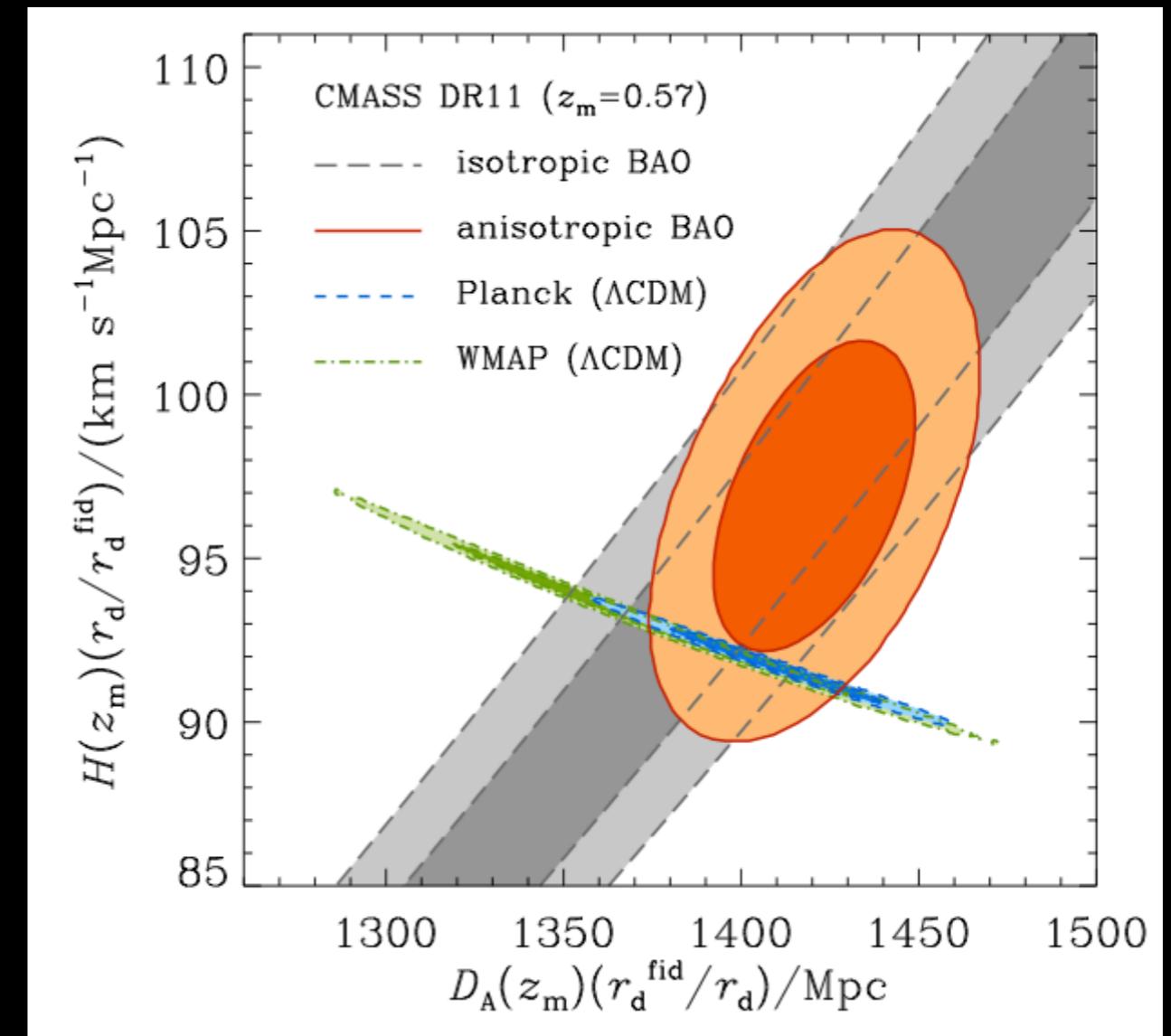
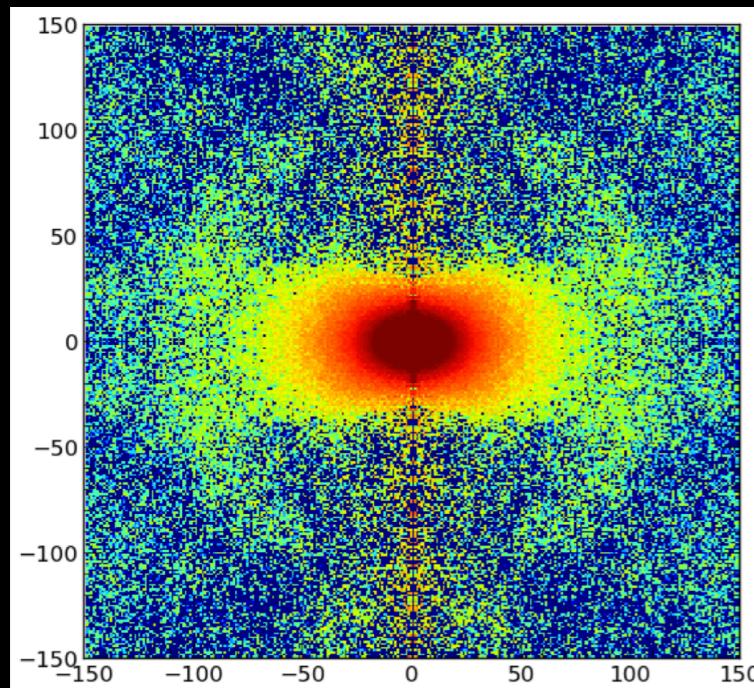
Planck: $\Omega_m h^2 = 0.1427 \pm 0.0024$

eWMAP: $\Omega_m h^2 = 0.1353 \pm 0.0035$

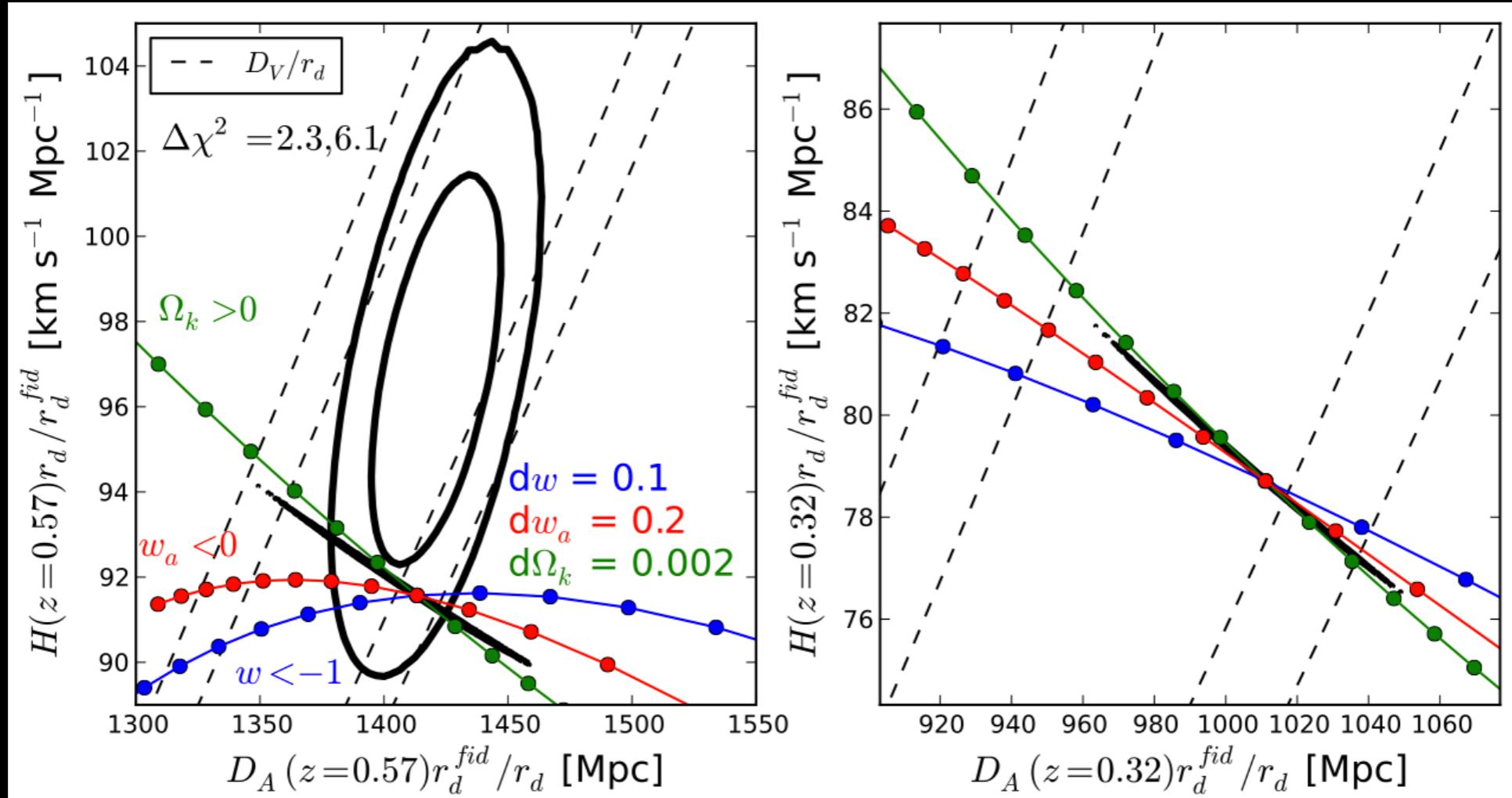
Λ CDM predictions from different CMB data sets differ because of $\Omega_m h^2$ difference; unfortunately BAO data straddle those predictions!

BOSS, arXiv:1312.4877

Anisotropic BAO fit results

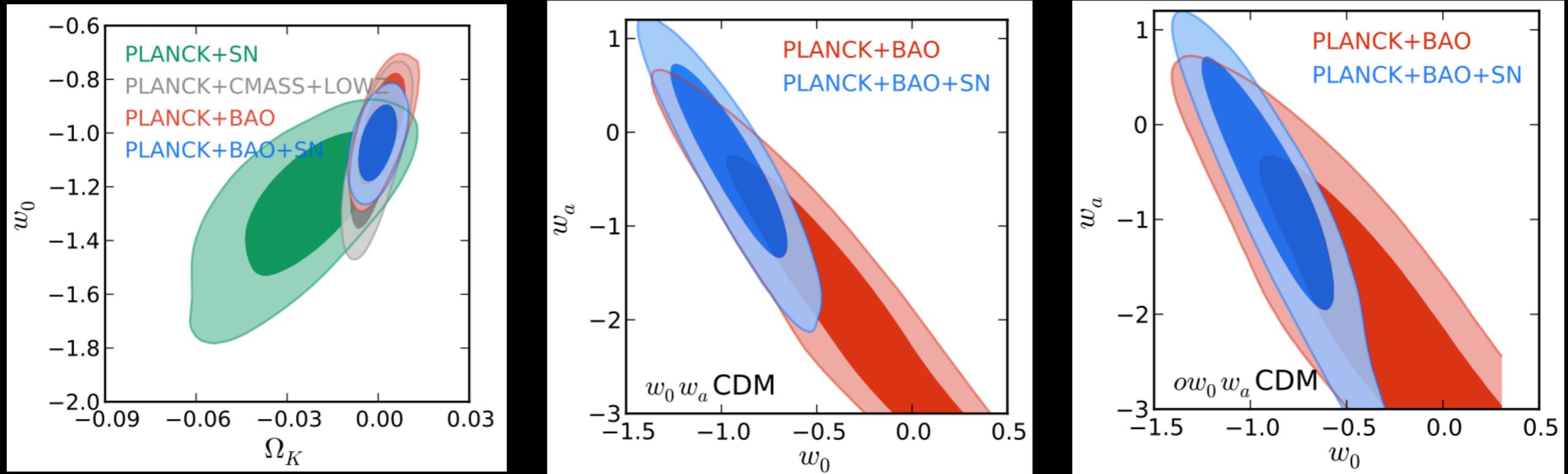


LOWZ and anisotropic CMASS BAO fits



- D_A constraint improved
- $D_A * H \sim 1.5\sigma$ high; no standard parameters predict observed value.
- $D_V(z=0.32)$ measured to 2%
- Lower redshift increases sensitivity to w_0, w_a

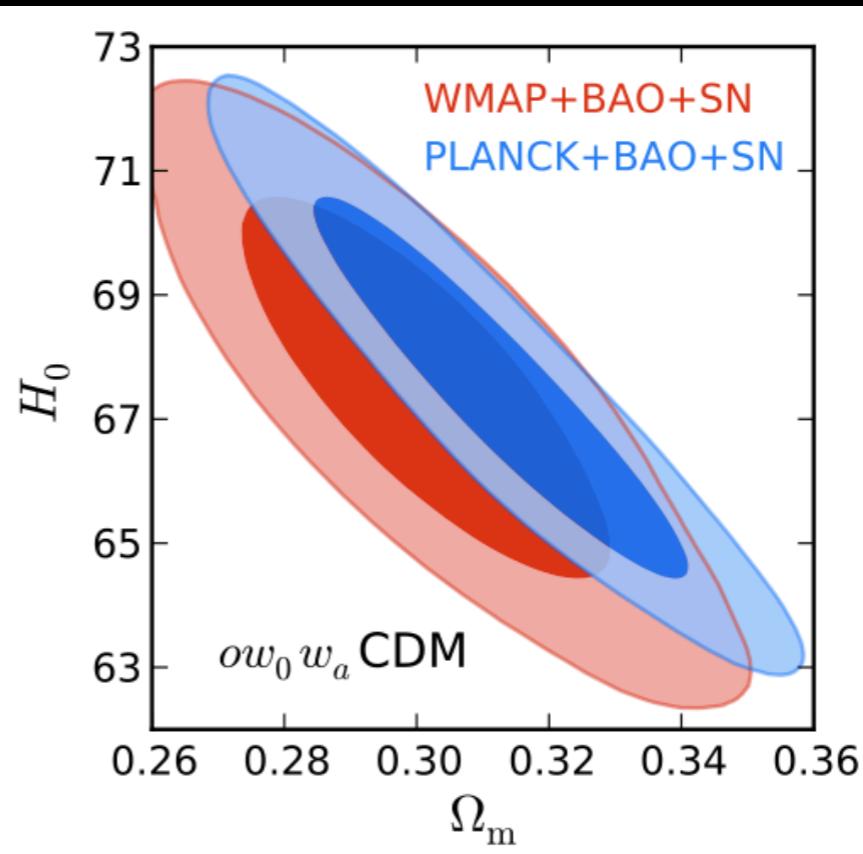
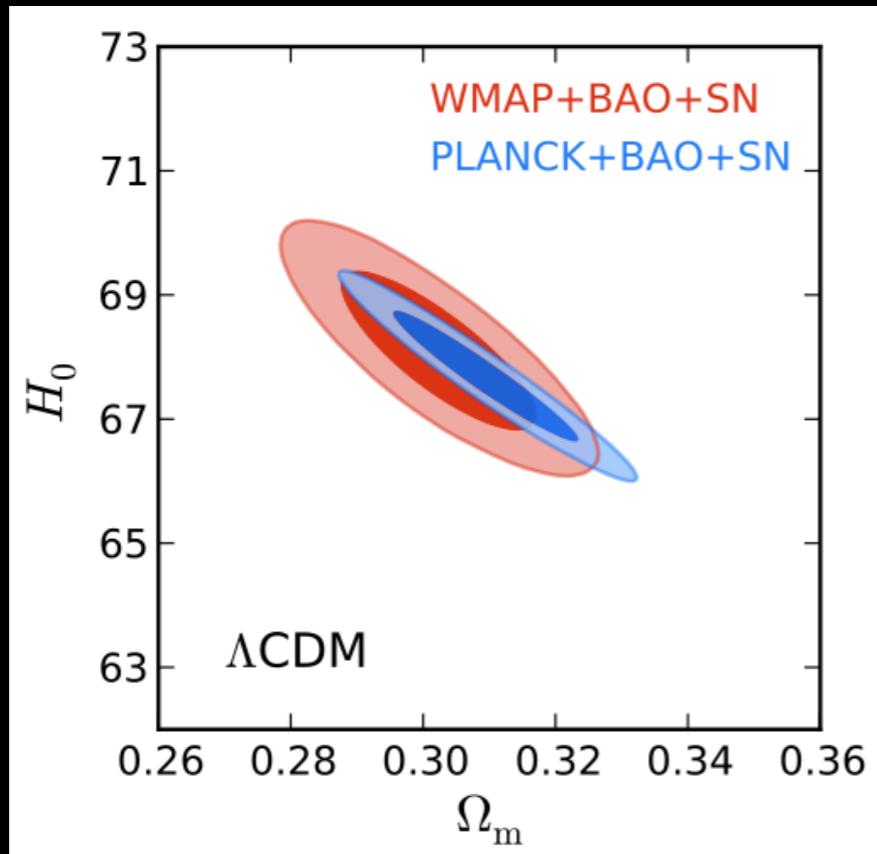
Galaxy BAO Cosmological Implications



- $\Omega_k = 0.0002 \pm 0.0033$
- $w = -1.04 \pm 0.07$
- $w_0 = -0.94 \pm 0.17$
- $w_a = -0.37 \pm 0.60$
- $\Omega_k = 0.0027 \pm 0.0042$
- $w_0 = -0.87 \pm 0.19$
- $w_a = -0.73 \pm 0.80$

[Planck + BAO + Union2 SN]

Galaxy BAO Cosmological Implications



- $\Omega_m = 0.309 \pm 0.008$
- $H_0 = 67.7 \pm 0.6$

[Planck + BAO + Union2 SN]

- $\Omega_m = 0.312 \pm 0.016$
- $H_0 = 67.5 \pm 1.7$; works in even more general DE models thanks to BAO absolute calibration and SN coverage to $z=0$ [BOSS + SNLS in prep]

Summary

- We've been through the gory detail of how we constrain cosmological parameters from LSS surveys, including all the real world complications
- We focused on the BAO standard ruler to constrain the expansion history of the universe and its cosmological implications
- Next time -- Redshift Space Distortions

EXTRAS

Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- There are n_d real galaxies (“data”) in your survey, and you generate n_r random galaxies (“randoms”). You can make n_r as big as you want so it does not contribute to your error in measuring ξ . I typically use $n_r = 50n_d$.
- We estimate ξ by counting up the number of unique data-data (DD), data-random (DR) and random-random (RR) pairs.

Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- The total number of unique DD, DR, and RR pairs in the survey is $n_d(n_d - 1)/2$, $n_d(n_r - 1)/2$, $n_d n_r$, and $n_r(n_r - 1)/2$ respectively.

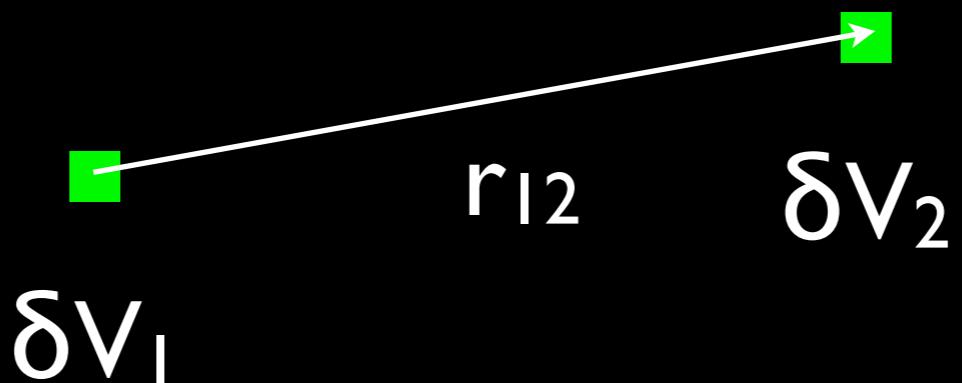
Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- Remember, n_d is a random variable as well -- if you surveyed a different region of the sky with the exact same survey configuration, we would have seen a different number of galaxies!
- Our best guess at $\langle n_d \rangle$ is just the observed n_d . The error caused by not knowing the true value of $\langle n_d \rangle$ is the “integral constraint.” More later.

Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- We will use what we learned about the Poisson process and the definition of ξ from Lecture 1:

$$P(1,2) = n_d^2 (1 + \xi(r_{12})) \delta V_1 \delta V_2$$



Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- The DD, DR, and RR pair counts of all pairs with separations $r_{12} - dr/2 < r < r_{12} + dr/2$ are all random variables:

$$DD = \langle DD \rangle (1 + \alpha)$$

$$DR = \langle DR \rangle (1 + \beta)$$

Here $\langle \alpha \rangle = \langle \beta \rangle = 0$. We ignore variation in RR because it can be made arbitrarily small with larger n_r .

Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- Landy-Szalay work in the limit of weak correlations ($\xi \ll 1$). This is the correct limit for in the BAO region, where $\xi \sim 0.005$.
- We start by setting $\xi = 0$, so the galaxy distribution is a pure Poisson process.
- Divide the survey region up into K tiny cells hosting either 0 or 1 galaxies. The number of galaxies in each cell is V_i .

Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- $\langle DD \rangle = \left\langle \sum_{i < j} V_i V_j \Theta^{r_{12}}_{ij} \right\rangle = \sum_{i < j} \langle V_i V_j \rangle \Theta^{r_{12}}_{ij}$

Sum over all unique pairs of cells

$\Theta^{r_{12}}_{ij} = 1$ if cells i and j are separated by $r_{12} \pm dr/2$, 0 otherwise

For Poisson process, $\langle V_i V_j \rangle = n_d(n_d - 1)/[K(K-1)]$

$\sum_{i < j} \Theta^{r_{12}}_{ij} = [K(K-1)/2] G_p(r_{12})$. $G_p(r_{12})$ is the geometric factor specifying the fraction of pairs of cells in the survey within the separation bin r_{12} .

- $\langle DD \rangle = n_d(n_d - 1)/2 * G_p(r_{12})$.

Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- $\langle DD \rangle = n_d(n_d - 1)/2 * G_P(r_{12})$.
- Finally, average over all possible values of n_d if $\langle n_d \rangle = N$:
- $\langle DD \rangle = G_P(r_{12}) \sum_{nd} n_d(n_d - 1)/2 P(n_d) = G_P(r_{12}) N^2/2$



Poisson distribution

Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- Now let's recompute $\langle DD \rangle$ assuming $\xi \neq 0$. The calculation is almost the same, except:

$$\langle v_i v_j \rangle = n_d(n_d - 1)/[K(K-1)] [1 + \xi]$$

- We get the normalization by forcing the total number of pairs to be $n_d(n_d - 1)/2$ again
- $$\langle DD \rangle = n_d(n_d - 1)/2 * G_P(r_{12}) * [1 + \xi(r_{12})]/[1 + \int \xi dV_s].$$
- This is called the integral constraint. As long as the survey is large, $\int \xi dV_s \approx 0$ and we can ignore it.

Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- Finally, $\langle DR \rangle = n_d n_r G_P(r_{12})$.
- Therefore, we can combine our pair counts to get an estimate of ξ :

$$\xi = DD/DR [2n_r/(n_d - 1)]$$

- By computing $\langle DD * DD \rangle$, $\langle DR * DR \rangle$, and $\langle DD * DR \rangle$ in the Poisson limit (neglecting correlations) Landy-Szalay showed that the Landy-Szalay estimator is not biased and has a smaller variance than the simple one above.

Computing ξ : Landy-Szalay estimator [ApJ 412, 64 (1993)]

- The Landy-Szalay estimator is

$$\xi = [DD f_{dd} - 2*DR f_{dr} + RR]/RR$$

where f_{dd} , f_{dr} match the counts of DD and DR to RR:

$$f_{dd} = n_r(n_r - 1)/n_d(n_d - 1); f_{dr} = n_r(n_r - 1)/2n_d n_r$$