An Implementation of the two-stage Stochastic Unit Commitment Problem

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Introduction

A Unit Commitment problem is an optimization model that aims to develop a production schedule for a given time period to minimize the operating costs for energy generating units while satisfying the demand and reserve constraints. This project report presents a two-stage stochastic optimization model formulation for a Thermal Stochastic Unit Commitment (SUC) problem for a 24-hour time horizon. A thermal energy based generation unit is usually supplemented by a unit harnessing wind for energy generation. While the hourly load requirements for a given period can be estimated based on historical data available, the wind power generation is usually uncertain and cannot be accurately determined in advance. The model developed in this report attempts to capture this uncertainty by assuming that the wind power generation for the first twelve hour period is deterministic, while the wind power generation for the nlast twelve periods is stochastic. A Gaussian Copula method described in [] is used to generate a bivariate distribution between load and wind in order to generate random wind energy amounts. Since the model involves binary and integer constrained decision variables in the second stage, algorithms like the L-shaped method cannot be used since the Stochastic Program becomes non-convex. We therefore use Progressive Hedging, fully described in [], as the solution algorithm for our problem. The advantage of the PH algorithm is that it is a scenario-based algorithm. For any scenario, the thermal unit demands are deterministic and can be solved as a Mixed Integer Linear Program (MILP) or even a Mixed Integer Quadratic Program(MIQP) if necessary.

Model

Parameters

a_j, b_j, c_j - Coefficients of the quadraticd production cost function of unit j
$cc_j, hc_j, t_j^{\text{cold}}$ - Coefficients of the startup cost function of unit j.
C_j - Shutdown cost of unit j.
DT_j - Minimum down time of unit j.
G_j - Number of periods unit j must be initially online due to its minimum up time constraint.
K_t^t - cost of the interval t of the stairwise startup cost function of unit j.
L_j - Number of periods unit j must be initially offline due to its minumum down time constraint.
ND_j - Number of intervals of the stairwise startup cost function of unit j.
$\overline{P}_j$ - Capacity of unit j.
$\underline{P}_j$ - Minimum pwer output of unit j.
RD_j - Ramp-down limit of unit j.
RU_j - Ramp-up limit of unit j.
$S_j(0)$ - Number of periods unit j has been offline prior to the first period of the time span (end of period 0).
SD_j - Shutdown ramp limit of unit j.
SU_j - Startup ramp limit of unit j.
T- Number of periods of the time span.
U_j^0 - Number of periods unit j has been online prior to the first period of the time span (end of period 0).
UT_j - Minimum up time of unit j.
$V_j(0)$ - Initial commitment state of unit j (1 if it is online, 0 otherwise).

Random Parameters

R_k^ω - Spinning reserve requirement in period k.
D_k^ω - Load demand in period k.

Deterministic Decision Variables

$c_j^d(k)$ - Shutdown cost of unit j in period k.
$c_j^u(k)$ - Startup cost of unit j in period k.
$p_j(k)$ - Power output of unit j in period k.
$\overline{p}_j(k)$ - Maximum available power output of unit j in period k.
$v_j(k)$ - Binary variable that is equal to 1 if unit j is online in period k and 0 otherwise.

Stochastic Decision Variables

$c_j^{d,\omega}(k)$ - Shutdown cost of unit j in period k.
$c_j^{u,\omega}(k)$ - Startup cost of unit j in period k.
$p_j^\omega(k)$ - Power output of unit j in period k.
$\overline{p}_j^\omega(k)$ - Maximum available power output of unit j in period k.
$v_j^\omega(k)$ - Binary variable that is equal to 1 if unit j is online in period k and 0 otherwise.

Index Sets

I - Set of indices of the generating units.
K - Set of indices of the time periods.
Kstoch - Set of periods in which the variables are stochastic (13-24).
Kdet - Set of periods in which the variables are deterministic (1-12).

The stochastic model formulated for the Unit commitment problem is derived from/based on the deterministic Unit commitment model described in [1] and is elaborated below. The optimization problem is to minimize the total expected operating cost, which consists of three components: 1. Production cost, 2. Startup cost and 3. Shutdown cost. The decision variables for of time periods in $Kdet$ consisting of $\{1,2,\dots,S\}$ are taken to be first stage, while those in $Kstoch$ consisting of $\{S+1, S+2,\dots,T\}$ are modeled as second stage decision variables and are therefore, scenario-dependent.

$$Minimize \sum_{k\in Kdet}\sum_{j\in J} c_j^p(k) + c_j^u(k) + c_j^d(k) + \frac{1}{|\Omega|}\sum_{k\in Kstoch}\sum_{j\in J} c_j^{p,\omega}(k) + c_j^{u,\omega}(k) + c_j^{d,\omega}(k) \tag{1}$$

Subject to

$$\sum_{j\epsilon J} p_j(k) = D(k), \qquad \forall k \in Kdet \tag{2.1}$$
$$\sum_{j\epsilon J} \bar{p}_j(k) \geq D(k) + R(k), \qquad \forall k \in Kdet \tag{3.1}$$
$$p_j(k) \in \Pi_j(k), \qquad \forall j \in J,\ \forall k \in Kdet \tag{4.1}$$

$$\sum_{j\epsilon J} p_j^{\omega}(k) = D^{\omega}(k), \qquad \forall k \in Kstoch,\ \forall \omega \in \Omega \tag{2.2}$$
$$\sum_{j\epsilon J} \bar{p}_j^{\omega}(k) \geq D^{\omega}(k) + R^{\omega}(k), \qquad \forall k \in Kstoch, \forall \omega \in \Omega \tag{3.2}$$
$$p_j^{\omega}(k) \in \Pi_j^{\omega}(k), \qquad \forall j \in J,\ \forall k \in Kstoch, ,\forall \omega \in \Omega \tag{4.2}$$

Objective function

The objective function as mentioned above includes production cost, startup cost and shutdown cost. The production cost function (5.1 and 5.2) in the model is assumed to be a quadratic function of thermal power being generated. The startup cost function (6.1 and 6.2) , which is usually represented as an exponential function of the offline time prior to being started, is simplified as a stairwise function while the shutdown cost (7.1 and 7.2) is modeled as constant.

$$c_j^p(k) = a_j v_j(k) + b_j p_j(k) + c_j p_j^2(k), \qquad \forall j \in J, \forall k \in Kdet \tag{5.1}$$

$$c_j^{p,\omega}(k) = a_j v_j^{\omega}(k) + b_j p_j^{\omega}(k) + c_j {p_j^{\omega}}^2(k), \qquad \forall j \in J, \forall k \in Kstoch, \forall \omega \in \Omega \tag{5.2}$$

$$c_j^u(k) \geq K_j^t\left[v_j(k) - \sum_{n=1}^{t} v_j(k-n)\right], \qquad \forall j \in J, \forall k \in Kdet, \forall t = 1,\dots,ND_j$$
$$c_j^u(k) \geq 0, \qquad \forall j \in J, \forall k \in Kdet \tag{6.1}$$

$$c_j^{u,\omega}(k) \geq K_j^t\left[v_j^{\omega}(k) - \sum_{n=1}^{t} v_j^{\omega}(k-n)\right], \qquad \forall j \in J, \forall k \in Kstoch, \forall t = 1,\dots,ND_j, \forall \omega \in \Omega$$
$$c_j^{u,\omega}(k) \geq 0\ , \qquad \forall j \in J, \forall k \in Kstoch, \forall \omega \in \Omega \tag{6.2}$$

$$c_j^d(k) \geq C_j\left[v_j(k-1) - v_j(k)\right], \qquad \forall j \in J, \forall k \in Kdet$$
$$c_j^d(k) \geq 0\ , \qquad \forall j \in J, \forall k \in Kdet \tag{7.1}$$

$$c_j^{d,\omega}(k) \geq C_j\left[v_j(S) - v_j^{\omega}(S+1)\right], \qquad \forall j \in J, k = S+1, \forall \omega \in \Omega$$
$$c_j^{d,\omega}(k) \geq C_j\left[v_j^{\omega}(k-1) - v_j^{\omega}(k)\right], \qquad \forall j \in J, \forall k \in Kstoch - \{S+1\}, \forall \omega \in \Omega$$
$$c_j^{d,\omega}(k) \geq 0, \qquad \forall j \in J, \forall k \in Kstoch, \forall \omega \in \Omega \tag{7.2}$$

Constraints

There are three different types of constraints that have been incorporated in the model

1. *Generation Limit Constraints*
   These constraints impose restriction on the minimum and maximum power that can be produced by generator *j* in any given time period, *k*.

$$\underline{P}_j v_j(k) \le p_j(k) \le \overline{p}_j(k), \qquad \forall j \in J,\ \forall k \in Kdet$$
$$0 \le \overline{p}_j(k) \le \overline{P}_j v_j(k), \qquad \forall j \in J,\ \forall k \in Kdet \qquad (8.1)$$
$$\underline{P}_j(k) v_j^{\omega}(k) \le p_j^{\omega}(k) \le \bar{p}_j^{\omega}(k), \qquad \forall j \in J,\ \forall k \in Kstoch, \forall \omega \in \Omega$$
$$0 \le \bar{p}_j^{\omega}(k) \le \overline{P}_j v_j^{\omega}(k), \qquad \forall j \in J,\ \forall k \in Kstoch, \forall \omega \in \Omega \qquad (8.2)$$

2. *Ramp up, Start up and Ramp down rate constraints*
   The power generated is also constrained by the ramp up and startup rates of the generator (eq. 9.1, 9.2, 10.1, 10.2, 11.1, 11.2)

$$\bar{p}_j(k) \le p_j(k-1) + RU_j v_j(k-1) + SU_j[v_j(k) - v_j(k-1)] + \overline{P}_j[1 - v_j(k)],$$
$$\forall j \in J,\ \forall k \in Kdet \qquad (9.1)$$

$$\bar{p}_j^{\omega}(k) \le p_j(S) + RU_j v_j(S) + SU_j[v_j^{\omega}(S+1) - v_j(S)] + \overline{P}_j[1 - v_j^{\omega}(S+1)],$$
$$\forall j \in J,\ k = S+1, \forall \omega \in \Omega$$
$$\bar{p}_j^{\omega}(k) \le p_j^{\omega}(k-1) + RU_j v_j^{\omega}(k-1) + SU_j[v_j^{\omega}(k) - v_j^{\omega}(k-1)] + \overline{P}_j[1 - v_j^{\omega}(k)],$$
$$\forall j \in J,\ \forall k \in Kstoch - \{S+1\}, \forall \omega \in \Omega \qquad (9.2)$$

$$\bar{p}_j(k) \le \overline{P}_j v_j(k+1) + SD_j[v_j(k) - v_j(k+1)], \qquad \forall j \in J,\ \forall k \in Kdet\text{-}\{\mathrm{S}\}$$
$$\bar{p}_j(k) \le \overline{P}_j v_j(k+1) + SD_j\left[v_j(S) - v_j^{\omega}(S+1)\right], \qquad \forall j \in J,\ k = \mathrm{S} \qquad (10.1)$$
$$\bar{p}_j^{\omega}(k) \le \overline{P}_j v_j^{\omega}(k+1) + SD_j[v_j^{\omega}(k) - v_j^{\omega}(k+1)],$$
$$\forall j \in J,\ \forall k \in Kstoch - \{T\}, \forall \omega \in \Omega \qquad (10.2)$$

$$p_j(k-1) - p_j(k) \le RD_j v_j(k) + SD_j[v_j(k-1) - v_j(k)] + \overline{P}_j[1 - v_j(k-1)]$$
$$\forall j \in J,\ \forall k \in Kdet \qquad (11.1)$$
$$p_j^{\omega}(k-1) - p_j^{\omega}(k) \le RD_j v_j^{\omega}(k) + SD_j[v_j^{\omega}(k-1) - v_j^{\omega}(k)] + \overline{P}_j[1 - v_j^{\omega}(k-1)]$$
$$\forall j \in J,\ \forall k \in Kstoch, \forall \omega \in \Omega \qquad (11.2)$$

3. *Minimum up and down time constraints*
   A thermal generator, if switched on, must be kept on for a minimum up time period. These are modeled in the following constraints. (12) constrains the generator to be switched on for G_j number of initial periods, where $G_j = Min\{T, \left[UT_{j,} U_j^0\right] V_j(0)\}$, whereas (13.1), (13.2) and (13.3) specifies the generator j to be switched on for UT_j time periods if it was switched on, at or before time period $T - UT_j + 1$. (14) restricts the generators to be in on state till time period T if they were switched in the last $UT_j - 1$ periods.

$$\sum_{k=1}^{\min(G_j,S)}\left[1-v_j(k)\right]+\sum_{k=S+1}^{G_j}\left[1-v_j^{\omega}(k)\right]=0, \qquad \forall j\in J \tag{12}$$

$$\sum_{n=k}^{\min(k+UT_j-1,S)} v_j(n)+\sum_{n=S+1}^{k+UT_j-1} v_j^{\omega}(n)\geq \mathrm{UT}_j\left[v_j(k)-v_j(k-1)\right], \quad \forall k=G_j+1,\dots,\min(S,T-UT_j+1) \tag{13.1}$$

$$\sum_{n=S+1}^{\mathrm{k}+UT_j-1} v_j^{\omega}(n)\geq \mathrm{UT}_j\left[v_j^{\omega}(S+1)-v_j(S)\right], \quad k=S+1 \ and \ G_j+1\leq k\leq T-UT_j+1 \tag{13.2}$$

$$\sum_{n=k}^{k+UT_j-1} v_j^{\omega}(n)\geq \mathrm{UT}_j[v_j^{\omega}(k)-v_j^{\omega}(k-1)], \quad \forall k=\max(G_j+1,S+2),\dots,T-UT_j+1 \tag{13.3}$$

$$\sum_{n=k}^{\min(T,S)}\left\{v_j(n)-\left[v_j(k)-v_j(k-1)\right]\right\}+\sum_{n=S+1}^{T}\left\{v_j^{\omega}(n)-\left[v_j(k)-v_j(k-1)\right]\right\}\geq 0 \quad \forall k=T-UT_j+2,\dots,\min(S,T) \tag{14.1}$$

$$\sum_{n=S+1}^{T}\left\{v_j^{\omega}(n)-\left[v_j^{\omega}(S+1)-v_j(S)\right]\right\}\geq 0, \ k=S+1 \ and \ T-UT_j+1\leq k\leq T \tag{14.2}$$

$$\sum_{n=S+1}^{T}\left\{v_j^{\omega}(n)-\left[v_j^{\omega}(k)-v_j^{\omega}(k-1)\right]\right\}\geq 0, \quad \forall k=\max(T-UT_j+2,S+2),\dots,T-UT_j+1 \tag{14.3}$$

Similar to the minimum up time constraints, the minimum down time constraints are also constructed below. Constraint (15) is analogous to (12), while sets of constraints (16) and (17) are analogous to (13) and (14).

$$\sum_{k=1}^{\min(L_j,S)} v_j(k)+\sum_{k=S+1}^{L_j} v_j^{\omega}(k)=0 \tag{15}$$

$$\sum_{n=k}^{\min(k+DT_j-1,S)}[1-v_j(n)]+\sum_{n=S+1}^{k+DT_j-1}[1-v_j^{\omega}(n)]\geq \mathrm{DT}_j\left[v_j(k-1)-v_j(k)\right], \quad \forall k=L_j+1,\dots,\min(S,T-DT_j+1) \tag{16.1}$$

$$\sum_{n=S+1}^{\mathrm{k}+DT_j-1}\left[1-v_j^{\omega}(n)\right]\geq \mathrm{DT}_j\left[v_j(S)-v_j^{\omega}(S+1)\right], \quad k=S+1 \ and \ L_j+1\leq k\leq T-UT_j+1 \tag{16.2}$$

$$\sum_{n=k}^{k+DT_j-1}[1-v_j^{\omega}(n)]\geq \mathrm{DT}_j[v_j^{\omega}(k-1)-v_j^{\omega}(k)], \quad \forall k=\max(L_j+1,S+2),\dots,T-DT_j+1 \tag{16.3}$$

$$\sum_{n=k}^{\min(T,S)}\left\{1-v_j(n)-\left[v_j(k-1)-v_j(k)\right]\right\}+\sum_{n=S+1}^{T}\left\{v_j^{\omega}(n)-\left[v_j(k)-v_j(k-1)\right]\right\}\geq 0 \quad \forall k=T-DT_j+2,\dots,\min(S,T) \tag{17.1}$$

$$\sum_{n=S+1}^{T}\left\{1-v_j^{\omega}(n)-\left[v_j(S)-v_j^{\omega}(S+1)\right]\right\}\geq 0, \quad k=S+1 \ and \ T-DT_j+1\leq k\leq T \tag{17.2}$$

$$\sum_{n=S+1}^{T}\left\{1-v_j^{\omega}(n)-\left[v_j^{\omega}(k-1)-v_j^{\omega}(k)\right]\right\}\geq 0, \quad \forall k=\max(T-DT_j+2,S+2),\dots,T-DT_j+1 \tag{17.3}$$

Results

Our implementation of the SUC problem is too large for us too see convergence in a reasonable amount of time. Therefore, we settled for using an upper bound obtained after 300 iterations of the PH algorithm operating on the SUC problem. This particular instance only had two scenarios and six periods. The upper bound was obtained by simply centering the weights, setting rho to 0, solving the modified problem for each scenario, then evaluating the cost function for the solution provided for each scenario. The upper bound is simply the average of these objective values since the distribution is uniform. The results of this experiment are summarized in the table below. The lower bound along with a solution for each scenario is provided

Rho	Scenario 1 Solution	Scenario 2 Solution	Upper Bound
10	94261	97364	106807
100	94625	94603	109001
1000	95211	98975	108934

Though we were not able to know how well the PH algorithm worked on the SUC problem, the nurse scheduling problem from homework 6 provided us with the opportunity to see how the algorithm performed with different values of rho since we know the optimal solution. In this experiment, the tolerance for convergence is fixed to 0.1. The results are summarized in the table below. Any value of rho above 200 yielded a quick suboptimal solution, and any value of rho below 100 yielded either a slow suboptimal solution or no convergence in a fixed amount of time.

Rho	Number of Iterations	Optimality	Solution
200	10	Suboptimal	20766
150	18	Suboptimal	20777
100	23	Optimal	20744

Justification of Techniques to Solve/Generate Data for Model

PROGRESSIVE HEDGING ALGORITHM

Since the two-stage stochastic model that we formulated has integer restrictions on decision variables in both stages, a standard L-shaped algorithm cannot be used to solve the stochastic model. We, therefore, use Progressive Hedging (PH), described in [2], for our problem.

A two stage stochastic model can be written in the following general extensive form:

$$Minimize \quad c.x + \sum_{s \in S} \Pr(S)\,(f_s.y_s)$$

$$s.t. \quad (x, y_s) \in Q_s, \quad \forall s \in S$$

In the formulation above, x are the first stage decision variables, while y_s are the second decision variables (dependent on the scenarios). Progressive Hedging solves this two stage stochastic model by considering each scenario separately i.e. for each scenario, it finds optimal x_s and y_s values. The algorithm then incorporates non-anticipativity $x_s = x$ by including Lagrangian multipliers w_s

Algorithm (Taken from [2])

1. $k \coloneqq 0$
2. For all $s \in S, x_s^{(k)} \coloneqq$ argmin $(c.x + f_s.y_s)$: $(x, y_s) \in Q_s$
3. $\bar{x}^{(k)} \coloneqq \sum_{s \in S} \Pr(s)\, x_s^{(k)}$
4. For all $s \in S, w_s^{(k)} \coloneqq \rho(x_s^{(k)} - \bar{x}^{(k)})$
5. $k \coloneqq k + 1$
6. For all $s \epsilon S$,

   $$x_s^{(k)} \text{:=argmin} \left(c.x + w_s^{(k-1)}x + \rho/2\, \left\|x - \bar{x}^{(k-1)}\right\|^2 + f_s.y_s\right) : (x, y_s) \in Q_s$$

7. $\bar{x}^{(k)} \coloneqq \sum_{s \in S} \Pr(s)\, x_s^{(k)}$
8. For all $s \in S$, $w_s^{(k)} \coloneqq w_s^{(k-1)} + \rho(x_s^{(k)} - \bar{x}^{(k)})$
9. $g^{(k)} \coloneqq \sum_{s \in S} \Pr(s) \left\| x_s^{(k)} - \bar{x}^{(k)} \right\|$
10. If $g^{(k)} < \varepsilon$, then go to Step 5. Otherwise, terminate.

Once the solutions are generated, we check for their quality, by generating upper and lower bounds for the objective function value. The upper bound is evaluated as U= $c\bar{x}^{(t)} + \sum_{s \in S} \Pr(s)\, f_s.y_s^{(t)}$, where t is the iteration number for which the algorithm converged while for the lower bounds the following procedure was used

1. First the weights was centered using, $\overline{w}^s = w_s^{(t)} - \sum_{s \in S} \Pr(s)\, w_s^{(t)}$
2. Lower bound, $L = \frac{1}{|S|} \sum_{s \in S}((c - \overline{w}^s)\, \bar{x}^{(t)} + f_s.y_s^{(t)})$

Several modifications, based on the discussions made in [2],were introduced in the algorithm to make the algorithm more computationally efficient. These modifications are been detailed as under:

1. *ρ values for fast convergence*
   Effective ρ values are especially desired in the algorithm as they determine to a great extent (see step 8) how quickly a "good value" for the multiplier, w^* is obtained. In our algorithm, based on [2], the first stage decision were set to be proportional to their corresponding cost coefficients. In particular, for the integer variables, the ρ value was

$$\rho(i) = \frac{c(i)}{(x^{max} - x^{min} + 1)}$$

   where $x^{max} = max_{s \in S} x_s^{(0)}$ and $x^{min} = min_{s \in S} x_s^{(0)}$.

   For the continuous variables, the

$$\rho(i) := \frac{c(i)}{\max\left(\left(\sum_{s \in S} \Pr(s) \left| x_s^{(0)} - \bar{x}^{(0)} \right|, 1\right)\right)}$$

   In both the cases, $c(i)$ was divided by a factor so that the desired w^* value is achieved from below. This prevent the possibility of oscillations or thrashing, which may occur in the algorithm if ρ values are changed very aggressively.

2. *Cycle detection*
   As discussed in [2], for cycle detection, the weights $w_s(i)$ are tracked. If the same $w_s(i)$ is repeated over and over, then there might be a possibility for cycling. The $x_s(i)$ and $\bar{x}(i)$ are additionally checked to ascertain cyclic behavior.

SIMULATION OF WIND POWER GENERATION DATA

The Gaussian Copula technique for modeling a bivariate distribution between load and wind power generation has been described in detail in [3], where a truncated lognormal distribution was used to model load and a truncated bounded Johnson-system distribution was used to model wind power. In [3], the parameters for the two distributions and the correlation coefficient, σ_ζ were calculated based on Nov. 2009 to Oct. 2010 data available from the Electric Reliability Council of Texas (ERCOT).

For our project, we made use of the parameters calculated for these two distributions and σ_z, which is the correlation coefficient for the underlying zero mean and unit variance normal random variables. We then modified the algorithm provided in [3] to generate wind power generation data based on the data provided in [1]. We assumed the data provided in [1] to be given historical data for load and appropriately scaled it (by a factor of 40) so that the data fits well for the model in [3]. The scaling can be justified on the basis that the Thermal Energy Unit for which we are optimizing satisfies only a fraction of the total Texas load demand.

Algorithm

Input:

1. 24-Hour load vector, taken from data in [1]
2. Marginal cdfs F_L (truncated lognormal distribution) and F_W(truncated bounded Johnson-system distribution) for load and wind respectively, taken from data in [3]
3. Correlation coefficient, σ_z between underlying zero mean and unit variance normal r.v.s, taken from data in [3]

Output: 24- Hour Wind Power vector, ζ

For each load, l in the 24-hour load vector,

1. Calculate $F_L\,(l)$ and compute $W_1 = \Phi^{-1}(F_L\,(l)\,)$
2. Generate a normal random variable, $W_2 \sim N(0,1)$
3. Let $Z_2 = W_1 + \sqrt{1 - \sigma_z^2}\, W_2$
4. Compute $\zeta_l = F_W^{-1}(\Phi(Z_2))$

References

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