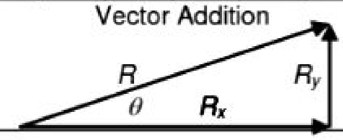
Physics Formulas and Constants

Metric Prefixes	Astronomy	Subatomic Parts	Conversions	Constants	Common Angles			
giga, G, 10^9	$m_e = 5.97 \times 10^{24}$ kg	$m_p = 1.67 \times 10^{-27}$ kg	$\rho_{kg/m3} = \rho_{g/cm3} \times 10^3$	$g = 9.8$ m/s^2	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
mega, M, 10^6	$r_e = 6.38 \times 10^6$ m	$m_n = 1.67 \times 10^{-27}$ kg	$P_{Pa} = P_{atm} \times 10^5$	$G = 6.67 \times 10^{-11}$ N•m^2/kg^2	0^o	0	1	0
kilo, k, 10^3	$m_m = 7.35 \times 10^{22}$ kg	$m_e = 9.11 \times 10^{-31}$ kg	$T_K = T_{oC} + 273$	$R = 8.31$ J/(mol•K)	30^o	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
centi, c, 10^{-2}	$r_m = 1.74 \times 10^6$ m	$q_p = 1.60 \times 10^{-19}$ C	$E_J = E_{eV} \times 1.60 \times 10^{-19}$	$k = 9.0 \times 10^9$ N•m^2/C^2	37^o	3/5	4/5	3/4
milli, m, 10^{-3}	$r_{e\text{-}m} = 3.84 \times 10^8$ m	$q_e = -1.60 \times 10^{-19}$ C	$n_{mole} = N/6.02 \times 10^{23}$	$\epsilon_o = 8.85 \times 10^{-12}$ C/N•m^2	45^o	$\sqrt{2}/2$	$\sqrt{2}/2$	1
micro, μ, 10^{-6}	$m_s = 1.99 \times 10^{30}$ kg	**Water**	$m_{kg} = m_u \times 1.66 \times 10^{-27}$	$k' = 2 \times 10^{-7}$ T•m/A	53^o	4/5	3/5	4/3
nano, n, 10^{-9}	$r_s = 6.96 \times 10^8$ m	$\rho_{water} = 1000$ kg/m^3	**Percent**	$\mu_o = 4\pi \times 10^{-7}$ T•m/A	60^o	$\sqrt{3}/2$	1/2	$\sqrt{3}$
pico, p, 10^{-12}	$r_{e\text{-}s} = 1.50 \times 10^{11}$ m	melting point = 0^oC	$\% \Delta = 100\lvert mean\text{-}true\rvert/true$	$c = 3 \times 10^8$ m/s	90^o	1	0	∞
		boiling point = 100^oC	$\% \leftrightarrow = \dfrac{100\Sigma\lvert trial - mean\rvert}{N(mean)}$	$h = 6.63 \times 10^{-34}$ J•s				
		$c = 4180$ J/kg•K		$B_H = 13.6$ eV				

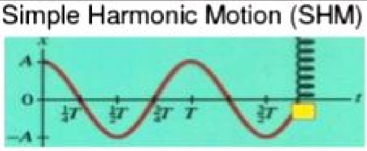
Displacement d in m	$d = x - x_o$
Average Velocity v_{av} in m/s	$v_{av} = \frac{1}{2}(v_o + v_t) = d/t$
Acceleration a in m/s^2	$a = (v_t - v_o)/t$
Uniform Circular Motion	$v = 2\pi r/T$
(circumference = $2\pi r$, T = period)	$a_c = v^2/r$
Kinematic Equations	$d = v_o t + \frac{1}{2}at^2$
	$d = \frac{1}{2}(v_t + v_o)t$
	$v_t = v_o + at$
	$v_t^2 = v_o^2 + 2ad$

	d	v_o	v_t	a	t
↓↑	d_y	v_{yo}	v_{yt}	$-g$	t
⇆	d_x	v_x			

Vector Addition	$R_x = R\cos\theta$
	$R_y = R\sin\theta$
	$R = (R_x^2 + R_y^2)^{1/2}$
	$\tan\theta = R_y/R_x$



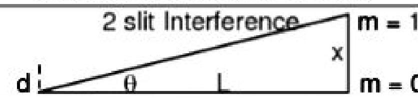
Force F in N	$F_{\parallel} = ma$
$\Sigma F_{\parallel} = ma$	$F_g = mg = GMm/r^2$
$\Sigma F_{\perp} = 0$	$F_f \le \mu F_n$
	$F_s = kx$
	$F_c = mv^2/r$
Torque τ in m•N	$\tau = r_{\perp}F$
Center of Mass cm in m	$cm = \Sigma(r_i m_i)/\Sigma(m_i)$
Work W in J	$W = F_{\parallel}d$
Power P in W = J/s	$P = W/t = Fv_{av}$
Kinetic Energy K in J	$K = \frac{1}{2}mv^2$
	$K_r = \frac{1}{2}\beta mv^2$
Potential Energy U in J	$U_g = mgh = -GMm/r$
	$U_s = \frac{1}{2}kx^2$
Conservation of Mechanical Energy	$K + U \pm W = K' + U'$
Momentum p in kg•m/s	$p = mv$
Impulse J in N•s = kg•m/s	$J = F\Delta t = m\Delta v = \Delta p$
$K \leftrightarrows p$	$K = p^2/2m$
Conservation of Momentum	$p_A + p_B = p_A' + p_B'$
Angular Momentum L in kg•m^2/s	$L = r\beta mv$
Simple Harmonic Motion (SHM)	$T_s = 2\pi(m/k)^{1/2}$
	$v_o = 2\pi A/T = A(k/m)^{1/2}$
	$a_A = v_o^2/A = A(k/m)$
	$K_o = \frac{1}{2}mv^2$
	$U_A = \frac{1}{2}kA^2$



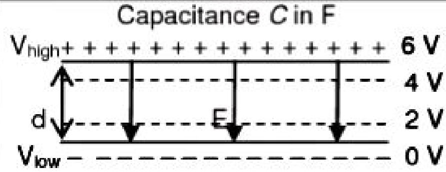
Pendulum	$T_p = 2\pi(L/g)^{1/2}$
Mass-Energy	$E = mc^2$
Nuclear Reaction	$m_{reactants} = m_{products} \pm m_{BE}$
Particle Wavelength	$\lambda_{particle} = h/p$
Photon in J or eV	$E_{photon} = hf = mc^2$
	$E_{photon} = 1240\ eV{\cdot}nm/\lambda_{nm}$
	$p_{photon} = mc = h/\lambda = E/c$
Electron Energy Levels in J or eV	$E_{n\text{-}electron} = -B/n^2$
Photoelectrons in J or eV	$K_{electron} = E_{photon} - \phi_{metal}$
Density ρ in kg/m^3	$\rho = m/V$
Pressure P in Pa = N/m^2	$P = F/A$
	$P = \rho gh$
Buoyancy F_b in N	$F_b = \rho_f V_o g$
Volume Flow Rate in m^3/s	$V/t = A_1 v_1 = A_2 v_2$

Bernoulli's Equation in Pa	$P + \rho gy + \frac{1}{2}\rho v^2 = C$
Specific Gravity	$s.g. = \rho_{object}/\rho_{fluid}$
Rate of Heat Transfer in W	$H = kA(T_H - T_L)/L$
Thermal Expansion in m	$\Delta L = \alpha L_o \Delta T$
Molecular Kinetic Energy in J	$K = {}^3/_2 RT = \frac{1}{2}Mv^2$
Ideal Gas Law	$PV = nRT$
Calorimetry in J	$Q = mc\Delta T$
Heat Engines in J	$\Delta U = Q_{in} + W_{in}$
	$W_{in} = -P\Delta V$
	$\Delta U = {}^3/_2\Delta(PV) = {}^3/_2 nR\Delta T$
	$e_c = (T_H - T_L)/T_H$
	$e = \lvert W_{in} - W_{out}\rvert/\Sigma Q_{in}$

Mechanical Waves	$T = 1/f$
	$v_w = \lambda/T = \lambda f$
	$v_w = [F_T/(m/L)]^{1/2}$ (string)
	$\lambda_n = 2L/n,\ f_n = nf_1$
	$f' = f(v_w \pm v_o)/(v_w \pm v_s)$
Radius of Curvature r	$r = 2f$
Refraction	$v_n = c/n,\ f_n = f_1,\ \lambda_n = \lambda_1/n$
	$n_1\sin\theta_1 = n_2\sin\theta_2$
	$\sin\theta_c = n_{low}/n_{high}$
Lens and Mirrors	$1/d_o + 1/\pm d_i = 1/\pm f$
convex lens = concave mirror	$M = h_i/h_o = -d_i/d_o$
2 slit Interference	$\tan\theta = x/L$
	$\sin\theta_c = m\lambda/d$
	$\sin\theta_d = (m + \frac{1}{2})\lambda/d$



Light Spot Width W	$W \approx 2\lambda L/D$
Electric Field E in N/C = V/m	$E = k\lvert Q\rvert/r^2$
Electric Force F_e in N	$F_e = k\lvert Qq\rvert/r^2 = \lvert q\rvert E$
Voltage V in J/C = V	$V = kQ/r$
Electric Energy U_e in J	$U_e = kQq/r = qV$
Capacitance C in F	$C = \epsilon_o A/d$
	$V = Ed$
	$Q = CV$
	$U_C = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}Q^2/C$
	$1/C_s = 1/C_1 + 1/C_2 + \ldots$
	$C_p = C_1 + C_2 + \ldots$



Battery in V	$V = \mathcal{E} - Ir$
	$\mathcal{E}_s = \mathcal{E}_1 \pm \mathcal{E}_2$
Current I in A = C/s	$I = Q/t$
Resistors in Ω	$R = \rho L/A$
	$V = IR$
	$P = IV = I^2R = V^2/R$
	$R_s = R_1 + R_2 + \ldots$
	$1/R_p = 1/R_1 + 1/R_2 + \ldots$
Magnetic Field B in T	$B_{straight} = k'I/r$
	$B_{loop} = \mu_o I(N/L)$
Magnetic Force F_B in N	$F_B = qvB = ILB$
Induced emf $\mathcal{E}$ in V	$\Phi_B = A \times B$
	$\mathcal{E} = \Delta\Phi_B/\Delta t = vLB$

EQUATIONS OF MOTION

$V = d/t$

$a = \Delta d / \Delta t = v - u / t$

$F = ma$

momentum $= mV = \Sigma F \Delta t = m \Delta V$

impulse $= F \Delta t = \Delta$ momentum (p)

$V_{av} = u + V / 2$ or add up the distance and divide by time

$V = u + at$

displacement $= t(u+v)/2$

displacement $= ut + (at^2/2)$

displacement $= vt - (at^2/2)$

$V^2 = u^2 + 2ax$ (disp.)

$\Sigma F = F_1 + F_2 \cdots$

$\Sigma F = F_g + F_N = F_g \sin\theta = mg\sin\theta$

$F_N = F_g \cos\theta$

$a = g\sin\theta$ (acceleration)

$\Sigma F = \Delta p / \Delta t$

$\Delta p = p_f - p_i$

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

weight $= mg$

work $= fx$ (also Δ kinetic energy)

(in a pulley system $\Rightarrow$) $a = \Sigma F / \Sigma m$

$\Sigma F = F_T + F_g$

kinetic energy $= 0.5\, mv^2$

gravitational potential $= mgh$

power (watts) $= E/t$

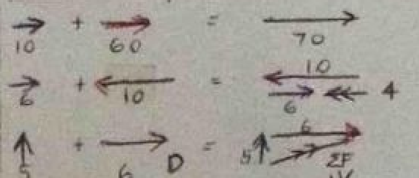
velocity	m/s
displacement	m
force	N
acceleration	m/s^2
mass	kg
momentum	kgm/s
impulse	Ns
weight	N
work	J
energy	J
power	W

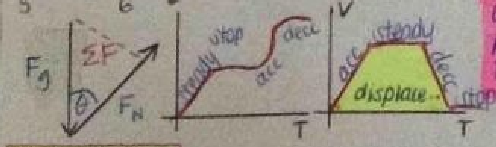
SUVAT

density $= \frac{\text{mass}}{\text{volume}}$

~~elastic potential~~ total energy = kinetic + grav

Add vector quantities head to tail!





Newton's laws

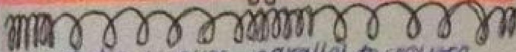
1. A body's speed remains constant if the net force acting on it is zero (balanced).
2. Acceleration is directly proportional to net force and inversely proportional to mass.
3. Forces exist in action/reaction pairs.

Remember that when gravity is acting **against** the sum of forces, it should be factored as a **negative** value.

$$m/s \xrightarrow[\div 3.6]{\times 3.6} km/h$$

WAVE PROPERTIES OF LIGHT

waves - transfer of energy but not matter



longitudinal waves - parallel to source

transverse waves - perpendicular

→ Displacement - distance graphs

→ Displacement - time graphs

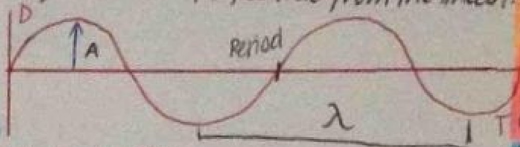
Frequency = cycles per second

$f = 1/T$

frequency	Hz (waves / second)
T: period	seconds (time per cycle)

wavelength: distance between points having the same displacement; two points in phase. λ

Amplitude: value of the maximum displacement of a particle from the mean.



wave equation:

$V = f\lambda = \lambda / T$

wavelength	m
speed of wave	m/s

wave reflecting from fixed end will undergo phase reversal: shift of $\lambda/2$

wave reflecting from free end will not undergo phase reversal.

superposition - when 2 or more waves travel in a medium the resulting wave is the sum of their displacements.

constructive interference

Destructive interference

if two continuous waves are in phase constructive interference will occur all along the wire.

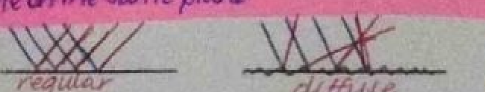
Light sources produce rays that diverge converge or run parallel.

rays represent direction of waves from point source

DIFFRACTION IS THE BENDING OF LIGHT AROUND OBSTACLES AND THROUGH APERTURES.

LAW OF REFLECTION:

The angle of incidence is equal to the angle of reflection. The incident and reflected beams and the normal, all lie on the same plane.



OPAQUE: doesn't allow light to pass thru

TRANSPARENT: allows most light to pass

TRANSLUCENT: allows light through but distorts its path to blur the image.

No material can allow 100% of incident light to pass through.

REFRACTION is the bending of the path of light due to a change in speed as it enters a medium of different optical density.

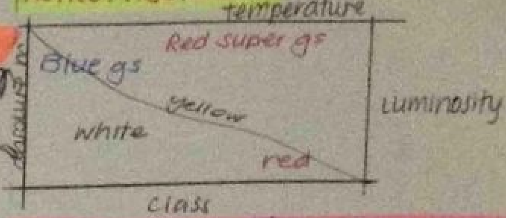
denser / high refractive index

slower medium = refracted toward normal

faster medium = refracted away from normal

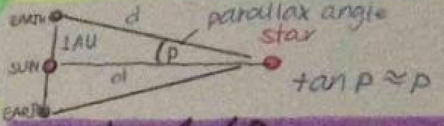
less dense

ASTROPHYSICS



$\theta = (2.1 \times 10^5)\lambda / D$

resolution	arcseconds
distance	same unit as wavelength



$\tan p \approx p$

$d = 1/p$

distance	parsecs ($1pc = 3.07 \times 10^3$ km)
parallax	arcseconds

Doppler shift

velocity of stars measured by shift.

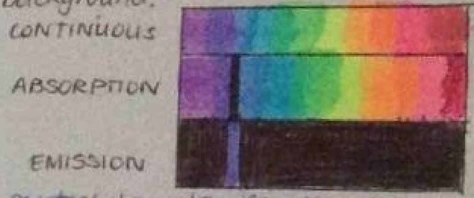
MOVING AWAY = LONGER WAVELENGTH ON EMISSION SPECTRUM, SHIFT TO RED END.

MOVING CLOSER = SHORTER WAVELENGTH ON EMISSION SPECTRUM, SHIFT TO BLUE END

Also helps detect binary stars. (spectroscopic binary system)

spectra

gases could produce continuous spectra or narrow bright lines on a black background.



spectral class classification scheme is based on temperature

O B A F G K M L hot → cool

cooler stars have more complex elements

magnitude

Apparent magnitude: magnitude it has as seen by an observer on earth

Absolute magnitude: magnitude it has if it were 10 parsecs from earth.

difference of 1 magnitude = 2.512x brighter.

luminosity

measure of the total amount of energy radiated by a star / second.

The hotter the star the greater its luminosity.

The bigger the star (surface, radius) the more luminous.

cepheids are stars that vary in brightness over periods of 1 - 70 days

Amplitude range: 0.5 - 2 magnitudes.

(the lower the magnitude the brighter the star)

The greater the period the greater the luminosity of the cepheid.

THE SUN IS AN ORDINARY G2 STAR

FUSION

$4H^+ \rightarrow {}^4_2He + 2e^+ + 2V +$ energy

Energy is created when two light atomic nuclei fuse to form a heavier nucleus. They release energy when breaking the strong nuclear force as heat and light.

stars on the main sequence use th... pure hydrogen.

star heats, shrinks then expand...

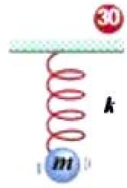
birth → yellow dwarf → today → red giant → ...

MECHANICS (continued)

K. Universal Gravitation

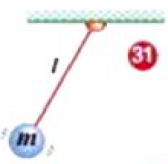
1. **Goal**: Examine gravitational energy and force fig 28

2. **Case 1**: Bodies of mass M_1 & M_2 separated by r
3. **Key equations**:
   a. Gravitational **Energy**: $U_r = \frac{GM_1M_2}{r}$
   b. Gravitational **force**: $F_r = \frac{GM_1M_2}{r^2}$
   c. **Acceleration due to gravity**: $g = G\,M(\text{earth})/r^2$
   For objects on the Earth's surface, g = **9.8 m/s^2**
   Sample: Verify "g" **at the Earth's surface**
   Equation: $g = G\,M(\text{earth})/r^2$
   Given: $M = 6{\times}10^{24}$ k, $r = 6.4{\times}10^6$ m
   Calculation: $= \frac{6.67\times10^{-11} m^3 kg^{-1} s^{-2} \times 6\times10^{24} kg}{(6.4\times10^6 m)^2}$ = **9.8 ms^2**

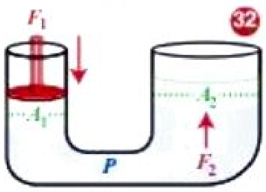


HOOKE'S LAW

4. **Case 2**: A body interacts with the Earth fig 29
5. **Key Equation**:
   a. **Gravitational potential energy**: $U_g = m\,g\,h$; object on the Earth's surface, $h = 0$; $U_g = 0$
   b. **Weight** = gravitational force; $F_g = m\,g$
   Sample: Calculate **escape velocity**, v_{esc}, for an orbiting rocket of mass m at altitude h

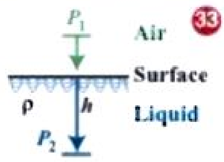


Hint: $K = U_g$ at point of escape; $r = h + r(\text{earth})$
$\frac{1}{2}m\,v_{esc}^2 = \frac{GmM}{r}$; therefore, $v_{esc} = \sqrt{\frac{2GM}{r}}$
Note: v_{esc} varies with altitude, but not rocket mass

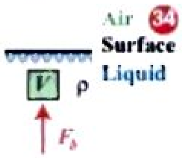


L. Oscillatory Motion

1. **Goal**: Study motion & energy of oscillating body
2. **Simple harmonic motion** (1-d)
   a. Force: $F = -k\Delta x$ (Hooke's Law)
   b. Potential Energy: $U_k = \frac{1}{2}k\Delta x^2$
   c. Frequency $= \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ fig 30



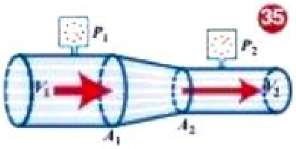
3. **Simple Pendulum**
   a. Period: $T = 2\pi\sqrt{\frac{l}{g}}$
   b. Potential energy: $U_g = m\,g\,h$
   c. Frequency $= \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ fig 31



4. **For both cases**:
   a. **Kinetic energy**: $K = \frac{1}{2}m\,v^2$
   b. **Conservation of Energy**: $E = U + K$

M. Forces in Solids and Liquids

1. **Goal 1**: Examine properties of solids & liquids



Flow Through a Hose

   a. **Density of a solid or liquid**: $\rho = \frac{\text{mass}}{\text{volume}}$
   •Common unit: g/cm^3; g/L; kg/m^3
   • **Sample**: A piece of metal, 1.5 cm × 2.5 cm × 4.0 cm, has a mass of 105.0 g; determine ρ
   Equation: $\rho = \frac{m}{V}$
   Data: m = 105.0 g, V = 1.5 × 2.5 × 4.0 cm^3 = 15 cm^3
   Calculate: ρ = 105.0/15.0 g/cm^3 = 7.0 g/cm^3
   b. Pressure exerted by a fluid: $P = \frac{\text{force}}{\text{area}}$
   c. **Pascals's Law**: For an enclosed fluid, pressure is equal at all points in the vessel
   Sample: **Hydraulic press**: $F = P/A$ for enclosed liquid; A is the surface area of the piston inserted into the fluid
   Equation: $A_1F_1 = A_2F_2$; cylinder area determines force fig 32
   d. A column of water generates pressure, P increases with depth;
   Equation: $P_2 = P_1 + \rho gh$ fig 33
   e. **Archimedes' Principle**: Buoyant force, F_b, on a object of volume V submerged in liquid of density ρ: $F_b = \rho Vg$ fig 34
2. **Goal 2**: Examine fluid motion & fluid dynamics
   a. Properties of an **Ideal fluid**: Non-viscous, incompressible, steady flow, no turbulence At any point in the flow, the product of area and velocity is constant: $A_1v_1 = A_2v_2$
   b. Variable density: $\rho_1A_1v_1 = \rho_2A_2v_2$; illustrations: gas flow through a smokestack, water flow through a hose fig 35
   c. **Bernoulli's Equation**: For any point y in the fluid flow, $P + \frac{1}{2}\rho v^2 + \rho\,g\,y$ = constant
   •**Special case**: Fluid at rest $P_1 - P_2 = \rho\,g\,h$

WAVE MOTION

A. Descriptive Variables

1. **Types**: Transverse, longitudinal, traveling, standing, harmonic
   a. General form for transverse traveling wave: $y = f(x - vt)$ (to the right) or $y = f(x + vt)$ (to the left)
   b. General form of **harmonic wave**: $y = A\sin(kx - \omega t)$ **or** $y = A\cos(kx - \omega t)$
   c. **Standing wave**: Integral multiples of $\frac{\lambda}{2}$ fit the length of the oscillating material
   d. **General wave equation**: $\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$
   e. **Superposition Principle**: Overlapping waves interact => constructive and destructive interference

Harmonic Wave Properties

Wavelength	λ (m)	Distance between peaks
Period	T (sec)	Time to travel one λ
Frequency	f (Hz)	$f = \frac{1}{T}$
Angular Frequency	ω (rad/s)	$\omega = \frac{2\pi}{T} = 2\pi f$
Wave Amplitude	A	Height of wave
Speed	v (m/s)	$v = \lambda f$
Wave number	k (m^{-1})	$k = \frac{2\pi}{\lambda}$

2. **Sample**: Determine the velocity and period of a wave with λ = 5.2 m and f = 50.0 Hz
   Equations: $v = \lambda f$ $\quad T = \frac{1}{f}$
   Data: λ = 5.20 m; f = 50.0 Hz
   Calculations: $v = \lambda f$ = 5.20 m × 50.0 = 260 m/s
   $T = \frac{1}{f} = \frac{1}{50}$ Hz = 0.02 s

B. Sound Waves

1. **Wave nature of sound**: Compression wave displaces the medium carrying the wave
2. **General speed of sound**: $v = \sqrt{\frac{B}{\rho}}$;
   note: B = **Bulk Modulus** (measure of volume compressibility)
   For a gas: $v = \sqrt{\frac{\gamma RT}{M}}$; note: $\gamma = \frac{C_p}{C_v}$ (ratio of gas heat capacities)
   Sample: Calculate speed of sound in Helium at 273 K
   Helium: Ideal gas, γ = 1.66; M = 0.004 kg/mole
   $v = \sqrt{\frac{\gamma RT}{M}}$
   $= \sqrt{\frac{1.66 \times 8.314\,\text{kg m}^2/\text{s}^2 \times 273\text{K}}{0.004\text{kg}}}$
   $= \sqrt{941{,}900\ \text{m}^2/\text{s}^2}$ = **971 m/s** **note**: $\sqrt{\ }$ applies to the units
3. **Loudness as intensity and relative intensity**
   a. **Absolute Intensity** (I = Power/Area) is an inconvenient measure of loudness
   b. Relative loudness: **Decibel scale (dB)**: $\beta = 10 \log \frac{I}{I_0}$; I_0 is the threshold of hearing; $\beta(I_0) = 0$
   c. **Samples**: Jet plane: 150 dB; Conversation: 50 dB; a change in 10 dB represents a 10-fold increase in I
4. **Doppler effect**: The sound frequency shifts $\frac{f'}{f}$ due to relative motion of source and listener;
   v_0 - listener speed; v_s - source speed; v - speed of sound
   $\frac{f'}{f} = \frac{v + v_0}{v - v_s}$ $\qquad \frac{f'}{f} = \frac{v - v_0}{v + v_s}$

Key: Identify **relative** speed of source and listener

THERMODYNAMICS

A. Goal: Study of work, heat and energy of a system fig 36

Key Variables

Heat: Q	$+Q$ added to the system
Work: W	$+W$ done by the system
Energy: E	System internal E
Enthalpy: H	$H = E + PV$
Entropy: S	Thermal disorder
Temperature: T	Measure of thermal E
Pressure: P	Force exerted by a gas
Volume: V	Space occupied

Electric Field & Force

$$F_e = k\frac{|q_1||q_2|}{R^2}$$
$$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$$
$$F_e = |q|E$$
$$E = \frac{k|q|}{R^2}$$
$$\Phi_e = EA\cos\theta$$
$$\Phi_{net} = \frac{q_{enc}}{\epsilon_0}$$
$$\epsilon_0 = \frac{1}{4\pi k}$$

Electric Potential

$$V = \frac{\mathbf{kq}}{R}$$
$$PE_e = k\frac{q_1 q_2}{R}$$
$$W_e = q\Delta V$$
$$E = \frac{\Delta V}{d}$$

Capacitance

$$C = \frac{Q}{\Delta V} \quad , \quad Q = C\Delta V \quad , \quad \Delta V = \frac{Q}{C}$$
$$U = \frac{1}{2}Q\Delta V \quad , \quad U = \frac{1}{2}C\Delta V^2 \quad , \quad U = \frac{Q^2}{2C}$$
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}$$
$$C_p = C_1 + C_2 + \cdots + C_N$$
$$C = \frac{\kappa\epsilon_0 A}{d}$$

Magnetic Field & Force

$$F_m = |q|vB\sin\theta$$
$$F_m = ILB\sin\theta$$
$$\tau_{net} = IAB\sin\theta$$
$$B = \frac{\mu_0 I}{2\pi r_c}$$
$$B = \frac{\mu_0 I}{2a}$$
$$B = \frac{\mu_0 NI}{L} = \mu_0 nI$$
$$B_1\ell_1\cos\theta_1 + B_2\ell_2\cos\theta_2 + \cdots = \mu_0 I_{enc}$$

Faraday's Law

$$\Phi_m = BA\cos\theta$$
$$\varepsilon_{ind} = -N\frac{\Delta\Phi_m}{\Delta t}$$
$$\varepsilon_{ind} = -B\ell v$$

Resistance & Current

$$\Delta V = IR \quad , \quad I = \frac{\Delta V}{R} \quad , \quad R = \frac{\Delta V}{I}$$
$$P = I\Delta V \quad , \quad P = I^2 R \quad , \quad P = \frac{\Delta V^2}{R}$$
$$R_s = R_1 + R_2 + \cdots + R_N$$
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$
$$\sum_{entering} I_i = \sum_{exiting} I_j$$
$$\sum_{loop} \Delta V_i = 0$$
$$\varepsilon = \Delta V + Ir = I(R + r)$$
$$\rho = \frac{1}{\sigma} \qquad J = \sigma E$$
$$R = \frac{\rho L}{A} \qquad J = \frac{I}{A}$$
$$\rho = \rho_0(1 + \alpha\Delta T)$$
$$R \approx R_0(1 + \alpha\Delta T)$$
$$I = \frac{\Delta q}{\Delta t}$$

Inductance

$$\varepsilon_L = -L\frac{\Delta I}{\Delta t}$$
$$L = \frac{N\Phi_m}{I}$$
$$U_L = \frac{1}{2}LI^2$$

Time Constant

$$\tau = RC$$
$$t_{1/2} = \tau\ln(2)$$
$$\tau = \frac{L}{R}$$

Transformer

$$\frac{\Delta V_{in}}{\Delta V_{out}} = \frac{N_p}{N_s}$$

AC Circuits

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$
$$\Delta V_m = \sqrt{\Delta V_{Rm}^2 + (\Delta V_{Lm} - \Delta V_{Cm})^2}$$
$$\Delta V_m = I_m Z$$
$$X_L = \omega L$$
$$X_C = \frac{1}{\omega C}$$
$$\Delta V_{rms} = \frac{\Delta V_m}{\sqrt{2}}$$
$$I_{rms} = \frac{I_m}{\sqrt{2}}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0$$
$$Q_\circ = \frac{W_\circ}{\Delta w} = \frac{W_0 L}{R}$$

Gravitation

Newton's Law of Gravitation

$$\vec{F} = -\frac{GM_1M_2}{r^2}.\hat{r}$$

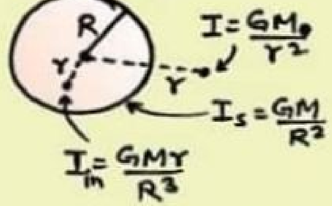
Gravitational Force is-

* Always attractive
* Independent of medium
* Conservative & Central Force

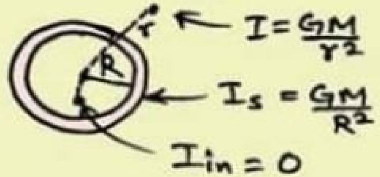
Gravitational Field Intensity

$$\vec{I} = -\frac{GM_e}{r^2}\hat{r} \Rightarrow \vec{I} = \frac{\vec{F}}{m}$$

Solid Sphere



Hollow sphere



Acceleration due to gravity

$$g = \frac{GM_e}{R_e^2} = \frac{4}{3}\pi\rho G R$$

$$g = 9.8\ m/s^2$$

M_e = Mass of earth
R_e = Radius of earth
ρ = density of earth
$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

Gravitational Potential

$$\boxed{V = -\frac{GM}{r}}$$

* It is scalar Quantity

Gravitational Potential difference

$$\boxed{V_B - V_A = \frac{W_{AB}}{m}}$$

Variation of Acceleration due to gravity

1.| Due to Altitude (h)

$$\boxed{g' = g\left[1 - \frac{2h}{R_e}\right]}$$

2.| Due to depth (d)

$$\boxed{g' = g\left[1 - \frac{d}{R_e}\right]}$$

3.| Due to Rotation of earth

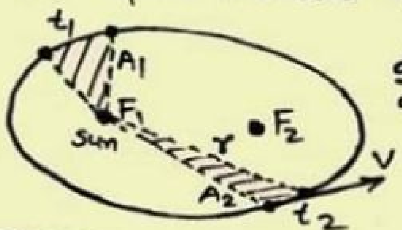
$$\boxed{g' = g - \omega^2 R_e \cos^2\lambda}$$

At equator $\lambda = 0°$
At poles $\lambda = 90°$

Kepler's Law

1.| Every planet revolve around the sun in an elliptical orbit with sun at one of the focus.

2.| The line Joining the sun to the planet sweeps out equal area in equal interval of time,



$$\frac{dA}{dt} = \frac{1}{2}rv = \text{Const.}$$

3.| $T^2 = K_s r^3$

T = Time period of revolution
r = Semimajor axis of the orbit
K_s = Kepler's Const.
$K_s = 2.97 \times 10^{-17}\ s^2/m^3$

Escape Velocity

$$V_e = \sqrt{2gR_e} = \sqrt{\frac{2GM_e}{R_e}}$$

ELECTRIC CHARGE & FIELD

by Umesh Rajoria

m_0 ◯ $- e \rightarrow$ ⊕$_m$ $m < m_0$

m_0 ◯ $+ e \rightarrow$ ⊖$_m$ $m > m_0$

Basic Properties of charge

- Two types (+ve & −ve)
- Charge is scalar
- Quantized $\boxed{Q = \pm ne}$
  $n = 0, 1, 2, 3, \ldots$
- Charge is Conserved
- Like charges Repel & Unlike charges attracts
- SI Unit – Coulomb (C)

Coulomb's Law

$$F_{vacuum} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

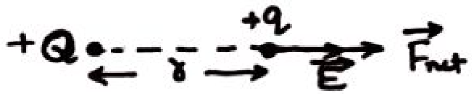
$$F_{medium} = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}$$

$$\frac{1}{4\pi\epsilon_0} = 9\times 10^9\ Nm^2/C^2$$

Dielectric Constant or relative electrical Permittivity

- $\epsilon_r = \frac{\epsilon}{\epsilon_0} = \frac{F_{vacuum}}{F_{medium}}$
- For Metals $\epsilon_r = \infty$
  For Air $\epsilon_r = 1$
- $\boxed{1 \leq \epsilon_r \leq \infty}$

Electric Field due to a point charge

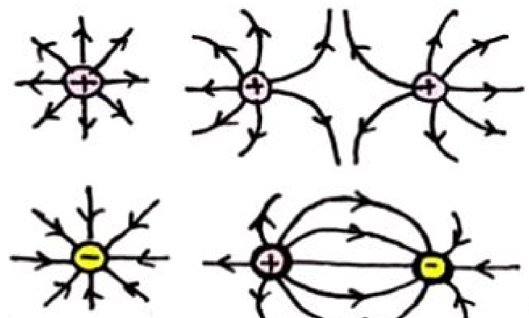


$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

$$\vec{E} = \frac{\vec{F}_{net}}{q}$$

Electric Field Lines

- Outward from +ve charge & Inward for −ve charge
- Do not intersect each other
- Do not form close loop



Electric dipole

$$\vec{P} = q \times 2\vec{a}$$

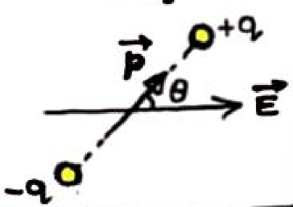
direction of $\vec{P}$ = −q to +q

Electric Field due to dipole

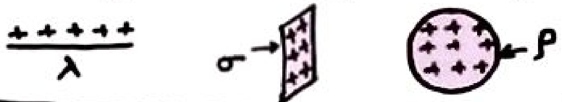
$$\vec{E}_{axis} = \frac{1}{4\pi\epsilon_0}\times\frac{2\vec{P}}{r^3}, \quad \vec{E}_{equatorial} = \frac{-1}{4\pi\epsilon_0}\times\frac{\vec{P}}{r^3}$$

Torque on dipole in Uniform Electric Field

$$\boxed{\begin{aligned}\tau &= pE\sin\theta \\ \vec{\tau} &= \vec{P}\times\vec{E}\end{aligned}}$$

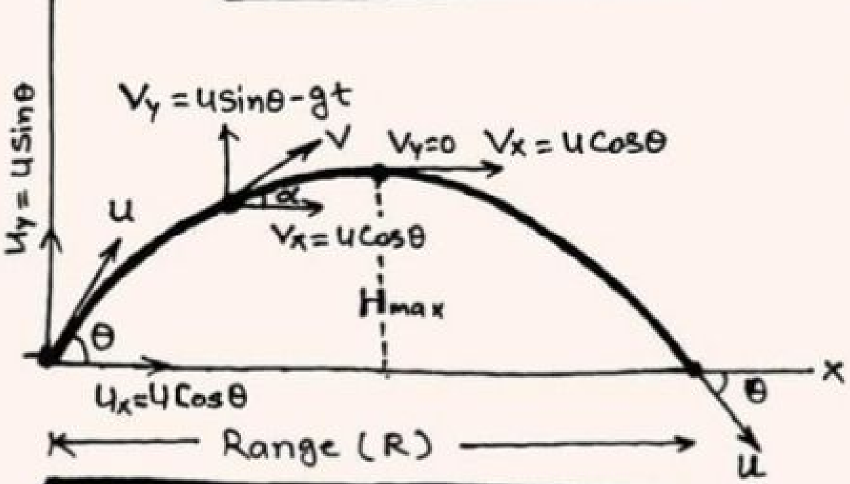


$$\lambda = \frac{Q}{l}, \quad \sigma = \frac{Q}{A}, \quad \rho = \frac{Q}{V}$$



Projectile Motion

Path → Parabolic



Horizontal Range

$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g}$$

Maximum Height

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g}$$

Time of Flight

$$T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$$

Equation of Trajectory

$$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

Coordinates at time 't'

$$x = (u\cos\theta)t$$
$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

Net displacement

$$s = \sqrt{x^2 + y^2}$$

Velocity at time 't'

$V_x = u\cos\theta$, $V_y = u\sin\theta - gt$

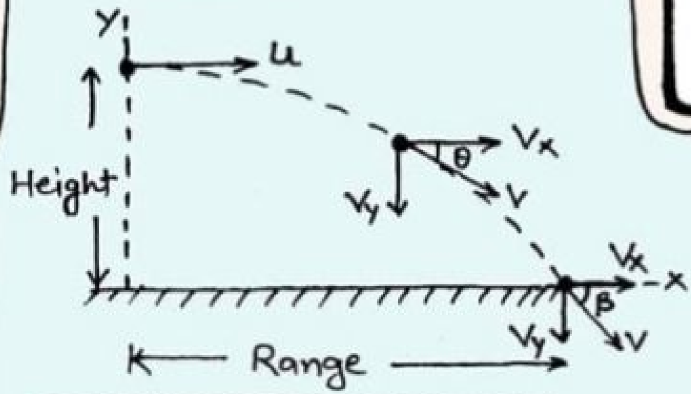
$$V = \sqrt{V_x^2 + V_y^2}$$

$$\tan\alpha = \frac{V_y}{V_x} \text{ [direction]}$$

In projectile Motion Total energy of projectile remains constant at every instant.

T.E. = P.E. + K.E.

Horizontal Projectile Motion



Displacement at time 't'

$x = ut$, $y = -\frac{1}{2}gt^2$

$$s = \sqrt{x^2 + y^2}$$

Velocity at time 't'

$V_x = u$, $V_y = gt$

$V = \sqrt{V_x^2 + V_y^2}$ $\tan\theta = \frac{V_y}{V_x}$

Horizontal Range

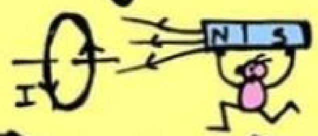
$$R = u\sqrt{\frac{2h}{g}}$$

Time of Flight

$$T = \sqrt{\frac{2h}{g}}$$

by Umesh Rajoria

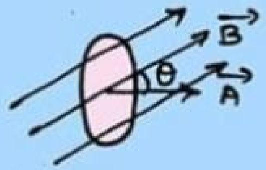
Electromagnetic Induction



Magnetic flux

$\phi = \vec{B} \cdot \vec{A}$

$\phi = BA\cos\theta$



Faraday's Law of EMI

(i) Reason of induced emf or current is change in magnetic flux

(ii) $e = -N\dfrac{d\phi}{dt}$

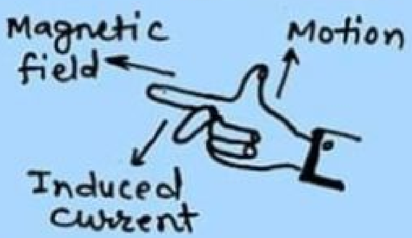
$I = -\dfrac{N}{R}\dfrac{d\phi}{dt}$

Lenz's Law

direction of induced emf or current is such that it always opposes the cause that produces it.

* Lenz's law based on the energy conservation law.

Fleming's Right hand Rule



Energy Stored in Inductor

$U = W = \frac{1}{2}LI^2$

Self Induction

$\phi = LI$

$e = -L\dfrac{dI}{dt}$

Mutual Induction

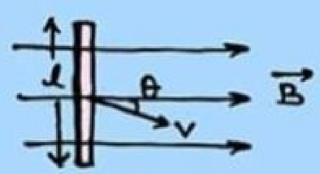
$\phi_2 = MI_1$

$e_2 = -M\dfrac{dI_1}{dt}$

Motional EMF due to

Translatory Motion

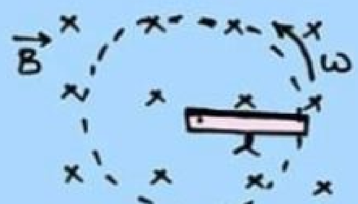
$e = Blv\sin\theta$



Rotational Motion

$e = \dfrac{Bl^2\omega}{2}$

$\omega = 2\pi\nu$



Eddy Current

It is induced current in a metal body due to change in magnetic flux

* Losses

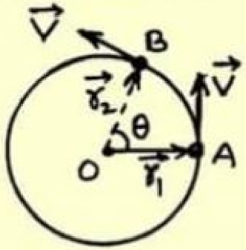
(i) Unnecessary Heat

(ii) Unnecessary Retardation

Circular Motion

Uniform Circular Motion

Particle Moves in a circle at a Constant Speed



Angular Velocity

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

$$\omega_i = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Relation between

* S and $\theta \Rightarrow S = r\theta$
* V and $\omega \Rightarrow \vec{V} = \vec{\omega} \times \vec{r}$

---

* As $F_c = \frac{mv^2}{r} \neq 0$, so the Particle is not in equilibrium
* $\tau_{net} = 0$ then Angular Momentum $L = $ Const. Momentum $p \neq$ Const.

Non-Uniform Circular Motion

* Tangential Acceleration $\vec{a}_t = \vec{\alpha} \times \vec{r}$

  Tangential Force $F_t = ma_t$
* Centripetal Acceleration $\vec{a}_c = \vec{\omega} \times \vec{V}$

  Centripetal Force $F_c = ma_c$
* Net Acceleration $a = \sqrt{a_t^2 + a_c^2}$

  Net Force $F = m\sqrt{a_t^2 + a_c^2}$

Equations of Circular Motion

$$\omega_2 = \omega_1 + \alpha t$$

$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$$

$$\omega_2^2 - \omega_1^2 = 2\alpha\theta$$

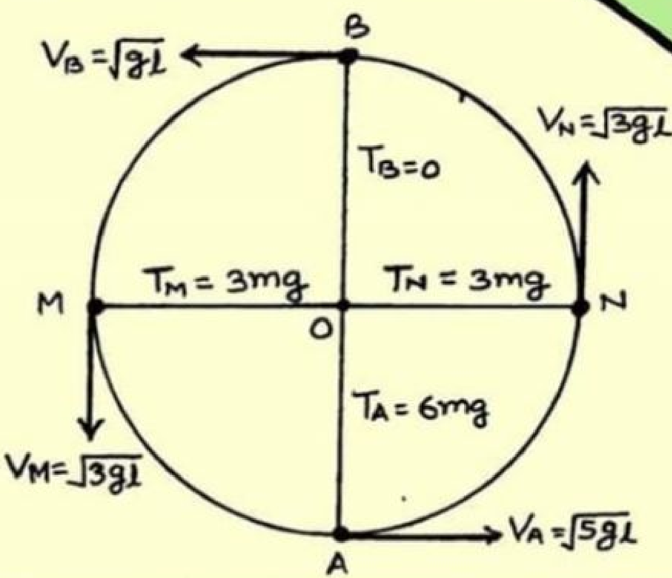
$$\theta_n = \omega_1 + \frac{\alpha}{2}(2n-1)$$

by- Umesh Rajoria

Motion in a Vertical Circle

Body is under the influence of gravity of earth and total Mechanical energy is conserved.

* Particle will complete the circle if $u > \sqrt{5gl}$
* Particle will Oscillate if velocity becomes zero and the tension in the string is not zero. $0 < u \leq \sqrt{2gl}$



Tension at any point on Vertical loop

$$T - mg\cos\theta = \frac{mv^2}{l}$$

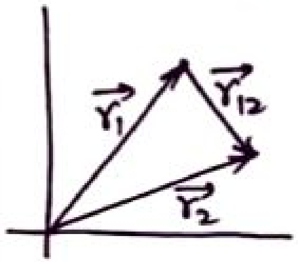
Motion in 1-D

Distance
- Scalar
- depends on path
- Always +ve

Displacement
- Vector
- depends on initial & Final Position
- May be +ve or −ve

Distance ≥ |Displacement|

Position Vector



$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

Speed
- Rate of distance Covered with time
- Scalar
- May be zero or +ve

Average Speed $V_{av} = \frac{\Delta s}{\Delta t}$

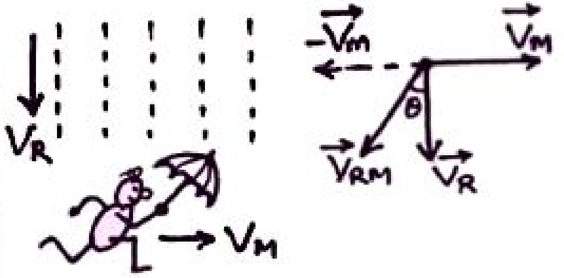
Instantaneous Speed $V = \frac{ds}{dt}$

Relative Velocity of Rain

$$\vec{V}_{RM} = \vec{V}_R - \vec{V}_M$$

$$\theta = \tan^{-1}\left(\frac{V_M}{V_R}\right)$$

Direction of Umberella is along the V_{RM} at angle θ from the vertical



Velocity
- Ratio of displacement to the time taken
- Vector
- May be zero, +ve & −ve

Average Velocity $\vec{V}_{av} = \frac{\Delta \vec{r}}{\Delta t}$

Instantaneous Velocity $\vec{V} = \frac{d\vec{r}}{dt}$

Acceleration

Its rate of change of velocity with time
- * Its vector
- * Its direction is along the change in Velocity

Average Acceleration $= \frac{\Delta \vec{V}}{\Delta t}$

Inst. Acceleration $= \frac{d\vec{V}}{dt} = \frac{d^2\vec{r}}{dt^2}$

Equations of Kinematics

$$\vec{V} = \vec{u} + \vec{a}t$$
$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$
$$V^2 = u^2 + 2as$$
$$\vec{S}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

$$\text{Displacement} \underset{\text{Integration}}{\overset{\text{differentiation}}{\rightleftarrows}} \text{Velocity} \underset{\text{Integration}}{\overset{\text{differentiation}}{\rightleftarrows}} \text{Acceleration}$$

Vector

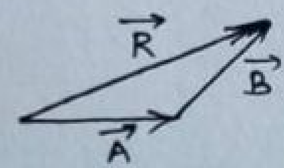
Unit Vector

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

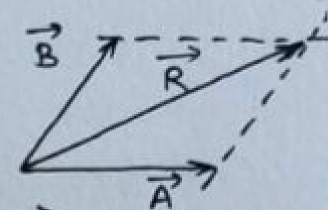
Vector addition

Graphical Method

Triangle Law

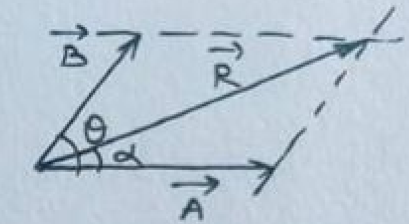


Parallelogram Law



$$\vec{R} = \vec{A} + \vec{B}$$

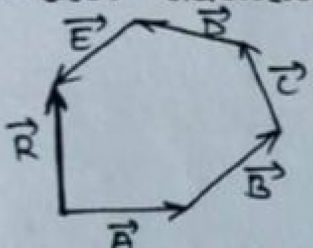
Analytical Method



$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

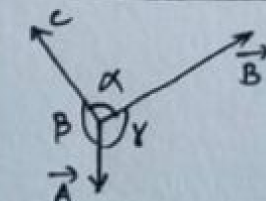
$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

Polygon Law of Vector addition



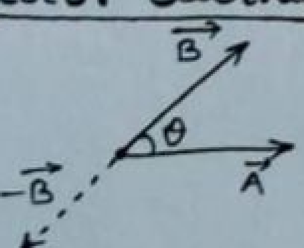
$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$

Lami's Theorem



$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Vector Subtraction

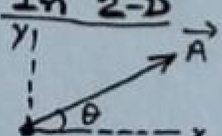


$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

Components of Vector

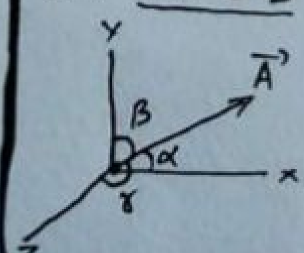
(i) In 2-D



$A_x = A\cos\theta$

$A_y = A\sin\theta$

(ii) In 3-D



$A_x = A\cos\alpha$

$A_y = A\cos\beta$

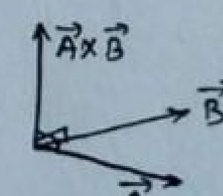
$A_z = A\cos\gamma$

Scalar Product

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

Vector Product

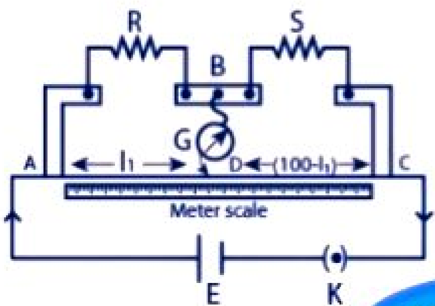
$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$



by Umesh Rajoria

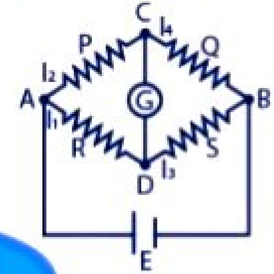
CURRENT ELECTRICITY

Meter Bridge



$R=S\frac{l_1}{100-l_1}$

Wheatstone Bridge



UNDER BALANCE CONDITION

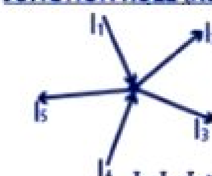
$\frac{P}{Q}=\frac{R}{S}$

Null point is not affected by resistance of G & E

Null point is not affected even if the positions of G & E are interchanged

Kirchoff's Rules

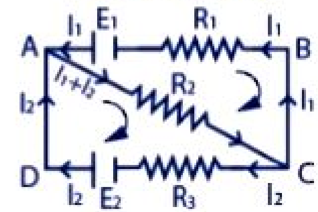
JUNCTION RULE (KCL)



$I_1-I_2-I_3+I_4-I_5=0$

$\Sigma I_{in}=\Sigma I_{out}$

LOOP RULE (KVL)



Loop ABCA

$-E_1+I_1R_1-(I_1+I_2)R_2=\infty$

$+E_1-I_1R_1-(I_1+I_2)R_2=0$

Loop ACDA

$-(I_1+I_2)R_2-I_2R_3+E_2=0$

$+(I_1+I_2)R_2+I_2R_3-E_2=0$

Potentiometer

(a) $\frac{E_1}{E_2}=\frac{l_1}{l_2}$

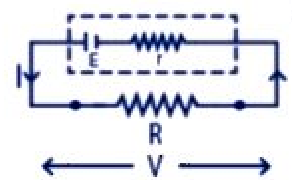
(b) $r=R\left(\frac{l_1}{l_2}-1\right)$

USE

(a) Comparing emfs of two cells

(b) Determine internal resistance of a cell

Cells, EMF, Internal Resistance



$I=\frac{E}{R+r}$

$E=IR+Ir$

$E=V+Ir$

$V=E-Ir$

Electric Current

$I=\frac{Q}{t}$

$I=\frac{dQ}{dT}$

$\frac{1\text{ Coulomb}}{1\text{ second}}=1\text{ ampere}$

Current is scalar quantity

Current density $J=\frac{I}{A}$

$I=neAv_d$ where v_d = drift velocity

$J=\frac{I}{A}=neAv_d=ne(\mu E)=\sigma E$ where μ = mobility

Where $\sigma=ne\mu$ is called conductivity

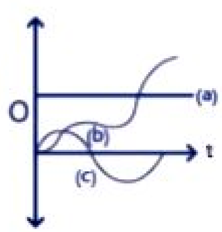
$P=\frac{1}{\sigma}$ is resitivity

DIFFERENT TYPES OF CURRENT

(a) Steady Current

(b) Varying Current

(c) Alternating Current



Resistance

$R\propto\frac{1}{A}$

$R=\rho\frac{l}{A}$

DEPENDENCE OF RESISTANCE ON TEMPERATURE

$R_T=R_0[1+\alpha(T-T_0)]$

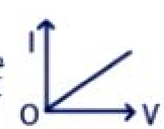
$\rho_T=\rho_0[1+\alpha(T-T_0)]$

where = temperature co-efficient of resistivity

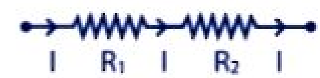
Ohm's Law

$V=RI$

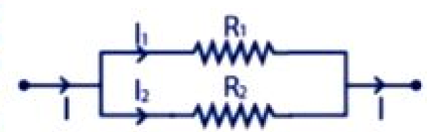
Electric Power $P=VI=I^2R=\frac{V^2}{R}$



COMBINATION OF RESISTANCES



$R=R_1+R_2$

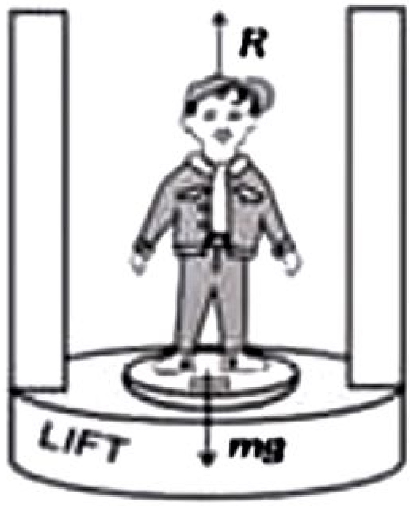


$I=\frac{R_1R_2}{R_1+R_2}$

Motion of a body in Lift

When a body of mass m is placed on a weighing machine which is placed in a lift, then actual weight of the body is mg

This force mg acts on a weighing machine which offers a reaction R given by the reading of weighing machine.



This reaction R exerted by the surface of contact on the body is the apparent weight of the body.

1.) When the lift is moving upwards -

Net upward force on lift is

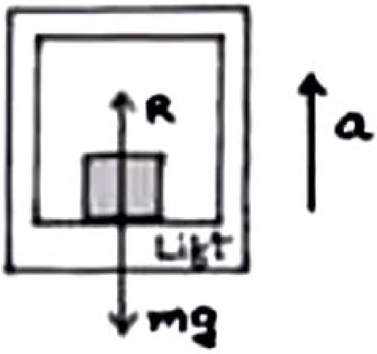
$$F = R - mg$$

or $$ma = R - mg$$

Hence apparent weight (R) is

$$\boxed{R = m(g+a)}$$

Apparent Weight > Actual Weight



2.) When the lift is moving downward -

Net downward force on lift is

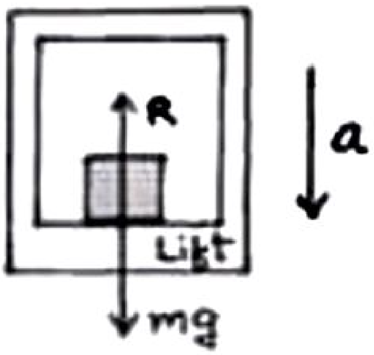
$$F = mg - R$$

or $$ma = mg - R$$

Hence apparent weight (R) is

$$\boxed{R = m(g-a)}$$

Apparent Weight < Actual Weight



Note - Here $m = m_1 + M_2$ [Mass of lift + Mass of body]

Note -

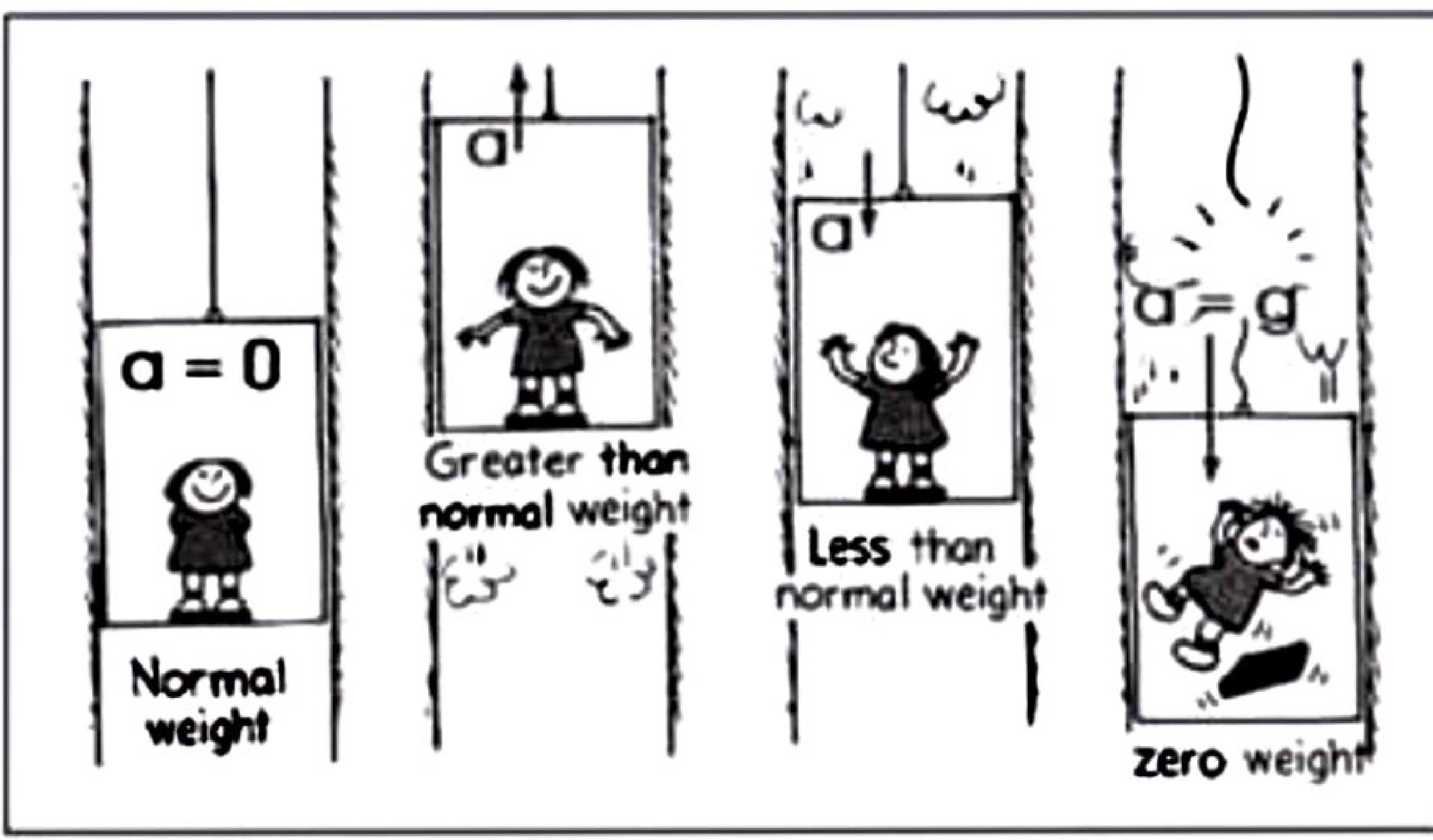
(i) If lift is at rest or moving upward or downward with constant velocity then $a = 0$ and $R = mg$.

Hence Apparent Weight = Actual Weight

(ii) If lift is accelerating downword at the rate of 'g' then $R = 0$.

Hence Apparent Weight = 0 (Weightlessness)

(iii) If lift is accelerating downword at the rate of $a > g$ then $R = -ve$.
Hence Apparent weight is -ve, which means the body will rise from the floor of the lift and stick to the ceiling of the lift.



Kinematics

Linear Motion

Vector quantity
- → Displacement (s)
- → velocity (v)

Average velocity
$= \frac{\Delta s}{\Delta t}$

- → Acceleration (a)

Average acceleration
$= \frac{\Delta v}{\Delta t}$

*sign of v = a, velocity ↑
sign of v ≠ a, velocity ↓

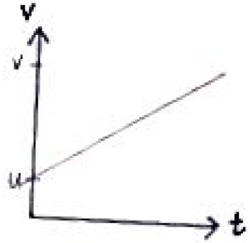
Scalar quantity
- → Distance (d)
- → Speed

Average speed
$= \frac{\text{total distance travelled}}{\text{total time taken}}$

*if speed not constant

Equations of Motion

* only applied to linear motion with constant acceleration



$v = u + at$

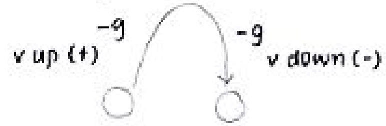
$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u+v)t$

$s = ut + \frac{1}{2}at^2$

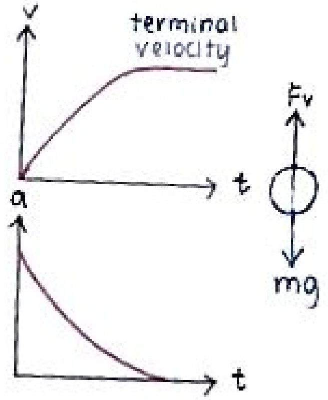
Free Falling Bodies

→ vertical motion under gravity without air resistance
· g = 9.81 ms^{-2}

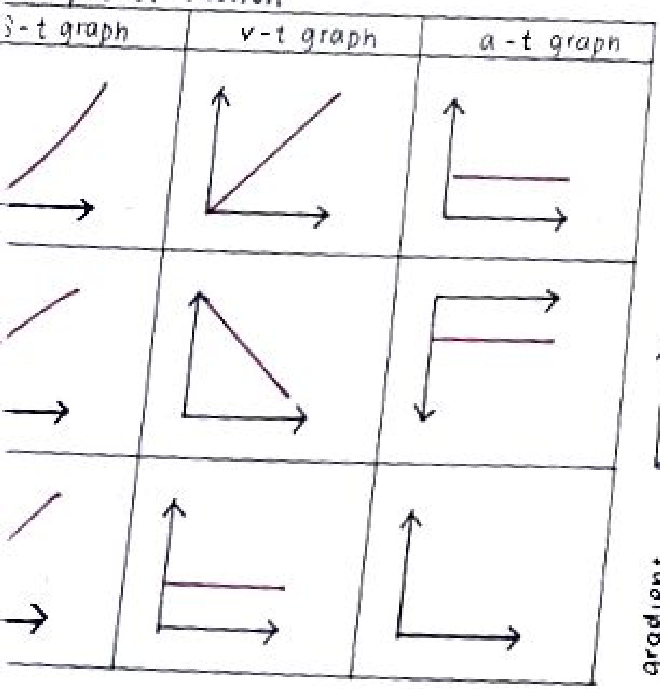


- if both (+), g acting downward because direction of g never changes (towards Earth)

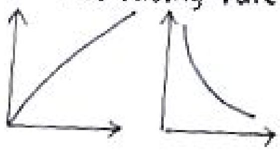
→ vertical motion under gravity with air resistance
· net force ma = mg - Fv
as $F \propto t$, eventually
mg - Fv = 0, a = 0, object fall at terminal velocity



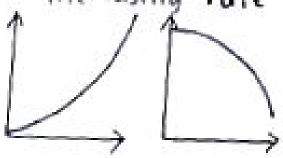
Graphs of Motion

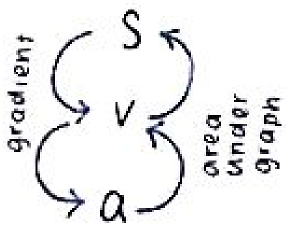


Bend towards y-axis
· decreasing rate

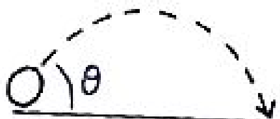


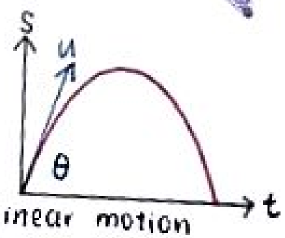
Bend away from y-axis
· increasing rate





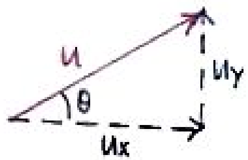
Projectile Motion



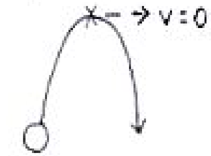


→ object undergoing linear motion
· in the horizontal direction
· in the vertical direction

Horizontal Component
· constant velocity
$u_x = u\cos\theta$
· horizontal distance
$S_x = u\cos\theta\, t$



Vertical component
· constant acceleration
$u_y = u\sin\theta$



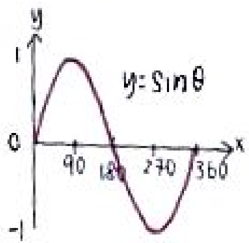
Maximum height
$H = \frac{u^2\sin^2\theta}{2g}$

Time taken, t_H
$t_H = \frac{u\sin\theta}{g}$

Total time taken,
$T = \frac{2u\sin\theta}{g}$

Horizontal Range, R
$R = \frac{u^2\sin 2\theta}{g}$

* Rmax when θ = 45°



when θ = 45°
$R_{max} = \frac{u^2\sin(2\times45)}{g}$

$R_{max} = \frac{u^2}{g}$