

CS381  
Project #2 Report  
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10/30/2015

Each dataset have five random starting points and the starting parameters given in the project speciation tested at last.

### Data set with missing rate being 10%

The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.82$$

$$P(w=0|g=0) = 0.37$$

$$P(w=0|g=1) = 0.88$$

$$P(h=0|g=0) = 0.83$$

$$P(h=0|g=1) = 0.77$$

The final conditional probability tables for hw2dataset\_10.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.642462 \quad P(\text{gender}=\text{F}) = 0.357538$$

Weight(w) given gender(g) table

$$P(w=0|g=0) = 0.794435 \quad P(w=1|g=0) = 0.205565$$

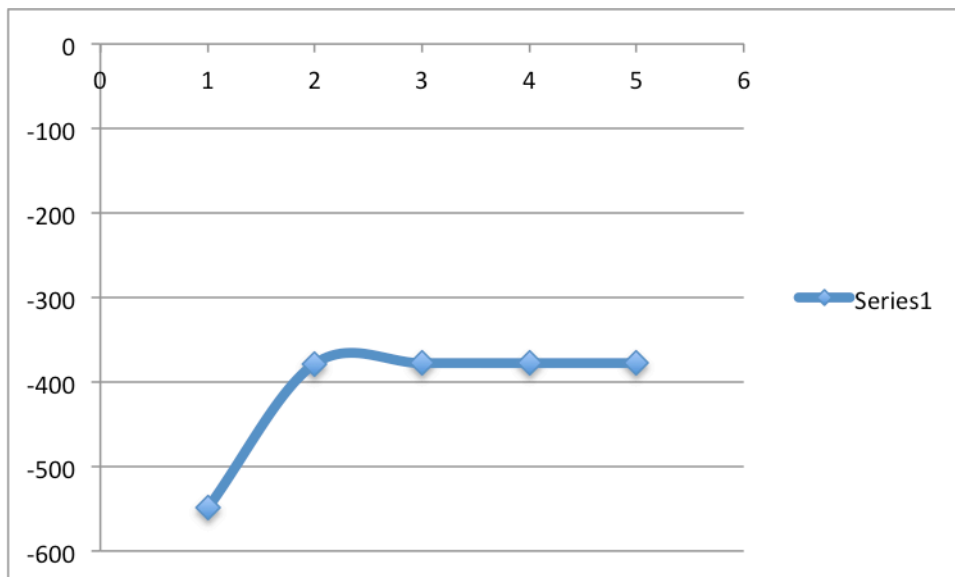
$$P(w=0|g=1) = 0.348511 \quad P(w=1|g=1) = 0.651489$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.661922 \quad P(h=1|g=0) = 0.338078$$

$$P(h=0|g=1) = 0.26498 \quad P(h=1|g=0) = 0.73502$$

Plots of the likelihood vs number of iterations



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.52$$

$$P(w=0|g=0) = 0.11$$

$$P(w=0|g=1) = 0.45$$

$$P(h=0|g=0) = 0.51$$

$$P(h=0|g=1) = 0.13$$

The final conditional probability tables for hw2dataset\_10.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.642461 \quad P(\text{gender}=\text{F}) = 0.357539$$

Weight(w) given gender(g) table

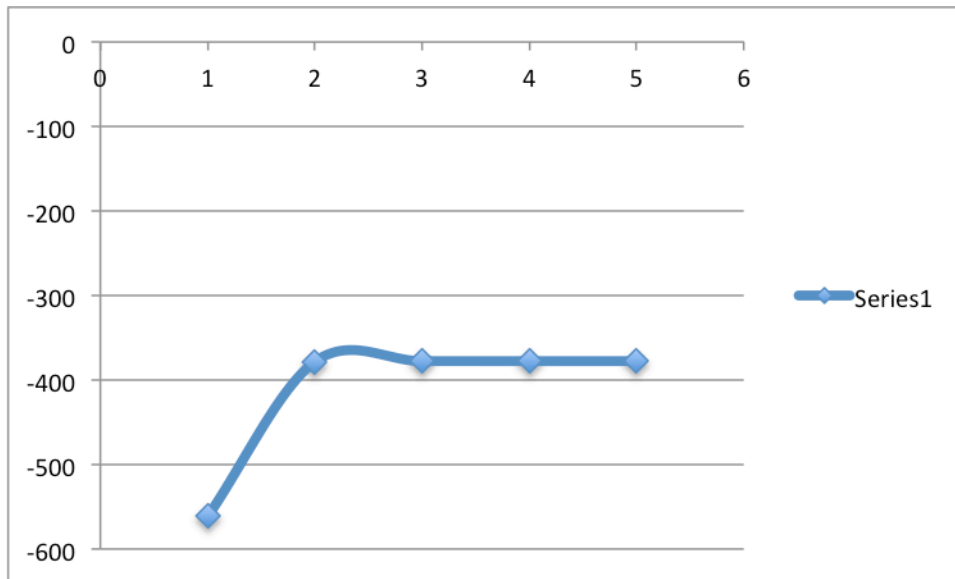
$$P(w=0|g=0) = 0.794435 \quad P(w=1|g=0) = 0.205565$$

$$P(w=0|g=1) = 0.348511 \quad P(w=1|g=1) = 0.651489$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.661922 \quad P(h=1|g=0) = 0.338078$$

$$P(h=0|g=1) = 0.264979 \quad P(h=1|g=1) = 0.735021$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.49$$

$$P(w=0|g=0) = 0.56$$

$$P(w=0|g=1) = 0.65$$

$$P(h=0|g=0) = 0.56$$

$$P(h=0|g=1) = 0.66$$

The final conditional probability tables for hw2dataset\_10.txt is:

Gender table

$P(\text{gender}=\text{M}) = 0.642462$   $P(\text{gender}=\text{F}) = 0.357538$

Weight(w) given gender(g) table

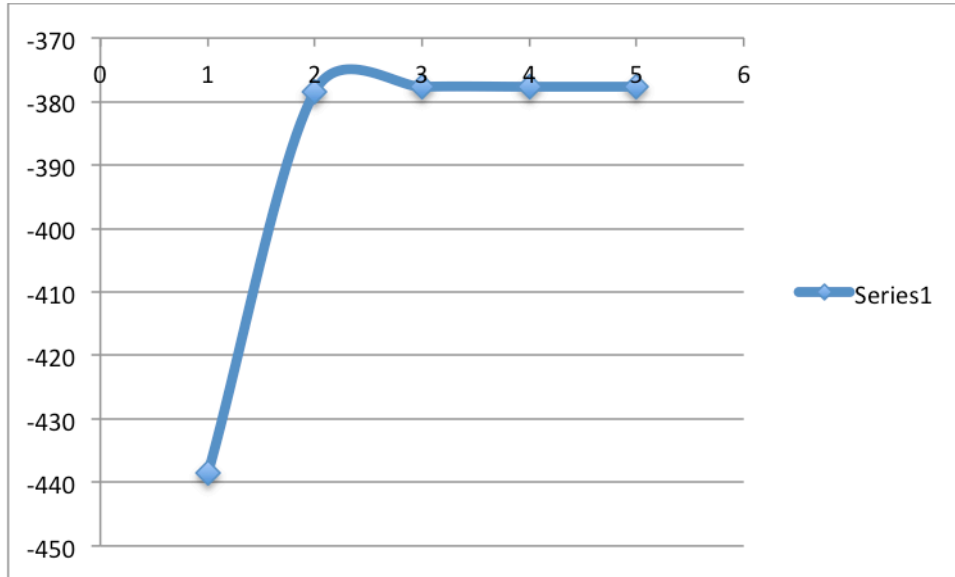
$P(w=0|g=0) = 0.794435$   $P(w=1|g=0) = 0.205565$

$P(w=0|g=1) = 0.348511$   $P(w=1|g=1) = 0.651489$

Height(h) given gender(g) table

$P(h=0|g=0) = 0.661922$   $P(h=1|g=0) = 0.338078$

$P(h=0|g=1) = 0.26498$   $P(h=1|g=0) = 0.73502$



The starting points of the learning:

$P(\text{gender}=\text{M}) = 0.96$

$P(w=0|g=0) = 0.33$

$P(w=0|g=1) = 0.21$

$P(h=0|g=0) = 0.89$

$P(h=0|g=1) = 0.41$

The final conditional probability tables for hw2dataset\_10.txt is:

Gender table

$P(\text{gender}=\text{M}) = 0.642463$   $P(\text{gender}=\text{F}) = 0.357537$

Weight(w) given gender(g) table

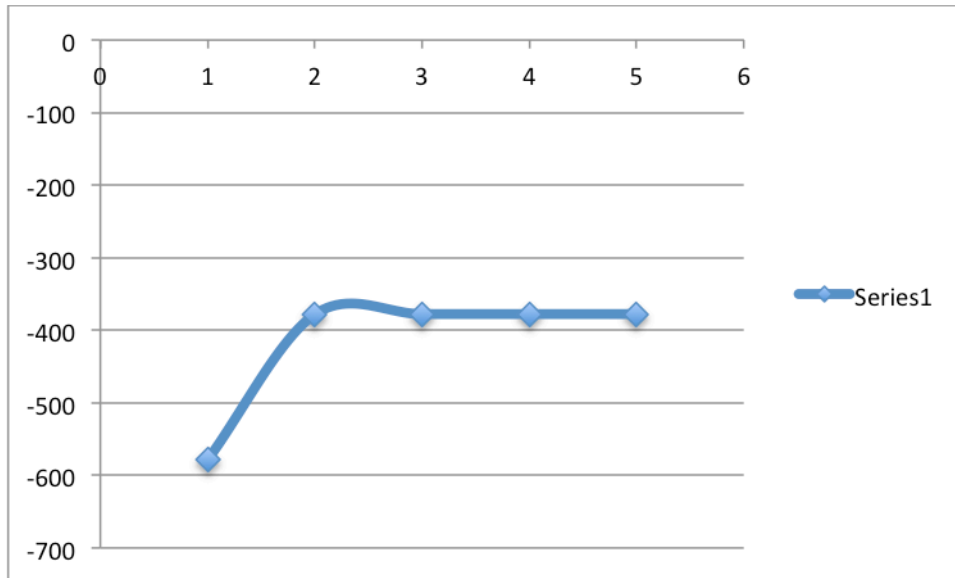
$P(w=0|g=0) = 0.794435$   $P(w=1|g=0) = 0.205565$

$P(w=0|g=1) = 0.34851$   $P(w=1|g=1) = 0.65149$

Height(h) given gender(g) table

$P(h=0|g=0) = 0.661922$   $P(h=1|g=0) = 0.338078$

$P(h=0|g=1) = 0.26498$   $P(h=1|g=0) = 0.73502$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.26$$

$$P(w=0|g=0) = 0.79$$

$$P(w=0|g=1) = 0.95$$

$$P(h=0|g=0) = 0.35$$

$$P(h=0|g=1) = 0.85$$

The final conditional probability tables for hw2dataset\_10.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.642462 \quad P(\text{gender}=\text{F}) = 0.357538$$

Weight(w) given gender(g) table

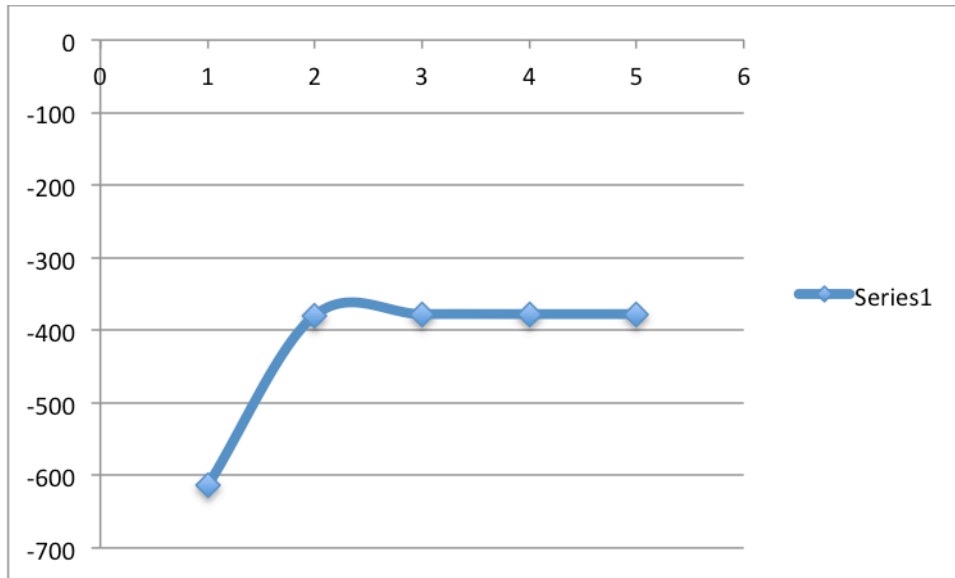
$$P(w=0|g=0) = 0.794435 \quad P(w=1|g=0) = 0.205565$$

$$P(w=0|g=1) = 0.348512 \quad P(w=1|g=1) = 0.651488$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.661922 \quad P(h=1|g=0) = 0.338078$$

$$P(h=0|g=1) = 0.26498 \quad P(h=1|g=1) = 0.73502$$



The final conditional probability tables for hw2dataset\_10.txt is:

Gender table

$P(\text{gender}=\text{M}) = 0.642462$   $P(\text{gender}=\text{F}) = 0.357538$

Weight(w) given gender(g) table

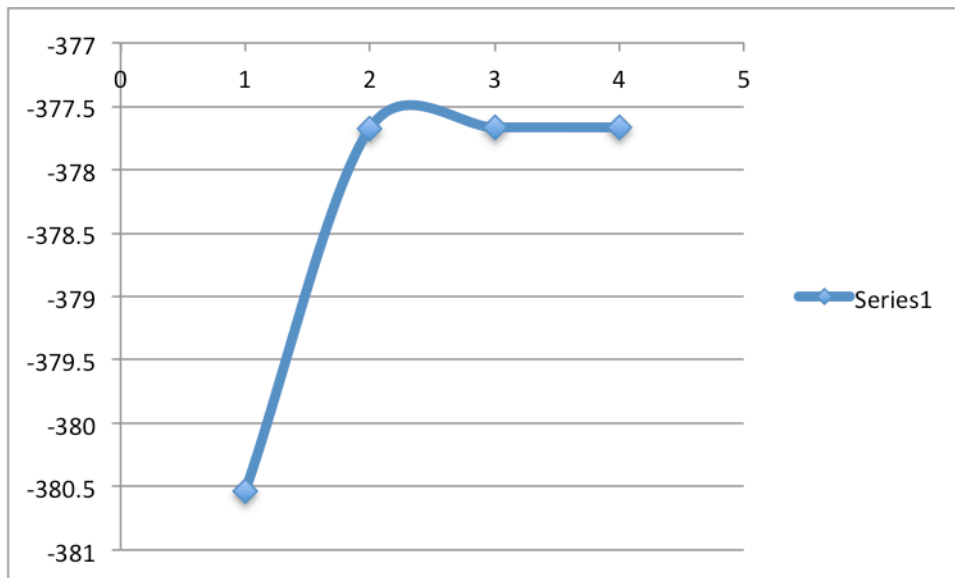
$P(w=0|g=0) = 0.794435$   $P(w=1|g=0) = 0.205565$

$P(w=0|g=1) = 0.34851$   $P(w=1|g=1) = 0.65149$

Height(h) given gender(g) table

$P(h=0|g=0) = 0.661922$   $P(h=1|g=0) = 0.338078$

$P(h=0|g=1) = 0.264979$   $P(h=1|g=1) = 0.735021$



### Data set with missing rate being 30%

The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.47$$

$$P(w=0|g=0) = 0.35$$

$$P(w=0|g=1) = 0.79$$

$$P(h=0|g=0) = 0.14$$

$$P(h=0|g=1) = 0.17$$

The final conditional probability tables for hw2dataset\_30.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.681544 \quad P(\text{gender}=\text{F}) = 0.318456$$

Weight(w) given gender(g) table

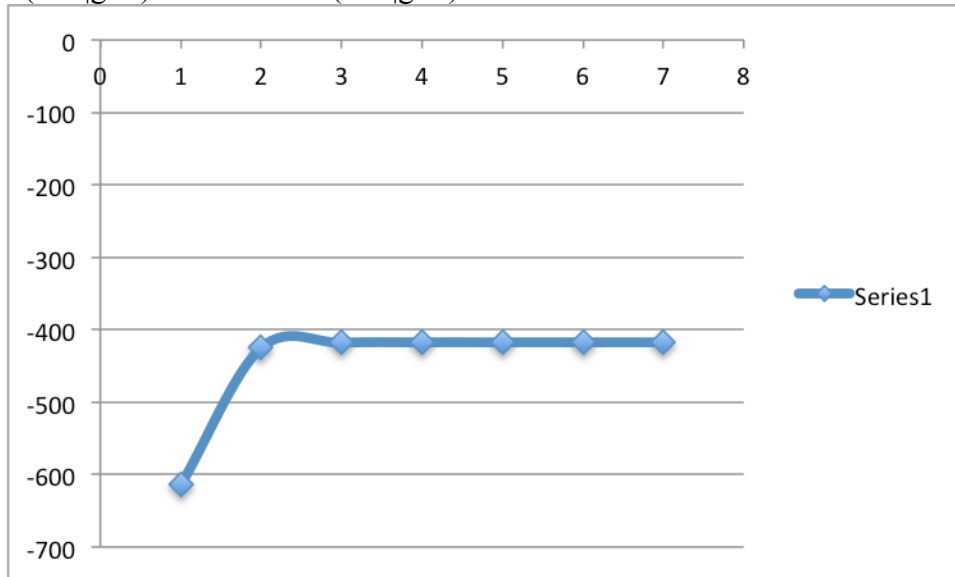
$$P(w=0|g=0) = 0.785886 \quad P(w=1|g=0) = 0.214114$$

$$P(w=0|g=1) = 0.374883 \quad P(w=1|g=1) = 0.625117$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.650897 \quad P(h=1|g=0) = 0.349103$$

$$P(h=0|g=1) = 0.349767 \quad P(h=1|g=1) = 0.650233$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.29$$

$$P(w=0|g=0) = 0.1$$

$$P(w=0|g=1) = 0.12$$

$$P(h=0|g=0) = 0.43$$

$$P(h=0|g=1) = 0.48$$

The final conditional probability tables for hw2dataset\_30.txt is:

Gender table

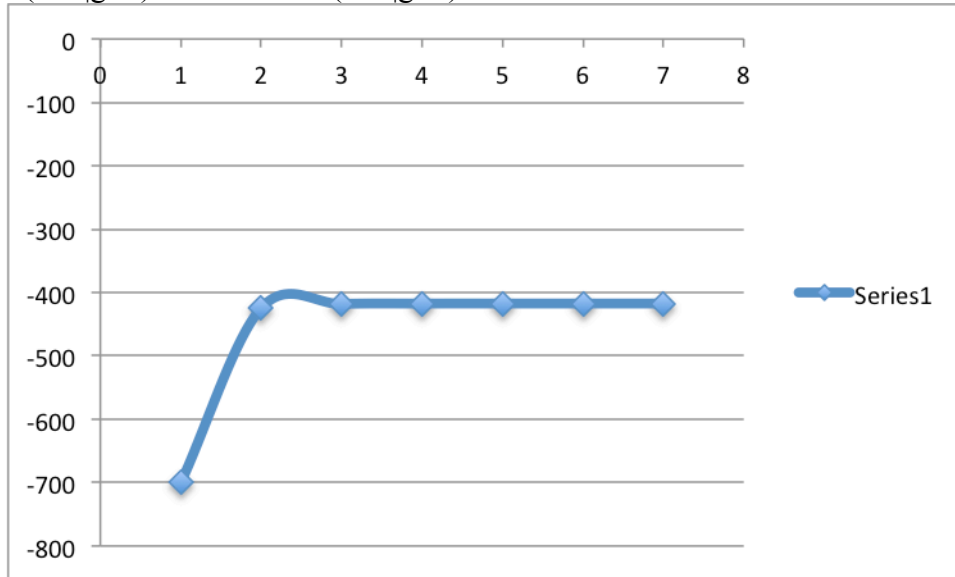
$$P(\text{gender}=\text{M}) = 0.681394 \quad P(\text{gender}=\text{F}) = 0.318606$$

Weight(w) given gender(g) table

$P(w=0|g=0) = 0.786044$   $P(w=1|g=0) = 0.213956$   
 $P(w=0|g=1) = 0.37474$   $P(w=1|g=1) = 0.62526$

Height(h) given gender(g) table

$P(h=0|g=0) = 0.650854$   $P(h=1|g=0) = 0.349146$   
 $P(h=0|g=1) = 0.349999$   $P(h=1|g=1) = 0.650001$



The starting points of the learning:

$P(\text{gender}=\text{M}) = 0.31$

$P(w=0|g=0) = 0.07$

$P(w=0|g=1) = 0.35$

$P(h=0|g=0) = 0.51$

$P(h=0|g=1) = 0.66$

The final conditional probability tables for hw2dataset\_30.txt is:

Gender table

$P(\text{gender}=\text{M}) = 0.681407$   $P(\text{gender}=\text{F}) = 0.318593$

Weight(w) given gender(g) table

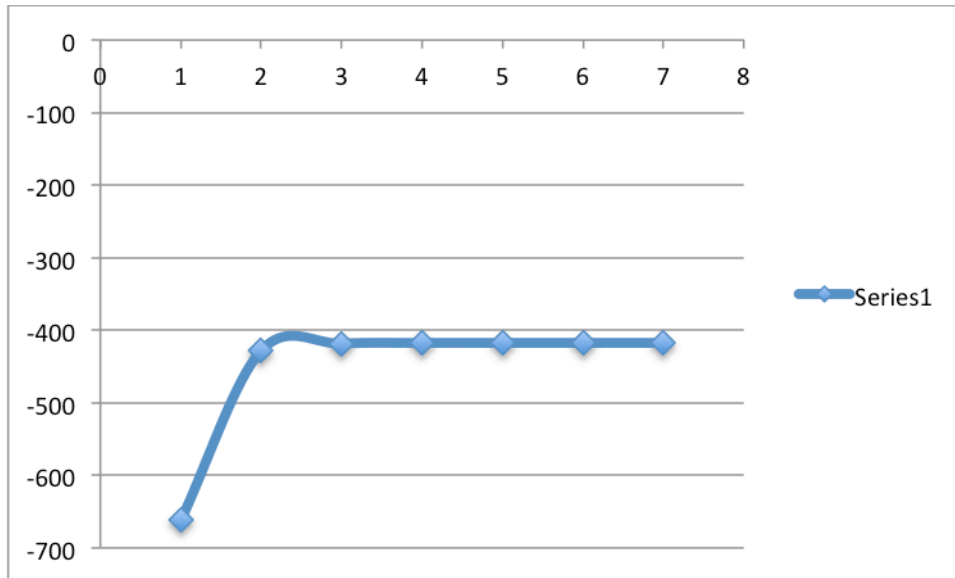
$P(w=0|g=0) = 0.786022$   $P(w=1|g=0) = 0.213978$

$P(w=0|g=1) = 0.374772$   $P(w=1|g=1) = 0.625228$

Height(h) given gender(g) table

$P(h=0|g=0) = 0.650858$   $P(h=1|g=0) = 0.349142$

$P(h=0|g=1) = 0.349979$   $P(h=1|g=1) = 0.650021$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0$$

$$P(w=0|g=0) = 0.67$$

$$P(w=0|g=1) = 0.29$$

$$P(h=0|g=0) = 0.8$$

$$P(h=0|g=1) = 0.84$$

The final conditional probability tables for hw2dataset\_30.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.70865 \quad P(\text{gender}=\text{F}) = 0.29135$$

Weight(w) given gender(g) table

$$P(w=0|g=0) = 0.718843 \quad P(w=1|g=0) = 0.281157$$

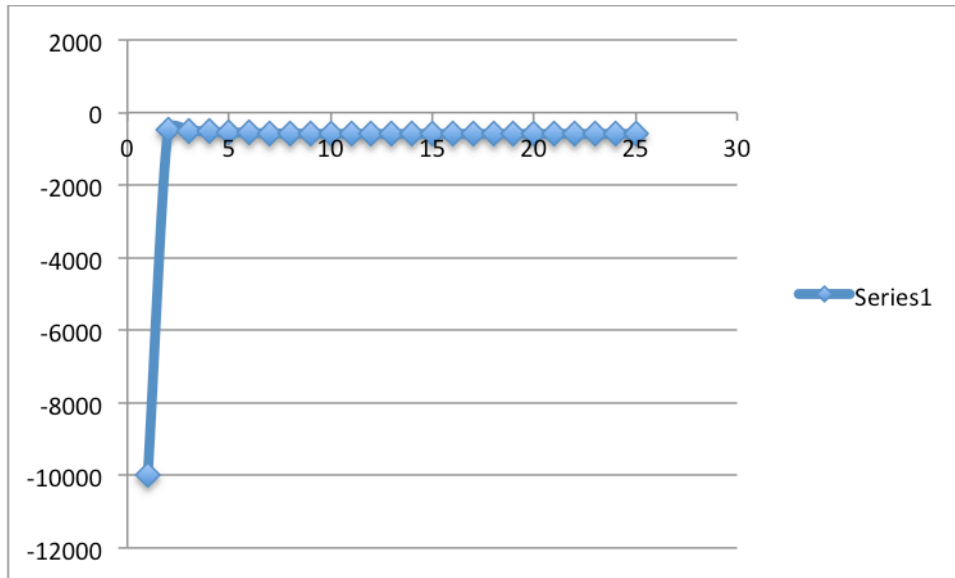
$$P(w=0|g=1) = 0.892396 \quad P(w=1|g=1) = 0.107604$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.600832 \quad P(h=1|g=0) = 0.399168$$

$$P(h=0|g=1) = 0.858073 \quad P(h=1|g=1) = 0.141927$$





The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.96$$

$$P(w=0|g=0) = 0.29$$

$$P(w=0|g=1) = 0.89$$

$$P(h=0|g=0) = 0.16$$

$$P(h=0|g=1) = 0.37$$

The final conditional probability tables for hw2dataset\_30.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.681712 \quad P(\text{gender}=\text{F}) = 0.318288$$

Weight(w) given gender(g) table

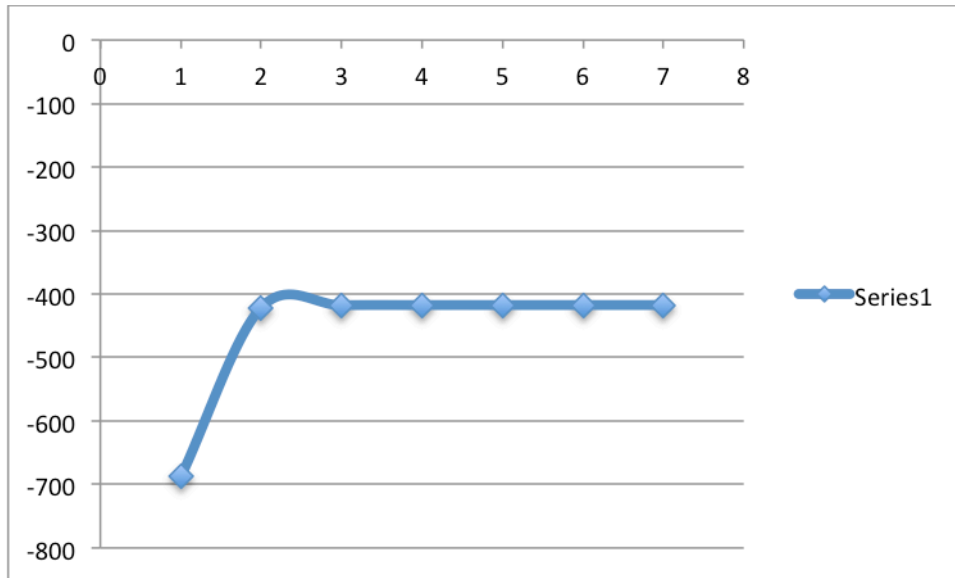
$$P(w=0|g=0) = 0.785739 \quad P(w=1|g=0) = 0.214261$$

$$P(w=0|g=1) = 0.374982 \quad P(w=1|g=1) = 0.625018$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.65094 \quad P(h=1|g=0) = 0.34906$$

$$P(h=0|g=1) = 0.349515 \quad P(h=1|g=1) = 0.650485$$



The final conditional probability tables for hw2dataset\_30.txt is:

Gender table

$P(\text{gender}=\text{M}) = 0.681612$   $P(\text{gender}=\text{F}) = 0.318388$

Weight(w) given gender(g) table

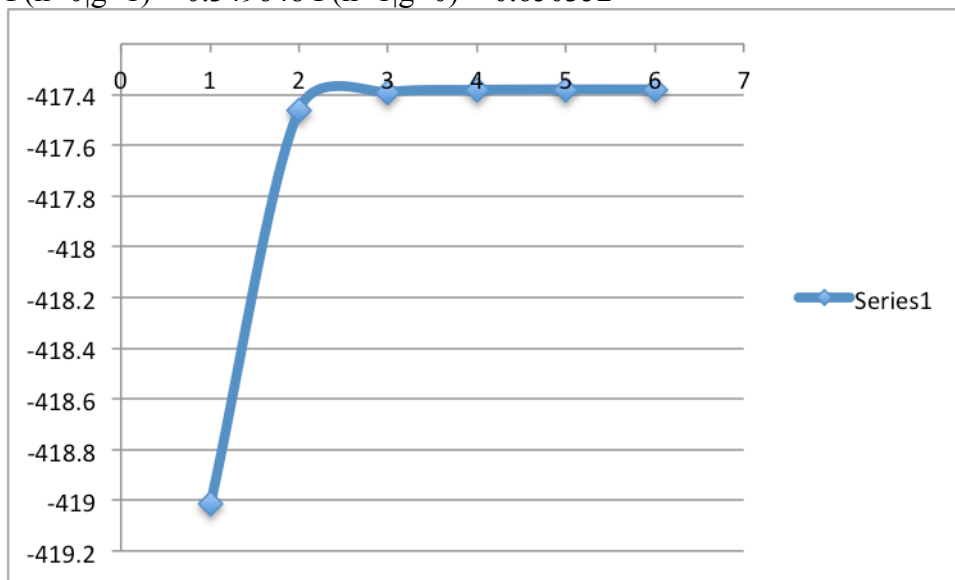
$P(w=0|g=0) = 0.785845$   $P(w=1|g=0) = 0.214155$

$P(w=0|g=1) = 0.374884$   $P(w=1|g=1) = 0.625116$

Height(h) given gender(g) table

$P(h=0|g=0) = 0.650922$   $P(h=1|g=0) = 0.349078$

$P(h=0|g=1) = 0.349648$   $P(h=1|g=0) = 0.650352$



### Data set with missing rate being 50%

The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.85$$

$$P(w=0|g=0) = 0.42$$

$$P(w=0|g=1) = 0.23$$

$$P(h=0|g=0) = 0.94$$

$$P(h=0|g=1) = 0.6$$

The final conditional probability tables for hw2dataset\_50.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.663596 \quad P(\text{gender}=\text{F}) = 0.336404$$

Weight(w) given gender(g) table

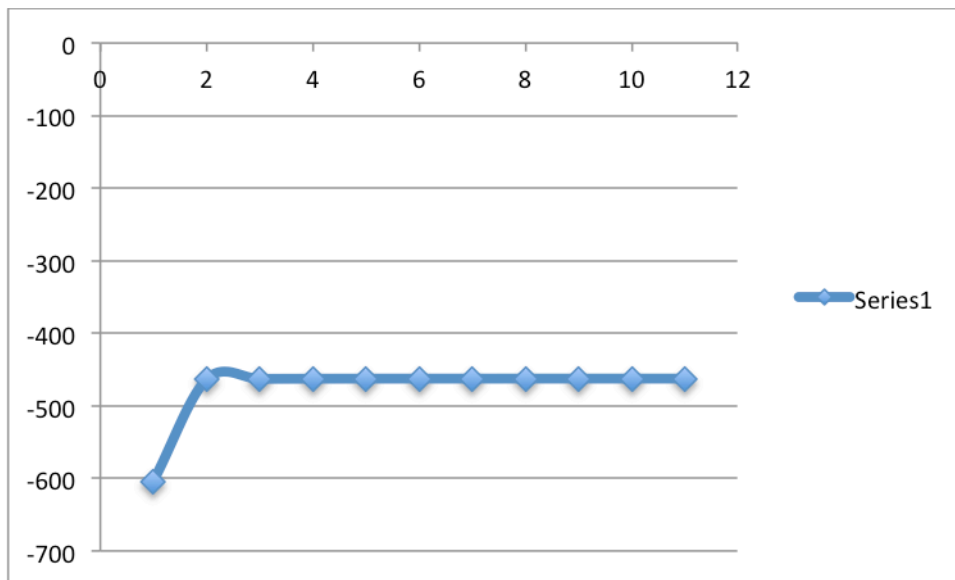
$$P(w=0|g=0) = 0.743306 \quad P(w=1|g=0) = 0.256694$$

$$P(w=0|g=1) = 0.480807 \quad P(w=1|g=1) = 0.519193$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.688703 \quad P(h=1|g=0) = 0.311297$$

$$P(h=0|g=1) = 0.335845 \quad P(h=1|g=0) = 0.664155$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.99$$

$$P(w=0|g=0) = 0.29$$

$$P(w=0|g=1) = 0.03$$

$$P(h=0|g=0) = 0.01$$

$$P(h=0|g=1) = 0.35$$

The final conditional probability tables for hw2dataset\_50.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.663445 \quad P(\text{gender}=\text{F}) = 0.336555$$

Weight(w) given gender(g) table

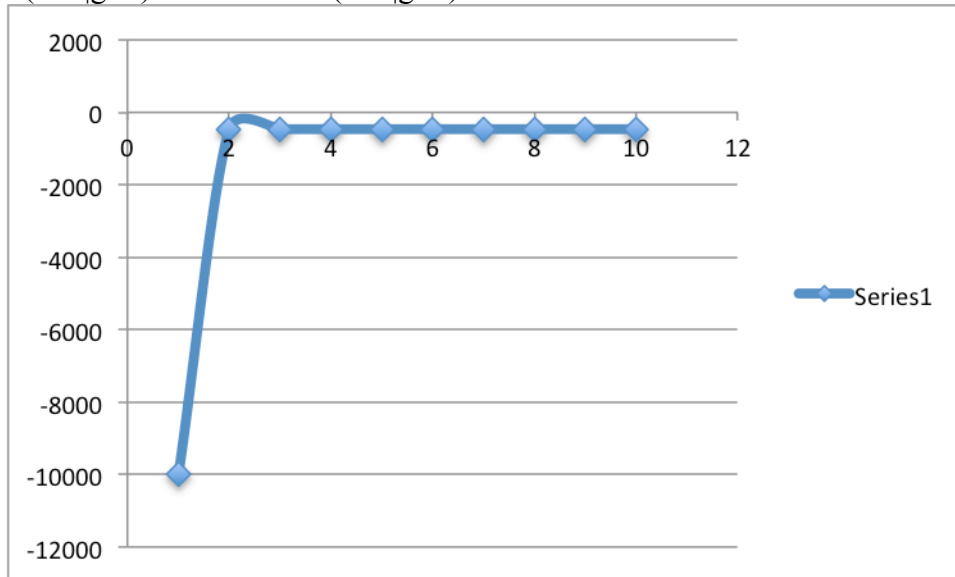
$P(w=0|g=0) = 0.743666$   $P(w=1|g=0) = 0.256334$

$P(w=0|g=1) = 0.480214$   $P(w=1|g=1) = 0.519786$

Height(h) given gender(g) table

$P(h=0|g=0) = 0.688383$   $P(h=1|g=0) = 0.311617$

$P(h=0|g=1) = 0.336634$   $P(h=1|g=1) = 0.663366$



The starting points of the learning:

$P(\text{gender}=\text{M}) = 0.55$

$P(w=0|g=0) = 0.63$

$P(w=0|g=1) = 0.8$

$P(h=0|g=0) = 0.15$

$P(h=0|g=1) = 0.96$

The final conditional probability tables for hw2dataset\_50.txt is:

Gender table

$P(\text{gender}=\text{M}) = 0.662266$   $P(\text{gender}=\text{F}) = 0.337734$

Weight(w) given gender(g) table

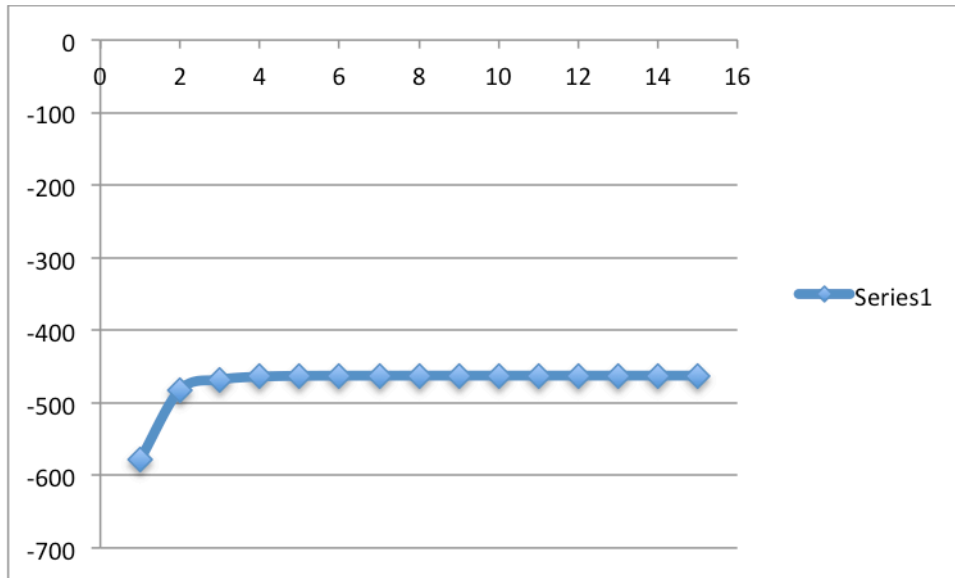
$P(w=0|g=0) = 0.744622$   $P(w=1|g=0) = 0.255378$

$P(w=0|g=1) = 0.479259$   $P(w=1|g=1) = 0.520741$

Height(h) given gender(g) table

$P(h=0|g=0) = 0.68798$   $P(h=1|g=0) = 0.31202$

$P(h=0|g=1) = 0.338651$   $P(h=1|g=1) = 0.661349$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.47$$

$$P(w=0|g=0) = 0.2$$

$$P(w=0|g=1) = 0.22$$

$$P(h=0|g=0) = 0.26$$

$$P(h=0|g=1) = 0.75$$

The final conditional probability tables for hw2dataset\_50.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.66226 \quad P(\text{gender}=\text{F}) = 0.33774$$

Weight(w) given gender(g) table

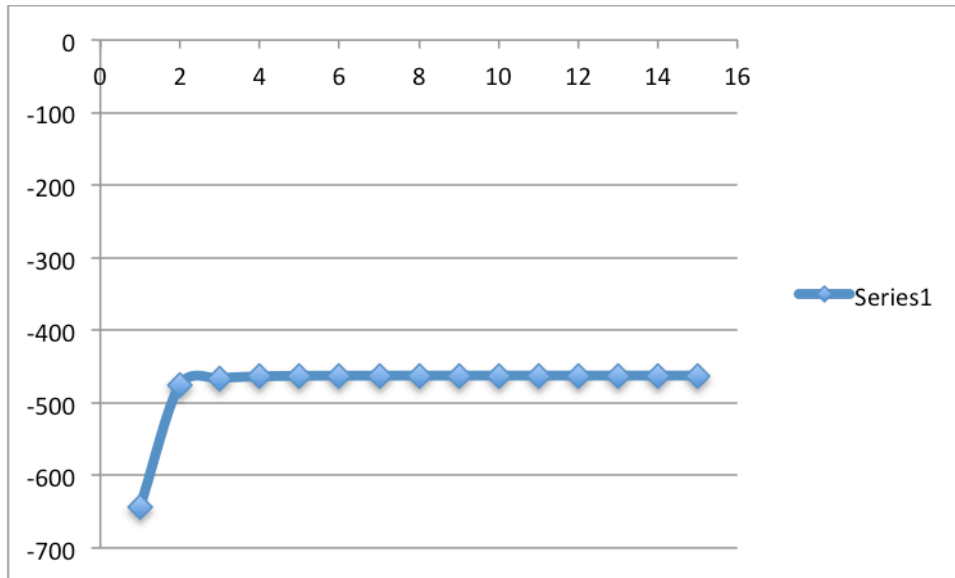
$$P(w=0|g=0) = 0.744626 \quad P(w=1|g=0) = 0.255374$$

$$P(w=0|g=1) = 0.479256 \quad P(w=1|g=1) = 0.520744$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.687979 \quad P(h=1|g=0) = 0.312021$$

$$P(h=0|g=1) = 0.338659 \quad P(h=1|g=1) = 0.661341$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.42$$

$$P(w=0|g=0) = 0.68$$

$$P(w=0|g=1) = 0.19$$

$$P(h=0|g=0) = 0.23$$

$$P(h=0|g=1) = 0.63$$

The final conditional probability tables for hw2dataset\_50.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.66234 \quad P(\text{gender}=\text{F}) = 0.33766$$

Weight(w) given gender(g) table

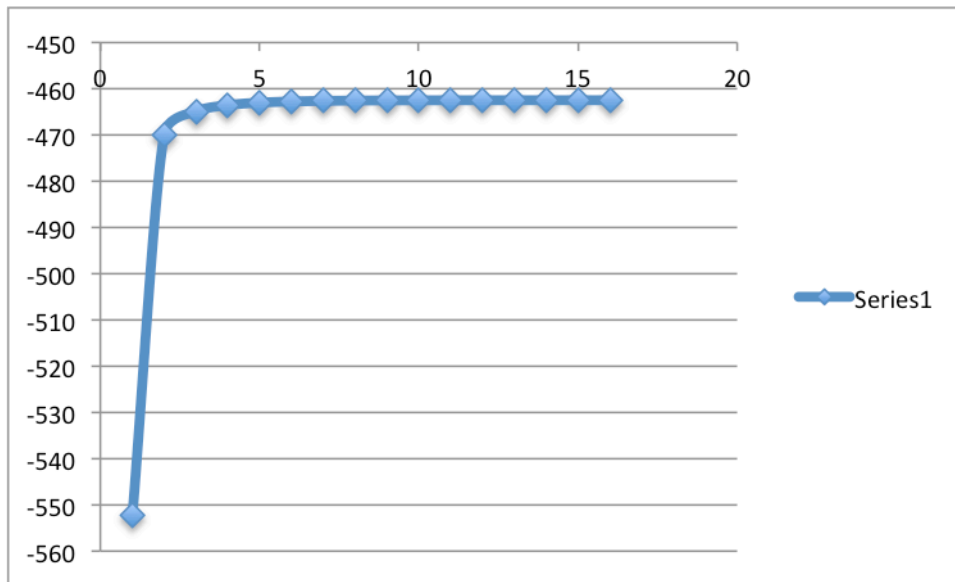
$$P(w=0|g=0) = 0.744551 \quad P(w=1|g=0) = 0.255449$$

$$P(w=0|g=1) = 0.479341 \quad P(w=1|g=1) = 0.520659$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.68802 \quad P(h=1|g=0) = 0.31198$$

$$P(h=0|g=1) = 0.338497 \quad P(h=1|g=1) = 0.661503$$



The final conditional probability tables for hw2dataset\_50.txt is:

Gender table

$P(\text{gender}=\text{M}) = 0.662702$   $P(\text{gender}=\text{F}) = 0.337298$

Weight(w) given gender(g) table

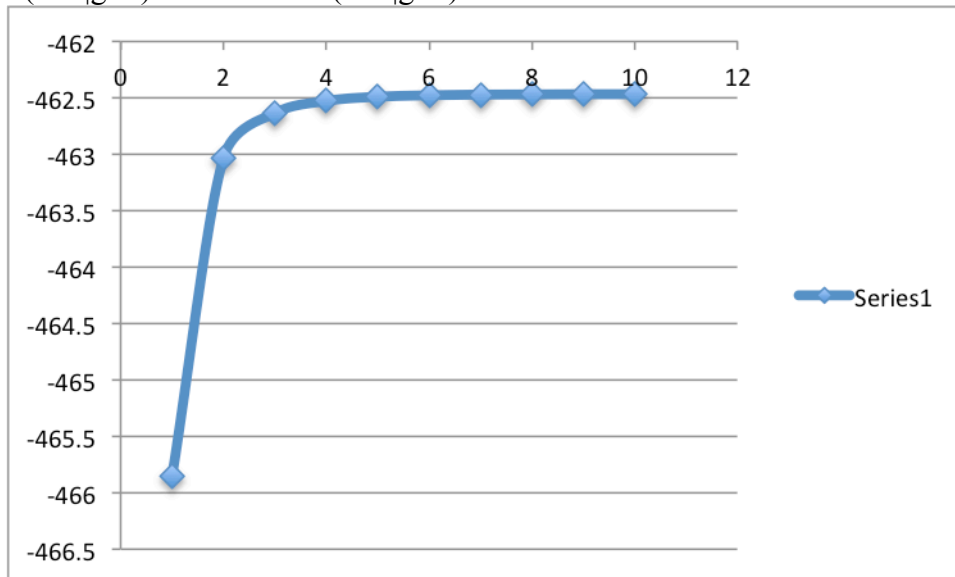
$P(w=0|g=0) = 0.744231$   $P(w=1|g=0) = 0.255769$

$P(w=0|g=1) = 0.479684$   $P(w=1|g=1) = 0.520316$

Height(h) given gender(g) table

$P(h=0|g=0) = 0.688183$   $P(h=1|g=0) = 0.311817$

$P(h=0|g=1) = 0.337801$   $P(h=1|g=1) = 0.662199$



### Data set with missing rate being 70%

The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.67$$

$$P(w=0|g=0) = 0.62$$

$$P(w=0|g=1) = 0.9$$

$$P(h=0|g=0) = 0.13$$

$$P(h=0|g=1) = 0.42$$

The final conditional probability tables for hw2dataset\_70.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.690218 \quad P(\text{gender}=\text{F}) = 0.309782$$

Weight(w) given gender(g) table

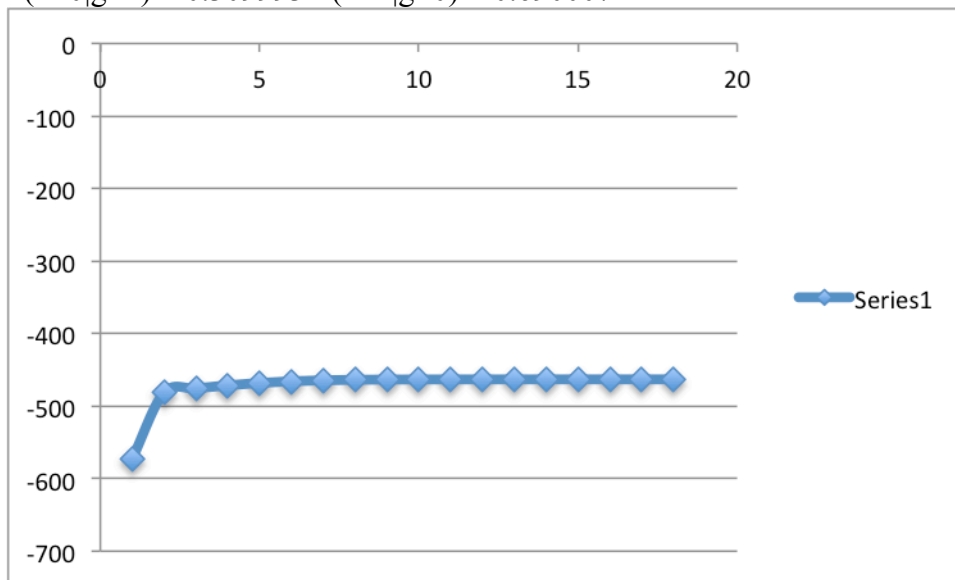
$$P(w=0|g=0) = 0.829367 \quad P(w=1|g=0) = 0.170633$$

$$P(w=0|g=1) = 0.4279 \quad P(w=1|g=1) = 0.5721$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.657719 \quad P(h=1|g=0) = 0.342281$$

$$P(h=0|g=1) = 0.309993 \quad P(h=1|g=1) = 0.690007$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.7$$

$$P(w=0|g=0) = 0.84$$

$$P(w=0|g=1) = 0.01$$

$$P(h=0|g=0) = 0.22$$

$$P(h=0|g=1) = 0.69$$

The final conditional probability tables for hw2dataset\_70.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.691768 \quad P(\text{gender}=\text{F}) = 0.308232$$

Weight(w) given gender(g) table

$$P(w=0|g=0) = 0.829539 \quad P(w=1|g=0) = 0.170461$$

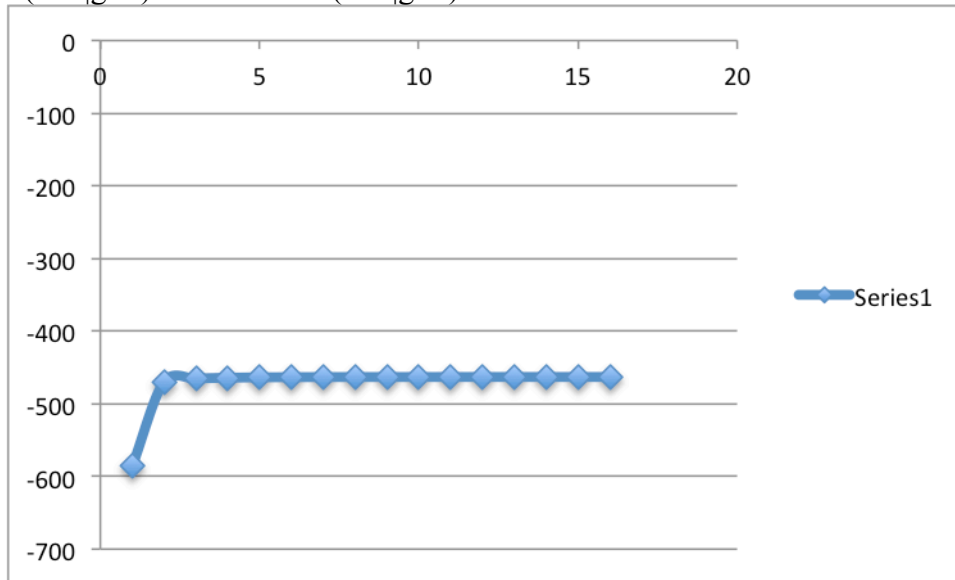


$$P(w=0|g=1) = 0.425494 \quad P(w=1|g=1) = 0.574506$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.656748 \quad P(h=1|g=0) = 0.343252$$

$$P(h=0|g=1) = 0.310424 \quad P(h=1|g=1) = 0.689576$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.63$$

$$P(w=0|g=0) = 0.24$$

$$P(w=0|g=1) = 0.03$$

$$P(h=0|g=0) = 0.02$$

$$P(h=0|g=1) = 0.95$$

The final conditional probability tables for hw2dataset\_70.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.690471 \quad P(\text{gender}=\text{F}) = 0.309529$$

Weight(w) given gender(g) table

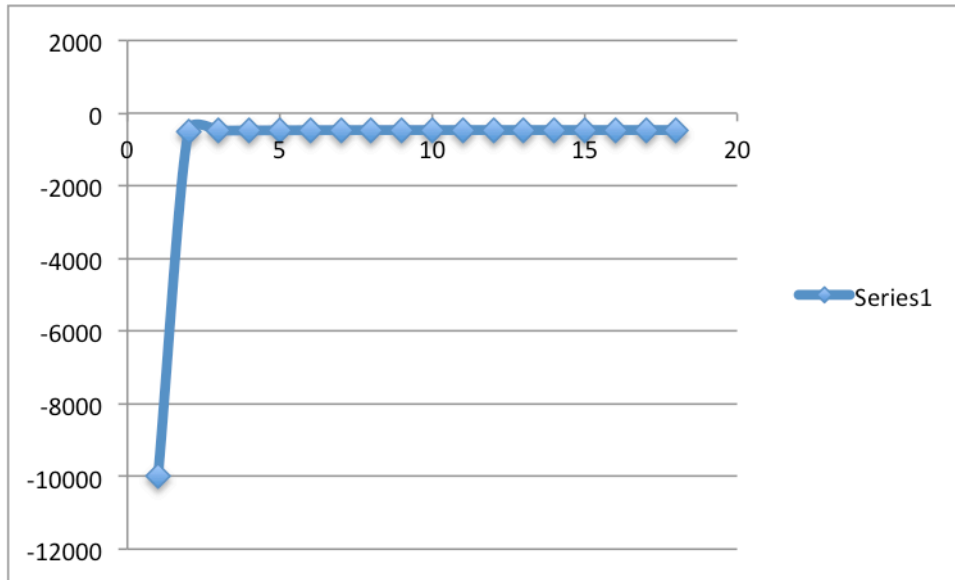
$$P(w=0|g=0) = 0.829383 \quad P(w=1|g=0) = 0.170617$$

$$P(w=0|g=1) = 0.427537 \quad P(w=1|g=1) = 0.572463$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.657549 \quad P(h=1|g=0) = 0.342451$$

$$P(h=0|g=1) = 0.310089 \quad P(h=1|g=1) = 0.689911$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.65$$

$$P(w=0|g=0) = 0.21$$

$$P(w=0|g=1) = 0.26$$

$$P(h=0|g=0) = 0.57$$

$$P(h=0|g=1) = 0.13$$

The final conditional probability tables for hw2dataset\_70.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.689193 \quad P(\text{gender}=\text{F}) = 0.310807$$

Weight(w) given gender(g) table

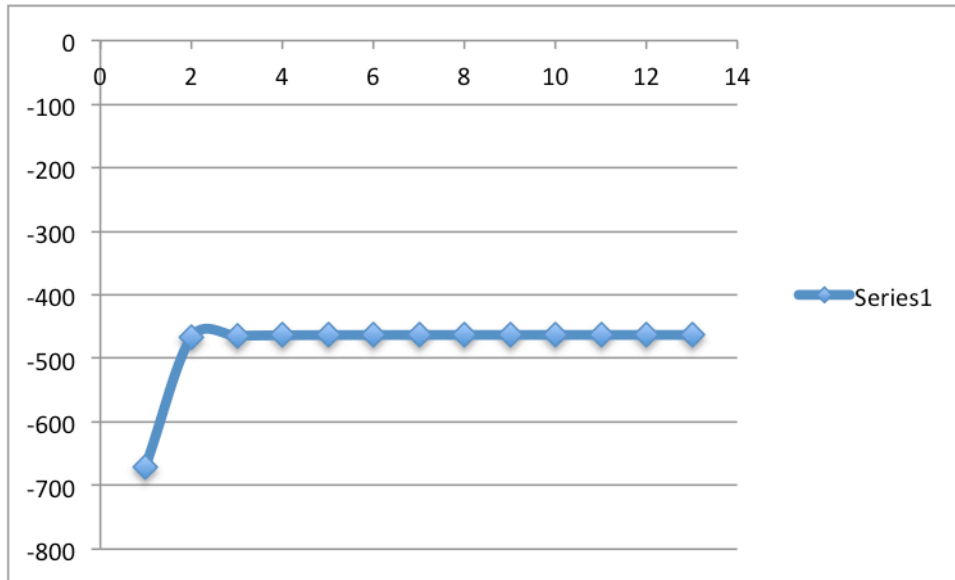
$$P(w=0|g=0) = 0.829232 \quad P(w=1|g=0) = 0.170768$$

$$P(w=0|g=1) = 0.429524 \quad P(w=1|g=1) = 0.570476$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.658431 \quad P(h=1|g=0) = 0.341569$$

$$P(h=0|g=1) = 0.309562 \quad P(h=1|g=1) = 0.690438$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.25$$

$$P(w=0|g=0) = 0.24$$

$$P(w=0|g=1) = 0.46$$

$$P(h=0|g=0) = 0.58$$

$$P(h=0|g=1) = 0.92$$

The final conditional probability tables for hw2dataset\_70.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.689672 \quad P(\text{gender}=\text{F}) = 0.310328$$

Weight(w) given gender(g) table

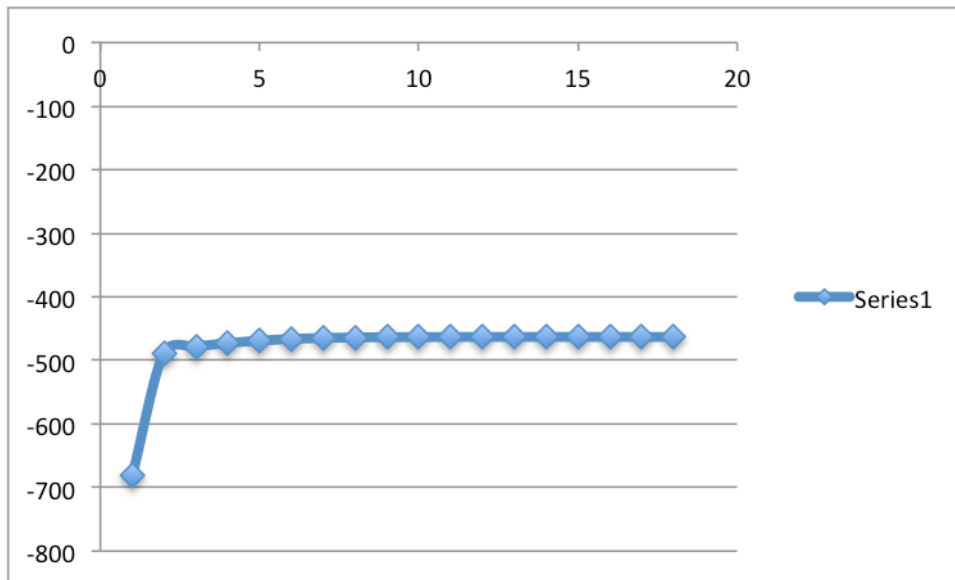
$$P(w=0|g=0) = 0.829321 \quad P(w=1|g=0) = 0.170679$$

$$P(w=0|g=1) = 0.42871 \quad P(w=1|g=1) = 0.57129$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.658064 \quad P(h=1|g=0) = 0.341936$$

$$P(h=0|g=1) = 0.30984 \quad P(h=1|g=1) = 0.69016$$



The final conditional probability tables for hw2dataset\_70.txt is:

Gender table

$P(\text{gender}=\text{M}) = 0.690582$   $P(\text{gender}=\text{F}) = 0.309418$

Weight(w) given gender(g) table

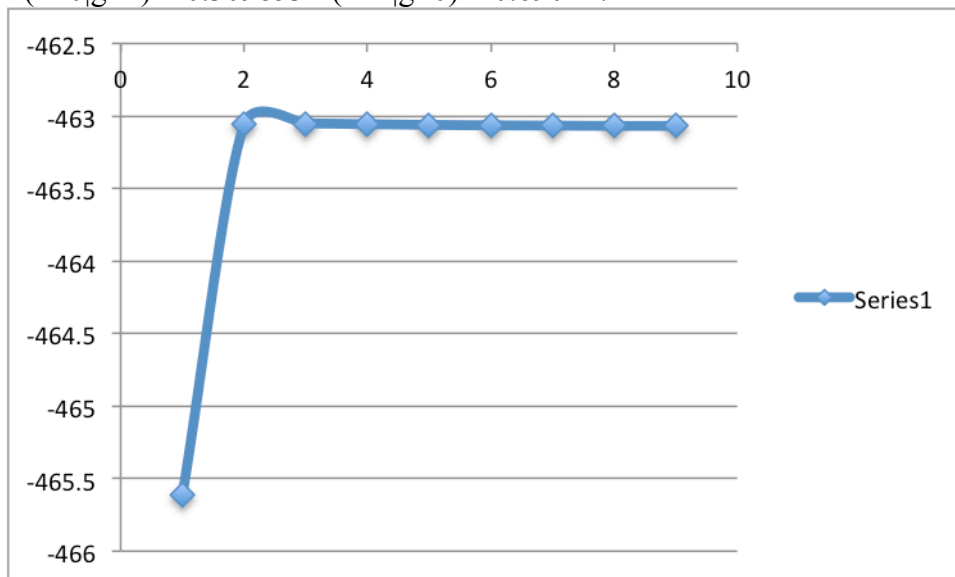
$P(w=0|g=0) = 0.829364$   $P(w=1|g=0) = 0.170636$

$P(w=0|g=1) = 0.427436$   $P(w=1|g=1) = 0.572564$

Height(h) given gender(g) table

$P(h=0|g=0) = 0.657599$   $P(h=1|g=0) = 0.342401$

$P(h=0|g=1) = 0.309853$   $P(h=1|g=1) = 0.690147$



### Data set with missing rate being 100%

The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.29$$

$$P(w=0|g=0) = 0.18$$

$$P(w=0|g=1) = 0.92$$

$$P(h=0|g=0) = 0.21$$

$$P(h=0|g=1) = 0.75$$

The final conditional probability tables for hw2dataset\_100.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.298406 \quad P(\text{gender}=\text{F}) = 0.701594$$

Weight(w) given gender(g) table

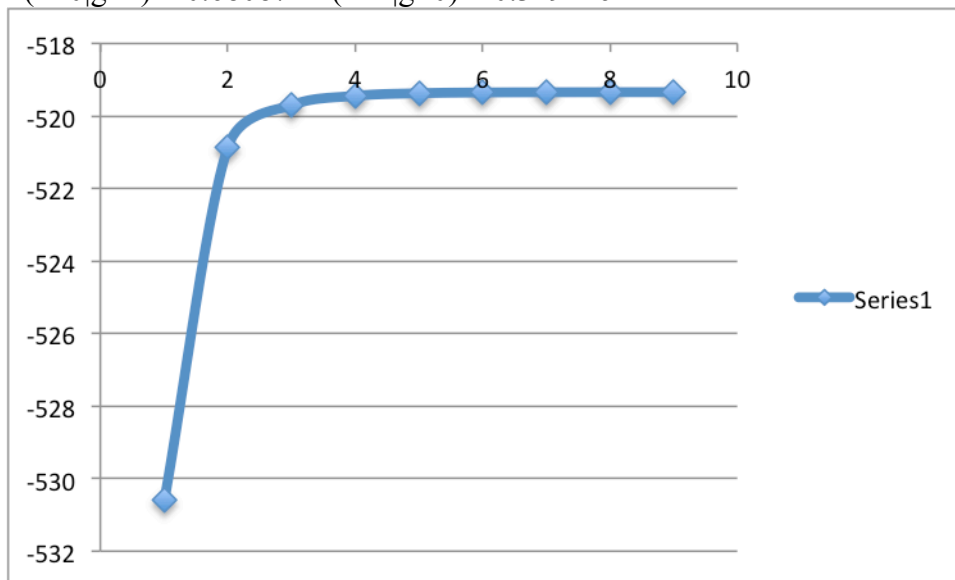
$$P(w=0|g=0) = 0.205357 \quad P(w=1|g=0) = 0.794643$$

$$P(w=0|g=1) = 0.860498 \quad P(w=1|g=1) = 0.139502$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.393098 \quad P(h=1|g=0) = 0.606902$$

$$P(h=0|g=1) = 0.680874 \quad P(h=1|g=0) = 0.319126$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.26$$

$$P(w=0|g=0) = 0.27$$

$$P(w=0|g=1) = 0.37$$

$$P(h=0|g=0) = 0.35$$

$$P(h=0|g=1) = 0.19$$

The final conditional probability tables for hw2dataset\_100.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.341385 \quad P(\text{gender}=\text{F}) = 0.658615$$

Weight(w) given gender(g) table

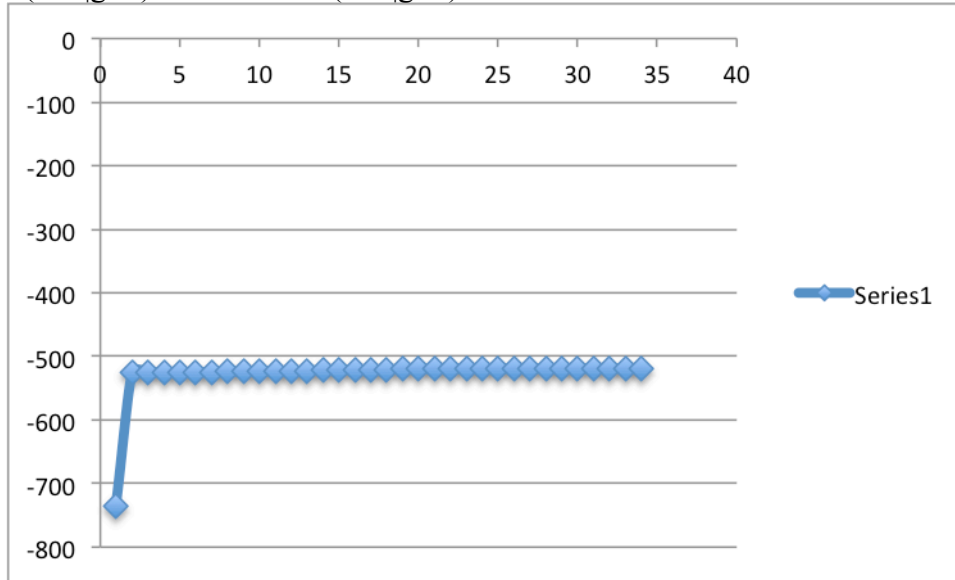
$$P(w=0|g=0) = 0.922035 \quad P(w=1|g=0) = 0.0779652$$

$$P(w=0|g=1) = 0.531769 \quad P(w=1|g=1) = 0.468231$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.8863 \quad P(h=1|g=0) = 0.1137$$

$$P(h=0|g=1) = 0.444009 \quad P(h=1|g=1) = 0.555991$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.24$$

$$P(w=0|g=0) = 0.08$$

$$P(w=0|g=1) = 0.17$$

$$P(h=0|g=0) = 0.56$$

$$P(h=0|g=1) = 0.72$$

The final conditional probability tables for hw2dataset\_100.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.216463 \quad P(\text{gender}=\text{F}) = 0.783537$$

Weight(w) given gender(g) table

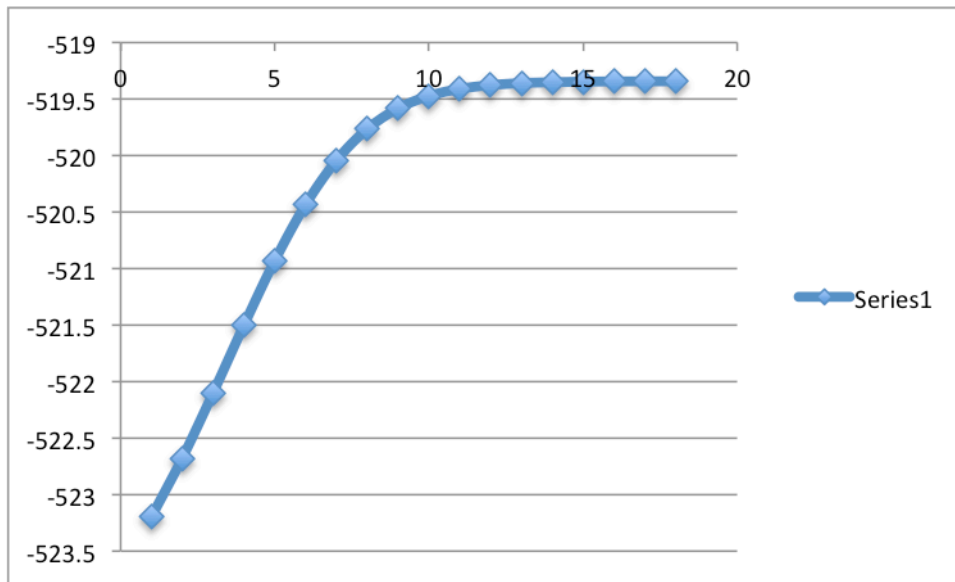
$$P(w=0|g=0) = 0.283082 \quad P(w=1|g=0) = 0.716918$$

$$P(w=0|g=1) = 0.77051 \quad P(w=1|g=1) = 0.22949$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.224495 \quad P(h=1|g=0) = 0.775505$$

$$P(h=0|g=1) = 0.697357 \quad P(h=1|g=1) = 0.302643$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.17$$

$$P(w=0|g=0) = 0.48$$

$$P(w=0|g=1) = 0.19$$

$$P(h=0|g=0) = 0.83$$

$$P(h=0|g=1) = 0.51$$

The final conditional probability tables for hw2dataset\_100.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.309605 \quad P(\text{gender}=\text{F}) = 0.690395$$

Weight(w) given gender(g) table

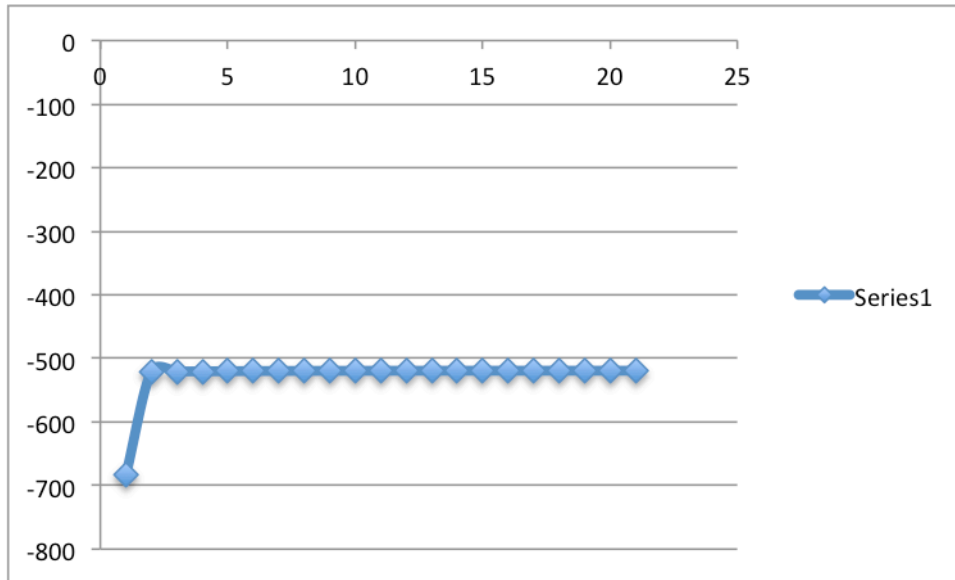
$$P(w=0|g=0) = 0.937714 \quad P(w=1|g=0) = 0.062286$$

$$P(w=0|g=1) = 0.542703 \quad P(w=1|g=1) = 0.457297$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.911686 \quad P(h=1|g=0) = 0.0883142$$

$$P(h=0|g=1) = 0.452984 \quad P(h=1|g=1) = 0.547016$$



The starting points of the learning:

$$P(\text{gender}=\text{M}) = 0.47$$

$$P(w=0|g=0) = 0.94$$

$$P(w=0|g=1) = 0.93$$

$$P(h=0|g=0) = 0.76$$

$$P(h=0|g=1) = 0.95$$

The final conditional probability tables for hw2dataset\_100.txt is:

Gender table

$$P(\text{gender}=\text{M}) = 0.550264 \quad P(\text{gender}=\text{F}) = 0.449736$$

Weight(w) given gender(g) table

$$P(w=0|g=0) = 0.518737 \quad P(w=1|g=0) = 0.481263$$

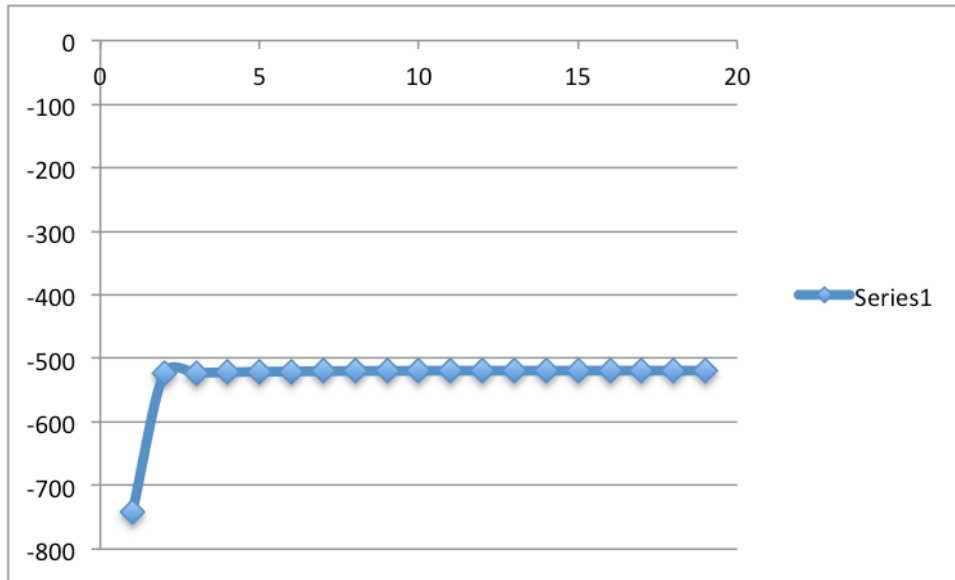
$$P(w=0|g=1) = 0.843957 \quad P(w=1|g=1) = 0.156043$$

Height(h) given gender(g) table

$$P(h=0|g=0) = 0.377 \quad P(h=1|g=0) = 0.623$$

$$P(h=0|g=1) = 0.861728 \quad P(h=1|g=1) = 0.138272$$





The final conditional probability tables for hw2dataset\_100.txt is:

Gender table

$P(\text{gender}=\text{M}) = 0.696858$   $P(\text{gender}=\text{F}) = 0.303142$

Weight(w) given gender(g) table

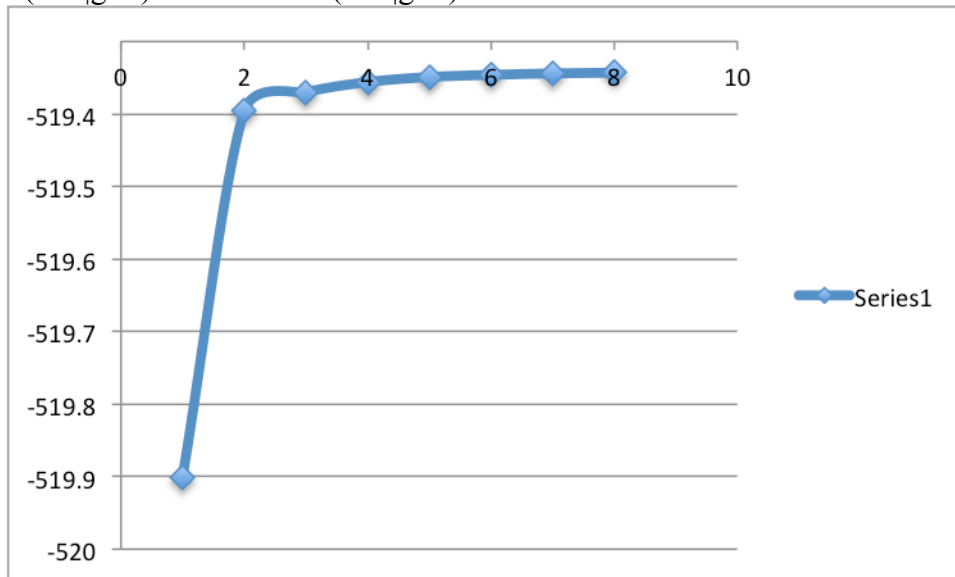
$P(w=0|g=0) = 0.796089$   $P(w=1|g=0) = 0.203911$

$P(w=0|g=1) = 0.363655$   $P(w=1|g=1) = 0.636345$

Height(h) given gender(g) table

$P(h=0|g=0) = 0.724454$   $P(h=1|g=0) = 0.275546$

$P(h=0|g=1) = 0.297413$   $P(h=1|g=1) = 0.702587$



## Analysis

1. Do multiple starting points help in finding better solutions?

Yes, multiple starting points help in finding better solutions. Because for EM Algorithm, it only guarantees local maxima, there might exist a better global maxima. From multiple starting points result above, though in general the solution falls into the same range, but the bad starting points occur, the solution is not accurate.

2. Do some of the different solutions have the same likelihood scores?

Yes. For example, for data set with missing rates being 30%, the third and fourth solutions are different but they have the same likelihood scores.

3. How does the data missing rate affect your algorithm and the results?

Generally, when data missing rate is low, it takes less time to converge and the results are more accurate. The higher the missing rate is, the less accurate results are. Worst case is when missing rate is 100%, there are many solutions.

4. When starting points are in reasonably guessing, they are faster to converge.

5. Avoiding bad local maxima by multiple restarts and early pruning of unpromising starting points.