To Infinity and Beyond: Forecasting with Dynamic Models

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Introduction

How do you forecast?

What models do you use?

The typical list includes:

- Exponential Smoothing (ETS) (Hyndman et al., 2008);
- Regression;
- ARIMA (Box and Jenkins, 1976);
- Your favourite method here;
- Other (e.g. SMA (Svetunkov and Petropoulos, 2018), ML methods).

Introduction

Almost all textbooks tell you that ETS, ARIMA and Regression are distinct models.

They are formulated differently.

In some cases some of them can be combined in one model: ETSX / ARIMAX.

But in general they are not comparable (e.g. via information criteria).

This is all lies!

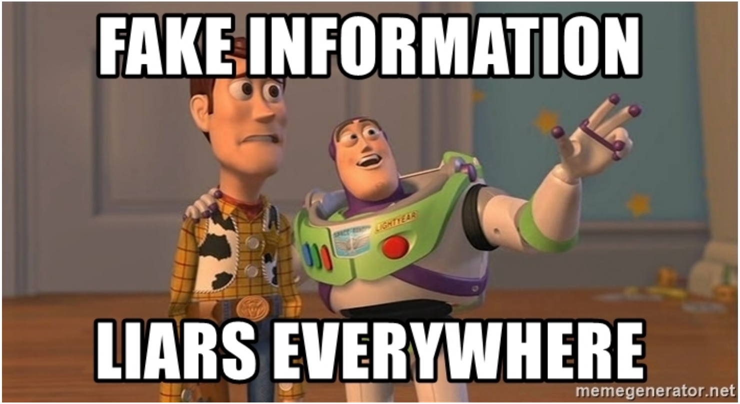


Figure: Don't believe your statistics textbooks!

Introduction

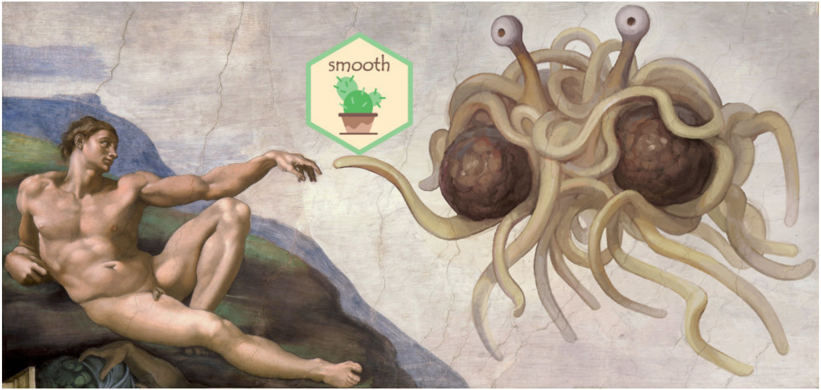
smooth package has implemented ARIMA and ETS in the state space form

The models `es()`, `ssarima()`, `msarima()`, `sma()` and `ces()` are directly comparable (via IC).

But they have limitaions:

- You cannot have ETS with ARIMA,
- You can only work with normal distribution,
- No time varying parameters,
- No multiple seasonalities,
- ...

Introducing ADAM



What is ADAM?

ADAM is Augmented Dynamic Adaptive Model.

It is a Single Source of Error state space model, implementing:

1. Exponential Smoothing (ETS);
2. ARIMA;
3. Regression and TVP regression;
4. Combination of (1), (2) and (3);
5. Components, variables and orders selection;
6. Normal and non-normal distributions;
7. Advanced and custom losses;
8. ...

What is ADAM?

ADAM is the instrument for:

- Benchmarking;
- Experimenting;
- Prototyping;
- Research.

All the details and technical aspects are summarised in the online textbook: `https://openforecast.org/adam/` (work in progress).

What is ADAM?

Instead of going through all the features, we will consider several case studies:

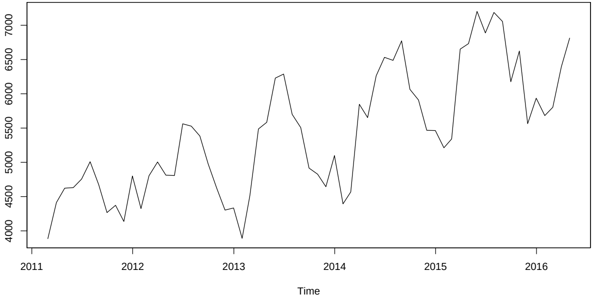
1. Fast Moving Consumer Goods (FMCG);
2. Promotional modelling;
3. Intermittent demand;
4. Multiple seasonal data.

Fast Moving Consumer Goods

Figure: When products sell fast enough

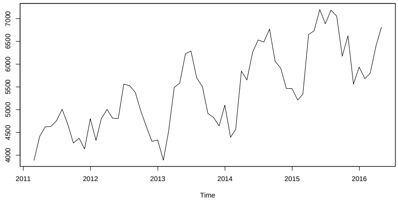
Fast Moving Consumer Goods

Sales of household product on a store level.



Use last year as the test set.

Fast Moving Consumer Goods



Things to note:

- The level changes over time,
- We have seasonality,
- Both seem to evolve over time

Use ETS Hyndman et al. (2008).

Fast Moving Consumer Goods

Use ETS with automatic components selection:

```
adamModel <- adam(y, h=12, holdout=TRUE, distribution="dnorm")
```

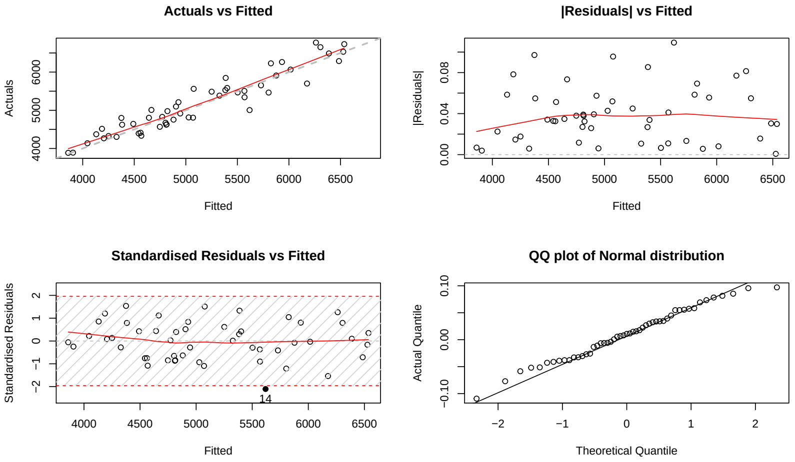
ETS(M,N,M):

- $\alpha = 0.882$, $\gamma = 0.003$;
- AICc=761.453
- For the holdout: MASE=0.790, RMSSE=0.739.

What about the analysis of the residuals?

```
plot(adamModel)
```

Fast Moving Consumer Goods



Fast Moving Consumer Goods

Observation number 14 is beyond the bounds.

This is fine, because 5% of values should lie outside.

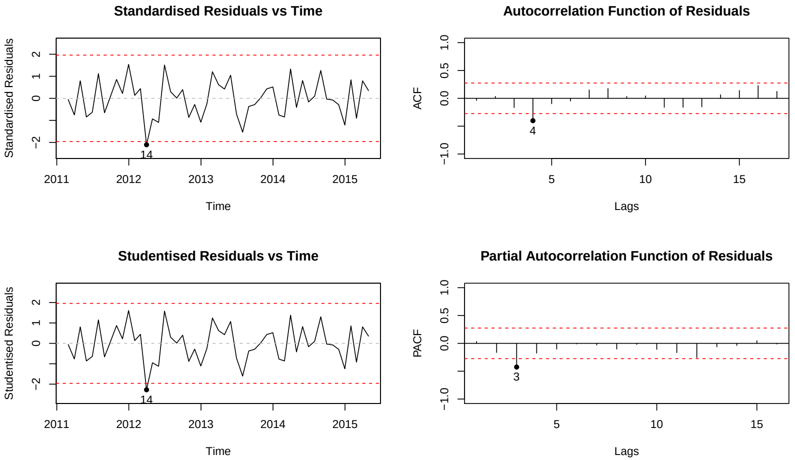
LOESS line on fitted vs Residuals is not straight.

But this could be because of randomness.

Further analysis:

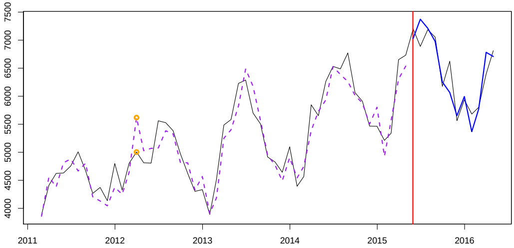
```
plot(adamModel, c(8,9,10,11))
```

Fast Moving Consumer Goods



Fast Moving Consumer Goods

- Observation 14 seems not random, it is followed by several other;
- Lags 3 and 4 PACF / ACF are significant, but this could be due to randomness;



Fast Moving Consumer Goods

Just to check, create the dummy variable for the outlier:

```
xreg <- cbind(y=y,x=0)
xreg$x[14] <- 1
adamModel2 <- adam(xreg, lags=12, h=12, holdout=T,
distribution="dnorm")
```

ETSX(M,N,M):

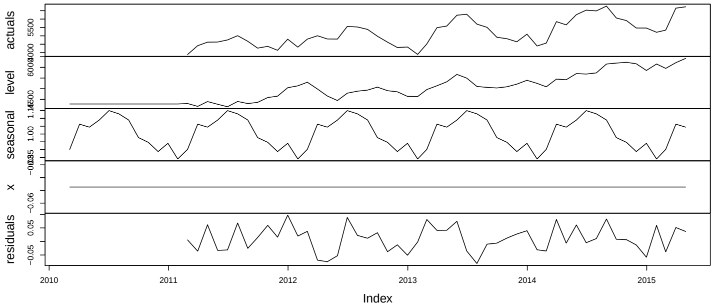
- $\alpha = 0.908$, $\gamma = 0.004$;
- AICc=768.888
- For the holdout: MASE=0.785, RMSSE=0.750.

Not worth it! ETS(M,N,M) is good enough.

Fast Moving Consumer Goods

Time series decomposition according to ETS(M,N,M):

```
plot(adamModel, 12)
```



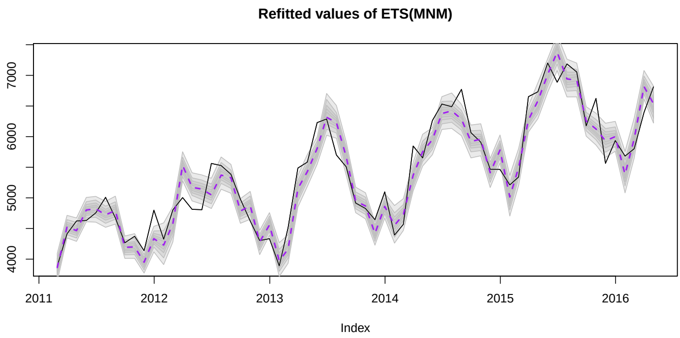
Fast Moving Consumer Goods, summary of the model

```
Model estimated using adam() function: ETS(MNM)
Response variable: data
Distribution used in the estimation: Normal
Loss function type: likelihood; Loss function value: 359.06
Coefficients:
             Estimate Std. Error Lower 2.5% Upper 97.5%
alpha          0.8816     0.1262     0.6259      1.0000
gamma          0.0031     0.1156     0.0000      0.1184
level       4282.1276   228.6296  3818.8800   4738.4743
seasonal_1     0.9025     0.0180     0.8661      0.9511
seasonal_2     1.0580     0.0194     1.0217      1.1067
seasonal_3     1.0445     0.0210     1.0082      1.0932
seasonal_4     1.0980     0.0223     1.0616      1.1466
seasonal_5     1.1486     0.0237     1.1122      1.1972
seasonal_6     1.1244     0.0244     1.0880      1.1730
seasonal_7     1.0916     0.0237     1.0552      1.1402
seasonal_8     0.9787     0.0210     0.9423      1.0273
seasonal_9     0.9490     0.0205     0.9126      0.9976
seasonal_10    0.8852     0.0182     0.8488      0.9338
seasonal_11    0.9394     0.0202     0.9030      0.9880

Sample size: 52
Number of estimated parameters: 15
Number of degrees of freedom: 37
Information criteria:
     AIC     AICc      BIC     BICc
748.1199 761.4533 777.3886 803.7302
```

Fast Moving Consumer Goods

Refitted line with the uncertainty:

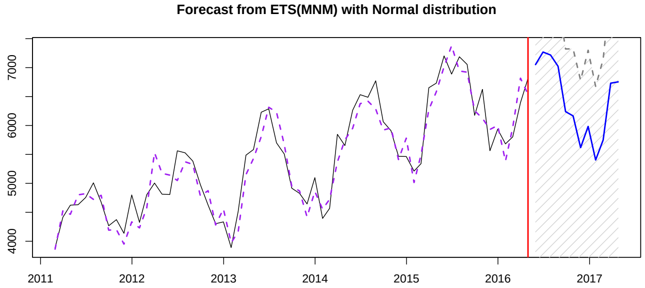
```
plot(refit(adamModel))
```



Fast Moving Consumer Goods

Forecast with the uncertainty of parameters:

```
plot(reforecast(adamModel, h=12, interval="prediction",
side="upper"))
```

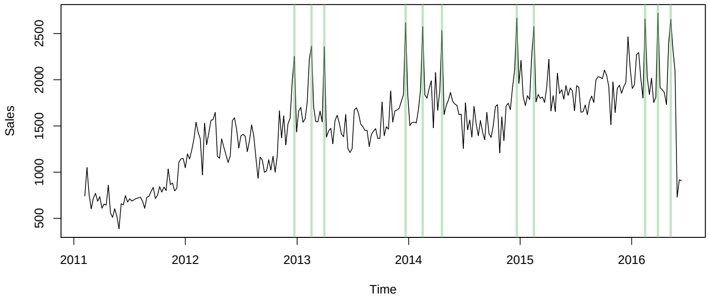


Promotional modelling

Figure: Back yard sale

Promotional modelling

Weekly sales of a food product with some promotions

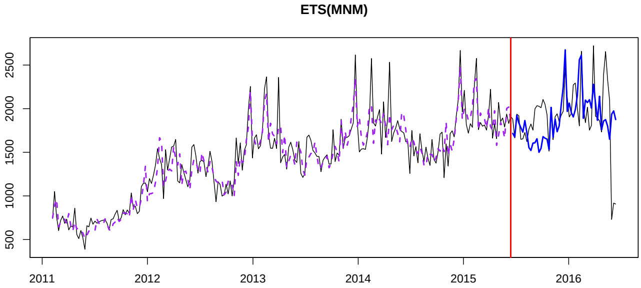


We'll use ETSX (similar to Koehler et al., 2012)

Promotional modelling

We will use automatic selection from pure models:

```
adamModel <- adam(y, "PPP", lags=c(1,52), h=52, holdout=TRUE)
```

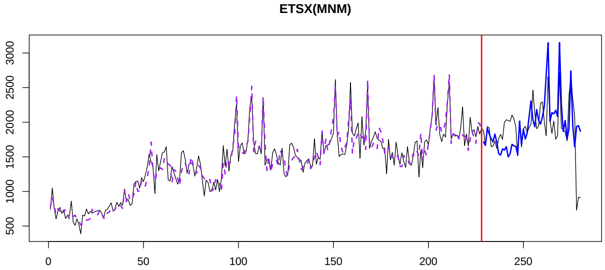


AICc=3072.125, MASE=1.508, RMSSE=1.453, Time=2.6 sec.

Promotional modelling

ETSX(M,N,M) with dummy for promotions and lags:

```
xreg <- data.frame(y, xregExpander(x, lags=-c(1,2), gaps="zero"))
adamModel <- adam(xreg, "MNM", lags=c(1,52), h=52, holdout=TRUE)
```

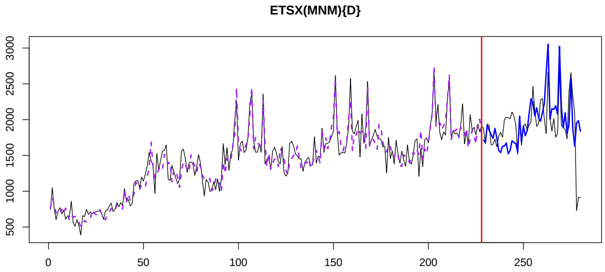


AICc=3040.972, MASE=1.526, RMSSE=1.446, Time=0.59 sec.

Promotional modelling

ETSX(M,N,M) with dynamic parameters:

```
adamModel <- adam(xreg, "MNM", lags=c(1,52), h=52, holdout=TRUE,
regressors="adapt")
```



AICc=3052.961, MASE=1.469, RMSSE=1.404, Time=0.59 sec.

Promotional modelling

Given AICc values, we should stick with the ETSX(M,N,M) with static regressors.

Dynamic regressors introduce additional uncertainty comming from smoothing parameters.

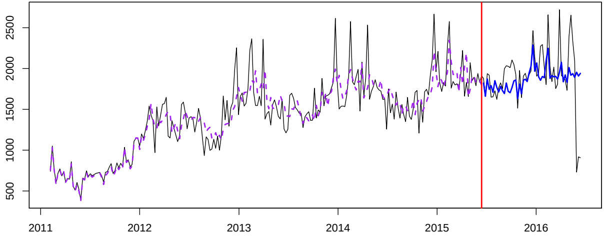
We could also use ARIMA and compare it with ETS:

```
adamModel <- adam(xreg, "NNN", lags=c(1,52),
orders=list(ar=c(3,2),i=c(2,1),ma=c(3,2),select=TRUE),
h=52, holdout=TRUE)
```

ARIMA is still work in progress!

Promotional modelling

SARIMA(0,1,3)(2,0,0)$_{52}$:

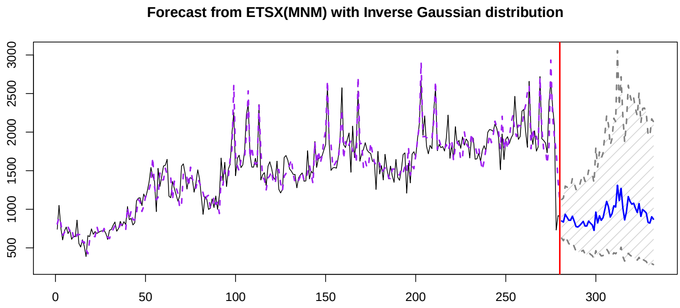


AICc=3493.406, MASE=1.242, RMSSE=1.312, Time=4.49 sec.

Worse than ETSX(M,N,M) based on AICc.

Promotional modelling

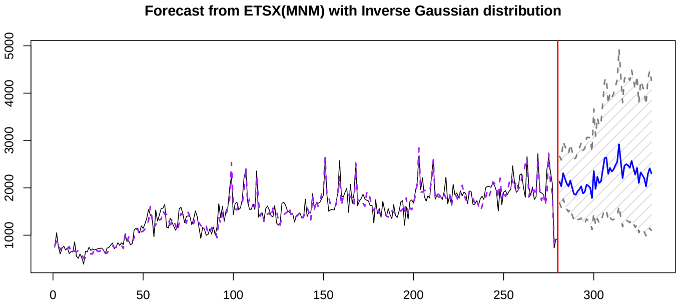
Final forecast from ETSX(M,N,M):



If the last three observations happened by chance, we could include dummies for them as well.

Promotional modelling

Alternative forecast from ETSX(M,N,M):

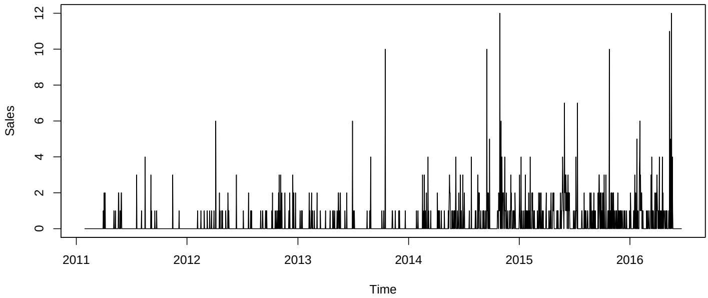


Intermittent demand

Figure: When demand is not continuous anymore

Intermittent demand

A “hobbies” product.



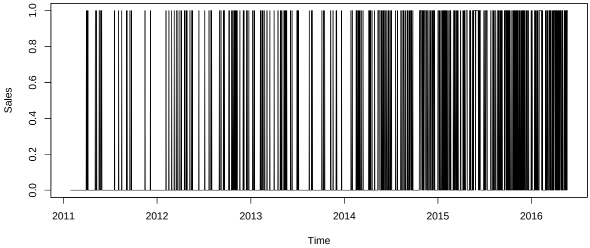
The iETS model is explained in Svetunkov and Boylan (2019)

Intermittent demand

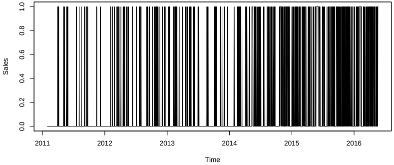
This data can be split into two parts:

1. Demand occurrence part (0 / 1);
2. Demand sizes part;

Here how occurrence changes over time:



Intermittent demand



We see that the occurrence part evolves over time,

The probability of occurrence increases.

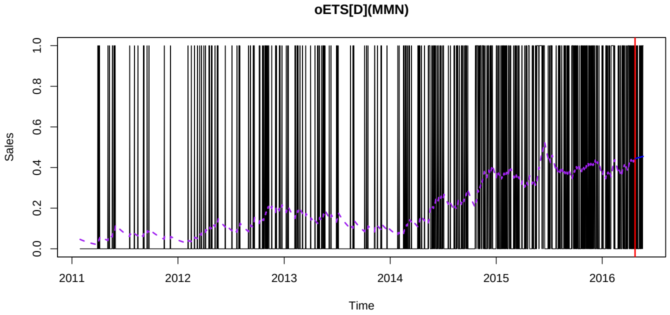
We can assume that it will increase in the holdout.

So we can use the model with trend to predict probability.

Intermittent demand

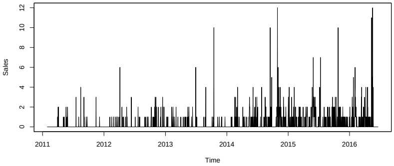
Use oETS(M,M,N) model for that:

```
oesModel <- oes(as.vector(y), "MMN", occurrence="direct",
h=28, holdout=TRUE)
plot(oesModel)
```



Intermittent demand

Now, what about the data itself?



The sizes increase over time. So trend would be suitable.

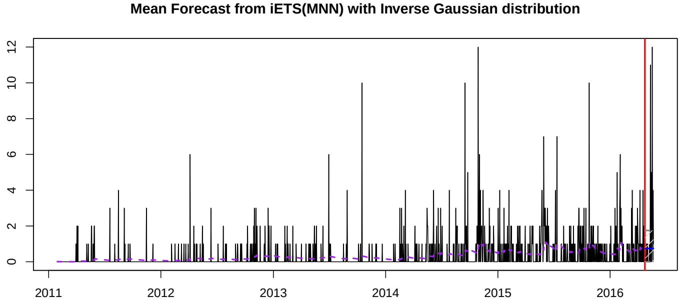
But let's select components automatically.

```
adamModel <- adam(y, "YYY", occurrence=oesModel,
h=28, holdout=TRUE)
```

Intermittent demand

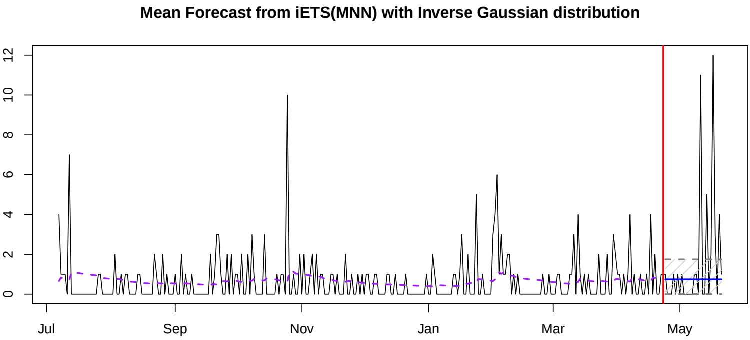
The final forecast (working and safety stocks):

```
plot(forecast(adamModel, h=28, interval="prediction",
nsim=10000, cumulative=TRUE, side="upper"))
```



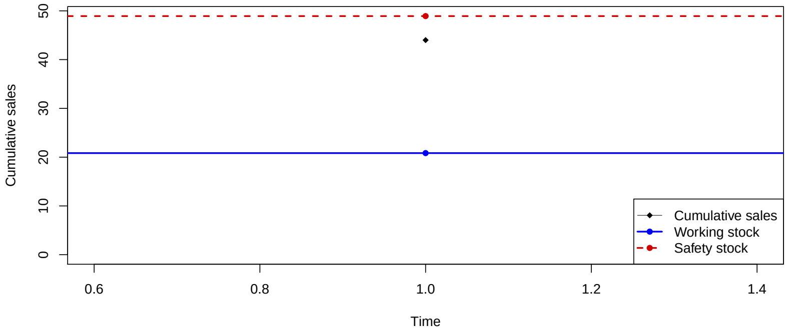
Intermittent demand

The final forecast zoomed in (mean over time values):



Intermittent demand

Still not very informative, so just compare cumulative sales over the 28 days with the forecasts

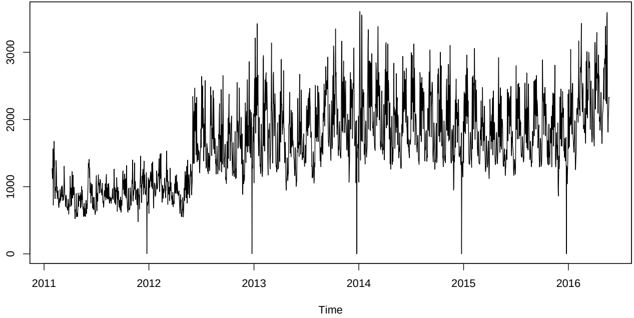


Multiple seasonal

Figure: When there are more than one... seasonals

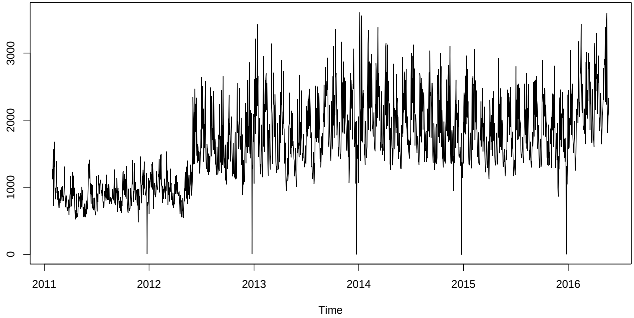
Multiple seasonal

Daily sales of a food product



Inspired by: Taylor (2008); Gould et al. (2008); Taylor (2010); Taylor and Snyder (2012)

Multiple seasonal

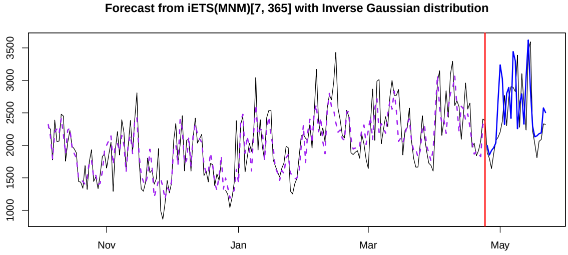


- Daily and yearly seasonality, lags=c(7,365);
- No work on Christmas!
- No obvious trend

Multiple seasonal

Flag Christmas as NA. Use pure multiplicative seasonal model:

```
adamModel <- adam(y, "MNM", lags=c(7,365),
h=28, holdout=TRUE)
```

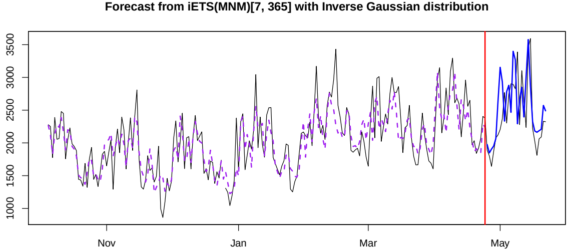


MASE=1.33, RMSSE=1.24, Time=12 seconds, k=375, AICc=26295.68

Multiple seasonal

Alterenative - use backcasting in the initialisation:

```
adamModel <- adam(y, "MNM", lags=c(7,365),
initial="backcasting", h=28, holdout=TRUE)
```



MASE=1.24, RMSSE=1.16, Time=0.2 seconds, k=4

AICc=25577.70 - not comparable with `initial="optimal"`

Multiple seasonal

We have days of week and days of year.

Why not have days of week and weeks of year?

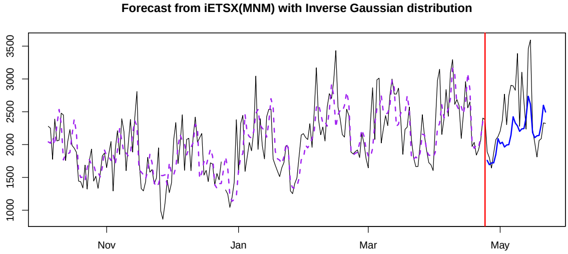
```
weekOfYear <- greybox::temporaldummy(y, type="week", of="year",
factors=TRUE)

xreg <- data.frame(y=y,x=factor(weekOfYear))
```

We could have models with static and dynamic parameters.

Multiple seasonal

Model with static parameters (deterministic weeks of year):

```
adamModel <- adam(xreg, "MNM", lags=c(7),
h=28, holdout=TRUE)
```

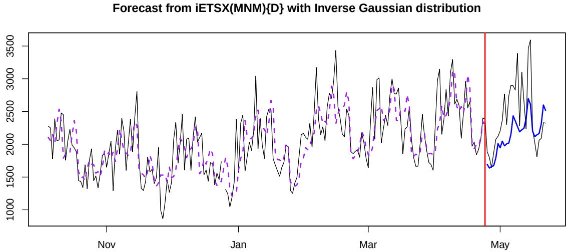


MASE=1.28, RMSSE=1.23, Time=11.2 seconds, k=62
AICc=26776.75

Multiple seasonal

Model with static parameters (stochastic weeks of year):

```
adamModel <- adam(xreg, "MNM", lags=c(7),
h=28, holdout=TRUE)
```

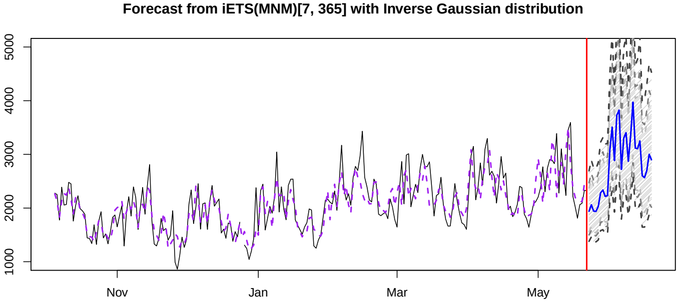


MASE=1.34, RMSSE=1.26, Time=11.4 seconds, k=63
AICc=26782.82

Multiple seasonal

So, double seasonal ETS(M,N,M) model is better.

```
plot(reforecast(adamModel, h=28, interval="prediction"))
```



Conclusions

Figure: Is this the end?

Conclusions

- ADAM is a new flexible model that supports many features;
- It was developed for demand forecasting, but can be used in other areas as well;
- It can be used for the standard problems instead of `es()` or `ets()`;
- It includes ARIMA in it;
- It can handle regressors;
- It can handle intermittent demand;
- It can handle multiple seasonal data;
- It can do a lot of things;
- And it probably contains tones of bugs :).

Thank you for your attention!

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