Generation of Time Series Scenarios: How to Do It and Make it Pay

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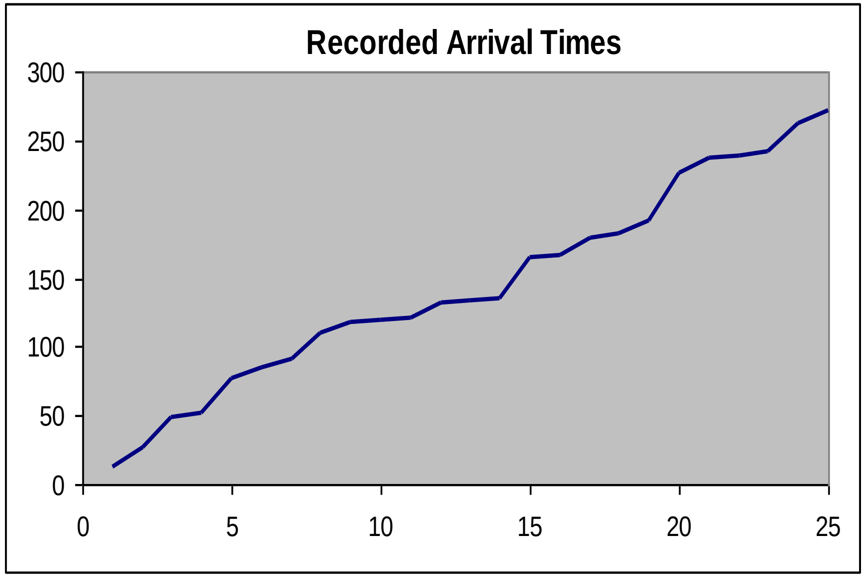
Scenarios:
Definition and Uses

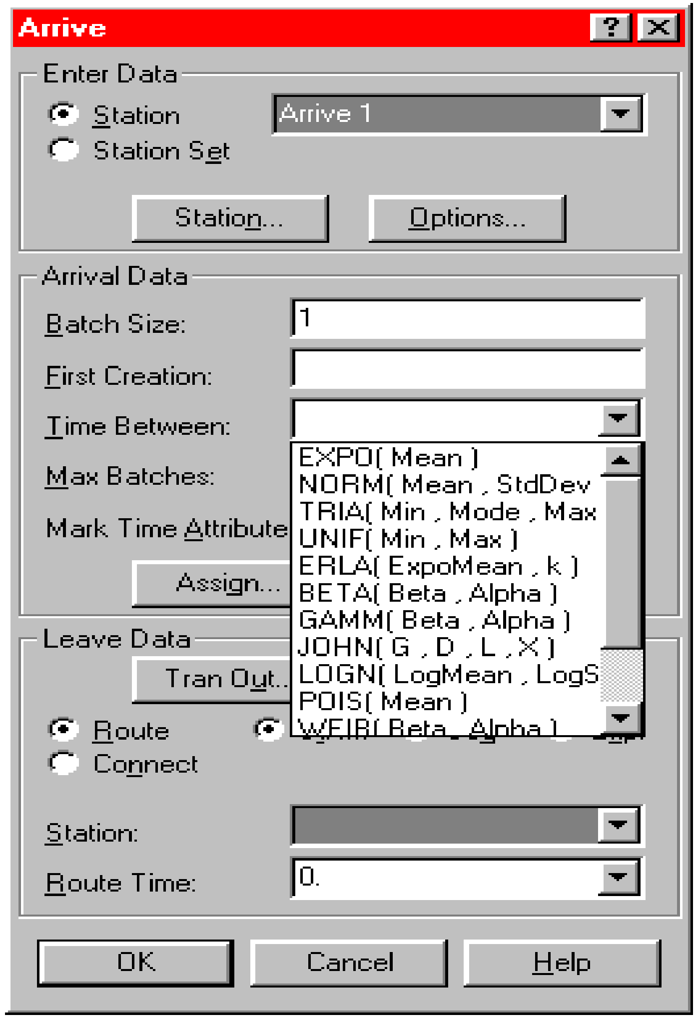
Scenario planning

"Scenario planning is **…** identifying a specific set of uncertainties, different 'realities' of what might happen in the future of your business."

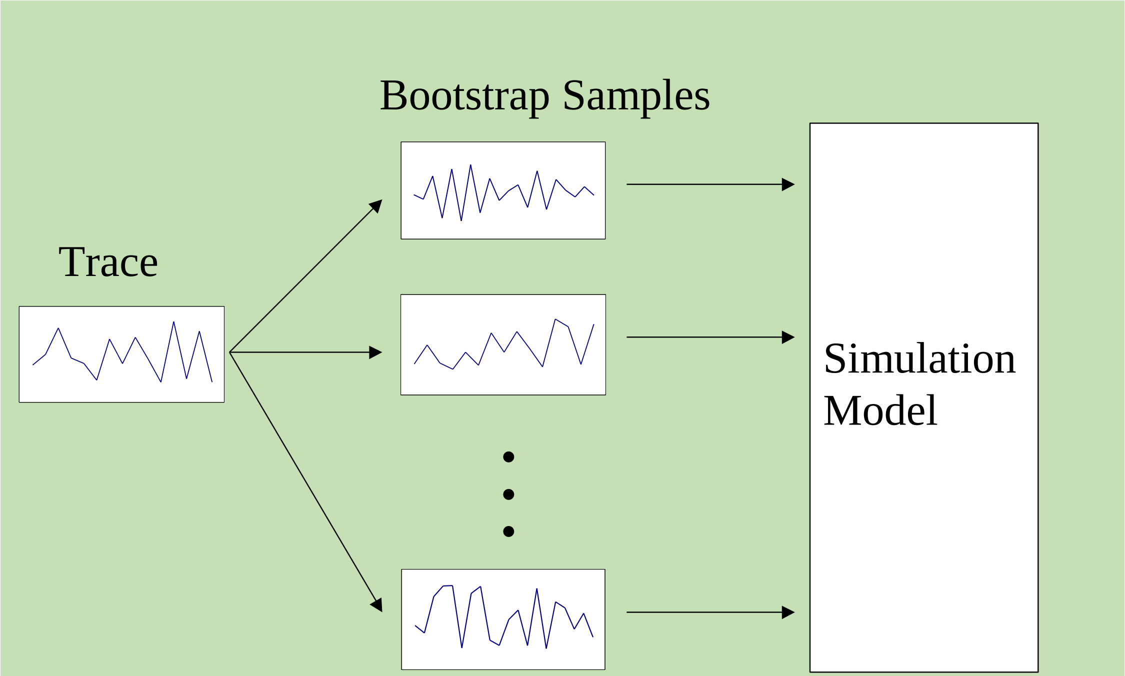
Jeremie Mariton

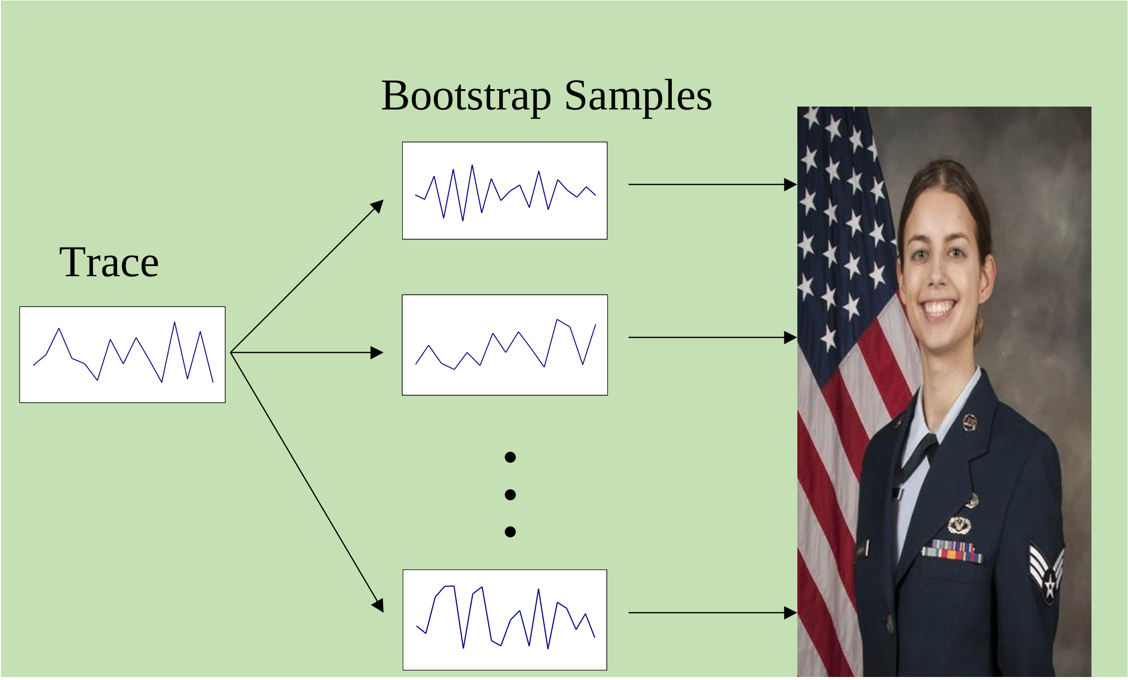
Input scenarios in discrete event simulation





Bootstrap scenarios: Two use cases





Scenarios:
Origins and Evaluation

Alternative sources of scenarios

1. Geppetto's Workshop: Hand-crafted
2. Groundhog Day: Replay
3. Stochastic process models: Poisson
4. Nonparametric time series bootstraps: 3 types

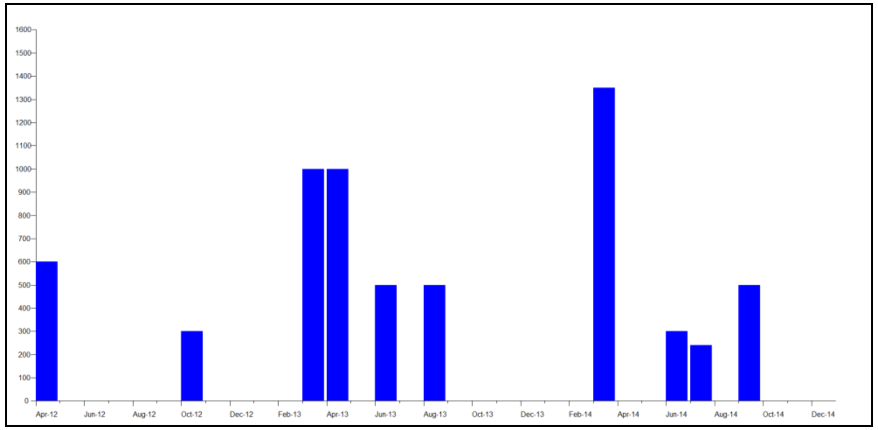
Four Criteria for Scenario Generators

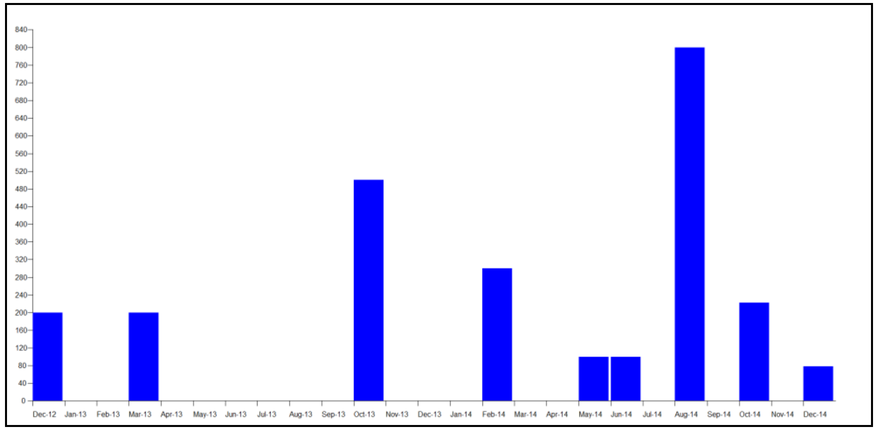
- Fidelity
    - Create scenarios that mimic (“look like”) real data streams.
    - “Don’t let the thunder precede the lightning.”
- Variety
    - Create scenarios that reflect natural variation across real data streams.
    - “Don’t give us the Groundhog Day scenario.”
- Quantity
    - Produce large numbers of scenarios.
    - “Don’t give us just two instances.”
- Cost
    - Produce each scenario with minimal computational and labor cost.
    - “Don’t hand-craft each scenario using a team of overpaid PhD’s.”

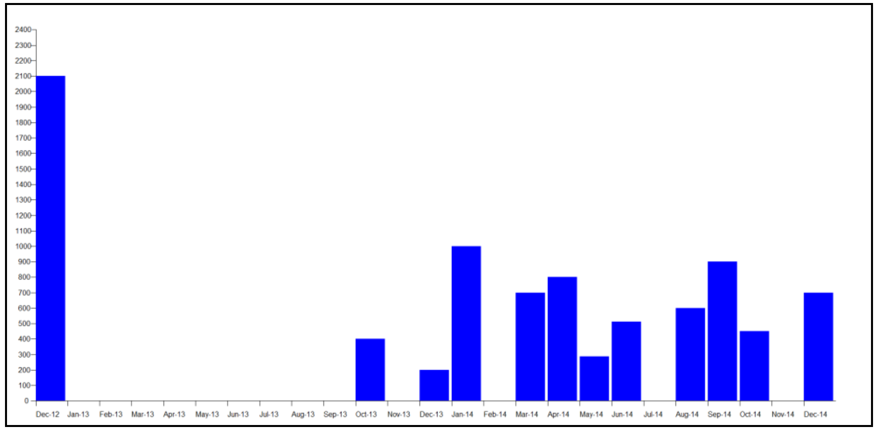
Generating Scenarios for Intermittent Demand

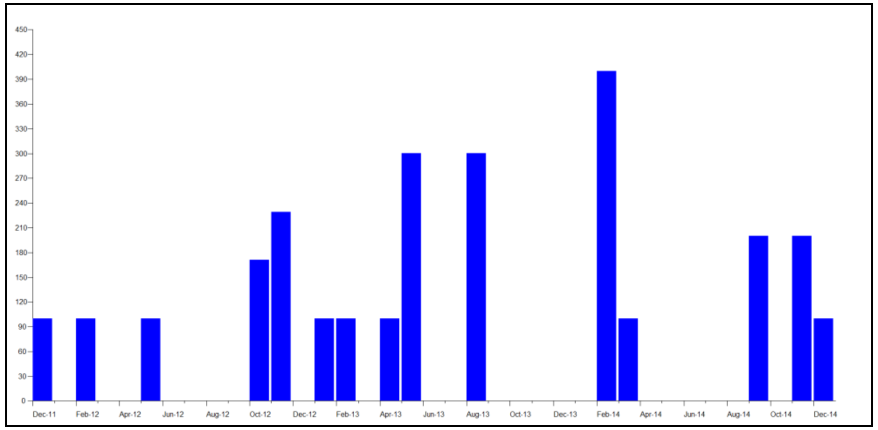
Markov Bootstrap

Forecasting Intermittent Demand

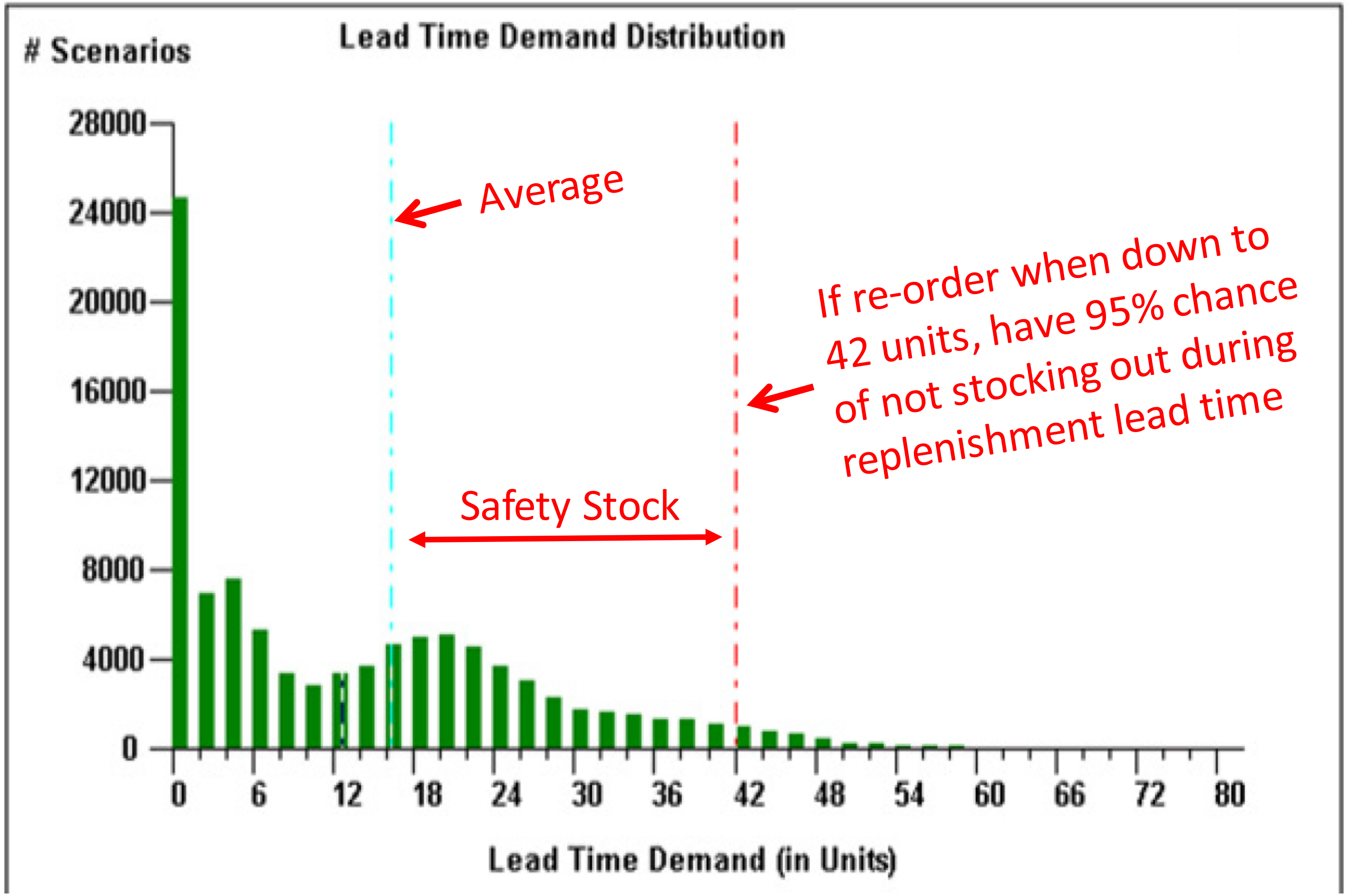








The answer we want to get to using scenarios



Step 1: Parse History into Two Pieces

Observation	1	2	3	4	5	6	7	8	9	10	11	12
Demand	4	0	0	9	3	2	0	0	8	3	0	5

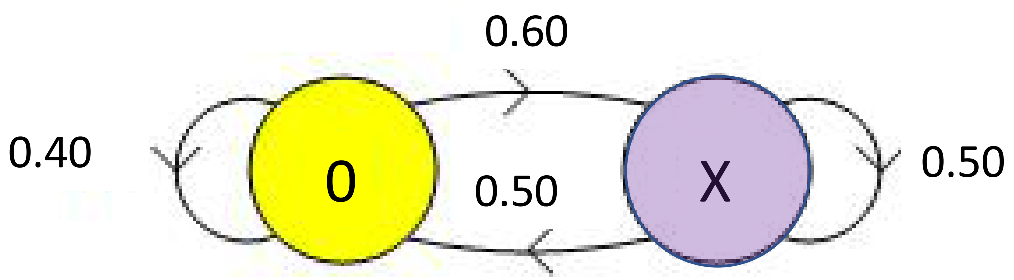
- Piece #1: Sequence of zero/nonzero demands.

Binary demand	X	0	0	X	X	X	0	0	X	X	0	X

- Piece #2: List of all nonzero demand values.

Non-zero demands	4	9	3	2	8	3	5

Step 2: Estimate Transition Probabilities



Markov model		Next	demand	
transition counts		0	X	
Current	0	2	3	5
demand	X	3	3	6
				11

Markov model		Next	demand	
transition probabilities		0	X	
Current	0	0.40	0.60	1.00
demand	X	0.50	0.50	1.00

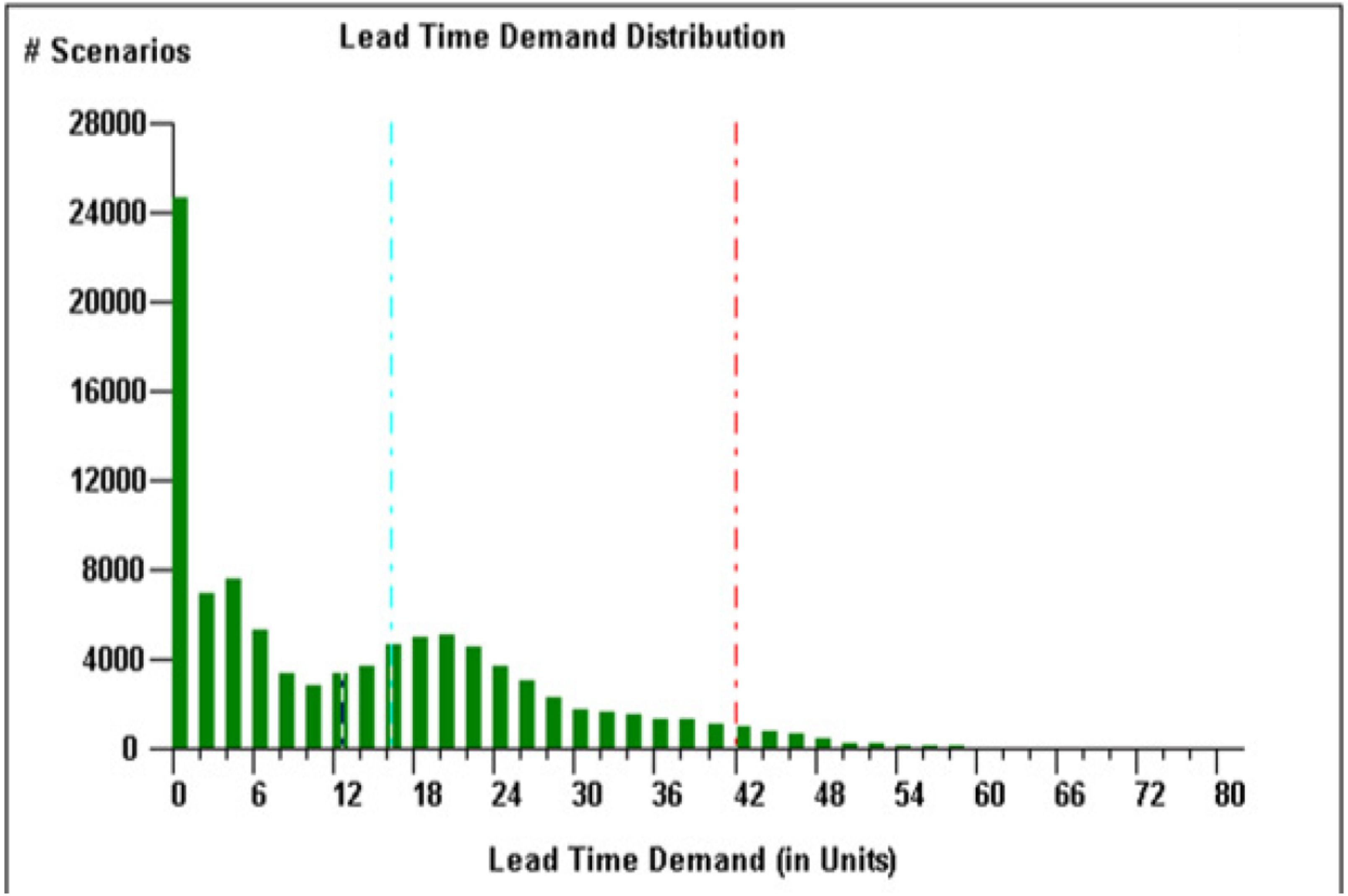
Step 3: Generate Demand Scenarios

Lead time scenarios

Future observation	13	14	15
Replication #1	0	0	X
Replication #2	0	X	X
Replication #3	X	X	0
Replication #4	0	0	0

Future observation	13	14	15	LTD
Replication #1	0	0	3	3
Replication #2	0	9	8	17
Replication #3	3	4	0	7
Replication #4	0	0	0	0

End result: Scenarios reveal the full scope of the randomness of demand



Generating Scenarios for Regular Univariate Demand

Moving Block Bootstrap

The Moving Blocks Bootstrap

Time:	1	2	3	4	5	6	7	8	
Data:	15	22	17	23	14	28	30	19	
Block 1:	15	22	17	23					
Block 2:		22	17	23	14				
Block 3:			17	23	14	28			
Block 4:				23	14	28	30		
Block 5:					14	28	30	19	
Boot sample 1:	22	17	23	14	14	28	30	19	(=Blocks 2&5)
Boot sample 2:	17	23	14	28	15	22	17	23	(=Blocks 3&1)
Boot sample 3:	14	28	30	19	14	28	30	19	(=Blocks 5&5)

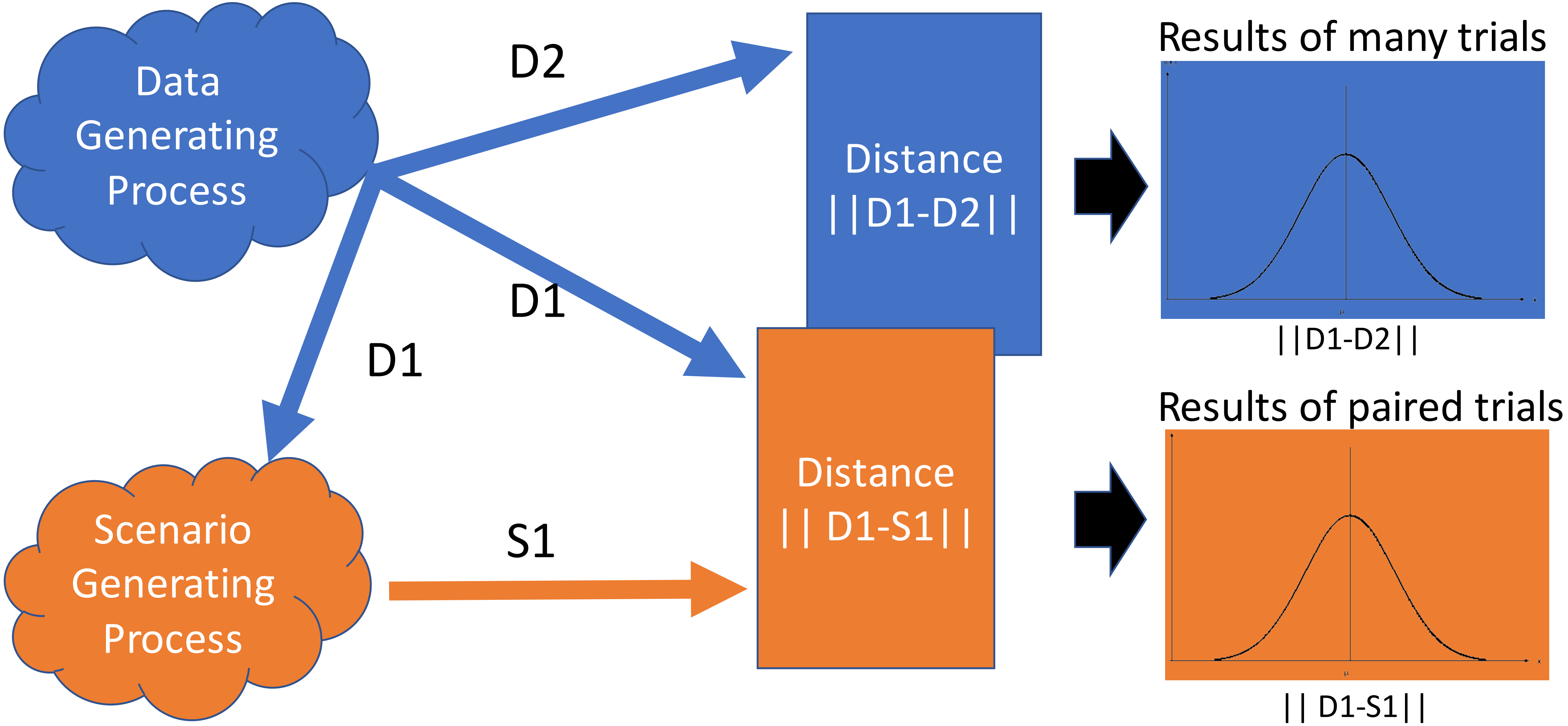
etc., etc.

Block Size = 4

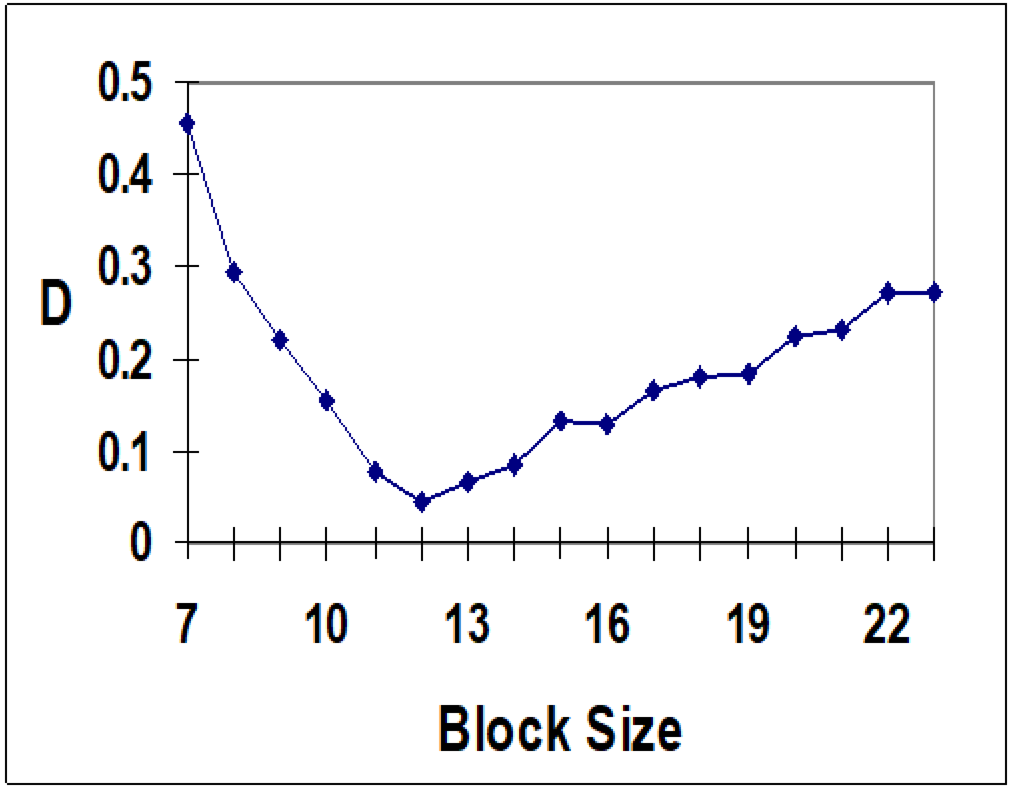
Choice of block size

- Optimal block size depends on the task
  - 2-sided confidence interval: $O(n^{1/5})$
  - 1-sided confidence interval: $O(n^{1/4})$
  - Bias and standard error: $O(n^{1/3})$
  - Time-series scenario: $O(n^{1/2})$ [new result]
- Computing specific block size
  - Optimize using subsampling, then scale up result using powers of n above.
- New metric combining fidelity and variety
  - Two samples from a DGP have a distribution of “discrepancy” D
  - Principle: One sample vs one scenario should have the same distribution of D.

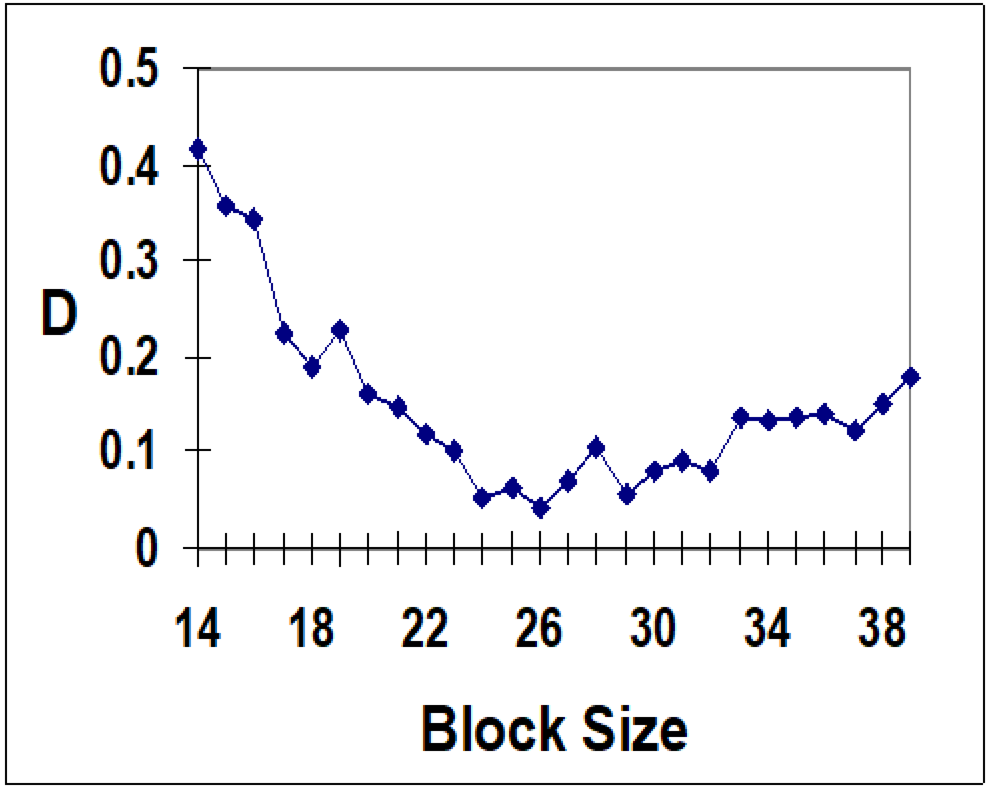
Mimic Inter-Sample Distances



Choose MBB Block Size to Minimize Discrepancy



AR(1) $\phi = 0.9$



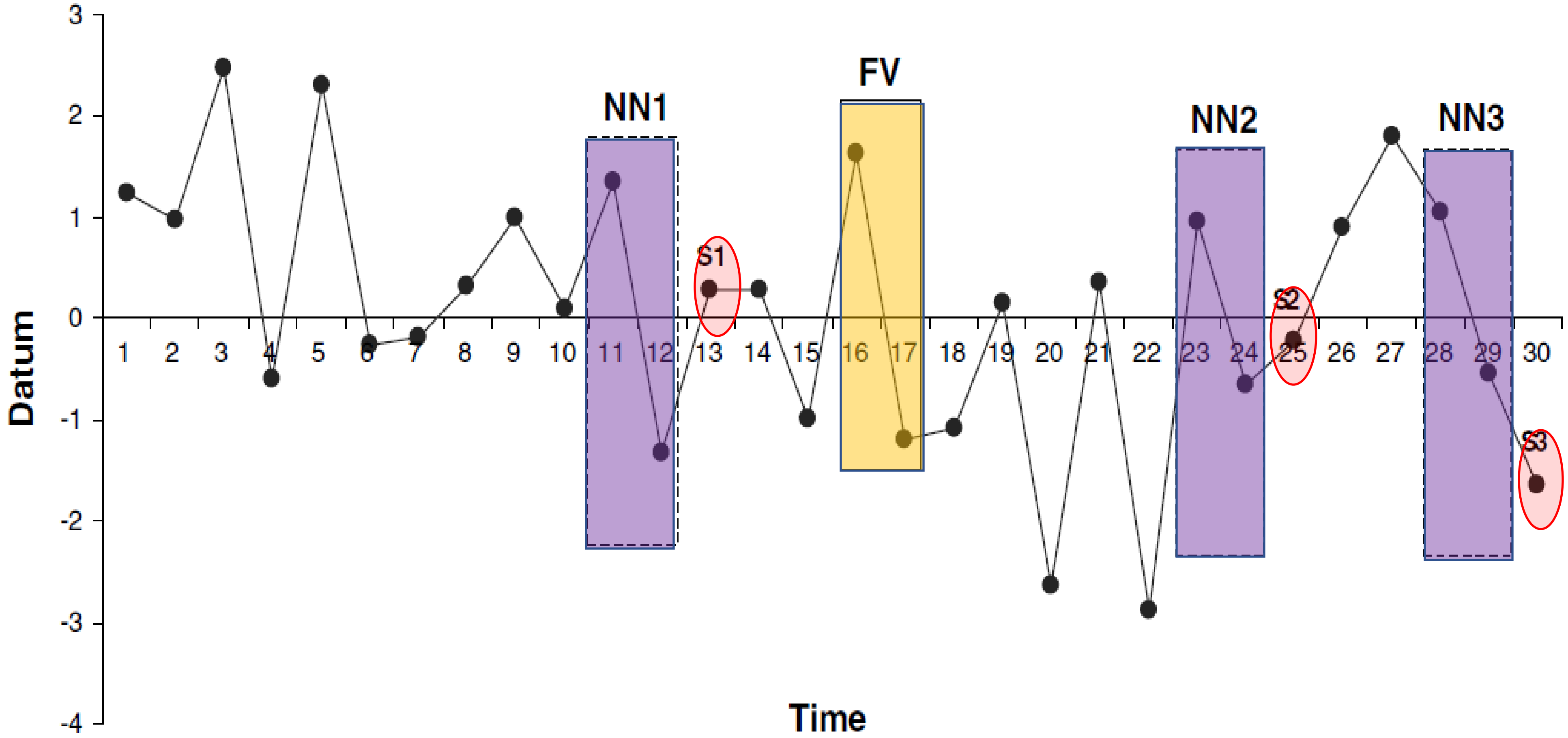
AR(1) $\phi = -0.9$

Generating Scenarios for Regular Multivariate Demand

Nearest Neighbor Bootstrap

The Nearest Neighbor Bootstrap

H Huang and TR Willemain—**Input modelling for financial simulations**



Ten-Dimensional Dynamic Scenarios

Real World Treasury Yields

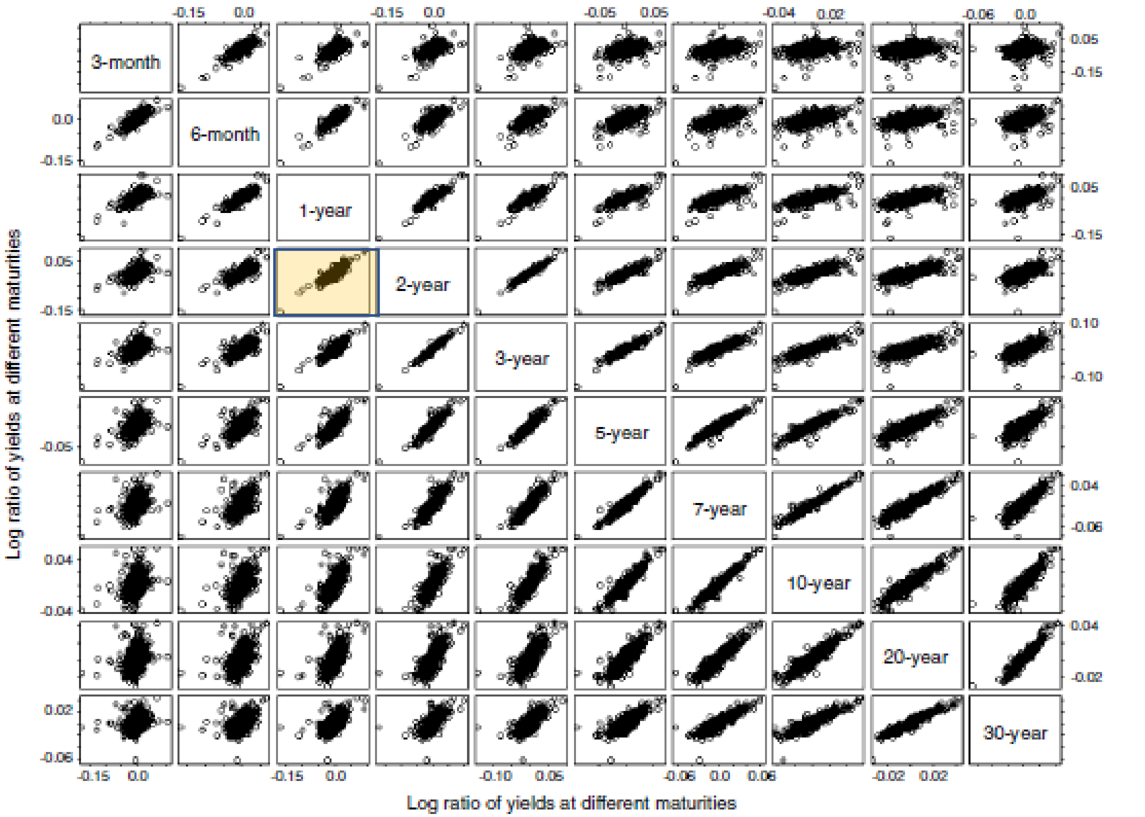


Figure 3 Scatter plot matrix for log ratios of the yields at 10 maturities. The positive correlations across the 10 dimensions can be observed easily. Looking at the plots along the diagonal, we can see the closer the two maturities, the stronger the correlation, and the narrower the spread of the data. Moving away from the diagonal, the correlations become weaker.

Simulation Using Nearest Neighbor Bootstrap

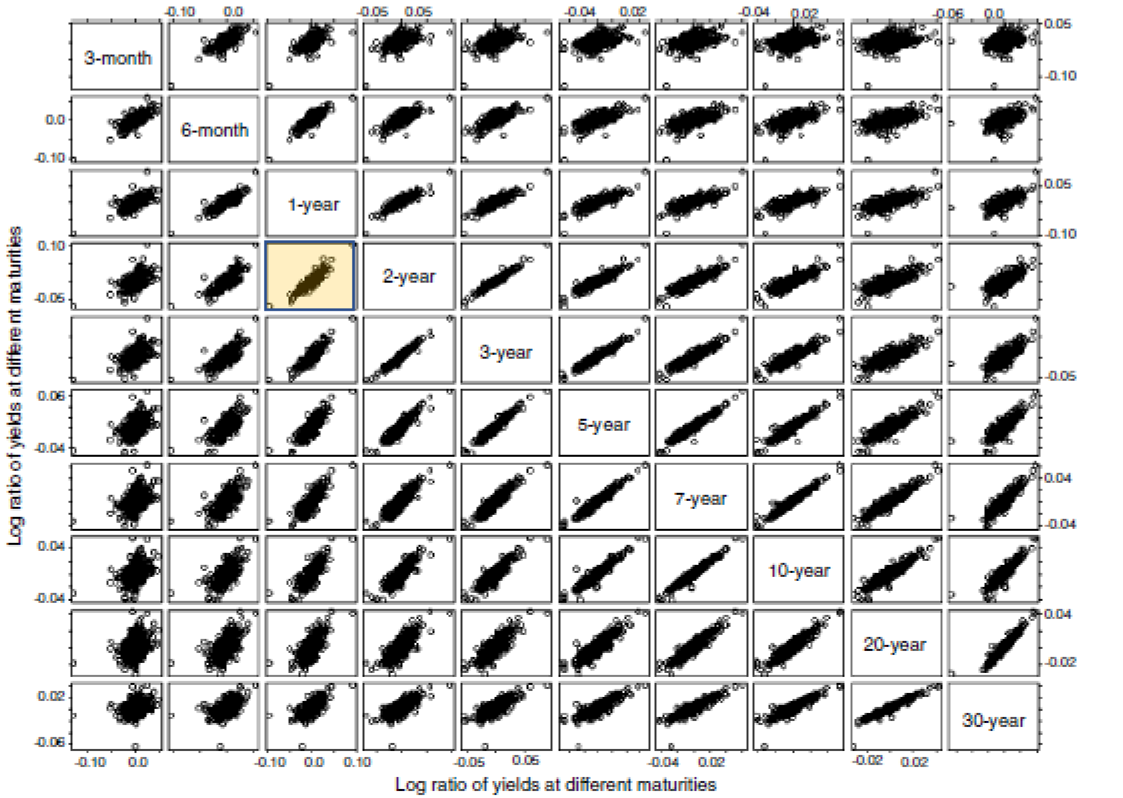


Figure 5 Scatter plot matrix of log ratios of the yields at 10 maturities in one simulated scenario. By comparison with Figure 3, the original correlation structure is well preserved in the simulated scenario. These simulated relationships are a good match to the actual ones shown in Figure 3, although the actual relationships show somewhat greater variability.

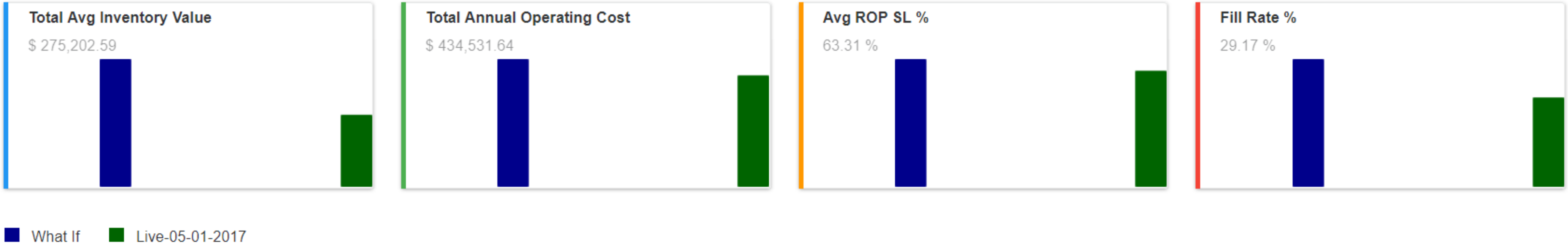
Scenarios in Inventory Optimization

Given a certain type of demand,
what are the best choices for Min and Max
(Min = reorder point, Max = order-up-to level)?

As-Is Scenario vs What-If Scenario

ALL ITEMS

Inventory Performance



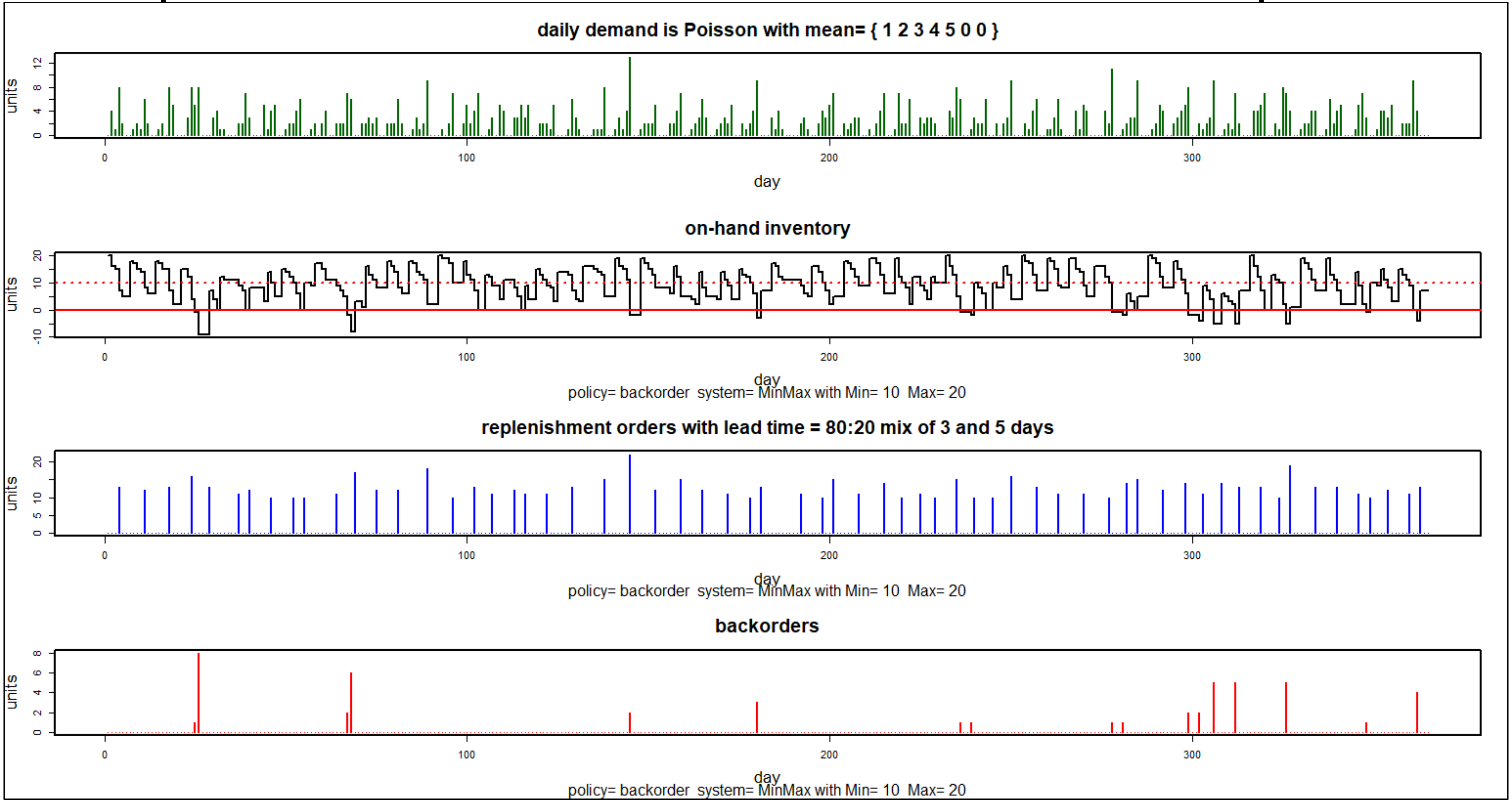
Summary

RECALCULATE | SAVE NEW | OVERWRITE | MAKE GOAL

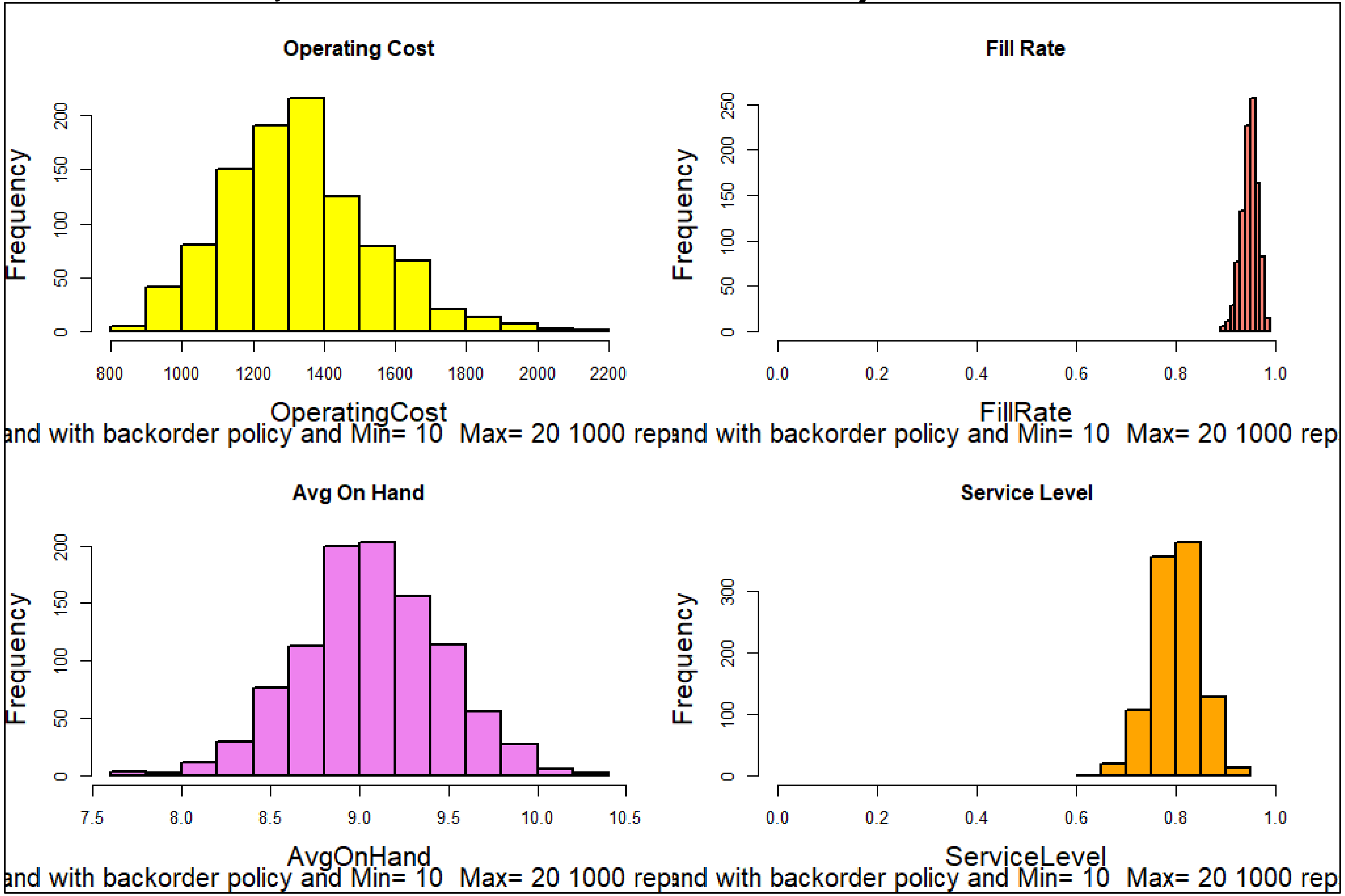
SUMMARY METRICS | DRIVERS | COSTS | DEMAND GROUPS | ERRORS

Scenario	Avg Full Cycle SL%	Total Avg Inventory Value	Total Annual Operating Cost	Total Orders/Yr	Fill Rate %	ROPValue	Avg Order Cycle
What If	92.71 %	$ 490,091.64	$ 498,452.25	8,627.93	41.43 %	$ 234,551.70	18.59
Live-05-01-2017	91.39 %	$ 275,202.59	$ 434,531.64	7,977.76	29.17 %	$ 85,688.44	8.62

Complex demand scenario and consequences



Results for 1,000 simulated years



Recap

- Two different targets for scenarios
  - System models or decision makers/operators.
- Where scenarios come from
  - Manual processes or parametric models or time series bootstrap.
- How to evaluate scenario generators
  - Fidelity, variety, quantity, cost.
- How to generate scenarios
  - Markov bootstrap for intermittent demand.
  - Moving block bootstrap, properly tuned, for univariate regular demand.
  - Nearest neighbor bootstrap, properly tuned, for multivariate regular demand.
- Payoffs
  - Scenarios are “fuel” for simulation models of systems.
  - First step in system optimization (search design space).

**Look, good against remotes is one thing.
Good against the living? That's something else.**

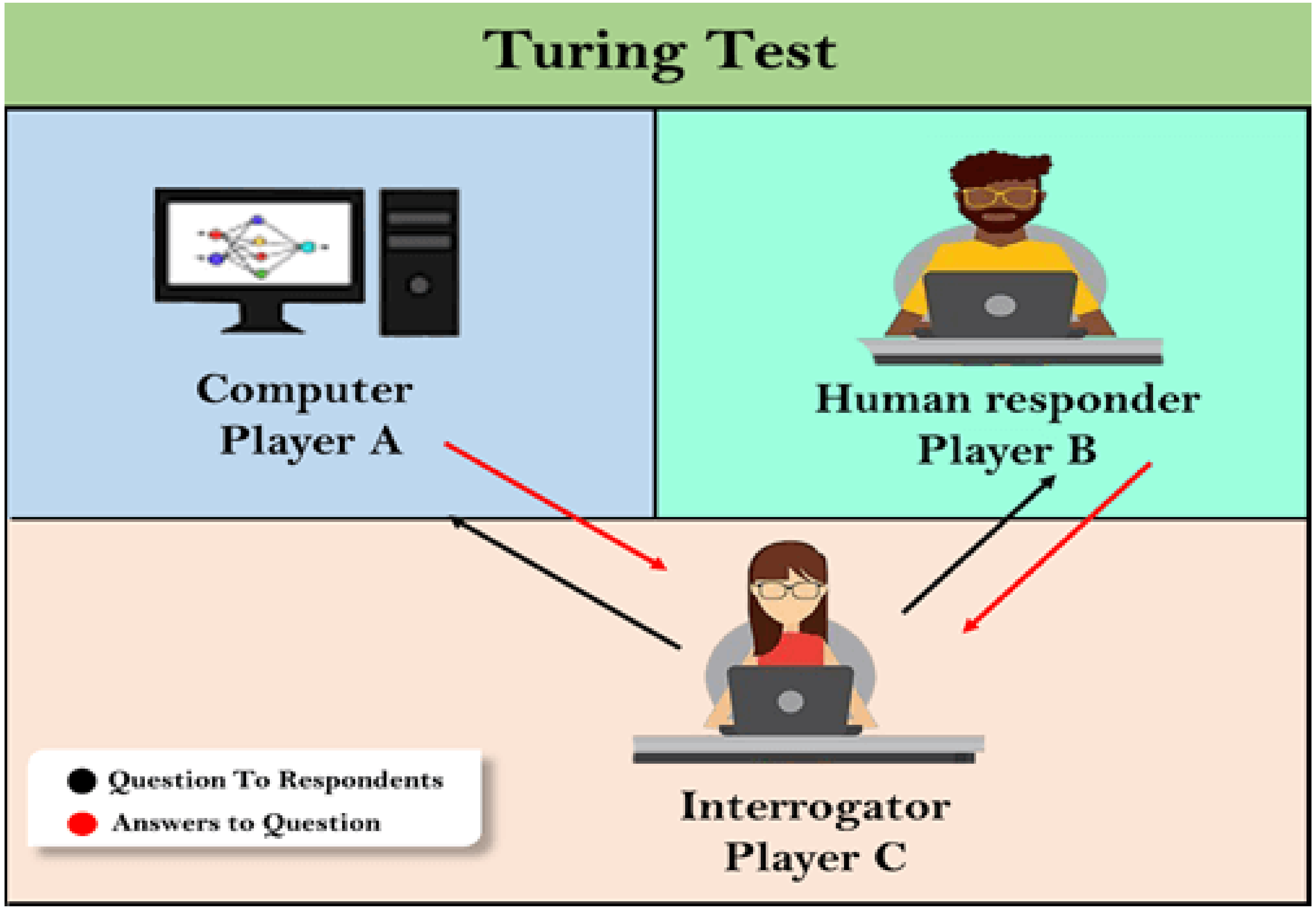
extra slides

Visual Turing Tests for scenarios going into human eyeballs

Evaluating scenario generators

Visual interfaces for scenario presentation

Original Turing Test



Visual Turing Test Experiment #1: Single series

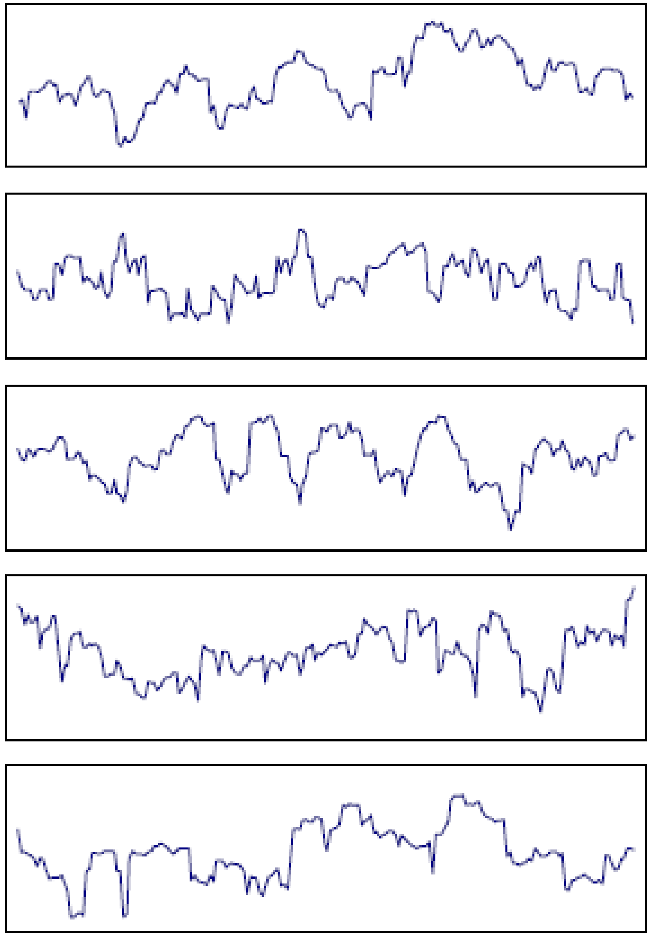


Figure 1 A sheet from the real data test (reduced from actual size).

Which one of the 5 samples is the bootstrap?
Six trials each from 12 experienced subjects.
Expect 1 in 5 successes by pure guessing.
Experts did worse than chance. Success!

Table 1 The number of bootstrap samples detected by each subject

Subject	A	B	C	D	E	F	G	H	I	J
# of Bootstrap Samples Identified (out of 6)	1	1	1	0	1	3	0	1	1	0

Visual Turing Test Experiment #2: Sets of 3

Which **set of 3** samples are bootstraps?
Repeated 4 times at 3 block sizes by 12 subjects.

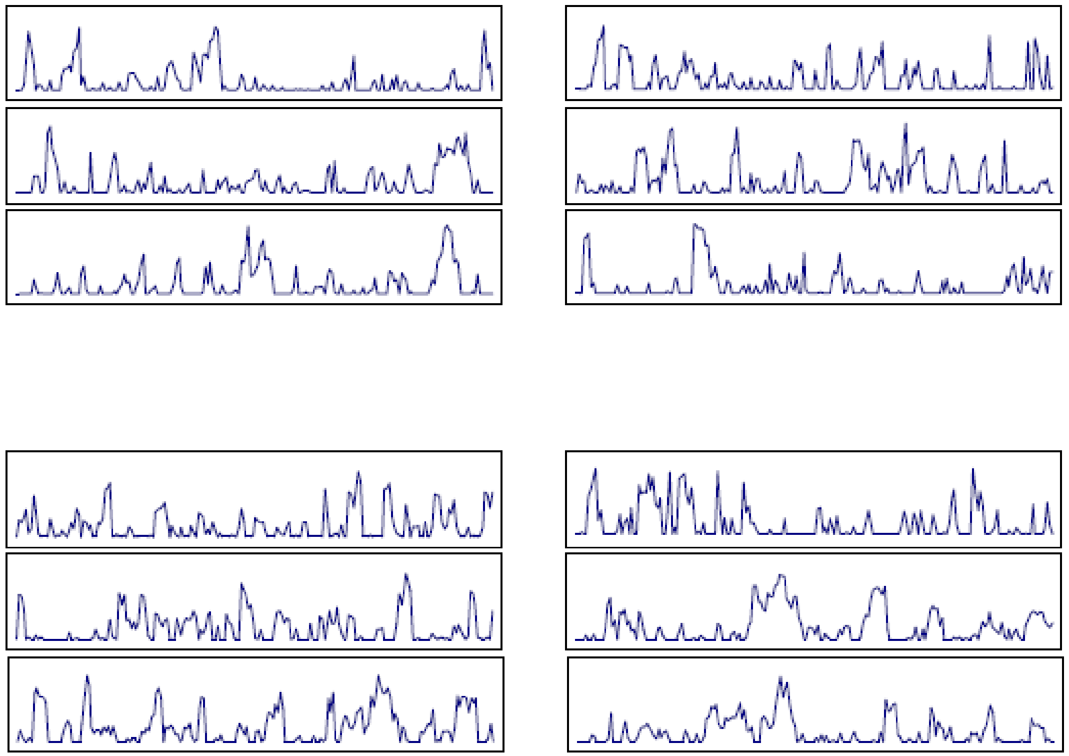


Figure 5 A sheet from the artificial data test (reduced from actual size).

Easier problem: compare sets of 3. Subjects did better than chance, but lowest-discrepancy MBB block size gave best results..

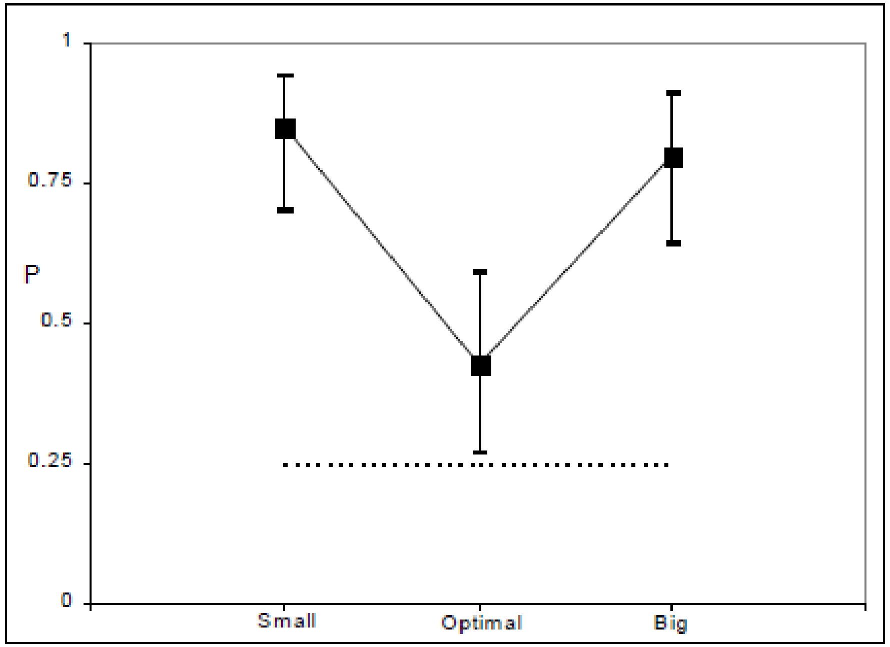


Figure 8 The 95 % CI for the proportions of bootstrap sets detected for each block size, the dotted line shows the probability value of 0.25.

Selected References

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