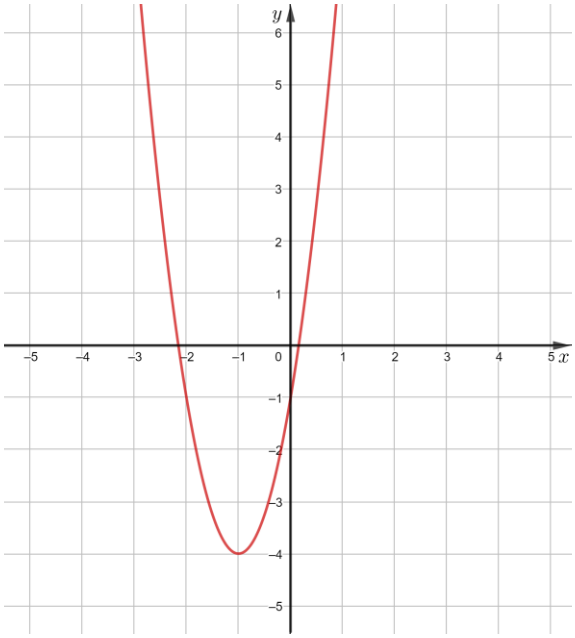
Vertex and general forms of a quadratic function - solutions

Task 1. (7p) A quadratic function f is defined by vertex form $f(x) = 3(x+1)^2 - 4$.

(a) Write down coordinates of the vertex V of parabola which is the graph of the f. $V = (-1, -4)$

(b) Write down coordinates of the point P, where the graph of f intersects the y-axis. $P = (0, -1)$

(c) Sketch the graph of f.



(d) Write down the equation of the line of symmetry of the graph of f. $x = -1$

(e) Write down the range of the function f.
$ZW_f = [-4, \infty)$

(f) Describe monotonicity intervals of the function f.

The function f increases in the interval $[-1, \infty)$.

The function f increases in the interval $(-\infty, -1]$.

(g) Write down the general form of the function f
$f(x) = 3x^2 + 6x - 1$

Task 2. (3p) Calculate coefficients b i c in the formula of a quadratic function $y = 4x^2 + bx + c$, knowing that the points $A(2, 27)$ and $B(-2, 3)$ lie on the graph of this function.

Solution

Let W_f be the graph of the function $y = 4x^2 + bx + c$.

$$\begin{cases} A(2,27) \in W_f \Rightarrow 27 = 16 + 2b + c \\ B(-2,3) \in W_f \Rightarrow 3 = 16 - 2b + c \end{cases}$$

So:

$$\begin{cases} 2b + c = 11 \\ -2b + c = -13 \end{cases} \qquad \begin{cases} c = 11 - 2b \\ -2b + 11 - 2b = -13 \end{cases} \qquad \begin{cases} c = 11 - 2b \\ -4b = -24 \end{cases} \qquad \begin{cases} c = -1 \\ b = 6 \end{cases}$$

Answer: $b = 6, c = -1$

Task 3. (4p) Work out the vertex form for the quadratic function f, knowing that the interval $[2, \infty)$ is the range of f and $f(-2) = f(-4) = 5$.

Solution

$f(x) = a(x-p)^2 + q$

1. $f(-2) = f(4) \;\Rightarrow\; p = \frac{-2+(-4)}{2} = -3 \;\Rightarrow f(x) = a(x+3)^2 + q$
2. $ZW_f = [2, \infty) \Rightarrow q = 2 \Rightarrow f(x) = a(x+3)^2 + 2$
3. $f(-2) = 5 \;\Rightarrow a(-2+3)^2 + 2 = 5 \;\Rightarrow a = 3 \Rightarrow f(x) = 3(x+3)^2 + 2$

Answer: $f(x) = 3(x+3)^2 + 2$.