

– Introduction

In this project, we are expected to implement a code in a parallel programming manner with C/C++ using MPI library. We need to implement a parallel algorithm for image denoising with the Ising model using Metropolis-Hastings algorithm. Throughout the project, while using the algorithm just mentioned, we follow up many statistical approaches and formulas. The main algorithm we need to use and the formula that gives us the necessary information are given to us a priori. We are given a black/white noisy image and our goal is to denoise it in a parallel programming manner.

In the project we are given the information about The Ising model and how to get use of it. The Ising model models things that have two states (for our case +1 or -1) and examines points' interact with its neighbors which means in our case if we take random black pixel from the image, it is more likely that this pixel is surrounded by black pixels (same for the white pixels). To use Ising model in Image denoising, we are given following formulas and algorithm.

Let the Z be a $I \times J$ binary matrix that describes an image where each Z_{ij} is either +1 or -1 (i.e. white or black). By randomly flipping some of the pixels of Z , we obtain the noisy image X which we are observing. We assume that the noise-free image Z is generated from an Ising model (parametrized with β) and X is generated by flipping a pixel of Z with a small probability π : (where β and π are defined by us, randomly (by fitting the best output))

$$Z \sim p(Z | \beta) \quad (1)$$

$$F_{ij} \sim \text{Be}(\pi) \quad (2)$$

$$X_{ij} = (-1)^{F_{ij}} Z_{ij} \quad (3)$$

where

$$p(Z | \beta) \propto e^{-E(Z|\beta)} \quad (4)$$

$$E(Z | \beta) = -\beta \sum_{(i,j) \sim (k,l)} Z_{ij} Z_{kl} \quad (5)$$

Then we want to find the posterior distribution of Z :

$$p(Z | X, \beta, \pi) = \frac{p(Z, X | \beta, \pi)}{p(X | \beta, \pi)} \quad (6)$$

$$\propto p(Z, X | \beta, \pi) \quad (7)$$

$$= p(Z | \beta) p(X | Z, \pi) \quad (8)$$

$$\propto \exp \left(\gamma \sum_{ij} Z_{ij} X_{ij} + \beta \sum_{(i,j) \sim (k,l)} Z_{ij} Z_{kl} \right) \quad (9)$$

where $\gamma = \frac{1}{2} \log \frac{1-\pi}{\pi}$. In this formulation, higher γ implies lower noise on the X . Similarly, we expect more consistency between the neighbouring pixels if β is higher.

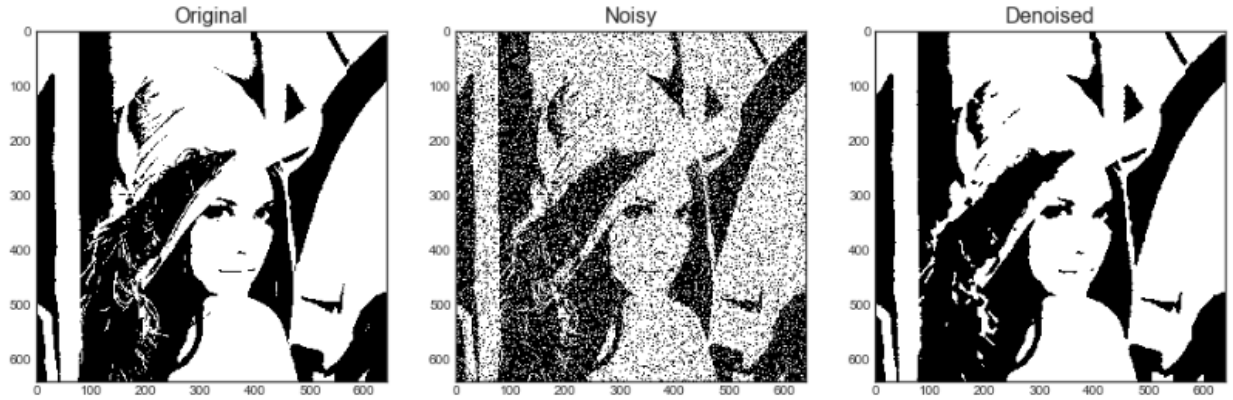
We want to achieve an image Z that is the original, not noisy image. (we want to get there)

Image X is the noisy version of Z which means some of the pixels of the image Z are flipped. Equations 2 and 3 model this noise relation. π is the probability of a random flip. We start with this noisy image X , and try to reach the noise-free image Z .

3. We assume that the image Z is generated from an Ising model.

4. We can statistically show our model as $p(Z | X, \beta, \pi)$.

It returns a score for a candidate Z image that we propose. The score is higher if the candidate image is more fit to the model. We want to find the Z image that gives the highest score from the posterior probability. Instead of finding the original image in an instant, we will approach to it step by step. As can be seen from above visually



To reach the noise free image Z we are given an algorithm to make some modification to X . Metropolis-Hastings algorithm claims that:

1. Choose a random pixel from the image X
2. Calculate an acceptance probability of flipping the pixel or not.
3. Flip this pixel with the probability of the acceptance probability that is calculated in the second step.
4. Repeat this process until it converges.

How to calculate the acceptance probability:

Assume a bit flip is proposed in pixel (i, j) at time step t , (a bit flip on $Z_{ij}^{(t)}$, i.e. $Z'_{ij} \leftarrow -Z_{ij}^{(t)}$), then

$$\alpha^{(t)} = \frac{p(Z' | X, \beta, \pi)}{p(Z^{(t)} | X, \beta, \pi)} \quad (10)$$

$$= \frac{\exp(\gamma Z'_{ij} X_{ij} + \beta \sum_{(i,j) \sim (k,l)} Z'_{ij} Z'_{kl})}{\exp(\gamma Z_{ij}^{(t)} X_{ij} + \beta \sum_{(i,j) \sim (k,l)} Z_{ij}^{(t)} Z_{kl}^{(t)})} \quad (11)$$

$$= \frac{\exp(-\gamma Z_{ij}^{(t)} X_{ij} - \beta \sum_{(i,j) \sim (k,l)} Z_{ij}^{(t)} Z_{kl}^{(t)})}{\exp(\gamma Z_{ij}^{(t)} X_{ij} + \beta \sum_{(i,j) \sim (k,l)} Z_{ij}^{(t)} Z_{kl}^{(t)})} \quad (12)$$

$$= \exp\left(-2\gamma Z_{ij}^{(t)} X_{ij} - 2\beta \sum_{(i,j) \sim (k,l)} Z_{ij}^{(t)} Z_{kl}^{(t)}\right) \quad (13)$$

1. Initialize your β and π priors. Then $\gamma = \frac{1}{2} \log \frac{1 - \pi}{\pi}$

2. Initialize $Z^{(0)} \leftarrow X$

3. At time step t :

3.1. Randomly choose a pixel (i, j) .

3.2. Propose a bit flip on $Z_{ij}^{(t)}$, i.e. $Z'_{ij} \leftarrow -Z_{ij}^{(t)}$.

3.3. Calculate acceptance probability $\alpha^{(t)} = \min \left\{ 1, \frac{p(Z'_{ij}|X, \beta, \pi)}{p(Z_{ij}^{(t)}|X, \beta, \pi)} \right\}$

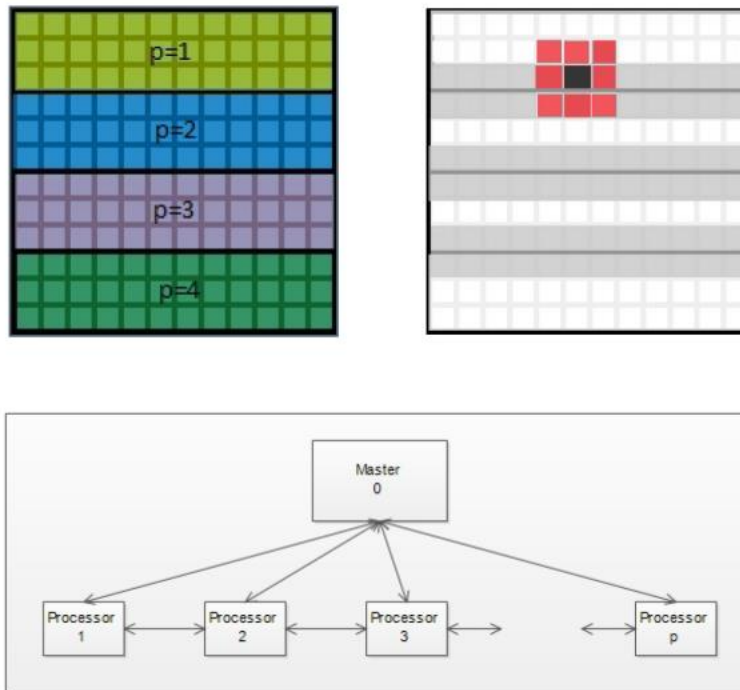
3.4. $Z^{(t+1)} \leftarrow Z'$, with probability $\alpha^{(t)}$

3.5. $Z^{(t+1)} \leftarrow Z^{(t)}$, otherwise

Parallel
Image

Denoising: (Also given with the project definition)

There will be 1 master and p slave processors (n is divisible by the number of slave processors p) each processor is responsible from a group of n/p adjacent rows. Each processor works on $(n/p \times n)$ pixels and these pixels are stored locally by the processor. When a pixel is inspected, the processor should inspect all of its neighbors. When the pixel is not on the boundary of two adjacent processors, information about the neighbor pixels are present to the processor. When a pixel on the boundary is inspected, the processor needs to obtain information from the adjacent processor. Consider the black pixel in the below figure. Processor 1 should communicate with Processor 2 in order to learn the status of the south neighbor of the black pixel. Therefore, each processor should communicate with the adjacent processor at every iteration. Information about those neighbor pixels of the boundary can be stored locally and updated at every iteration.



In order us to use the method we are also given “<https://github.com/suyunu/Markov-Chain-Monte-Carlo>” website where we can find a real working sequential version of this method.

– Program Interface

Installing Open MPI environment on Linux:

after downloading openmpi 1.4.4 from:

“<http://www.open-mpi.org/software/ompi/v1.4/downloads/openmpi-1.4.4.tar.gz>”

2. By these commands we build Open MPI

```
tar -xvf openmpi-1.4.4.tar.gz
```

```
./configure
```

```
make all install
```

How to compile my code with Open MPI:

```
mpic++ my_code.c -o my_program.out
```

– Program Execution

The noisy image’s text file is supplied to the program by user. The output of the program is a text file, which we can convert to png by a script

How to run my code with Open MPI:

```
mpiexec -n NUM_PROCESSORS ./your_program inputfile outputfile  $\beta$  value  $\pi$  value
```

How to check, text_to_image: We use a python script called text_to_image

```
python3 text_to_image.py my_input_text output_image.png
```

-Input and Output

The inputs are, a noisy image’s text file, β value and π value.

We don’t know the true values of β and π . According to our prior knowledge (by looking at the image or via our spidey sense) we are just guessing. These values represent:

- We expect more consistency between the neighbouring pixels if β is higher.
- π is our original prior to show the noise probability. So higher π implies higher noise on the X.

The output of the program is a text file, which we can convert to png by a script

– Program Structure

The program uses parallelism. Which means it runs concurrently, more than one processes. We have a master process and slaves. There are p slave processors (n (# of pixels) is divisible by the number of slave processors p) each processor is responsible for a group of n/p adjacent rows. Each processor works on $(n/p \times n)$ pixels and these pixels are stored locally by the processor. When a pixel is inspected, the processor should inspect all of its neighbors. When the pixel is not on the boundary of two adjacent processors, information about the neighbor pixels are present to the processor. When a pixel on the boundary is inspected, the processor needs to obtain information from the adjacent processor. Each processor communicates with the adjacent processor at every iteration. Information about those neighbor pixels of the boundary are stored locally and updated at every iteration.

At first we start the MPI, set the number of processes from the given input value and set the rank (kind of the ID of a process). Taking the input and printing the output are done in the master process. The master process sends the part of the image to the related slaves and each slave works on that specific part and informs each other. At each slave's partition, we pick a random pixel and apply the algorithm on it, and decide whether to flip the pixel or not, according to its surrounding pixels. We do that many times (500,000 for our case (for 200*200 image)). After the slave completes all iterations, it sends the final version of its partition to the master process. After all slaves do the same, the master process outputs the created final version of "denoised image". After completing printing the output, we call `MPI_Finalize()` and terminate the program. As we run our program, we have a text file as output, which is the denoised version of the initial noisy image. We can see the result as using a script which turns our text file into the png formatted image.

The `MPI_Recv` function used in the project is a blocking function, which means the related process waits until the receiving operation is held. Whereas the `MPI_Send` function is not a blocking function.

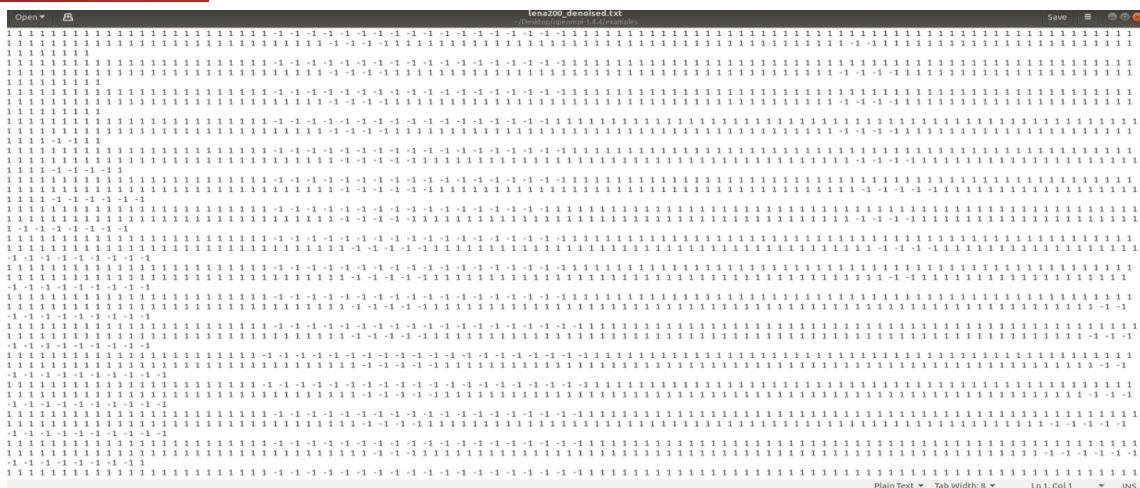
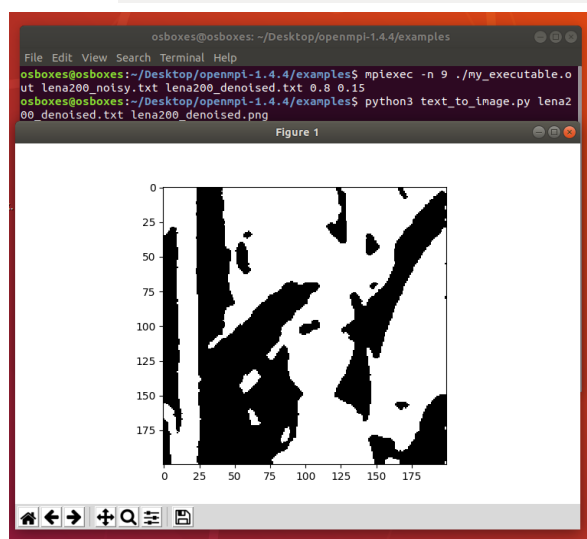
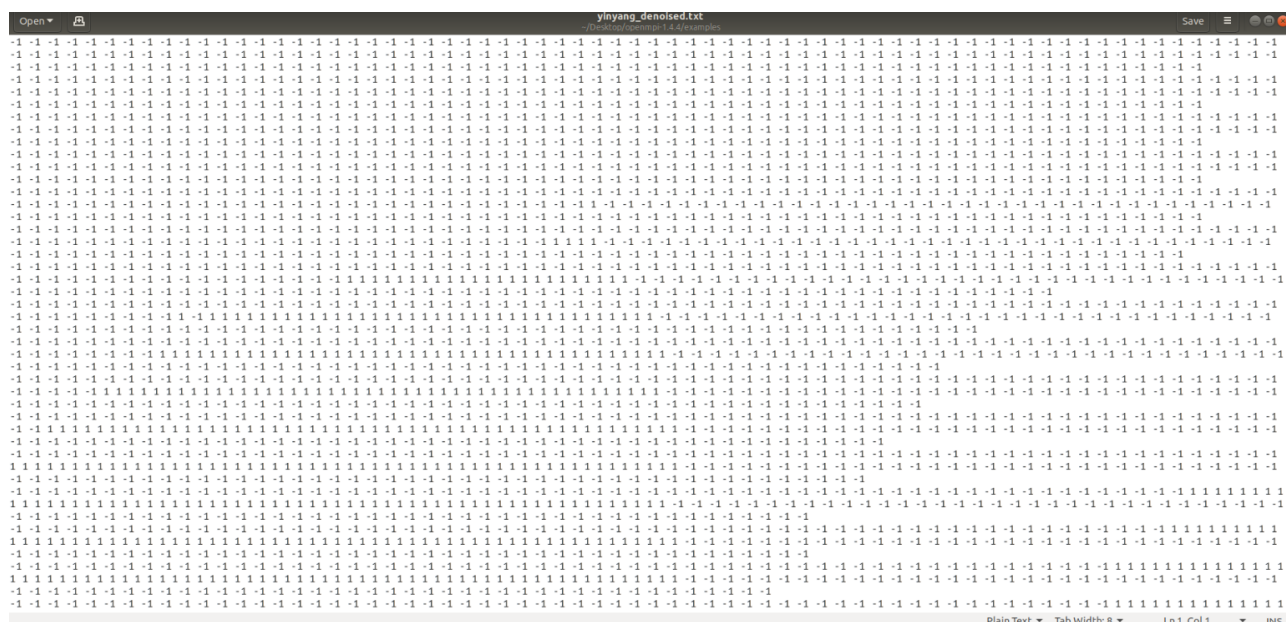
-Examples:

Some command line prompts and outputs of the program are given below:

```
osboxes@osboxes: ~/Desktop/openmpi-1.4.4/examples
File Edit View Search Terminal Help
osboxes@osboxes:~/Desktop/openmpi-1.4.4/examples$ mpic++ mpi_pro
ject_last.cpp -o my_executable.out
osboxes@osboxes:~/Desktop/openmpi-1.4.4/examples$ mpirun -n 9 ./
my_executable.out yinyang_noisy.txt yinyang_denoised.txt 0.8 0.15
osboxes@osboxes:~/Desktop/openmpi-1.4.4/examples$
```



yinyang_denoised.txt :



lena200 denoised.txt: