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1) b) $cn \leq \sqrt{10n^2+7n+3}$ positive c and no exists for all $n \geq n0$

$c \leq \sqrt{\frac{10n^2+7n+3}{n^2}}$

$= c \leq \sqrt{10 + \underbrace{\frac{7}{n} + \frac{3}{n^2}}}$

as n grows, this part goes to zero

$c = 1$

$n0 = 1$

True

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c) $n^2+n < cn^2$ for all $c>0$ there exists some $n0>0$ for all $n \geq n0$

for $c=1$

$\cancel{n^2}+n < \cancel{n^2}$

$n<0$, n must be positive

False

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d) $c_1 \log_2 n^2 \leq 3 \log_2^2 n \leq c_2 \log_2 n^2$ positive c_1 and c_2 exists for all $n \geq n0$

$2c_1 \log_2 n \leq 3 \log_2 n \log_2 n \leq 2c_2 \log_2 n$

$2c_1 \leq 3 \log_2 n \leq 2c_2$

$\frac{2c_1}{3} \leq \log_2 n$

$\log_2 n \leq \frac{2c_2}{3}$ → constant

$c_1 = 1$

$n0 = 2$

grows as n gets bigger

False

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a) $2^n + n^3 \leq c\,4^n$, positive c and n_0 exists for all $n \geq n_0$

$$\underbrace{\frac{2^n}{4^n} + \frac{n^3}{4^n}}_{\text{as } n \text{ gets larger this side goes to zero}} \leq c$$

$n_0 = 1$

$c = 5$

True ✓

e) $(n^3+1)^6 \leq c\,n^3$, positive c and n_0 exists for all $n \geq n_0$

$(n^{18} + \ldots) \leq c\,n^3$

for $n > 0$

$$\frac{n^{18} + \ldots}{n^3} \leq c$$

$$\underbrace{\frac{n^{15} + \ldots}{1}}_{\text{as } n \text{ gets larger this side goes to infinity}} \leq \underset{\text{constant}}{\underset{\downarrow}{c}}$$

False ✓

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2) b) $0.001n^4 + 3n^3 + 1$

Let's say

$c_1 g(n) \leq 0.001n^4 + 3n^3 + 1 \leq c_2 g(n)$

$g(n) = n^4$

$c_1 = 0.001$

$0.001n^4 \leq 0.001n^4 + 3n^3 + 1$

$3n^3 + 1 \geq 0$

$n_0 = 1$ ✓

$c_2 = 1.001$

$3n^3 + 1 \leq n^4$

$n_0 = 4$ ✓

a) $\underbrace{2n \log(n+2)^2}_{\Theta(n \log^2 n)} + \underbrace{(n+2)^2 \log\left(\frac{n}{2}\right)}_{\Theta(n^2 \log n)}$

$$\lim_{n \to \infty} \frac{n \log^2 n}{n^2 \log n} = \lim_{n \to \infty} \frac{\log n}{n} = \frac{1}{\infty} = 0$$

$n^2 \log n$ grows faster than $n \log^2 n$

$\Rightarrow g(n) = n^2 \log n$

3) b) $\lim_{n\to\infty} \frac{2^n}{n^2} = \lim_{n\to\infty} \frac{2^n \ln 2}{2n} = \lim_{n\to\infty} \frac{2^n \ln 2 \cdot \ln 2}{2} = \infty$

$\boxed{2^n > n^2}$

$\lim_{n\to\infty} \frac{n!}{2^n} = \lim_{n\to\infty} \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{2^n} = \lim_{n\to\infty} \sqrt{2\pi n} \cdot \left(\frac{n}{2e}\right)^n = \infty$

$n! > 2^n \Rightarrow n! > 2^n > n^2$ //

c) $\lim_{n\to\infty} \frac{n \log n}{\sqrt{n}} = \lim_{n\to\infty} \frac{\log n + \frac{1}{\ln 10}}{\frac{1}{2\sqrt{n}}} = \lim_{n\to\infty} 2\sqrt{n}\left(\log n + \frac{1}{\ln 10}\right)$

$= \infty$

$n \log n > \sqrt{n}$ //

d) $\lim_{n\to\infty} \frac{3^n}{2^n \cdot n} = \lim_{n\to\infty} \frac{(1.5)^n}{n} = \lim_{n\to\infty} \frac{(1.5)^n \ln(1.5)}{1}$

$= \infty$

$3^n > 2^n \cdot n$ //

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e) $\lim_{n\to\infty} \frac{n^3}{\sqrt{n+10}} = \lim_{n\to\infty} \frac{3n^2}{\frac{1}{2\sqrt{n+10}}} = \lim_{n\to\infty} 6n^2\sqrt{n+10} = \infty$

$n^3 > \sqrt{n+10}$ //

a) $\lim_{n\to\infty} \frac{\log n}{n^{1.5}} = \lim_{n\to\infty} \frac{1}{n\ln(n)} \cdot \frac{1}{(1.5)\sqrt{n}} = \frac{1}{\infty} = 0$

$n^{1.5} > \log n$

$$\lim_{n\to\infty} \frac{n^{\log n}}{n^{1.5}} = \lim_{n\to\infty} \frac{\left(\frac{\ln(n)}{n\ln 10} + \log n \ln 10\right) \cdot n^{\log n}}{1.5\, n^{0.5}}$$

$$= \lim_{n\to\infty} \frac{\left(\frac{\ln(n)}{n\ln 10} + \log n \ln 10\right) \cdot n^{\log n - \frac{1}{2}}}{1.5} = \infty$$

$y = n^{\log n}$

$\ln y = \log n \cdot \ln(n)$

$\frac{y'}{y} = \frac{\ln(n)}{n\ln 10} + \frac{\log n}{n}$

$y' = \left(\frac{\ln(n) + \log n \ln 10}{n\ln 10}\right) n^{\log n}$

$n^{\log n} > n^{1.5}$

$\Rightarrow n^{\log n} > n^{1.5} > \log n$

4) a) Basic operation is comparison

b) $\sum_{i=0}^{n-2} n-2-i = (n-2)+(n-3)+ \cdots + 0$

$\underbrace{1+ \cdots + n-2 + \overbrace{n-1+n}^{2n-1}}$

$\frac{n(n+1)}{2} - (2n-1)$

$A(n) = \frac{n^2+n-4n+2}{2} = \frac{n^2-3n+2}{2}$

c) Let's say $g(n) = n^2$

$\lim_{n\to\infty} \frac{n^2-3n+2}{2n^2} = \lim_{n\to\infty} \frac{\cancel{n^2}\left(1 - \overset{0}{\cancel{\frac{3}{n}}} + \overset{0}{\cancel{\frac{2}{n^2}}}\right)}{2\cancel{n^2}} = \frac{1}{2}$

$\Rightarrow A(n) = f(n) \in \Theta(n^2)$

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5) a) Basic operation is matrix multiplication

b) $\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1 = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n-1 = \sum_{i=0}^{n-1} (n-1)^2 = (n-1)^3$

$A(n) = n^3 - 3n^2 + 3n + 1$ //

c) Let's say $g(n) = n^3$

$$\lim_{n\to\infty} \frac{A(n)}{g(n)} = \lim_{n\to\infty} \frac{n^3 - 3n^2 + 3n + 1}{n^3} = \lim_{n\to\infty} \frac{\cancel{n^3}\overbrace{\left(1 - \frac{3}{n} + \frac{3}{n^2} + \frac{1}{n^3}\right)}^{0}}{\cancel{n^3}}$$

$= 1$

$\Rightarrow A(n) \in \Theta(n^3)$

6) algo (A[0, ..., n-1] , num)

```
for i to n-1 do
    for j to n-1 do
        if i == j
            continue                              ] Θ(1)
        if arr[i] * arr[j] == num
            print (arr[i], arr[j])                ] Θ(1)
```

$\Theta(n)$

$\Theta(n^2)$

$\Rightarrow \Theta(n^2)$ //