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1) $T(n) = T(n-1) + c \rightarrow$ constant time

a) $T(n-1) = T(n-2) + c$

$\vdots$

$T(1) = c$

$+$

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$T(n) = c \cdot n \in \Theta(n)$

b) $T(n) = 2T\left(\frac{n}{2}\right) + c$

from Master theorem

$a=2$

$b=2$

$\underbrace{n^{\log_2 2} = n = O(c)}_{\text{case - 1}} \Rightarrow T(n) = \Theta(n^{\log_a b})$

$T(n) = \Theta(n)$

Both algorithm's complexities are $\Theta(n)$ so it does not matter which algorithm to choose.

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2) $n \rightarrow$ number of costants

$$\sum_{i=1}^{n-1} \sum_{j=1}^{n-i} 1 = \sum_{i=1}^{n-1} n-i-1 = (n-2)+(n-3)+ \dots +0$$

$$\Rightarrow T(n) = \frac{(n-1)(n-2)}{2} = \frac{n^2 + \dots}{2} \in \Theta(n^2)$$

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3) Worst case happens if all the characters in the string are the first letter. And the best case happens if the list does not contain the first letter

$$T_{best} = \sum_{i=1}^{n} 1 = n \in \Theta(n)$$

$$T_{worst} = \sum_{i=1}^{n} \sum_{j=i}^{n} 1 = \sum_{i=1}^{n} (n-1) = (n-1)+(n-2)+ \cdots +0$$

$$\Rightarrow \frac{(n-1) \cdot n}{2} \in \Theta(n^2)$$

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4)

$$\text{Set of points} = \{ \overbrace{(x_1, x_2, x_3, \dots, x_k)}^{k \text{ dimensions}},$$
$$(x_1, x_2, x_3, \dots, x_k),$$
$$\vdots$$
$$(x_1, x_2, x_3, \dots, x_k) \}$$

n points

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function distance (p1[1,...,k], p2[1,...,k])
    total ← 0
    for i from 1 to k :
        total ← p1[i]*p1[i] - p2[i]*p2[i] + total    Θ(1)
    return sqrt (total)

function closestPair ( pointArr[1,....,n]):
    min ← inf
    current ← 0
    for i from 1 to n :
        for j from i+1 to n :
            current ← distance (pointArr[i], pointArr[j])
            if current < min :
                min ← current
    return min
```

Analysis of the function distance:

$$\sum_{i=1}^{k} 1 = \underbrace{1+1+\dots+1}_{k} = k \in \Theta(k)$$

constant time operations

Analysis of the function closestPair

$$\sum_{i=1}^{n} \sum_{j=i+1}^{n} k = \sum_{i=1}^{n} k(n-i-1) = (n-1)k + (n-2)k + \dots + k$$

$$\Rightarrow k(1 + 2 + \dots + n-1)$$

$$\frac{k(n-1)n}{2} = \frac{k(n^2-n)}{2} \in \Theta(kn^2)$$

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5)

a) $n \rightarrow$ number of regions

$$T(n) = \sum_{i=1}^{n} \sum_{j=i}^{n} 1 + \sum_{i=1}^{n} 1$$

adjusting the string cluster

$$T(n) = \sum_{i=1}^{n} n-i + n$$

$$T(n) = \underbrace{(n-1) + (n-2) + \dots + 0}\ + n$$

$$T(n) = \frac{n^2-n}{2} + n \in \Theta(n^2)$$

crossSum function

b) $T(n) = 2T\left(\frac{n}{2}\right) + M(n)$

$$M(n) = \sum_{i=1}^{n} 1 + \sum_{i=1}^{n} 1 = 2n$$

$$\Rightarrow T(n) = 2T\left(\frac{n}{2}\right) + 2n$$

from Master Theorem $a=2$, $b=2$

$\Rightarrow$ case-2 $\quad 2n = \Theta(n^{\log_2 2} \log^k n)$

$$T(n) = \Theta(n^{\log_2 2} \log^{k+1} n)$$

$$T(n) = \Theta(n \log n)$$

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