

$$1) \sum_{i=1}^n c = cn \in \Theta(n)$$

Homework-3 Divide & Conquer Approach:

$$T(n) = 2T\left(\frac{n}{2}\right) + M(n) \quad \xrightarrow{\text{cross sum}}$$

$$M(n) = \sum_{i=1}^n 1 + \sum_{i=1}^n 1 = 2n$$

$$\Rightarrow T(n) = 2T\left(\frac{n}{2}\right) + 2n \quad \text{from Master Theorem} \quad \begin{matrix} a=2 \\ b=2 \end{matrix}$$

$$\text{case-2} \rightarrow 2n = \Theta(n^{\log_2 2} \log^k n) \quad \checkmark$$

$$T(n) \in \Theta(n^{\log_2 2} \log^{k+1} n)$$

$$T(n) \in \Theta(n \log n)$$

Dynamic programming solves the problem more efficiently.

2) knapsack method : ($n \rightarrow$ size of input
 $m \rightarrow$ cap size)

$$T(n) = \underbrace{\sum_{i=1}^{m+1} \sum_{j=1}^{n+1} c}_{\text{table creation}} + \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} c = \sum_{i=1}^{m+1} (n+1)c + \sum_{i=1}^{n+1} (m+1)c$$

$$T(n) = (n+1)(m+1)c + (n+1)(m+1)c$$

$$T(n) = (mnc + nc + mc + c) \cdot 2 \Rightarrow T(n) \in \Theta(nm)$$

3) knapsack method : ($n \rightarrow$ size of input
 $m \rightarrow$ cap size)

$$T(n) = \text{insertionSort}(n) + \sum_{i=1}^n c = \text{insertionSort}(n) + n$$

worst case : insertion sort takes $\Theta(n^2)$ time

$$T(n) = n^2 + n \in \Theta(n^2)$$

Best case : insertion sort takes $O(n)$ time

$$T(n) = n + n \in O(n)$$

4) maxCourses method =

$$T(n) = \text{insertionSort}(n) + \sum_{i=1}^n c = \text{insertionSort}(n) + nn$$

worst-case $\rightarrow T(n) = n^2 + n \in \Theta(n^2), n$

best-case $\rightarrow T(n) = n + n = 2n \in O(n),$