

I hereby pledge on my honor that I will strictly adhere to academic integrity code and the work done on this examination is solely my own and I will not receive/give any help from/to anybody or source during this examination.

1) a) In array list get method takes constant time. And the for loop takes $\log_2 n$ times. Because we multiply by 2

$$\Theta(n) = \log_2 n$$

b) In linked list, getting an element takes linear time. And the for loop takes $\log_2 n$ times again

$$\Theta(n) = n \log_2 n$$

$$\begin{aligned} 2) \quad T(n) &= n + T(n-1) \\ T(n-1) &= n-1 + T(n-2) \end{aligned}$$

$$T(n) = \begin{cases} n + T(n-1), & n > 0 \\ 1, & n = 0 \end{cases}$$

$$\begin{aligned} & \vdots \\ & T(1) = 1 + T(0) \\ + & \hline & T(n) - T(n-1) = n \\ & T(n-1) - T(n-2) = n-1 \end{aligned}$$

$$T(1) - T(0) = 1$$

$$T(n) - T(0) = \frac{n(n+1)}{2}$$

n square
↑

$$T(n) = \frac{n(n+1)}{2} + 1 = \Rightarrow T\left(\frac{n^2+n}{2} + 1\right) = O(n^2)$$

3) a) Since strings are immutable in java, everytime we change the string we have to copy all of it's content to another String object.

And we do this process for 'n' times.
nodeRef -> nodeRef.next → we do this for 'n' times also,

$T(n) = 2n + 2$
 $n \log n \geq 2n + 2$
 $c = 2$
 $n \log n \geq 2n + 1$
 $2n \log n \geq 2n + 1$
 $n = 2$ for all $n \geq 2$
 and $c = 2$
 $O(n \log n)$ ✓ true

$n \cdot c_1 \geq 2n + 2 \geq n \cdot c_2$
 \downarrow
 $c_1 = 3$
 $3n \geq 2n + 2$
 $n = 2$ for all $n \geq 2$
 and $c_1 = 3$
 $\Theta(n)$ ✓ true

$2n + 2 \geq n \cdot c_2$
 $c_2 = 1$
 $2n + 2 \geq n$
 $n + 2 \geq 0$
 for all $n \geq 0$
 and $c_2 = 1$
 $\Theta(n)$ ✓ true

$cn^2 \leq 2n + 2$
 wrong $\Omega(n^2)$ ✗ false
 There is not a constant c and no

b) Loop iterates for index(n) times.

Let's say $T(n) = n$
 $n \log n \geq n$
 $c = 1$
 $n \log n \geq n$
 $n = 1$ for all $n \geq 1$
 $O(n \log n)$ ✓ true

$n \cdot c_1 \geq n \geq n \cdot c_2$
 \downarrow
 $c_1 = 1$ ✓ $c_2 = 1$
 $n = 1$ ✓ $n = 1$
 $\Theta(n)$ ✓ true
 for all $n \geq 1$
 and $c_1 = c_2 = 1$

$n^2 \cdot c < n$
 n is a positive int.
 $n \cdot c < 1$
 $\Omega(n^2)$ ✗ false