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1st Slide

Q4) a) and b)

p	q	$p \wedge q$	$(p \wedge q) \to p$	$p \vee q$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	T

c) and d)

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$	$p \wedge q$	$(p \wedge q) \to (p \to q)$
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	F	T
F	F	T	T	T	F	T

Q5) Let $P(x)$ be "x is perfect"

Let $F(x)$ be "x is your friend."

Let the domain be all people.

a) This means that everyone has the property of being not perfect.

This is represented by $\forall x \neg P(x)$.

This is "Everyone is perfect": $\neg \forall x P(x)$.

b) Negation of "Everyone is perfect": $\neg \forall x P(x)$.

c) If someone is your friend, then that person is perfect: $\forall x (F(x) \to P(x))$

d) $\exists x (F(x) \wedge P(x))$

e) $\forall x (F(x) \wedge P(x))$

f) $(\neg \forall x F(x)) \vee (\exists x \neg P(x))$

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Q6) Let $F(p)$ be "Printer p is out of service."

Let $B(p)$ be "Printer p is busy."

Let $L(j)$ be "Print job j is Lost."

Let $Q(j)$ be "Print job j is queued."

a) If there is a printer that is both out of service and busy, then some job has been lost / is lost.

b) If every printer is busy, then there is a job in the queue.

c) If there is a job that is both queued and lost, then some printers are out of service.

d) If every printer is busy and every job is queued, then some job is lost.

2nd Slide

Q3) Prove that 2 divides n^2+n whenever n is a positive integer.

Basis Step: Let n be $=1$, then $1^2+1=2$ which can be divided by 2 so we show that the basis step is correct.

Inductive Step: Let n be $=k$, then we accept that 2 divides k^2+k.

Let $n=k+1$, then we have to prove that 2 divides $(k+1)^2+(k+1)$.

$(k+1)^2+(k+1) = k^2+2k+1+k+1 = \underbrace{k^2+k}_{\text{Recall that we accept } k^2+k \text{ ca be divided by } 2!} + \underbrace{2(k+1)}_{\text{This part is also divided by } 2!}$ → So we proved that 2 divides n^2+n where $n \in \mathbb{Z}^+$

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2nd Slide

Q4) Prove that $1^2+3^2+5^2+\dots+(2n+1)^2 = \dfrac{(n+1)(2n+1)(2n+3)}{3}$

whenever n is a nonnegative integer.

Basis Step: Let $n=1$, then $\underbrace{1^2+3^2}_{10} = \dfrac{(1+1)(2.1+1)(2.1+3)}{3}$

$$10 = \frac{2.3.5}{3} = 10 \checkmark$$

We show that the statement is correct in the basis step when $n=1$.

Inductive Step: Let $n=k$, then we accept that:

$$\underbrace{1^2+3^2+5^2+\dots+(2k+1)^2}_{a} = \underbrace{\frac{(k+1)(2k+1)(2k+3)}{3}}_{a} \text{ is correct.}$$

Let $n=k+1$, then we have to prove that:

$$\underbrace{1^2+3^2+5^2+\dots+(2k+1)^2}_{a} + (2(k+1)+1)^2 = \frac{(k+1+1)(2(k+1)+1)(2(k+1)+3)}{3}$$

is correct.

Instead of part "a" in the equation above, we can write the right hand side of the equation for $n=k$. Hence,

$$\frac{(k+1)(2k+1)(2k+3)}{3} + (2(k+1)+1)^2 = \frac{(k+1+1)(2(k+1)+1)(2(k+1)+3)}{3}$$

$$\frac{(k+1)(2k+1)(2k+3)}{3} + (2k+3)^2 = \frac{(k+2)(2k+3)(2k+5)}{3}$$

$$(2k+3)^2 = \frac{(k+2)(2k+3)(2k+5) - (k+1)(2k+1)(2k+3)}{3}$$

$$(2k+3)^2 = \left(\frac{2k+3}{3}\right)\left((k+2)(2k+5) - (k+1)(2k+1)\right) \rightarrow (2k+3)^2 = \left(\frac{2k+3}{3}\right)(6k+9)$$

$$(2k^2+5k+4k+10) - (2k^2+k+2k+1) =$$

$$2k^2+9k+10-2k^2-3k-1 =$$

$$6k+9$$

$$(2k+3)^2 = \frac{(2k+3)^2 \cdot \cancel{3}}{\cancel{3}}$$

$$(2k+3)^2 = (2k+3)^2 \checkmark$$