CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.
Yes and it's shown
(Solution)

(b) Find the solution with $a_0 = 1$.

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$a_n^{(h)}$ ↓ homogeneous part

$a_n^{(p)}$ → particular part

$$a_n^{(h)} : a_n - 3a_{n-1} = 0$$

$$r - 3 = 0$$

$$\boxed{r=3}$$

$$a_n^{(p)} : 2^n \Rightarrow A.2^n \text{ (solution)}$$

$$a_n = A2^n$$

$$a_{n-1} = A2^{n-1}$$

$$\Rightarrow A2^n = 3A2^{n-1} + 2^n$$

$$A = \frac{3A}{2} + 1$$

$$\boxed{A = -2}$$

$$-2\,2^n = -2^{n+1}$$

↓ an solution

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$a_n = \alpha 3^n - 2^{n+1}$$

$$a(0) = \alpha - 2 = 1$$

$$\boxed{\alpha = 3} \rightarrow \boxed{a_n = 3^{n+1} - 2^{n+1}}$$

(Solution)

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.

$$f_n = f_n^{(h)} + f_n^{(p)}$$

$$f_n^{(h)} : r^2 - 4r + 4 = 0$$

$$(r-2)^2 = 0$$

$$\boxed{r_{1,2} = 2,2}$$

$$f_n^{(p)} = An^2 + Bn + C$$

$$f_{n-1} = A(n^2 - 2n + 1) + B(n-1) + C$$

$$f_{n-2} = A(n^2 - 4n + 4) + B(n-2) + C$$

$$f(n) = 4f(n-1) - 4f(n-2) + n^2$$

$$An^2 + Bn + C = 4(An^2 - 2An + Bn + A - B + C)$$
$$-4(An^2 - 4An + Bn + 4A - 2B + C) + n^2$$

$$\Rightarrow \boxed{A = 1}$$

$$Bn = -8An + 16An$$

$$\boxed{B = 8}$$

$$-12 + 4B = C$$

$$\boxed{C = 20}$$

$$f(n) = \alpha 2^n + \beta n 2^n + An^2 + 8n + 20$$

$$f(0) = \alpha + 20 = 2$$

$$\boxed{\alpha = -18}$$

$$f(1) = 2(\alpha + \beta) = -24$$

$$\Rightarrow \boxed{\beta = 6}$$

$$\boxed{f(n) = -18 \cdot 2^n + 6n2^n + n^2 + 8n + 20}$$

(Solution)

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.
(Solution)

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.
(Solution)

$a_n = 2a_{n-1} - 2a_{n-2}$

$r^2 - 2r + 2 = 0$

$\Delta = 4 - 4.2 = -4$

$x = \frac{-b \mp \sqrt{\Delta}}{2a} = \frac{2 \mp 2i}{2} = 1 \mp i$

roots

$a_n = \alpha(1+i)^n + \beta(1-i)^n$

$a_0 = \boxed{\alpha + \beta = 1}$

$a_1 = \alpha(1+i) + \beta(1-i) = 2$

$\alpha + \beta + i(\alpha - \beta) = 2$

$\alpha - \beta = \frac{1}{i} = -i$

$\alpha + \beta = 1$

$\alpha - \beta = -i$

$+$

$2\alpha = 1 - i$

$\alpha = \frac{1-i}{2}, \quad \beta = \frac{1+i}{2}$

$$\boxed{a_n = \left(\frac{1-i}{2}\right)(1+i)^n + \left(\frac{1+i}{2}\right)(1-i)^n}$$