Barış Ayyıldız 1901042252 CSE 241-502 Final Exam B. Ayyıldız

I hereby pledge on my honor that I will strictly adhere to academic integrity codes and the work done on this examination is solely my own and I will not receive/give any help from/to anybody or source during this examination.

a) Yes it is linear homogeneous recurrence. Because a_n only depends on it's previous values such as $a_{n-1}, a_{n-2}, a_{n-3}$.

$a_0 = 3$

$a_1 = -2$

$a_2 = 5$

$a_3 = -4$

Degree = 3

$$a_n = -a_{n-1} + a_{n-2} + a_{n-3}$$

$$a_n + a_{n-1} - a_{n-2} - a_{n-3} = 0$$

$$r^3 + r^2 - r - 1 = 0$$

$(r^2+2r+1)(r-1)$

$r^3 - r^2 + 2r^2 - 2r + r - 1$

$r^3 + r^2 - r - 1$

$$(r+1)^2(r-1) = 0$$

$$r = 1, -1, -1 \rightarrow a_n = \alpha_1 1^n + \alpha_2(-1)^n + \alpha_3 n(-1)^n$$

$$a_0 = \alpha_1 + \alpha_2 = 3$$

$$a_1 = \alpha_1 - \alpha_2 - \alpha_3 = -2$$

$$a_2 = \alpha_1 + \alpha_2 + 2\alpha_3 = 5 \Rightarrow 2\alpha_3 = 2$$

$\alpha_1 + \alpha_2 = 3$

$\alpha_1 - \alpha_2 = -1$

$\boxed{\alpha_1 = 1}$

$\boxed{\alpha_2 = 2}$

$\boxed{\alpha_3 = 1}$

$$a_n = 1^n + 2(-1)^n + n(-1)^n \ //$$

CamScanner ile tarandı

Barış Ayyıldız
1901042252
B. Ayyıldız

b) $a_0 = 1$ $a_2 = a_1 + a_0$
$a_1 = 1$ $a_3 = a_2 + a_1$

$$a_n = \begin{cases} a_{n-1} + a_{n-2} & , n > 1 \\ 1 & , 1 \\ 1 & , 0 \end{cases}$$

$a_n - a_{n-1} - a_{n-2} = 0$

$r^2 - r - 1 = 0$

$r = \dfrac{1 \mp \sqrt{5}}{2}$ $a_n = \alpha \left(\dfrac{1+\sqrt{5}}{2}\right)^n + \beta \left(\dfrac{1-\sqrt{5}}{2}\right)^n$

$a_0 = \alpha + \beta = 1$

$a_1 = \dfrac{\alpha + \alpha\sqrt{5} + \beta - \beta\sqrt{5}}{2} = 1$

$\Rightarrow \underbrace{\alpha + \beta}^{1} + \sqrt{5}(\alpha - \beta) = 2$

$\alpha - \beta = \dfrac{\sqrt{5}}{5}$

$\alpha + \beta = 1$

$2\alpha = \dfrac{\sqrt{5}+5}{5}$

$\alpha = \dfrac{\sqrt{5}+5}{10}$, $\beta = \dfrac{5-\sqrt{5}}{10}$

$$a_n = \frac{5+\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^n$$