CSE 211: Discrete Mathematics **(Due: 10/10/20)**

Problem Session #4

Instructor: Dr. Zafeirakis Zafeirakopoulos
Assistant: Gizem Süngü

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- They are the questions that I am going to solve during the next problem session.
- You should examine these questions before the lecture.
- I will write ONLY answers of the questions on the white board during the lecture.
- Since I have already shared the questions on Moodle, to obtain the questions is your responsibility.
- Please attend the lecture because I will take your attendances.

Problem 1: Cartesian Product of Sets

(0 points)

Let A = {a, b, c }, B = {x, y}, and C = { 0, 1}. Find

(a) A × B × C.

(b) C × B × A.

(c) C × A × B.

(d) B × B × B.

Problem 2: Set Operations

(0 points)

Let A, B, and C be sets. Show that

(a) (A ∪ B) ⊆ (A ∪ B ∪ C).

(b) (A - B) - C ⊆ A - C.

Problem 3: Relations

(0 points)

For each relation below determine if they are reflexive, symmetric, anti-symmetric, and transitive. Ifa relation has a property, indicate this. If it does not, show why.

(a) X = {a, b, c, d, e}, R_1 = {(a, a), (a, b), (a, e), (b, b), (b, e), (c, c), (c, d), (d, d), (e, e)}

(b) X = {a, b, c, d, e}, R_2 = {(a, a), (a, b), (b, a), (b, b), (c, c), (c, d), (d, c), (d, d)}

(c) X = {a, b, c, d, e}, R_3 = {(a, a), (b, b), (c, c), (d, d), (e, e)}

Problem 4: Sets (0 points)

Which of the following sets are equal? Show your work step by step.

(a) {t : t is a root of x^2 − 6x + 8 = 0}

(b) {y : y is an integer number in the closed interval [2, 3]}

(c) {4, 2, 5, 4}

(d) {4, 5, 7, 2} - {5, 7}

(e) {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}

(Solution)

(a) x^2 − 6x + 8 = (x - 4)(x - 2)=0 so the roots are t = 4 or t = 2 so the set is { 2, 4 }.

(b) the set is { 2, 3 }

(c) Obviously the set is { 4, 2, 5, 4 }

(d) { 4, 2 }

(e) The number of sides of a rectangle is 4 or the number of digits in any integer between 11 and 99 is 2. Since or means union for the sets, { 4 } U { 2 } = { 2, 4 }.

Consequently, **(a)**, **(d)** and **(e)** are equal.

Problem 5: Cartesian Product of Sets (0 points)

Explain why (A × B) × (C × D) and A × (B × C) × D are not the same.

(Solution)

- Assume that a, b, c, d are elements from the sets A, B, C and D, respectively.
- The cartesian product of the elements for (A × B) × (C × D) is ((a, b), (c, d)).
- On the other hand, the cartesian product of the elements for A × (B × C) × D is (a, (b, c), d)
- Since ((a, b), (c, d)) ≠ (a, (b, c), d), we can say that (A × B) × (C × D) and A × (B × C) × D are not the same. Hence, the proposition is true.

Problem 6: Relations (0 points)

Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where (x, y) $\in$ R if and only if

(a) x $\neq$ y.

(Solution)

- **Reflexive:** Since a relation R on a set A is called reflexive when (a, a) $\in$ R for every element a $\in$ A, the relation R on the set cannot be reflexive with x $\neq$ y.

- **Symmetric:** Since for all (x, y) values, there must be (y, x) values by satisfying y $\neq$ x, the relation is symmetric.

- **Antisymmetric:** Since a relation R on a set A is antisymmetric if $\forall$ a $\forall$ b(((a, b) $\in$ R $\wedge$ (b, a) $\in$ R) $\Rightarrow$ (a = b)), the relation on the set is not antisymmetric due to x $\neq$ y.

- **Transitive:** Since a relation R on a set A is called transitive if whenever (a, b) $\in$ R and (b, c) $\in$ R, then (a, c) $\in$ R, for all a, b, c $\in$ A and a $\neq$ b $\neq$ c, the relation on the set is transitive.

(b) xy $\geq$ 1.

(Solution)

- **Reflexive:** The constraint xy $\geq$ 1 provides (1, 1) and (a, a) where a $\in$ $\mathbb{Z}$.Hence, the relation on the set is reflexive.

- **Symmetric:** Since for all (x, y) values, there must be (y, x) values, the relation is symmetric.

- **Antisymmetric:** Since a relation R on a set A is not antisymmetric if $\forall$ a $\forall$ b(((a, b) $\in$ R $\wedge$ (b, a) $\in$ R) $\Rightarrow$ (a $\neq$ b)), the relation on the set is not antisymmetric.

- **Transitive:** The relation on the set is transitive.

(c) x = y + 1 or x = y - 1.

(Solution)

- **Reflexive:** Since a relation R on a set A is called reflexive when (a, a) $\in$ R for every element a $\in$ A, the relation R on the set cannot be reflexive because x $\neq$ y.

- **Symmetric:** Assume that there is a (x, y) relation by satisfying x = y + 1 so the relation can be written as (y+1, y). The symmetry of (y+1, y) is (y, y+1) which is the relation by satisfying x = y - 1. Hence, the relation is symmetric.

- **Antisymmetric:** Since a relation R on a set A is antisymmetric if $\forall$ a $\forall$ b(((a, b) $\in$ R $\wedge$ (b, a) $\in$ R) $\Rightarrow$ (a = b)), the relation on the set is not antisymmetric due to x $\neq$ y for both the two constraints.

- **Transitive:**
  - Assume that y has a value y'.
  - There are two possible relations with y' value of y, (y'+1, y') and (y'-1, y').
  - In order to check transitivity, we should also obtain the relations whose x have y' value.
  - There are two possible relations with y' value of x, (y', y'-1) and (y', y'+1).

- Firstly, we can check (y'+1, y') and (y', y'-1). Since the difference between y'+1 and y'-1 is 2, there is no relation such as (y'+1, y'-1) with respect to the given constraints.
- Secondly, we can check (y'+1, y') and (y', y'+1). Since x and y values cannot be equal according to the given constraints, there is no relation such as (y'+1, y') and (y', y'+1).
- Thirdly, we can check (y'-1, y') and (y', y'+1). There is the same situation with (y'+1, y') and (y', y'+1).
- Consequently, the relation on the set is not transitive.