Barış Ayyıldız 1901042252   CSE 211-502 Final Exam   B. Ayyıldız

I hereby pledge on my honor that I will strictly adhere to academic integrity codes and the work done on this examination is solely my own and I will not receive/give any help from/to anybody or source during this examination.

a) Prove that "$P \to Q \iff \neg Q \to \neg P$" by using truth tables

b) Prove that $\neg(a \wedge \neg b) \vee b = a \to b$ using Logical equivalences

c) Prove that $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ using induction

d) What is cardinality of $S = \{\{1\}, \{1,2\}, \{3\}, \{4,5\}, \{6\}\}$

e) If $A = \{1, 2, \{3\}, a\}$ and $B = \{\{1,2\}, 3, a\}$ compute sets $A \cup B$, $A \cap B$ and $A \setminus B$. Which of the following are functions. If they are functions, are they one-to-one, onto, bijections?

---

b) $\neg(a \wedge \neg b) \vee b \overset{?}{=} a \to b$

$\underbrace{(\neg a \vee b)}_{b} \vee b \overset{?}{=} a \to b$

$\neg a \vee b = a \to b$

$a \to b = a \to b$ ✓

d) Cardinality of a set means number of element. And it is denoted as $|S|$

$|S| = 5$, There are 5 elements

Boris Ayyıldız
190104?752
Ş. Ayyıldız

e) $A \cup B = \{1, 2, \{3\}, a, \{1,2\}, 3\}$

$A \cap B = \{a\}$ $A \setminus B = \{1, 2, \{3\}\}$

---

a)

P	Q	$\neg P$	$\neg Q$	$P \rightarrow Q$	$\neg Q \rightarrow P$	$(P \rightarrow Q) \Leftrightarrow (\neg Q \rightarrow P)$
1	1	0	0	1	1	1
1	0	0	1	0	1	0
0	1	1	0	1	1	1
0	0	1	1	1	0	0

---

c) for $n = 1$: $\dfrac{1}{1 \cdot 2} = \dfrac{1}{1 \cdot 2}$ ✓ proved

for $n = k$ $\dfrac{1}{1 \cdot 2} + \dfrac{1}{2 \cdot 3} + \cdots + \dfrac{1}{k(k+1)} = \dfrac{k}{k+1}$ let's suppose this is true
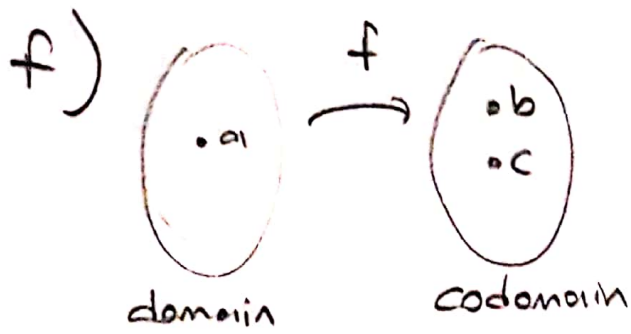
for $n = k+1$ $\underbrace{\dfrac{1}{1 \cdot 2} + \dfrac{1}{2 \cdot 3} + \cdots + \dfrac{1}{k(k+1)}}_{\frac{k}{k+1}} + \dfrac{1}{(k+1)(k+2)} = \dfrac{k+1}{k+2}$

$\Rightarrow \dfrac{1}{(k+1)(k+2)} = \dfrac{k+1}{k+2}_{(k+1)} - \dfrac{k}{k+1}_{(k+2)}$

$\Rightarrow \dfrac{1}{(k+1)(k+2)} = \dfrac{k^2 + 2k + 1 - k^2 - 2k}{(k+1)(k+2)}$

$\Rightarrow \dfrac{1}{(k+1)(k+2)} = \dfrac{1}{(k+1)(k+2)}$ ✓

f)

Barış Ayyıldız 190104??52 ŞiAyiu

domain   codomain

If the (a,b) exists then (a,c) cannot. And every element en domain should have a corresponding value in codomein

	Function	One-to-one	onto	Bijection
$f(x) = x^2 + 2$	✓	X	X	X
$f(x) = \dfrac{1}{x}$	✓	✓	X	X
$f(x) = \dfrac{1}{x} + e^x$	X	X	X	X
$f(x) = x + 156$	✓	✓	✓	✓