CSE 211: Discrete Mathematics **(Due: 24/11/20)**

Problem Session #5

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- They are the questions that I am going to solve during the next problem session.
- You should examine these questions before the lecture.
- I will write ONLY answers of the questions on the white board during the lecture.
- Since I have already shared the questions on Moodle, to obtain the questions is your responsibility.
- Please attend the lecture because I will take your attendances.

Problem 1: Functions (0 points)

If f and f ∘ g are one-to-one, does it follow that g is one-to-one? Justify your answer.
(Solution)

- Since we know f and f ∘ g are one-to-one, we can define f and g functions as g: A ⇒ B and f: B ⇒ C.
- Assume that b_1 and b_2 are the elements ∈ B and since we know that f is one-to-one, their images are not equal, f(b_1) ≠ f(b_2).
- Assume that a_1 and a_2 are the elements ∈ A and since we know that f ∘ g is one-to-one, we can say that f(g(a_1)) ≠ f(g(a_2)).
- Using these two equations, we can suppose that g(a_1) = b_1 and g(a_2) = b_2.
- Since g(a_1) ≠ g(a_2), we can see that g is one-to-one.

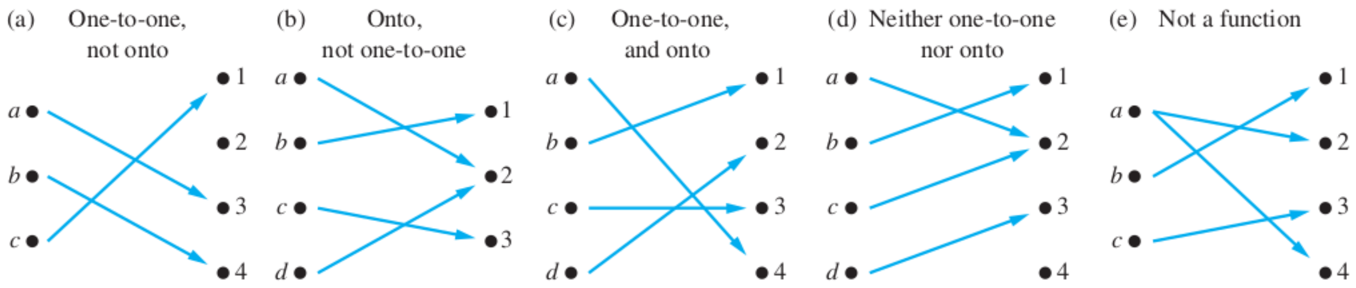
Problem 2: Inverse of Functions (0 points)

Let f be the function from $\mathbb{R}$ to $\mathbb{R}$ defined by f(x) = x^2. Find
(a) f^{-1} ({ x | 0 < x < 1 })
(Solution) { x | - 1 < x < 0 ∨ 0 < x < 1 }

(b)f^{-1} ({ x | x > 4 })
(Solution) {x | x > 2 ∨ x < -2}

Problem 3: Function Properties (0 points)

Give an example of a function N → N which is:



(a) onto but not one-to-one;
(Solution) The function defined by f (1) = 1 and for n > 1, f (n) = n - 1, for example.
(b) neither one-to-one nor onto;
(Solution) the constant function f (n) = 107 for all n, for example.
(c) both one-to-one and onto.
(Solution) the identify function F (n) = n, for all n, for example.

Problem 4: Functions (0 points)

Let S = {1, 2, 3, 4, 5} and let f , g, h : S → S be the functions defined by

- f = {(1, 2), (2, 1), (3, 4), (4, 5), (5, 3)}
- g = {(1, 3), (2, 5), (3, 1), (4, 2), (5, 4)}
- h = {(1, 2), (2, 2), (3, 4), (4, 3), (5, 1)}

(a) Explain why f and g have inverses but h does not. Find f^{-1} and g^{-1}.
(Solution)
f^{-1} = {(1, 2), (2, 1), (3, 5), (4, 3), (5, 4)};
g^{-1} = {(1, 3), (2, 4), (3, 1), (4, 5), (5, 2)}

Functions f and g have inverses because they are one-to-one and onto while h does not have an inverse because it is not one-to-one(equally because it is not onto).

(b) Show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1} \neq f^{-1} \circ g^{-1}$.

(Solution)

- $(f \circ g)^{-1} = f(g(x))^{-1}$ = {(1, 4), (2, 3), (3, 2), (4, 1), (5, 5)}
  - x $\in \{1, 2, 3, 4, 5\}$ for $f(g(x))^{-1}$
  - when x = 1:
    - g(1) = 3 so f(g(1)) = f(3) = 4. If f(g(1)) = 4, then $f(g(4))^{-1} = 1$.
  - when x = 2:
    - g(2) = 5 so f(g(2)) = f(5) = 3. If f(g(2)) = 3, then $f(g(3))^{-1} = 2$.
  - when x = 3:

        - g(3) = 1 so f(g(3)) = f(1) = 3. If f(g(2)) = 3, then $f(g(3))^{-1} = 2$.
    - when x = 4:
        - g(4) = 2 so f(g(4)) = f(2) = 1. If f(g(4)) = 1, then $f(g(1))^{-1} = 4$.
    - when x = 5:
        - g(5) = 4 so f(g(5)) = f(4) = 5. If f(g(5)) = 5, then $f(g(5))^{-1} = 5$.

- $g^{-1} \circ f^{-1} = g^{-1}(f^{-1}(x)) = \{(1,\ 4),\ (2,\ 3),\ (3,\ 2),\ (4,\ 1),\ (5,\ 5)\} = (f \circ g)^{-1}$
    - x $\in \{1, 2, 3, 4, 5\}$ for $g^{-1}(f^{-1}(x))$
    - when x = 1:
        - $f^{-1}(1) = 2$ so $g^{-1}(f^{-1}(1)) = g^{-1}(2) = 4 = (1, 4)$
    - when x = 2:
        - $f^{-1}(2) = 1$ so $g^{-1}(f^{-1}(2)) = g^{-1}(1) = 3 = (2, 3)$
    - when x = 3:
        - $f^{-1}(3) = 5$ so $g^{-1}(f^{-1}(3)) = g^{-1}(5) = 2 = (3, 2)$
- $f^{-1} \circ g^{-1} = \{(1,\ 5),\ (2,\ 3),\ (3,\ 2),\ (4,\ 4),\ (5,\ 1)\} \neq (f \circ g)^{-1}$