Gebze Technical University
Computer Engineering Department
MATH118 Probability and Statistics

HOMEWORK I

Due April,14 2024 23:59

1. What is an equiprobable sample space? Define and give an example of a sample space which is not equiprobable?
2. Define probability formally (mathematically). How is probability defined in equiprobable and non-equiprobable sample spaces?
3. Prove that two events A and B are independent if and only if $P(A \cap B) = P(A)P(B)$.
4. Barış and Erdem are playing a game with some game cards. Each card is equally likely to show either ´1´, ´2´ or ´3´. Erdem picks 10 cards from the deck (without showing that to Barış) and Barış tries to guess the numbers on each card correctly. What is the probability that Barış correctly guesses at least one card?
5. A sequence of interviews is required to be hired by an institution. Corrupted by their power, the jury running the hiring process by taking bribes from many of the candidates. Each set of interviews, a given candidate is either allowed to stay on the list or is eliminated. If the candidate has been bribing the judges, she will be allowed to stay with probability 1. If the candidate has not been bribing the jury, she will be allowed to stay with probability 1/3. Suppose that 1/5 of the candidates have been bribing the jury. The same candidates bribe the jury in both rounds, i.e., if a candidate bribes them in the first round,she bribes them in the second round too, if a candidate is honest then she keeps being honest.
   - (a) If you pick a random candidate who was allowed to stay during the first round, what is the probability that she was bribing the jury?
   - (b) If you pick a random candidate, what is the probability that she is allowed to stay during both of the first two interviews?
   - (c) If you pick random candidate who was allowed to stay during the first interview, what is the probability that she is eliminated during the second round?
6. A rare disease exists with which only 1 in 500 is affected. A test for the disease exists, but of course it is not infallible. A correct positive result (patient actually has the disease) occurs 95% of the time, while a false positive result (patient does not have the disease) occurs 1% of the time. If a randomly selected individual is tested and the result is positive, what is the probability that the individual has the disease?

7. There is a 50-50 chance that the queen carries the gene of hemophilia. If she is a carrier, then each prince has a 50-50 chance of having hemophilia independently. If the queen is not a carrier, the prince will not have the disease. Suppose the queen has had three princes without the disease. What is the probability the queen is a carrier?

8. Let W be a random variable giving the number of heads minus the number of tails in three tosses of a coin. List the elements of the sample space S for the three tosses of the coin and to each sample point assign a value w of W.

9. Suppose a certain type of small data processing firm is so specialized that some have difficulty making a profit in their first year of operation. The probability density function that characterizes the proportion Y that make a profit is given by

$$f(y) = \begin{cases} ky^4(1-y)^3, & 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

   (a) What is the value of k that renders the above a valid density function?

   (b) Find the probability that at most 50% of the firms make a profit in the first year.

   (c) Find the probability that at least 80% of the firms make a profit in the first year.

10. Prove the following:
Let $X_1, X_2, ..., X_n$ be n random variables, discrete or continuous, with joint probability distribution $f(x_1, x_2, ..., x_n)$ and marginal distribution $f_1(x_1), f_2(x_2), ..., f_n(x_n)$, respectively. The random variables $X_1, X_2, ..., X_n$ are said to be mutually statistically independent if and only if

$$f(x_1, x_2, ..., x_n) = f_1(x_1)f_2(x_2)...f_n(x_n)$$

for all $(x_1, x_2, ..., x_n)$ within their range.