Math 118 HW1

1) Equiprobable sample space is in Probability where each outcome in a finite sample space has an equal change of occurring.

Example

- Unfair die. Sample space is same $S=\{1,2,3,4,5,6\}$ but probabilidies of each outcome is different.

2) Probability is between 0 that indicates impossibility and 1 that indicates certanity.

$$P = \frac{\text{Number of desired outcomes}}{\text{Total number of possible outcomes}}$$

- Equiprobable sample space where each outcome has an equal chanse of occurring.

$$P(E) = \frac{m \to \text{Event that can occur}}{n \to \text{All equally outcomes}}$$

- Non-equiprobable sample space the outcomes have different probabilities of occurring.

$$P(E) = \sum_{i=1}^{n} P_i \to \text{Each probability}$$

3)

① $P(A \cap B) = P(A) \cdot P(B) \to$ Assume

② $P(A|B) = \frac{P(A \cap B)}{P(B)} \to$ Conditional Probability

③ Put ① to ②

$P(A|B) = P(A) \to$ Prob. A has occurred is the same Prob B

④ $P(B|A) = P(B) \to$ Reverse is possible

- Thus we provided both directions of statement, two events are independed and $P(A \cap B) = P(A) \cdot P(B)$ is correct.

4) The Probability of guessing a card is $\frac{1}{3}$.

The Probability of not guessing a card is $1-\frac{1}{3}=\frac{2}{3}$

10 guesses $\rightarrow \left(\frac{2}{3}\right)^{10}$ not guess correctly in 10 guesses

$\boxed{1-\left(\frac{2}{3}\right)^{10}} \rightarrow$ At least 1 correct guess

5) If candidate bribes $\rightarrow 1$

If canditate not bribe $\rightarrow \frac{1}{3}$

$\frac{1}{5}$ of candidates bribing $\rightarrow B$

a) $P(B)=\frac{1}{5}$ $\quad P(B')=\frac{4}{5}$ $\quad P(B|A)=?$

$P(A|B)=1$ $\quad P(A|B')=\frac{1}{3}$

$$P(B|A)=\frac{P(B).P(A|B)}{P(A)}$$

$$P(A)=P(A|B).P(B)+P(A|B').P(B')$$

$$P(B|A)=\frac{P(B).P(A|B)}{P(A|B).P(B)+P(A|B').P(B')}$$

$$=\frac{\frac{1}{5}\cdot 1}{1.\frac{1}{5}+\frac{1}{3}\cdot\frac{4}{5}}=\frac{1}{5}\cdot\frac{15}{7}=\frac{3}{7}$$

b) $P(C) \rightarrow$ stay both rounds

$$P(C)=\underbrace{P(B).1}_{\text{Bribing}}+\underbrace{P(B').\frac{1}{9}}_{\text{Non-bribing two rounds}}$$

$$P(C)=\frac{1}{5}.1+\frac{4}{5}\cdot\frac{1}{9}$$

$$P(C)=\frac{13}{45}$$

2) $E \to$ Candita stays after the first interview.

$P(B) \to \frac{1}{5}$ If candita bribes, probability is $P(E|B)=1$

$P(B') \to \frac{4}{5}$ 〃 〃 not bribe, 〃 〃 $P(E|B')=\frac{1}{3}$

$$P(B'|E) = \frac{P(E|B') \cdot P(B')}{P(E)}$$

$$P(E) = P(E|B) \cdot P(B) + P(E|B') \cdot P(B')$$

$$= 1 \cdot \frac{1}{5} + \frac{1}{3} \cdot \frac{4}{15} = \frac{7}{15}$$

$$P(B'|E) = \frac{\frac{1}{3} \cdot \frac{4}{15}}{\frac{7}{15}} = \frac{4}{7}$$

$$P(\text{Eliminated in the second} | E) = P(B'|E) \cdot (1 - P(E|B)$$

〃 $$= \frac{4}{7} \cdot \left(1 - \frac{1}{3}\right) = \boxed{\frac{8}{21}}$$

6) $A \to$ Has disease $\quad B \to$ Positive result

$A' \to$ Hasn't disease $\quad B' \to$ Negative result

$P(B|A) = 0.95 \quad P(A) = \frac{1}{500} = 0.002$

$P(B|A') = 0.01 \quad P(A') = 0.998$

$$P(B) = P(B|A) \cdot P(A) + P(B|A') \cdot P(A')$$

$$P(B) = (0.95) \cdot (0.002) + (0.01)(0.998)$$

$$P(B) = 0.0019 + 0.00998$$

$$P(B) = 0.01188$$

$$P(A|B) = ? = \frac{P(A) \cdot P(B|A)}{P(B)} = \frac{(0.002) \cdot (0.95)}{(0.01188)}$$

$$\boxed{P(A|B) = 0.1599}$$

7) A → Queen Carries gene

A' → Queen does not carry gene

B → All 3 prince dont have hemophilia

$P(A) = 0.5$ $\quad P(B|A) = (0.5)^3 = 0.125$ → Queen carrier 3 prince dont hemo

$P(A') = 0.5$ $\quad P(B|A') = 1$ → Quena dont carry gene 3 prince dont hemophilia

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)} = \frac{P(B|A).P(A)}{P(B|A).P(A) + P(B|A').P(A')}$$

$$= \frac{(0.125).(0.5)}{(0.125).(0.5) + 1.(0.5)} = \frac{(0.5)^4}{(0.5)^3 + 0.5}$$

$$= \boxed{0.111}$$

8) HHH → $W = 3-0 = 3$

HHT → $W = 2-1 = 1$

HTH → $W = 2-1 = 1$

HTT → $W = 1-2 = -1$

THH → $W = 2-1 = 1$

TTH → $W = 1-2 = -1$

THT → $W = 1-2 = -1$

TTT → $W = 0-3 = -3$

$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

$W = \{3, 1, 1, -1, 1, -1, -1, -3\}$

9)

$$f(y) = \begin{cases} k y^4 (1-y)^3, & 0 \le y \le 1 \\ 0, & \text{others} \end{cases}$$

a) $\int_0^1 k y^4 (1-y)^3 \, dy = 1$

$= \int_0^1 k y^4 (1 - 3y + 3y^2 - y^3) \, dy = 1$

$= k \int_0^1 y^4 - 3y^5 + 3y^6 - y^7 \, dy = 1$

$= k \left(\frac{y^5}{5} - \frac{3y^6}{6} + \frac{3y^7}{7} - \frac{y^8}{8} \right) \Big|_0^1 = 1$

$= k \left(\frac{1}{5} - \frac{1}{2} + \frac{3}{7} - \frac{1}{8} \right) - 0 = 1$

(56) (140) (40) (35)

$56 - 140 + 120 - 35 = 1$

$= k \cdot \left(\frac{1}{280} \right) = 1$

$\boxed{k = 280}$

b) $P(0<y<0.5) = \int_0^{0.5} 280 (y^4 - 3y^5 + 3y^6 - y^7) \, dy$

$P(0<y<0.5) = 280 \left(\frac{y^5}{5} - \frac{3y^6}{6} + \frac{3y^7}{7} - \frac{y^8}{8} \right) \Big|_0^{0.5}$

$= 280 \left(\frac{1}{160} - \frac{1}{128} + \frac{3}{896} - \frac{1}{2048} \right)$

$= 1.75 - 2.19 + 0.938 - 0.137$

$\boxed{P(0<y<0.5) = 0.36}$

c) $P(0.8<y<1) = 280 \left(\frac{y^5}{5} - \frac{y^6}{2} + \frac{3y^7}{7} - \frac{y^8}{8} \right) \Big|_{0.8}^{1}$

$= 280 \left(\frac{1}{5} - \frac{1}{2} + \frac{3}{7} - \frac{1}{8} \right) - 280 \left(\frac{(0.8)^5}{5} - \frac{(0.8)^6}{2} + \frac{3(0.8)^7}{7} - \frac{(0.8)^8}{8} \right)$

$= 1 - 280(0.0655 - 0.131 + 0.0899 - 0.0210)$

$\boxed{P(0.8<y<1) = 0.056}$

If Part

10) a) Assume $X_1, X_2, \ldots, X_n$ are mutually independent

$$P(X_1=x_1, X_2=x_2, \ldots, X_n=x_n) = P(X_1=x_1) \cdot P(X_2=x_2) \ldots P(X_n=x_n)$$

b) For continuous random Variables Probability Density func

$$f(x_1, x_2 \ldots x_n) = f_1(x_1) \cdot f_2(x_2) \ldots f_n(x_n)$$

c) For Discrete random Variables Probability Mass Func applies same as above

Only If Part

a) Conversely Assume that

$$f(x_1, x_2, \ldots x_n) = f_1(x_1) f_2(x_2) \ldots f_4(x_n)$$

for all $x_1, x_2 \ldots x_n$ within their range

b) Any two random variables X_i, X_J from set $(X_1, X_2 \ldots X_n)$

If the joint distrubition function can be expressed as the product of:

$$f(x_i, x_J) = f_i(x_i) \cdot f_J(x_J)$$

c) Above implies $P(X_i=x_i)$ and $P(X_J=x_J)$ is the product of $P(X_i)$ and $P(X_J)$ takins on those values independently. Therefore X_i and X_J are independed as any Pair of set.

Result

$(f(x_1, x_2 \ldots x_n) = f_1(x_1) f_2(x_2) \ldots f_n(x_n))$ if and only if $X_1, X_2 \ldots X_n$ are mutually statistically independent.