**CSE 211: Discrete Mathematics** **(Due: 30/10/22)**

# Homework #1

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**Course Policy**: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to bkarakas2018@gtu.edu.tr
- Use LaTeX. You can work on the tex file shared with you in the assignment document.
- Submit both the tex and pdf files into Homework1. Name of the files should be "*SurnameName_Id.tex*" and "*SurnameName_Id.pdf*".

**Problem 1: Sets** (3+3+3+3+3=15 points)

Which of the following sets are equal? Show your work step by step.

**(a)** {t : t is a root of $x^2$ − 6x + 8 = 0}

**(b)** {y : y is a real number in the closed interval [2, 3]}

**(c)** {4, 2, 5, 4}

**(d)** {4, 5, 7, 2} - {5, 7}

**(e)** {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}

***(Solution)***

(a) For identifying the set(a) we need to find of $x^2 - 6x + 8 = 0$

$$\begin{aligned} x^2 - 6x + 8 &= 0 \\ x^2 - 4x - 2x + 8 &= 0 \\ x(x-4) - 2(x-4) &= 0 \\ (x-2)(x-4) &= 0 \\ x = 2, x &= 4 \end{aligned}$$

According to steps I wrote above the results of $x^2 - 6x + 8 = 0$ are 2 and 4.

And set of (a) $= \{2, 4\}$.

The sets of b, c and d are certain.

$$\begin{aligned}(b) &= [2,3]\\ (c) &= \{4,2,5,4\} \equiv \{2,4,5\}\\ (d) &= \{4,5,7,2\} - \{5,7\} = \{2,4\}\end{aligned}$$

(e)

- Number of sides of a rectangle = 4

- Number of digits in any integer between 11 and 99 = 2

The set of (e) = 2,4

**Thus**

| | |
|---|---|
| (a) ≡ (d) ≡ (e) | a = {2,4},<br>d = {2,4},<br>e = {2,4}, |

**Problem 2: Cardinality of Sets** (2+2+2+2=8 points)

What is the cardinality of each of these sets? Explain your answers.

**(a)** $\{\emptyset\}$

**(b)** $\{\emptyset, \{\emptyset\}\}$

**(c)** $\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$

**(d)** $\{\emptyset, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}\}$

***(Solution)***

(a) Cardinality is 1. This set containes an empty set($\emptyset$).

(b) Cardinality is 2. This set containes an empty set($\emptyset$) and another set that contains empty set($\{\emptyset\}$).

(c) Cardinality is 2. This set containes an empty set($\emptyset$) and ($\{\emptyset, \{\emptyset\}\}$).$\{\emptyset, \{\emptyset\}\}$ consider as 1.

(d) Cardinality is 2. This set containes an empty set($\emptyset$) and ($\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$). $\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$ consider as 1.

**Problem 3: Cartesian Product of Sets** (15 points)

Explain why (A × B) × (C × D) and A × (B × C) × D are not the same.

***(Solution)***

Let $A \times B = K$ and $C \times B = L$

$$\begin{aligned} K \times L &= \{(k, l); k \in K, l \in L\} \\ k &= (a, b), l = (c, d) \\ (A \times B) \times (C \times D) &= \{((a, b), (c, d)); a \in A, b \in B, c \in C, d \in D\} \end{aligned}$$

Let $B \times C = M$

$$\begin{aligned} M &= \{m; m \in M\} \\ A \times M \times D &= \{(a, m, d); a \in A, m \in M, d \in D\} \\ m &= (b, c) \\ A \times (B \times C) \times D &= \{(a, (b, c), d); a \in A, b \in B, c \in C, d \in D\} \end{aligned}$$

$$((a, b), (c, d)) \neq (a, (b, c), d)$$

**Result**

$$(A \times B) \times (C \times D) \neq A \times (B \times C) \times D$$

**Problem 4: Cartesian Product of Sets in Algorithms** (25 points)

Let A, B and C be sets which have different cardinalities. Let (p, q, r) be each triple of A × B × C where p ∈ A, q ∈ B and r ∈ C. Design an algorithm which finds all the triples that are satisfying the criteria: p ≤ q and q ≥ r. Write the pseudo code of the algorithm in your solution.

For example: Let the set A, B and C be as A = { 3, 5, 7 }, B = { 3, 6 } and C = { 4, 6, 9 }. Then the output should be : { (3, 6, 4), (3, 6, 6), (5, 6, 4), (5, 6, 6) }.

(Note: Assume that you have sets of A, B, C as an input argument.)

***(Solution)***

**Algorithm 1:** Pseudo Code of Your Algorithm

```
Input: The sets of A, B, C
if write a condition then
    Statements
else
    Statements
end
When you want to write a for loop, you can use:
 for write a condition do

end
When you want to write a while loop, you can use:
 while write a condition do
    If you need to return, use return
end
For any additional things you have to do while writing your pseudo code, Google "How to use
 algorithm2e in Latex?".
```

**Algorithm 2:** Solution

```
Input: The sets of A, B, C
for p in A do
    for q in B do
        for r in C do
            if p ≤ q and q ≥ r then
                print (p, q, r),
            else
                continue
            end
        end
    end
end
```

**Problem 5: Functions** (16 points)

If f and f ∘ g are one-to-one, does it follow that g is one-to-one? Justify your answer.

***(Solution)***

- Assume g is not one-to-one then $a \neq b$ in such that $g(a) = g(b)$
- f and $f \circ g$ are one-to-one $f \circ g(a) = f \circ g(b)$ then $g(a) = g(b)$
- With $a \neq b$,which contradicts $f \circ g$ one-to-one
- Then $g$ must be one-to-one

**Problem 6: Functions** (7+7+7=21 points)

Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if

**(a)** $f(m, n) = 2m - n$

**(b)** $f(m, n) = m^2 - n^2$

**(c)** $f(m, n) =| m | - | n |$

***(Solution)***

The function $f : A \to B$ is onto if range of function same as domain. $\Rightarrow f(A) = B$

(a) For any pair $(0, n) \in \mathbb{Z} \times \mathbb{Z}$

$$\begin{aligned} f(0, n) &= 2(0) - n \\ f(0, n) &= n \\ Therefore, \forall n &\in \mathbb{Z} \end{aligned}$$

So $f$ is **onto**.

(b) $f(m, n) = m^2 - n^2$ is a difference of two perfect squares.

$$\begin{aligned} f(1, 0) &= 1^2 - 0^2 = 1 \\ f(2, 1) &= 2^2 - 1^2 = 3 \\ f(3, 2) &= 3^2 - 2^2 = 5 \\ &\vdots \end{aligned}$$

According to the above, some elements in the image set do not have a pre image in the domain.(For ex. 2) So $f$ is **not onto**.

(c) For any pair $(m, 0) = \mathbb{Z} \times \mathbb{Z}$

$$\begin{aligned} f(m, 0) &=| m | - | 0 | \\ &=| m | \\ &= \pm m \\ Therefore, \forall \pm m \in \mathbb{Z} \end{aligned} \tag{0.1}$$

So f is **onto**

**Problem 7: Functions** (Bonus 20 points)

Suppose that $f$ is a function from $A$ to $B$, where $A$ and $B$ are finite sets with $\mid A \mid = \mid B \mid$. Show that $f$ is one-to-one if and only if it is onto.

***(Solution)***