Midterm-Q1
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Barış Büyüktaş

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1 Solution

Let graph $G = (V, E)$ consists of set of vertices V and set of edges E. An edge a_{ij} connects v_i with v_j where $a_{ij} \in E$, $v_i \in V$, $v_j \in V$.

According to the [1], local clustering coefficient C_i of a vertex v_i:

$$C_i = A_i/T_i \tag{1}$$

where A_i is number of edges between neighbors of v_i, T_i is maximum number of edges between neighbors of v_i. Clustering coefficient of the graph is the average of the local clustering of graph:

$$C = (1/N) * \sum_{n=1}^{N} C_i \tag{2}$$

where N is the number of vertices. The degree of a vertex v_i is k_i, which represents the number of edges connected to v_i. Since all vertices have same number of links in regular lattice [2], the degree of each vertex can be represented as k.

The vertex v_i has k neighbors. The maximum number of edges between k vertices is $\binom{k}{2}$. Therefore, the maximum number of edges between neighbors of v_i is:

$$T_i = \binom{k}{2} \tag{3}$$

Given that a 1-D ring lattice, where $k = 2r$. It means that there are r vertices connected to the left and r vertices connected to the right of the v_i. To calculate the number of edges between neighbors of v_i, we assume that the graph is divided into 3 parts, which are left, right and medium. We can see the example of a regular ring lattice in Fig. 1. The blue edges represent the edges between the neighbors to the right of v_i, the green edges represent the edges between the neighbors to the left of v_i, the orange edges represent the edges between right and left of v_i.

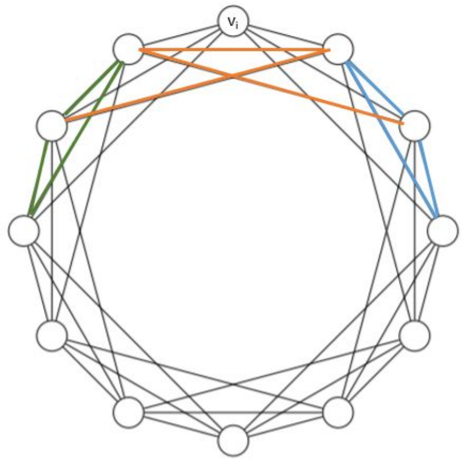

Figure 1: A regular ring lattice with N=12, k=6, r=3.

The number of edges between the neighbors to the right of v_i is $A_{ir} = \binom{r}{2}$, the number of edges between the neighbors to the left of v_i is $A_{il} = \binom{r}{2}$. However, the left and right side of the vertices could also connect to each other, therefore, we should take into account the medium part A_{im} as well. We can say that:

$$A_i = A_{ir} + A_{il} + A_{im} \tag{4}$$

The vertex v_{i+1} (the vertex that is located 1 vertex right to the v_i) can connect $r-1$ vertices to the left side of v_i, the vertex v_{i+2} (the vertex that is located 2 vertex right to the v_i) can connect $r-2$ vertices to the left side of v_i, if we sum the edges of vertices between v_{i+1} to v_{i+r}, we can obtain:

$$A_{im} = \sum_{n=0}^{r-1} n = \binom{r}{2} \tag{5}$$

Hence, A_i becomes:

$$A_i = 3 * \binom{r}{2} \tag{6}$$

Since we know that $k = 2r$, (6) can be written as:

$$A_i = 3 * \binom{k/2}{2} \tag{7}$$

Eventually, if we combine (1), (3), (7), we obtain:

$$C_i = \frac{3 * \binom{k/2}{2}}{\binom{k}{2}} \tag{8}$$

Lastly, clustering coefficient C of a graph is:

$$C = C_i = \frac{1}{N} * \sum_{n=1}^{N} \frac{3 * \binom{k/2}{2}}{\binom{k}{2}} = \frac{3}{4} * \frac{k-2}{k-1} \tag{9}$$

References

[1] D. J. Watts and S. H. Strogatz, “Collective dynamics of ‘small-world’networks,” *nature*, vol. 393, no. 6684, pp. 440–442, 1998.

[2] C. Montaña, “Generalization of clustering coefficient on lattice networks applied to criminal networks,” *International Journal of Computer and Information Engineering*, vol. 11, no. 7, pp. 868–872, 2017.