Midterm-Q2
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1 Question

Let $\mathbf{A}$ be the adjacency matrix of an undirected network and $\mathbf{1}$ be the column vector whose elements are all 1. In terms of these quantities write expressions for:

1. the vector $\mathbf{k}$ whose elements are the degrees k_i of the vertices;
2. the number m of edges in the network;
3. the matrix $\mathbf{N}$ whose element N_{ij} is equal to the number of common neighbors of vertices i and j;
4. the total number of triangles in the network, where a triangle means three vertices, each connected by edges to both of the others.

1.1 Solution

1) Let A be the adjacency matrix and b be the column vector whose elements are all 1. The elements of adjacency matrix A_{ij} is 1 if there is an edge between v_i and v_j. The degree of a vertex is sum of row or column in its corresponding index. Therefore, k=Axb will give us the degree vector of vertices.

2) Since we write each connection twice to the adjacency matrix, $m = \sum\limits_{i,j} A_{ij}/2$, which means that m is sum of each element in the matrix over 2 or $m = \sum\limits_{i} k_i/2$, which means that sum of degree vector over 2.

3) The number of common neighbors between i and j is the number of nodes connected to both i and j. We can find it with the formula of $\sum\limits_{z} A_{iz} * A_{zj}$. It is also equal to A^2. Hence, we can say that $N = A^2$, which N_{ij} is equal to the number of common neighbors between vertex i and j.

4) According to the [1], the number of paths of length r that start and end at the same vertex i is $[A^r]_{ii}$. In order to find the triangles, r should be 3. A^3_{ii} gives the 6 times the number of triangles. Because, it counts for the all 3 nodes and 2 directions (i.e, 1-2-3, 1-3-2, 2-1-3, 2-3-1, 3-1-2, 3-2-1). Hence, the total number of triangles is $\sum\limits_{i} A^3_{ii}/6$.

References

[1] M. Newman, *Networks.* Oxford university press, 2018.