



# BSM307

## İşaretler ve Sistemler

Dr. Seçkin Arı

Sürekli Zaman Fourier Dönüşümü

- Fourier Dönüşümü
- Ters Fourier Dönüşümü
- Fourier Dönüşüm Özellikleri
  - ◆ Doğrusallık
  - ◆ Zamanda
    - Öteleme
    - Ters çevirme
    - Türev ve İntegral
  - ◆ Zamanda ve Frekansta Ölçekleme
  - ◆ Frekansta Türev
  - ◆ Çift Taraflılık
  - ◆ Konvolüsyon
  - ◆ Çarpma

# Fourier Dönüşümü

- Sürekli Zaman Periyodik Olmayan İşaretler
  - ♦ Frekans spektrumu
- $x(t)$  biliniyor  $X(\omega)$  bulunuyor.
- $X(z) = \sum_{n=-\infty}^{\infty}$
- $X(\omega) =$

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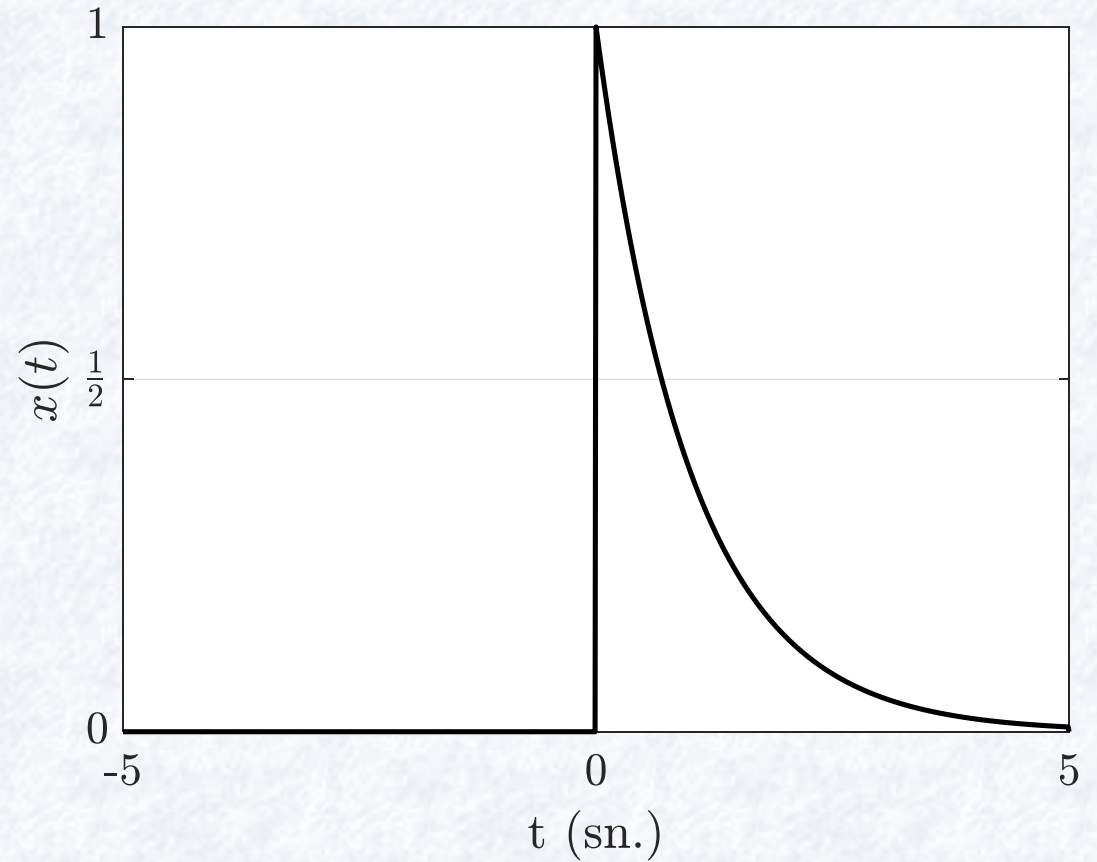


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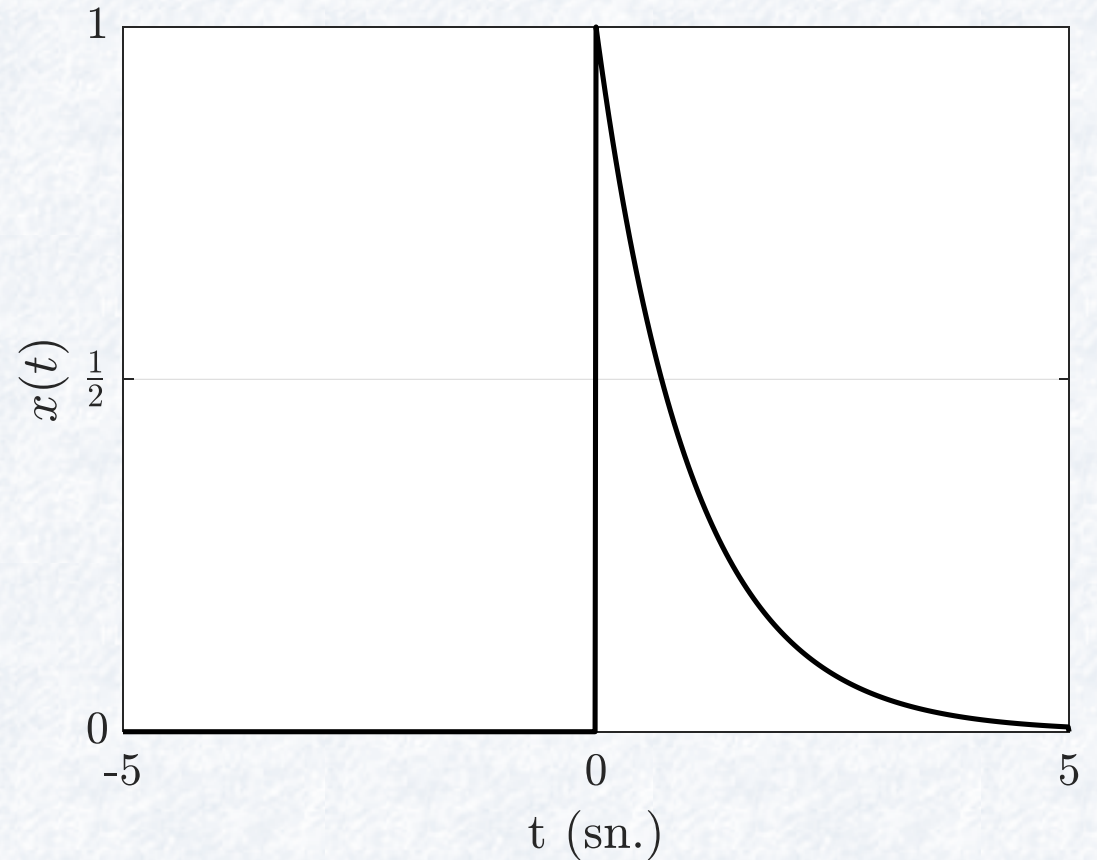
# Örnek 1

- $x(t) = e^{-at}u(t)$  ise  $X(\omega) = ?$



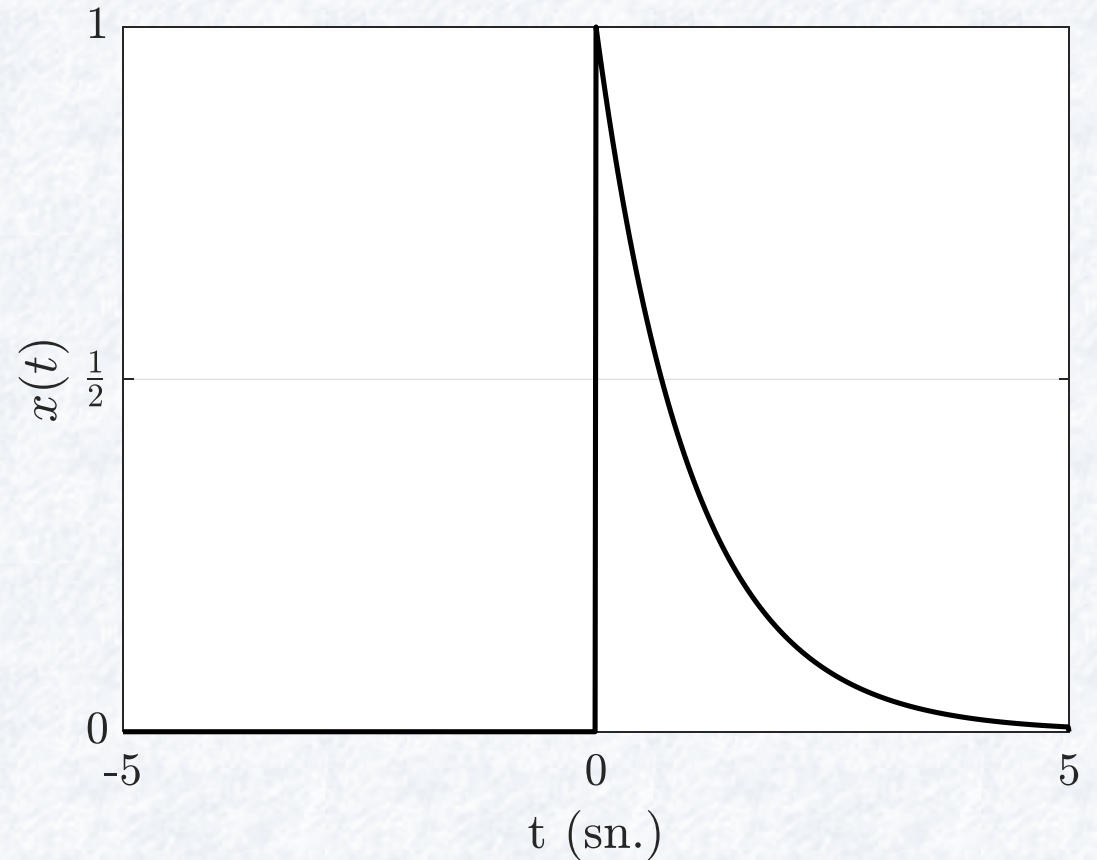
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- $x(t) = e^{-at}u(t)$  ise  $X(\omega) = ?$
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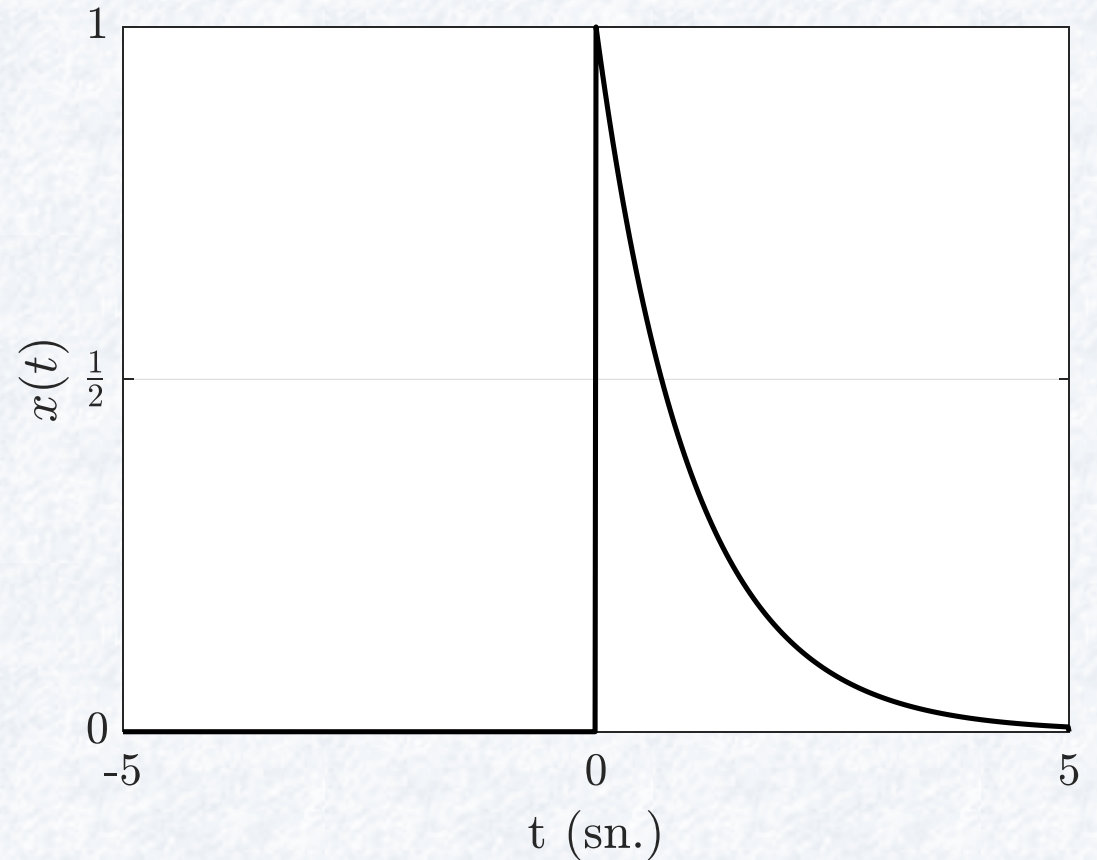
# Örnek 1

- $x(t) = e^{-at}u(t)$  ise  $X(\omega) = ?$
- $X(\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt$
- $X(\omega) = \int_0^{\infty} e^{-(a+j\omega)t} dt$
- $X(\omega) =$



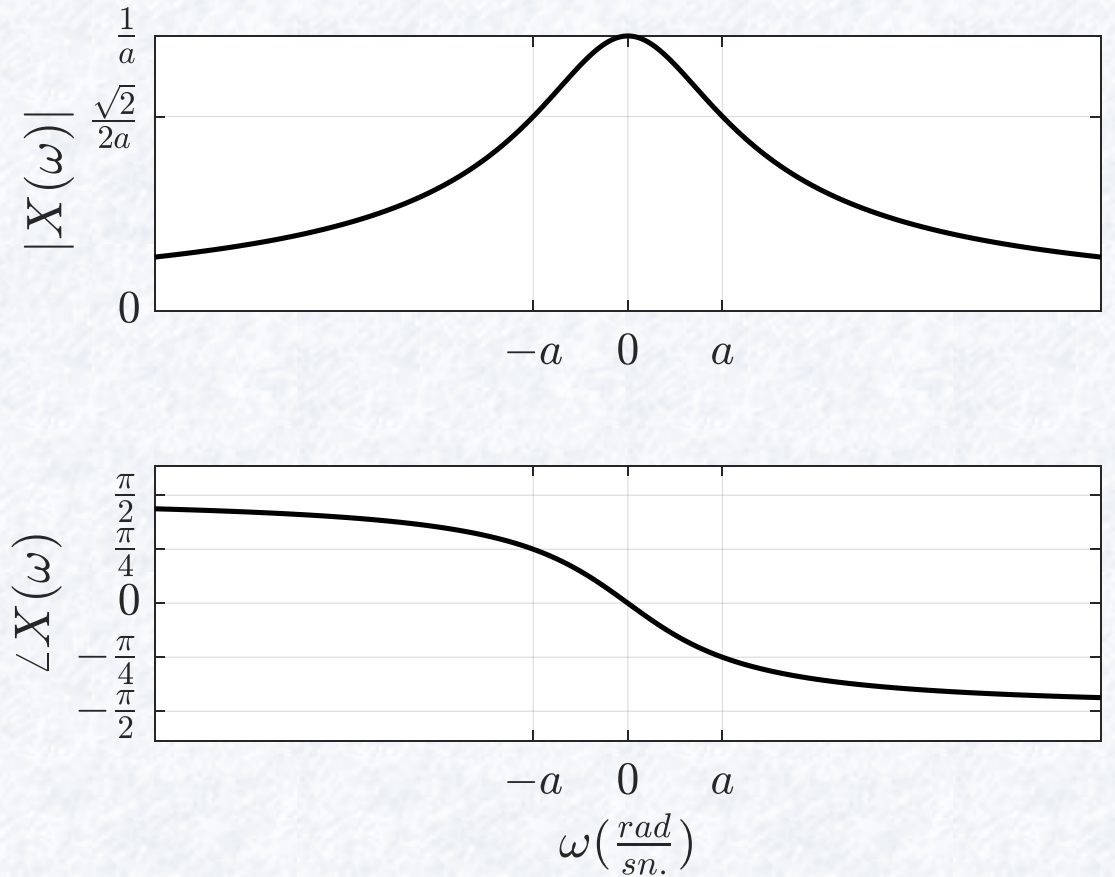
# Örnek 1

- $x(t) = e^{-at}u(t)$  ise  $X(\omega) = ?$
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- $X(\omega) = -\frac{1}{a+j\omega} e^{-(a+j\omega)t} \Big|_0^{\infty}$
- $X(\omega) =$



# Örnek 1

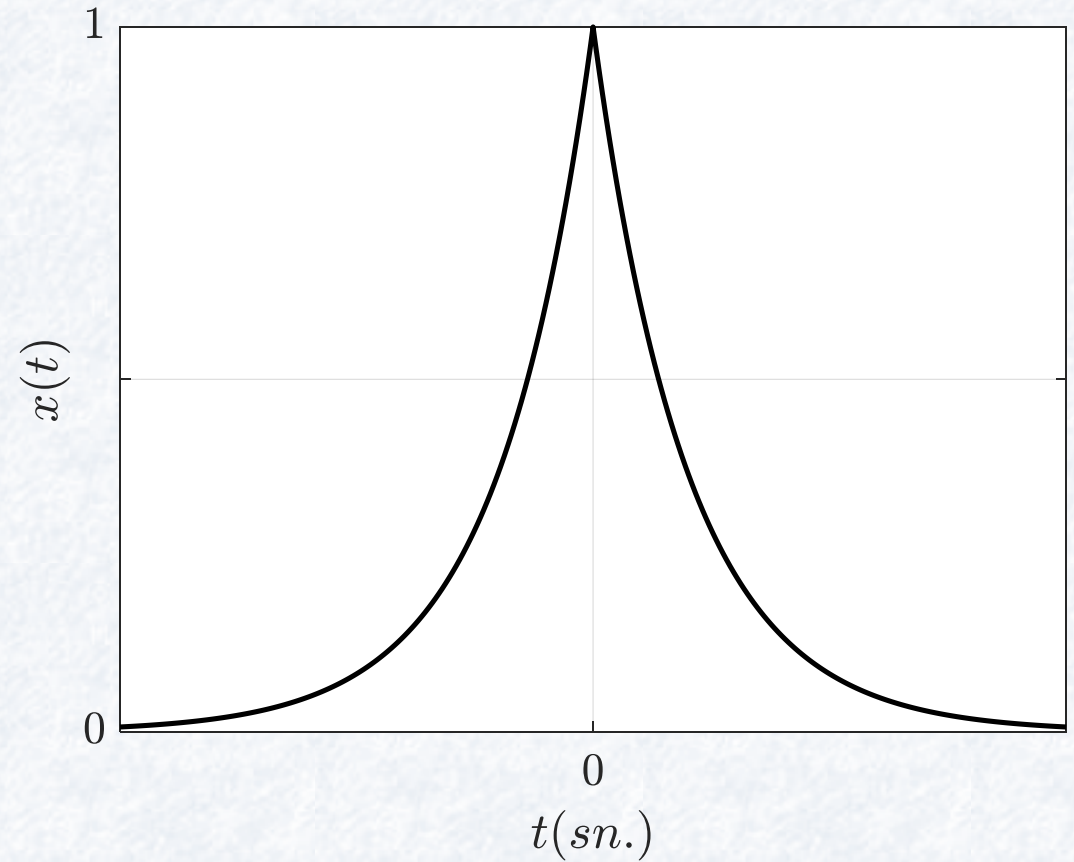
- $x(t) = e^{-at}u(t)$  ise  $X(\omega) = ?$
- $X(\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt$
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- $X(\omega) = -\frac{1}{a+j\omega} e^{-(a+j\omega)t} \Big|_0^{\infty}$
- $X(\omega) = -\frac{1}{a+j\omega} (0 - 1) = \frac{1}{a+j\omega}$





## Örnek 2

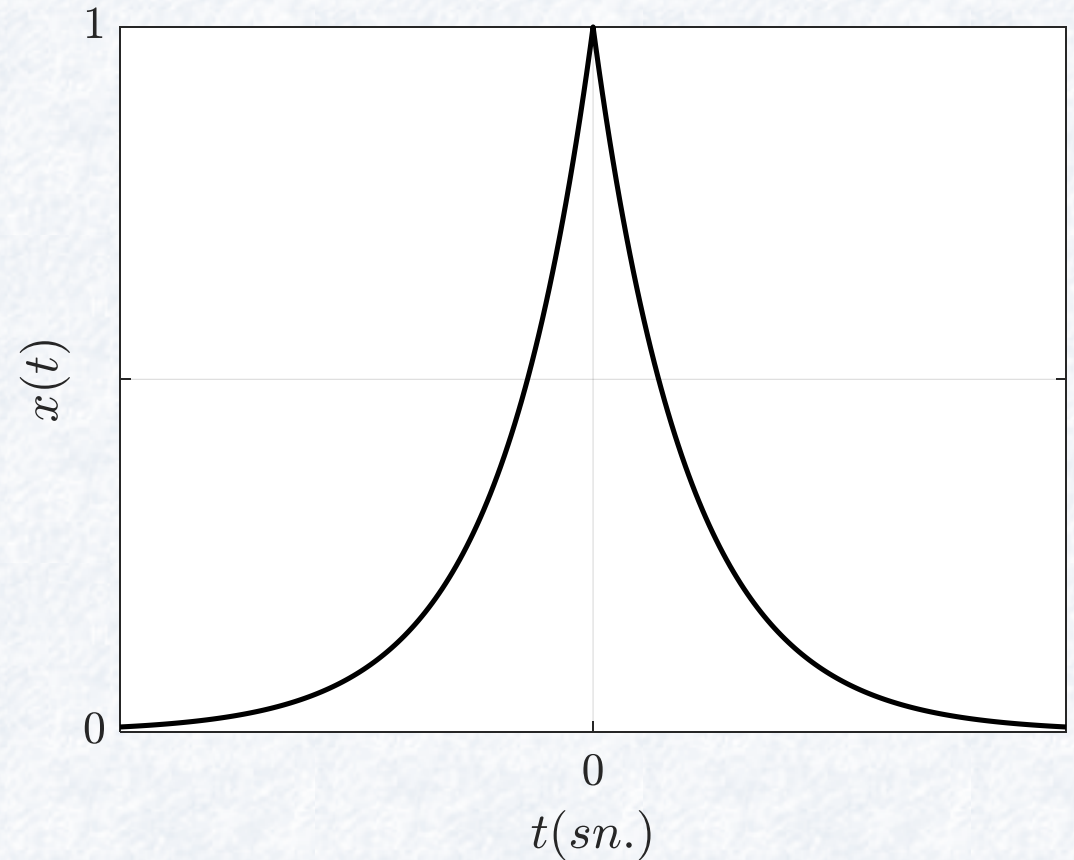
- $x(t) = e^{-a|t|}$  ise  $X(\omega) = ?$



## Örnek 2

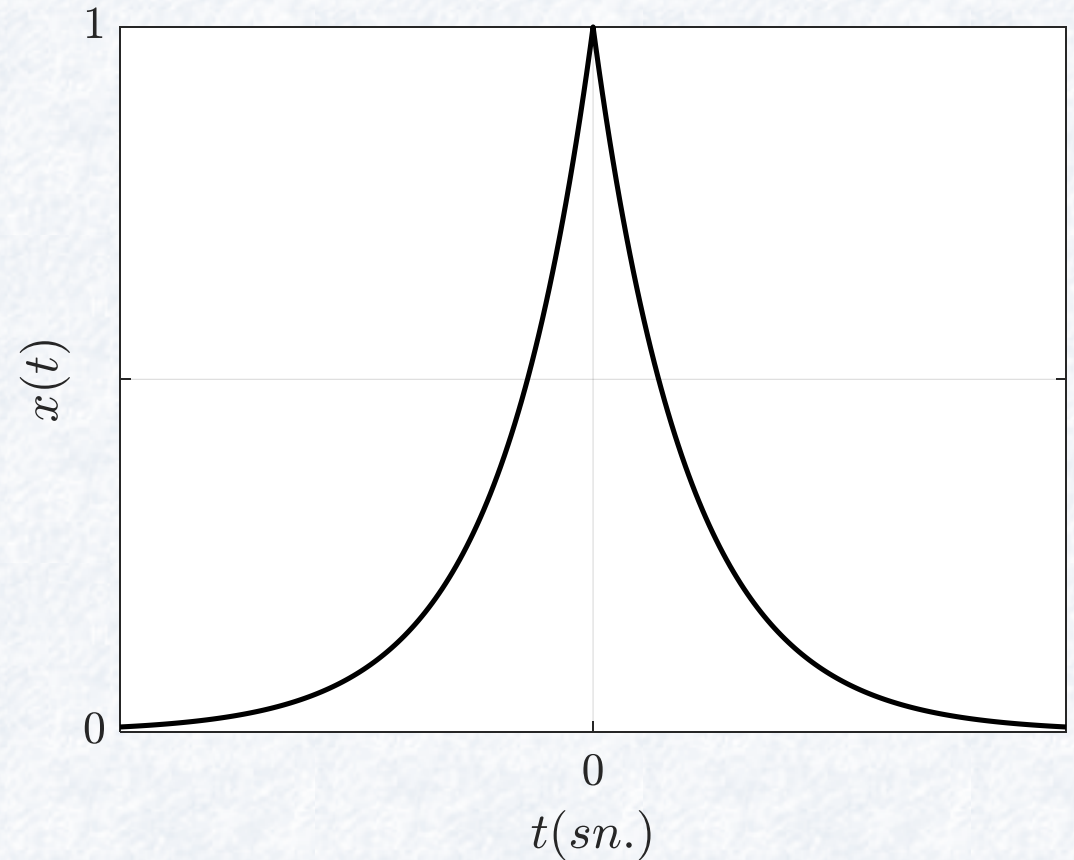
- $x(t) = e^{-a|t|}$  ise  $X(\omega) = ?$

- $x(t) = \begin{cases} \square, & t \geq 0 \\ \square, & t < 0 \end{cases}$



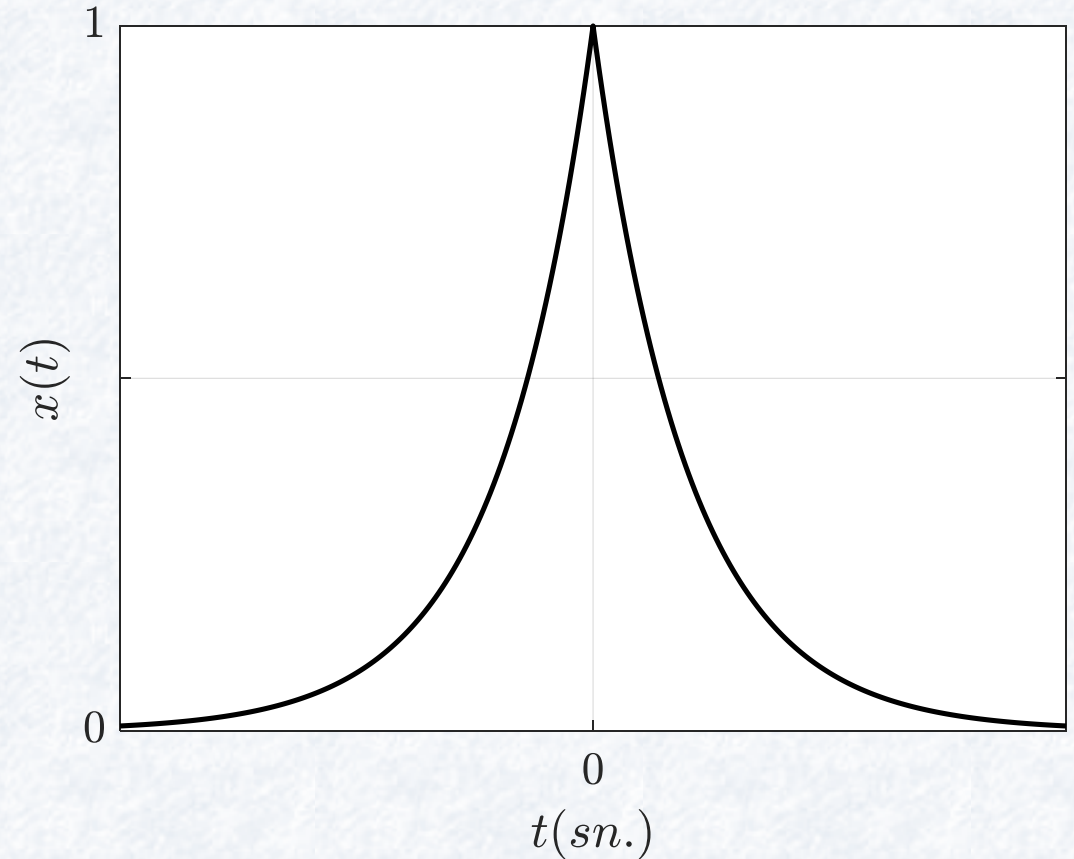
## Örnek 2

- $x(t) = e^{-a|t|}$  ise  $X(\omega) = ?$
- $x(t) = \begin{cases} e^{-at}, & t \geq 0 \\ e^{at}, & t < 0 \end{cases}$
- $X(\omega) =$



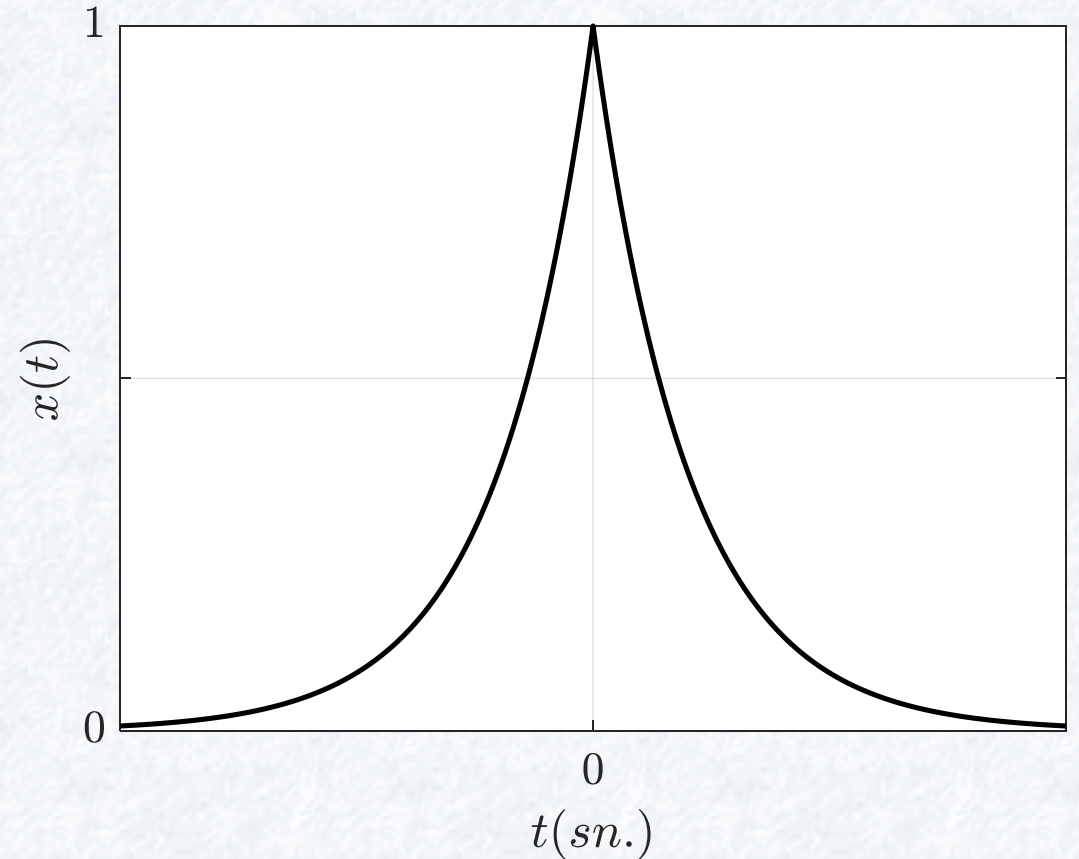
## Örnek 2

- $x(t) = e^{-a|t|}$  ise  $X(\omega) = ?$
- $x(t) = \begin{cases} e^{-at}, & t \geq 0 \\ e^{at}, & t < 0 \end{cases}$
- $X(\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt +$



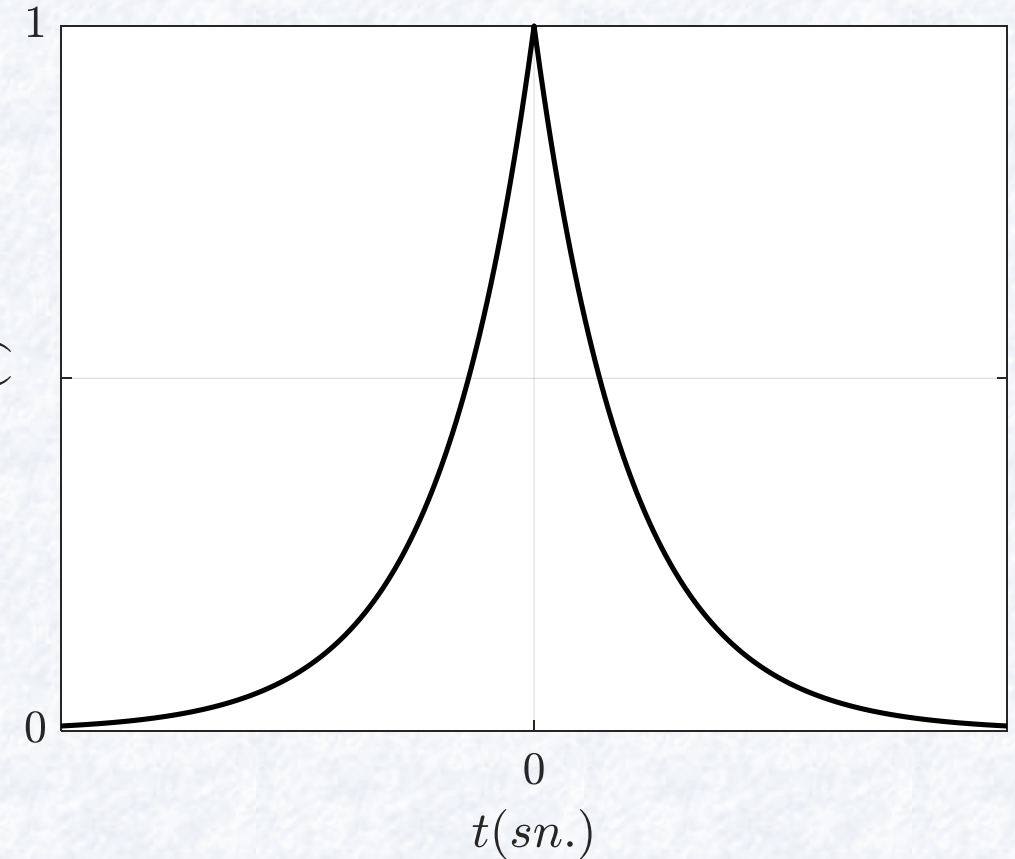
## Örnek 2

- $x(t) = e^{-a|t|}$  ise  $X(\omega) = ?$
- $x(t) = \begin{cases} e^{-at}, & t \geq 0 \\ e^{at}, & t < 0 \end{cases}$
- $X(\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt + \int_{-\infty}^0 e^{at} e^{-j\omega t} dt$



## Örnek 2

- $x(t) = e^{-a|t|}$  ise  $X(\omega) = ?$
- $x(t) = \begin{cases} e^{-at}, & t \geq 0 \\ e^{at}, & t < 0 \end{cases}$
- $X(\omega) = \frac{1}{a+j\omega} + \int_{-\infty}^{-0} e^{at} e^{-j\omega t} dt_{x(t)}$
- $\int_{-\infty}^{-0} e^{at} e^{-j\omega t} dt =$





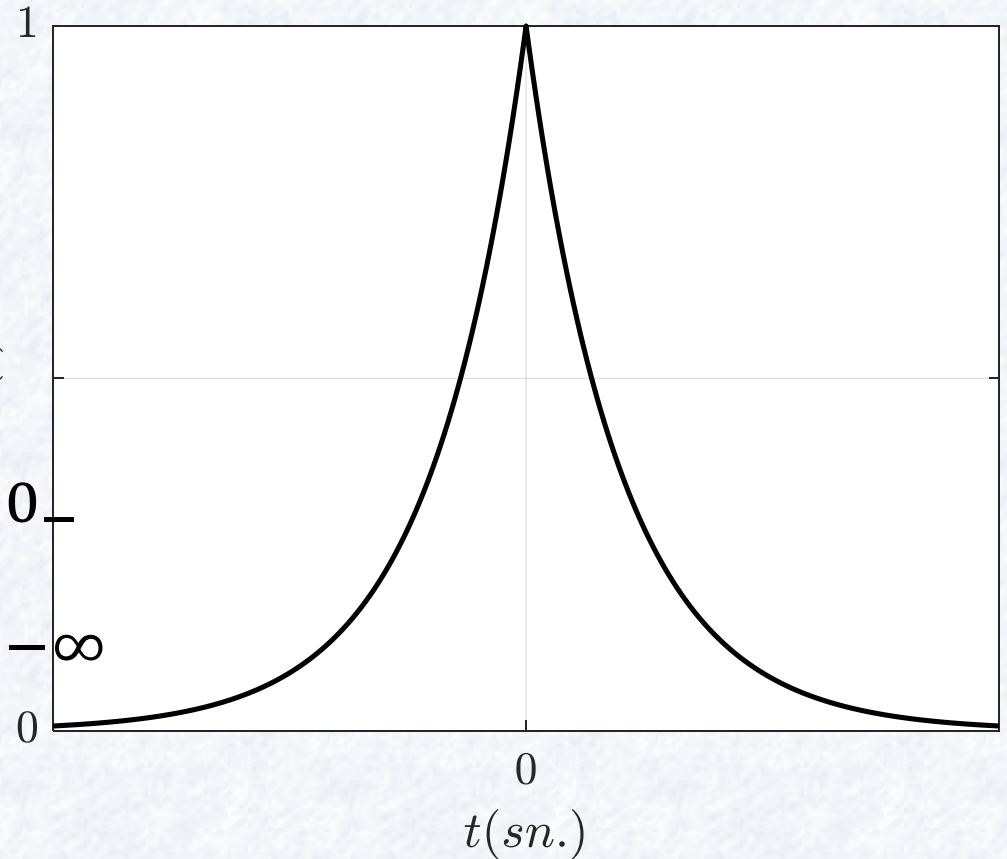
## Örnek 2

- $x(t) = e^{-a|t|}$  ise  $X(\omega) = ?$

- $x(t) = \begin{cases} e^{-at}, & t \geq 0 \\ e^{at}, & t < 0 \end{cases}$

- $X(\omega) = \frac{1}{a+j\omega} + \int_{-\infty}^{-0} e^{at} e^{-j\omega t} dt$

- $\int_{-\infty}^{-0} e^{at} e^{-j\omega t} dt = \frac{1}{a-j\omega} e^{(a-j\omega)t} \Big|_{-\infty}^{-0}$   
 $=$



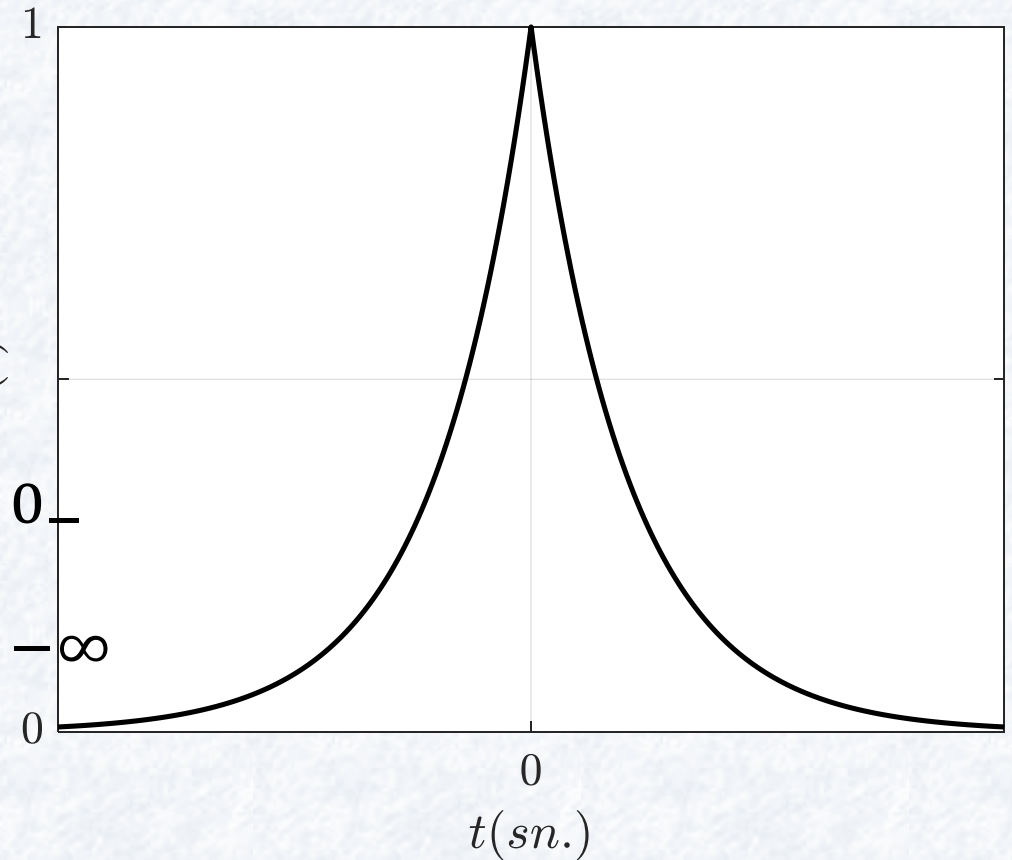
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- $X(\omega) = \frac{1}{a+j\omega} + \int_{-\infty}^{-0} e^{at} e^{-j\omega t} dt$

- $\int_{-\infty}^{-0} e^{at} e^{-j\omega t} dt = \frac{1}{a-j\omega} e^{(a-j\omega)t} \Big|_{-\infty}^{-0}$   
 $= \frac{1}{a-j\omega} (1 - 0) = \frac{1}{a-j\omega}$



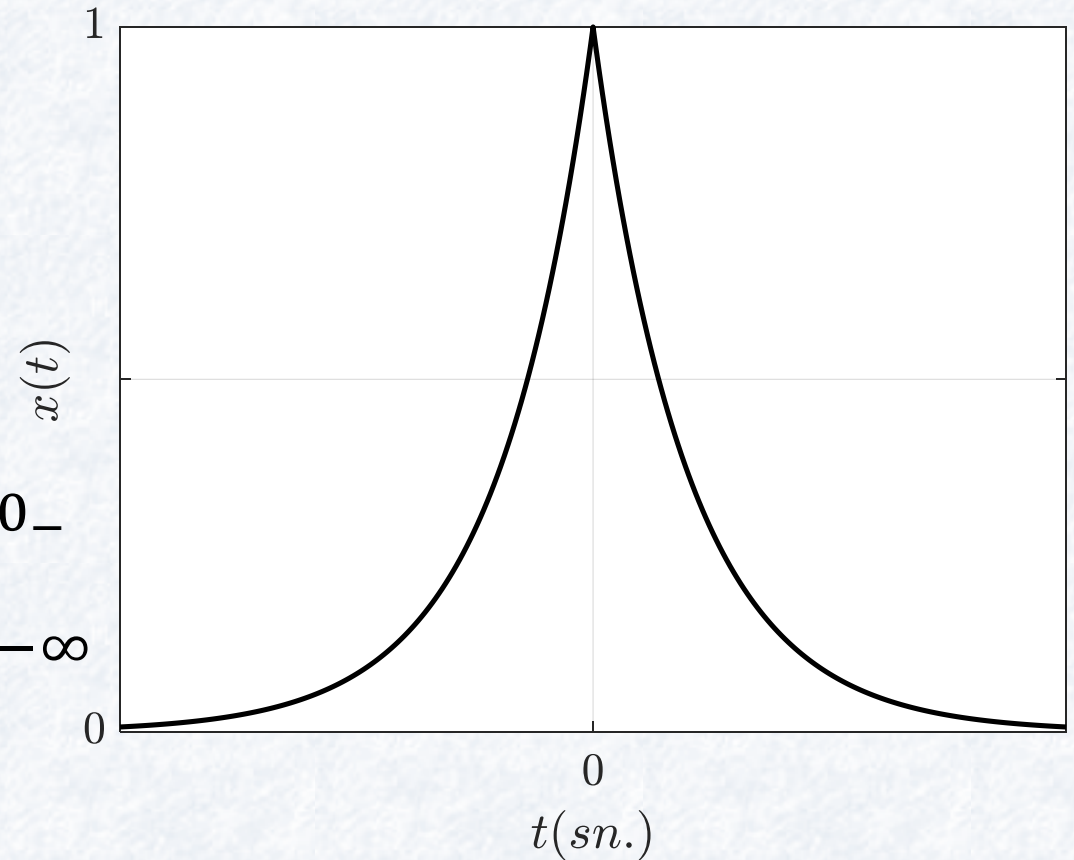
# Örnek 2

- $x(t) = e^{-a|t|}$  ise  $X(\omega) = ?$

- $x(t) = \begin{cases} e^{-at}, & t \geq 0 \\ e^{at}, & t < 0 \end{cases}$

- $X(\omega) = \frac{1}{a+j\omega} + \frac{1}{a-j\omega}$

- $\int_{-\infty}^{0-} e^{at} e^{-j\omega t} dt = \frac{1}{a-j\omega} e^{(a-j\omega)t} \Big|_{-\infty}^{0-}$   
 $= \frac{1}{a-j\omega} (1 - 0) = \frac{1}{a-j\omega}$



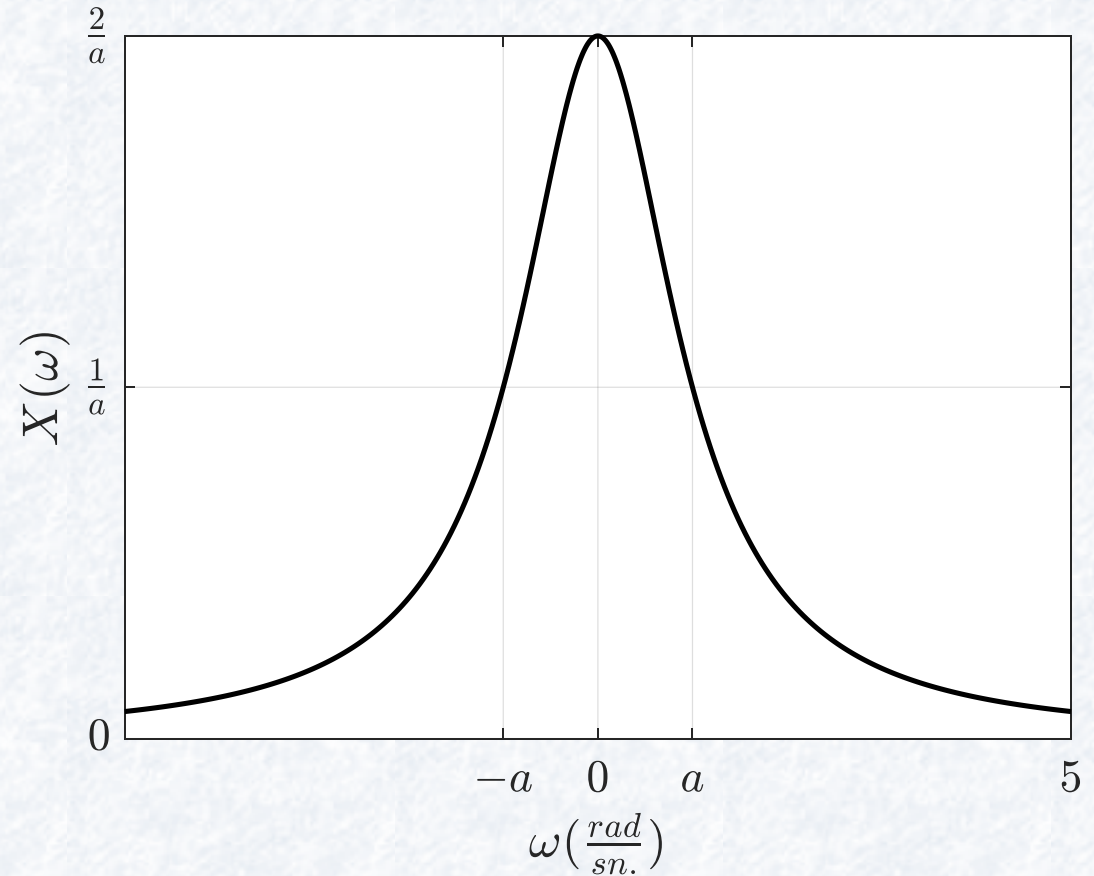
## Örnek 2

- $x(t) = e^{-a|t|}$  ise  $X(\omega) = ?$

- $x(t) = \begin{cases} e^{-at}, & t \geq 0 \\ e^{at}, & t < 0 \end{cases}$

- $X(\omega) = \frac{1}{a+j\omega} + \frac{1}{a-j\omega}$

- $X(\omega) = \frac{2a}{a^2 + \omega^2}$



## Örnek 3

- $x(t) = \delta(t)$  ise  $X(\omega) = ?$
- $X(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt =$

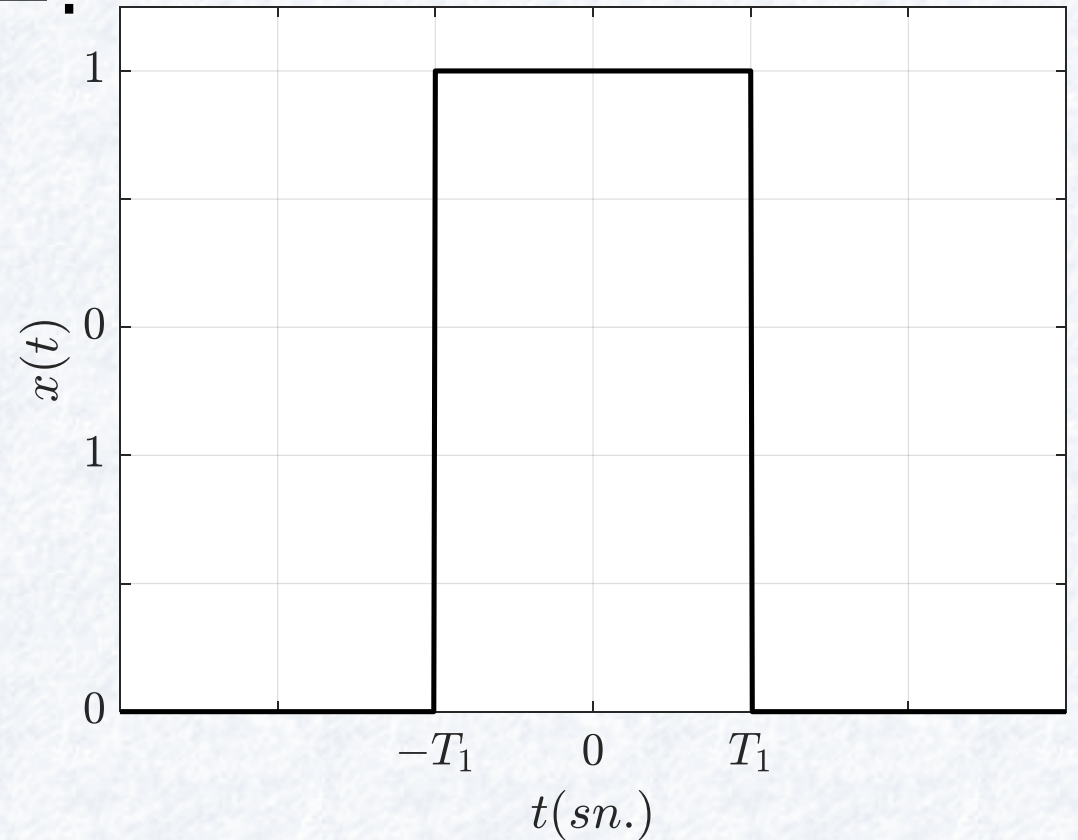
## Örnek 3

- $x(t) = \delta(t)$  ise  $X(\omega) = ?$
- $X(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1$



# Örnek 4

- $x(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases}$  ise  $X(\omega) = ?$
- $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$
- $X(\omega) =$



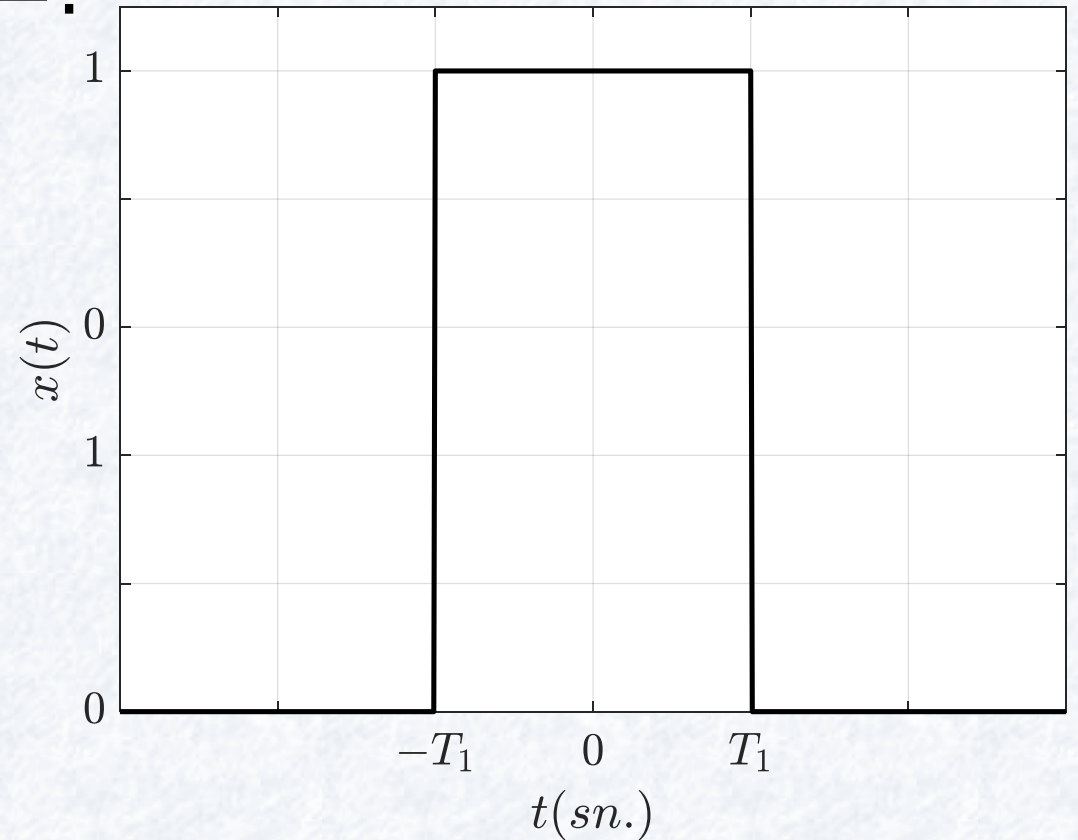
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- $x(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases}$  ise  $X(\omega) = ?$

- $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$

- $X(\omega) = \int_{-\infty}^{-T_1} + \int_{-T_1}^{T_1} + \int_{T_1}^{\infty}$

- $X(\omega) =$



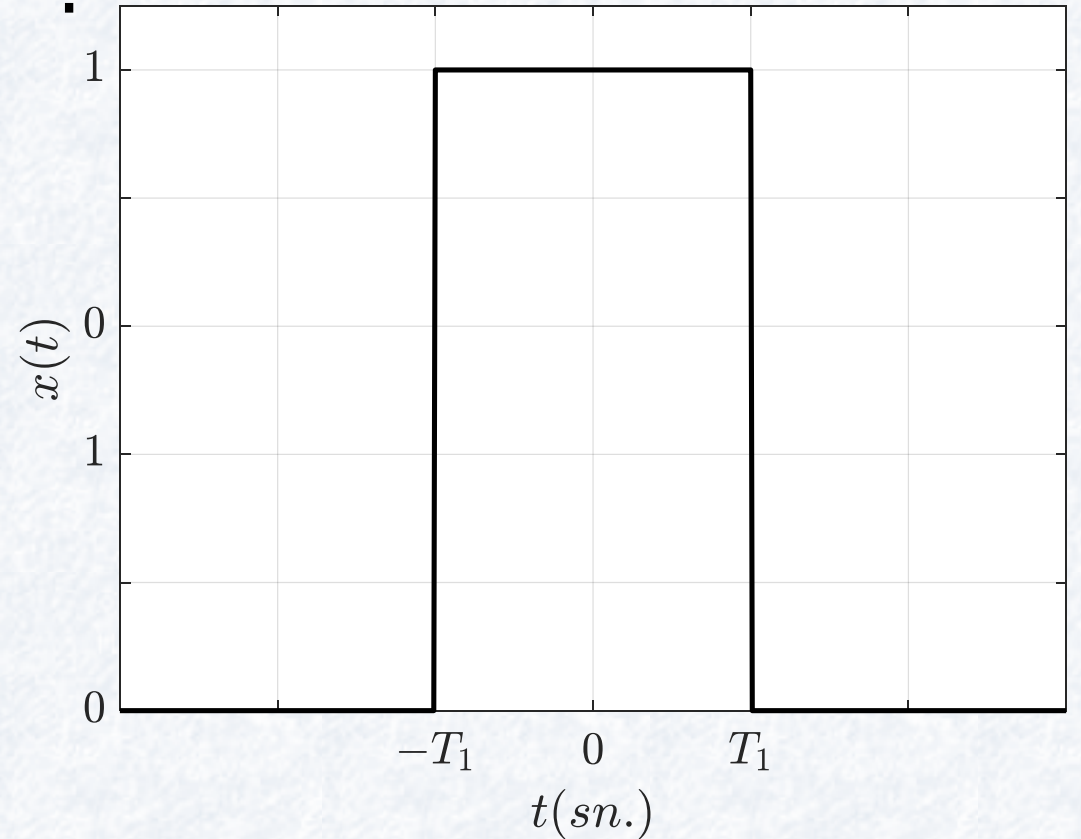
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- $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

- $X(\omega) = \int_{-\infty}^{-T_1} + \int_{-T_1}^{T_1} + \int_{T_1}^{\infty}$

- $X(\omega) = \int_{-T_1}^{T_1} e^{-j\omega t} dt$



# Örnek 4

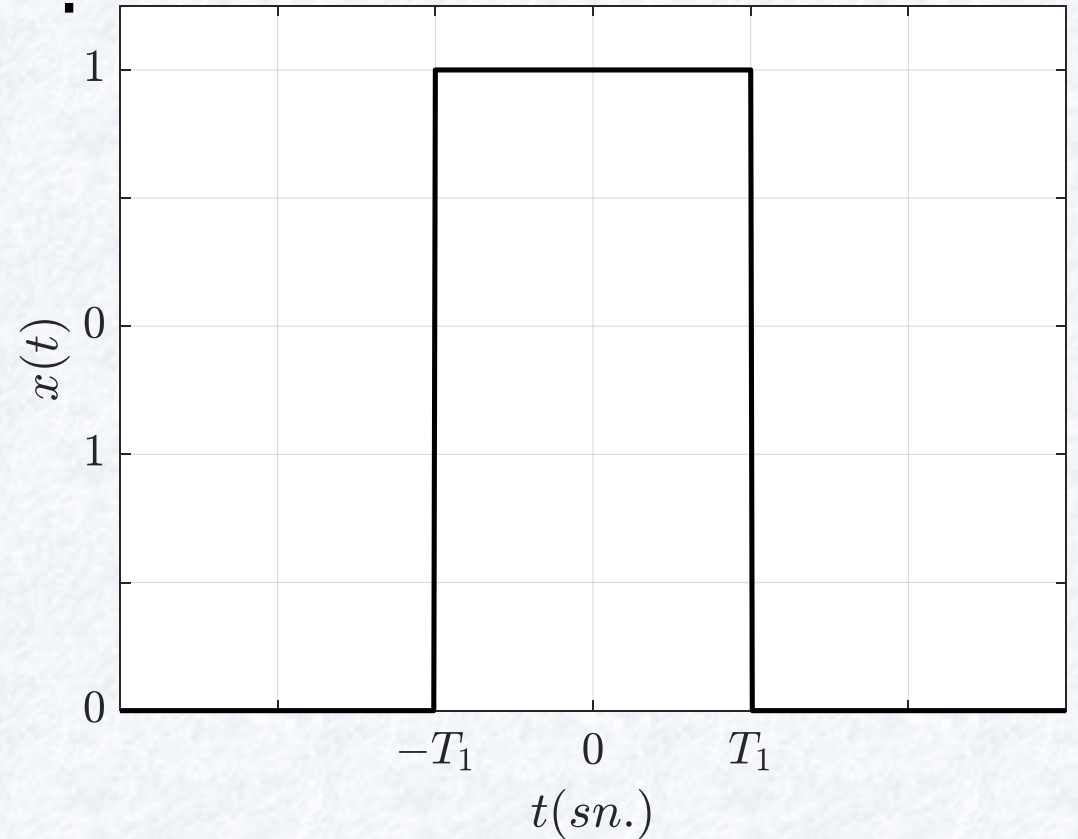
- $x(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases}$  ise  $X(\omega) = ?$

- $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

- $X(\omega) = \int_{-\infty}^{-T_1} + \int_{-T_1}^{T_1} + \int_{T_1}^{\infty}$

- $X(\omega) = \int_{-T_1}^{T_1} e^{-j\omega t} dt$

- $X(\omega) = -\frac{1}{j\omega} e^{-j\omega t} \Big|_{-T_1}^{T_1}$

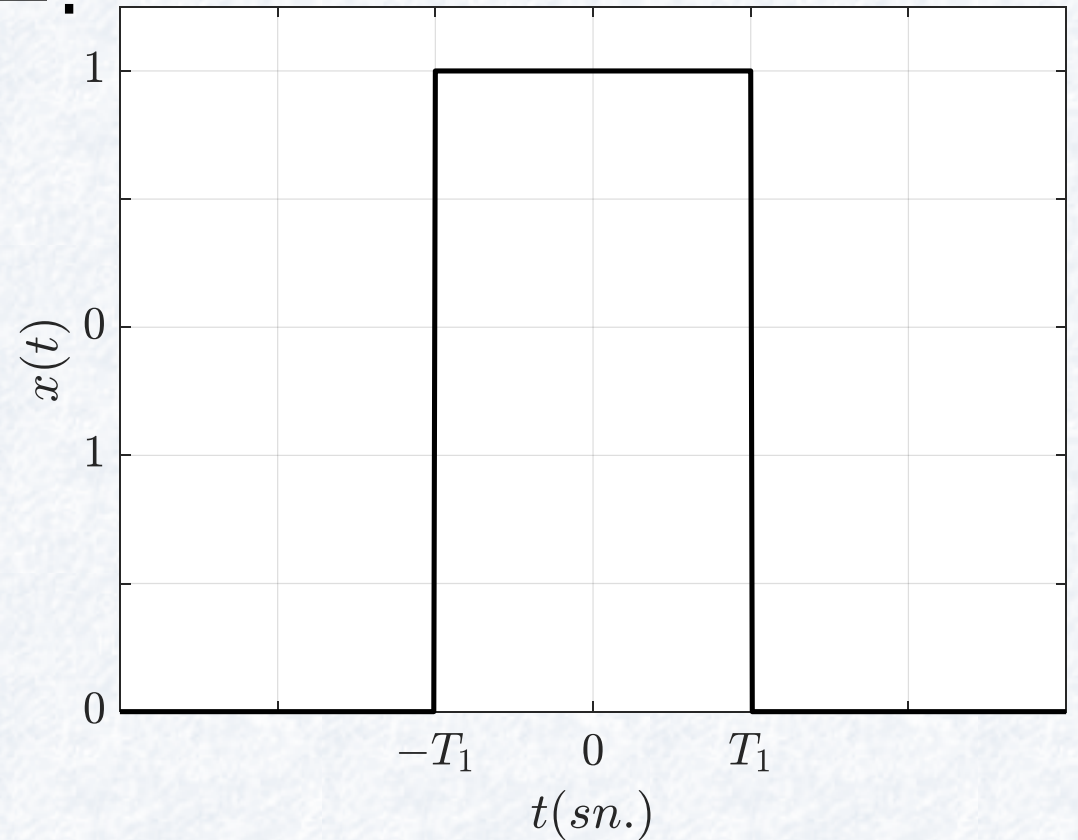


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- $X(\omega) =$



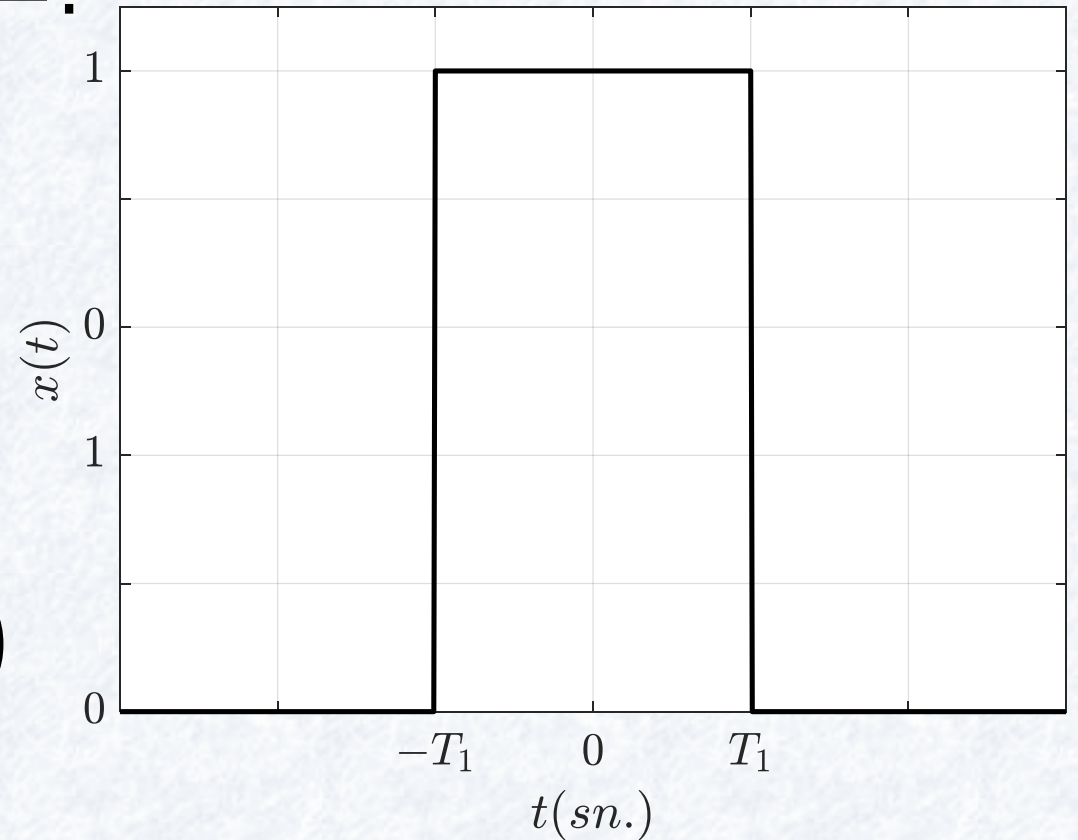
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- $x(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases}$  ise  $X(\omega) = ?$

- $X(\omega) = -\frac{1}{j\omega} e^{-j\omega t} \Big|_{-T_1}^{T_1}$

- $X(\omega) = -\frac{1}{j\omega} (e^{-j\omega T_1} - e^{j\omega T_1})$

- $X(\omega) = \frac{2}{2j\omega} (-e^{-j\omega T_1} + e^{j\omega T_1})$





# Örnek 4

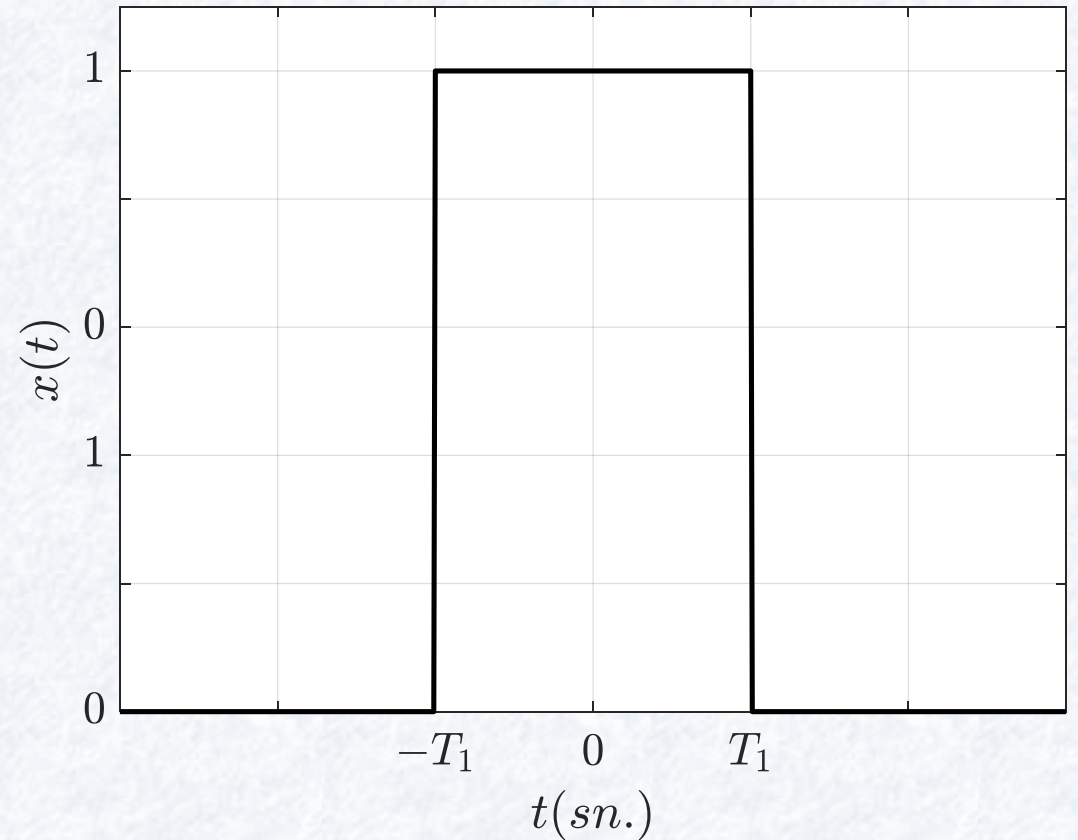
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- $X(\omega) = -\frac{1}{j\omega} e^{-j\omega t} \Big|_{-T_1}^{T_1}$

- $X(\omega) = -\frac{1}{j\omega} (e^{-j\omega T_1} - e^{j\omega T_1})$

- $X(\omega) = \frac{2}{2j\omega} (-e^{-j\omega T_1} + e^{j\omega T_1})$

- $X(\omega) = \frac{2}{\omega} \frac{-e^{-j\omega T_1} + e^{j\omega T_1}}{2j} =$



# Örnek 4

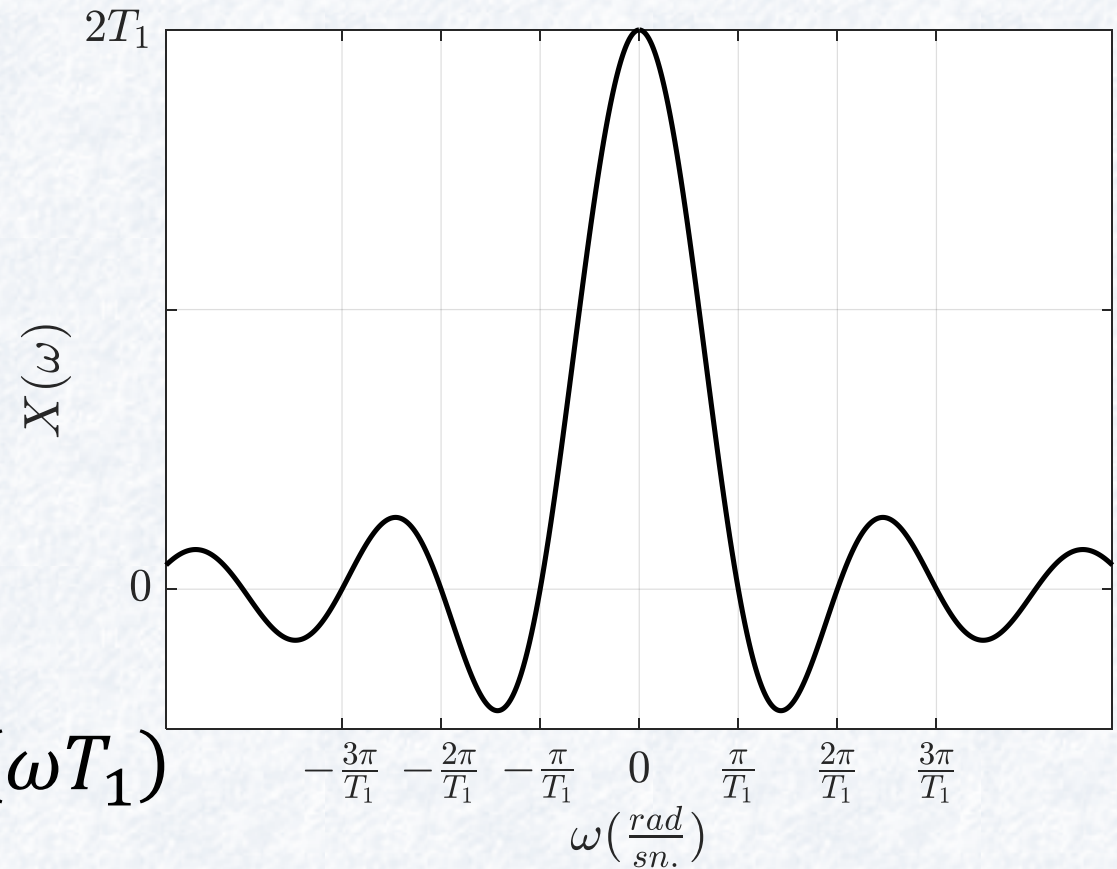
- $x(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases}$  ise  $X(\omega) = ?$

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- $X(\omega) = -\frac{1}{j\omega} (e^{-j\omega T_1} - e^{j\omega T_1})$

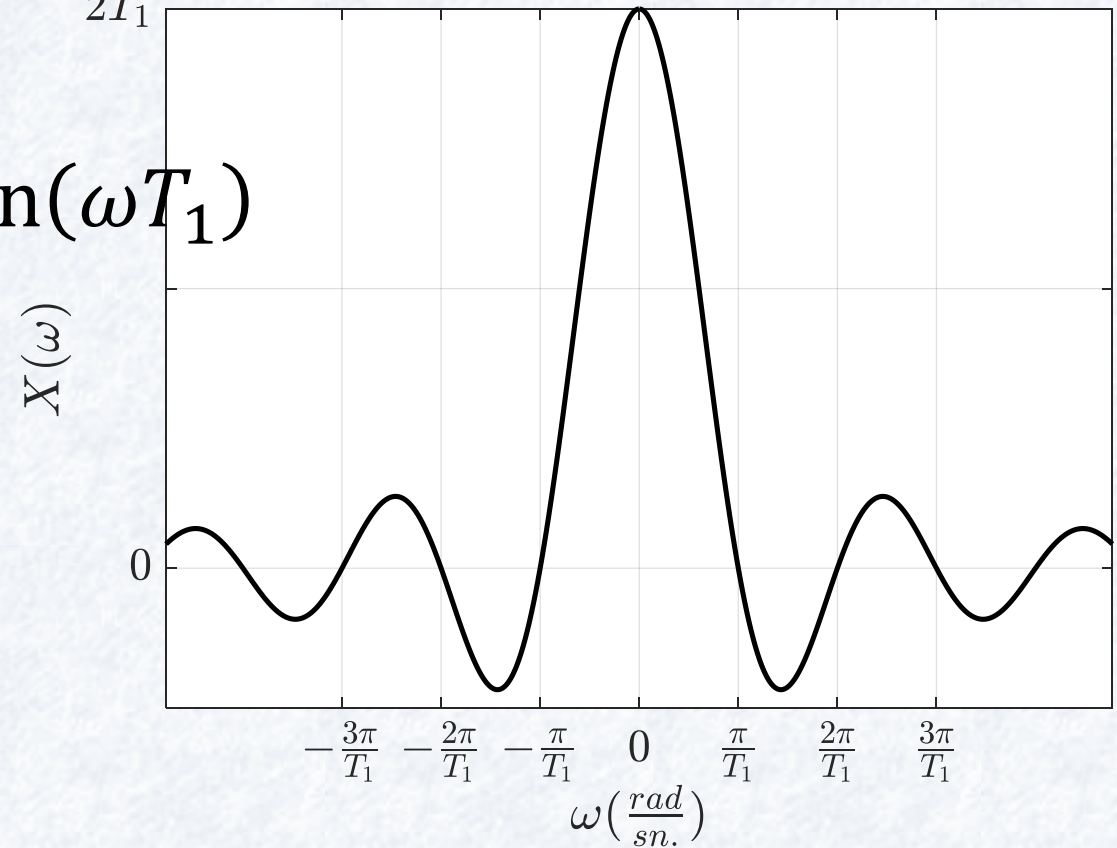
- $X(\omega) = \frac{2}{2j\omega} (-e^{-j\omega T_1} + e^{j\omega T_1})$

- $X(\omega) = \frac{2}{\omega} \frac{-e^{-j\omega T_1} + e^{j\omega T_1}}{2j} = \frac{2}{\omega} \sin(\omega T_1)$



# Örnek 4

- $x(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases}$  ise  $X(\omega) = ?$
- $X(\omega) = \frac{2}{\omega} \frac{-e^{-j\omega T_1} + e^{j\omega T_1}}{2j} = \frac{2}{\omega} \sin(\omega T_1)$
- Zamanda sınırlı
- Frekansta sınırsız
- Keskin geçişler
  - ♦ Yüksek frekans



# Örnek 4

- Zamanda sınırlı
- Frekansta sınırsız
- Keskin geçişler
  - ◆ Yüksek frekans
- Görüntü de kenarlar
  - ◆ Yüksek frekanslı bileşenler

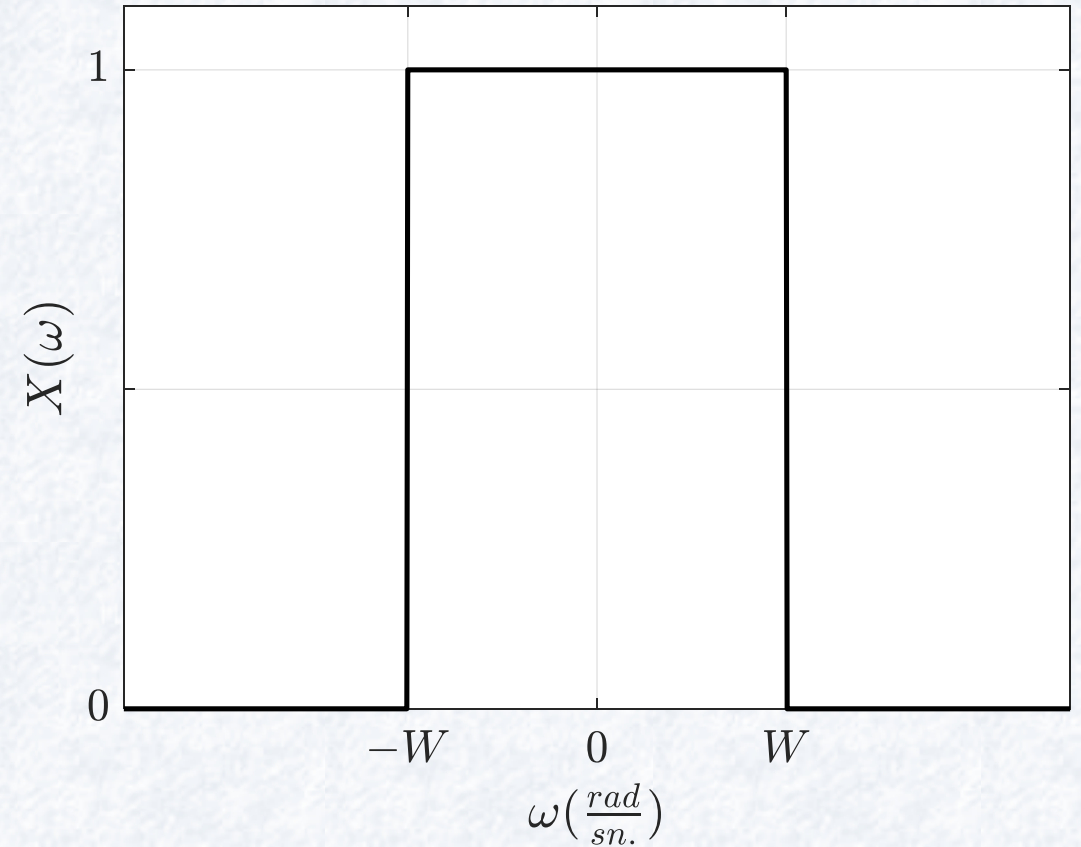
# Ters Fourier Dönüşümü

- $X(\omega)$  biliniyor  $x(t)$  bulunuyor.

- $$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

# Örnek 5

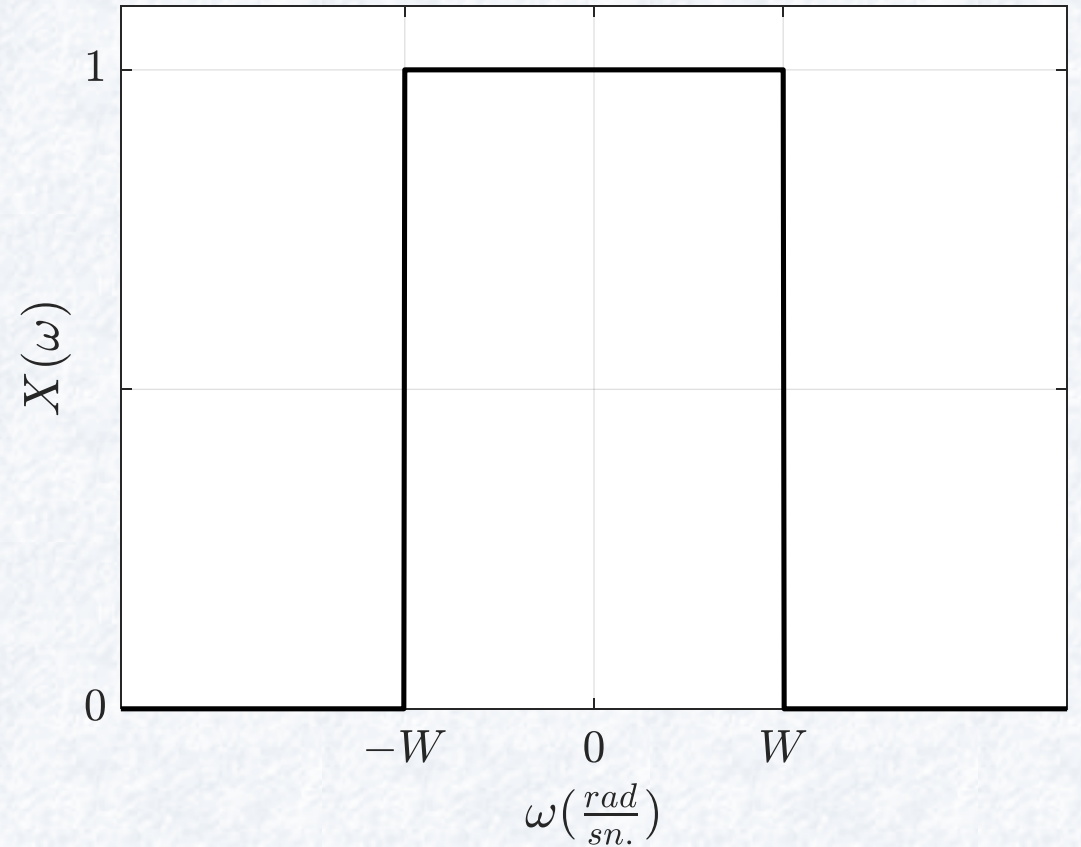
- $X(\omega) = \begin{cases} 1, & |\omega| \leq W \\ 0, & |\omega| > W \end{cases}$  ise  $x(t) = ?$
- $x(t) =$





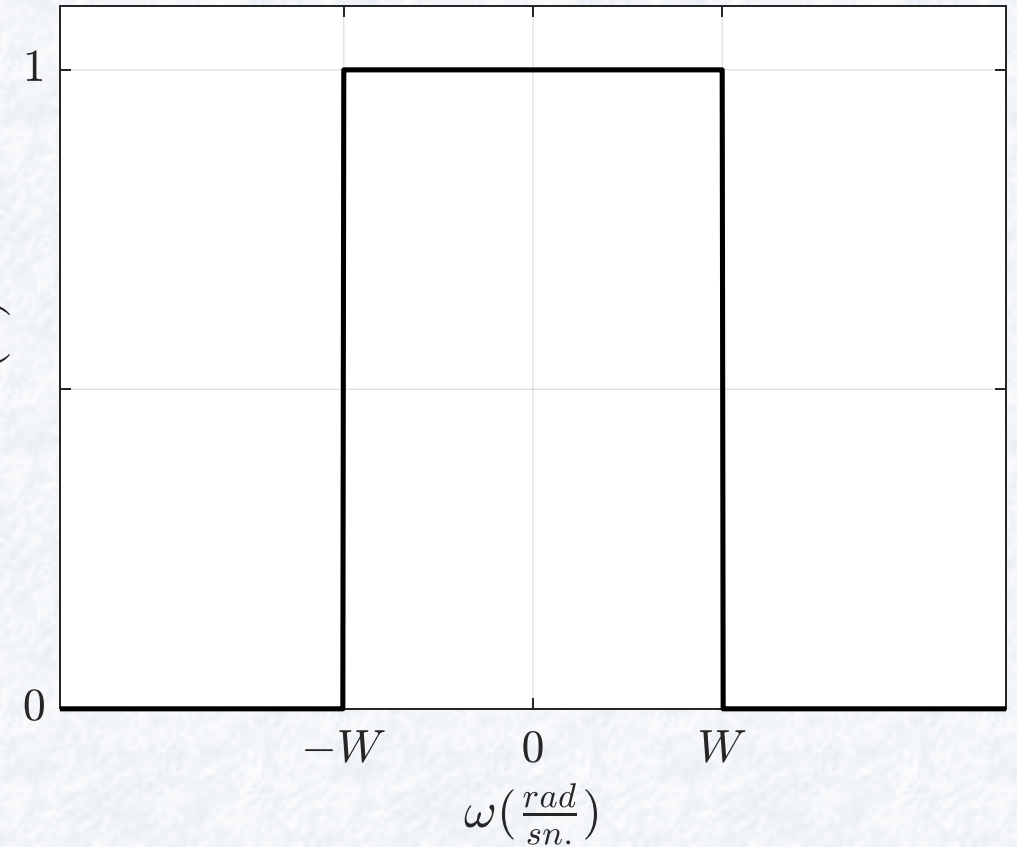
# Örnek 5

- $X(\omega) = \begin{cases} 1, & |\omega| \leq W \\ 0, & |\omega| > W \end{cases}$  ise  $x(t) = ?$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$
- $x(t) = \frac{1}{2\pi} ( \quad )$



# Örnek 5

- $X(\omega) = \begin{cases} 1, & |\omega| \leq W \\ 0, & |\omega| > W \end{cases}$  ise  $x(t) = ?$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$
- $x(t) = \frac{1}{2\pi} \left( \int_{-\infty}^{-W} + \int_{-W}^W + \int_W^{\infty} \right) \underbrace{\quad}_{X(\omega)}$
- $x(t) =$



# Örnek 5

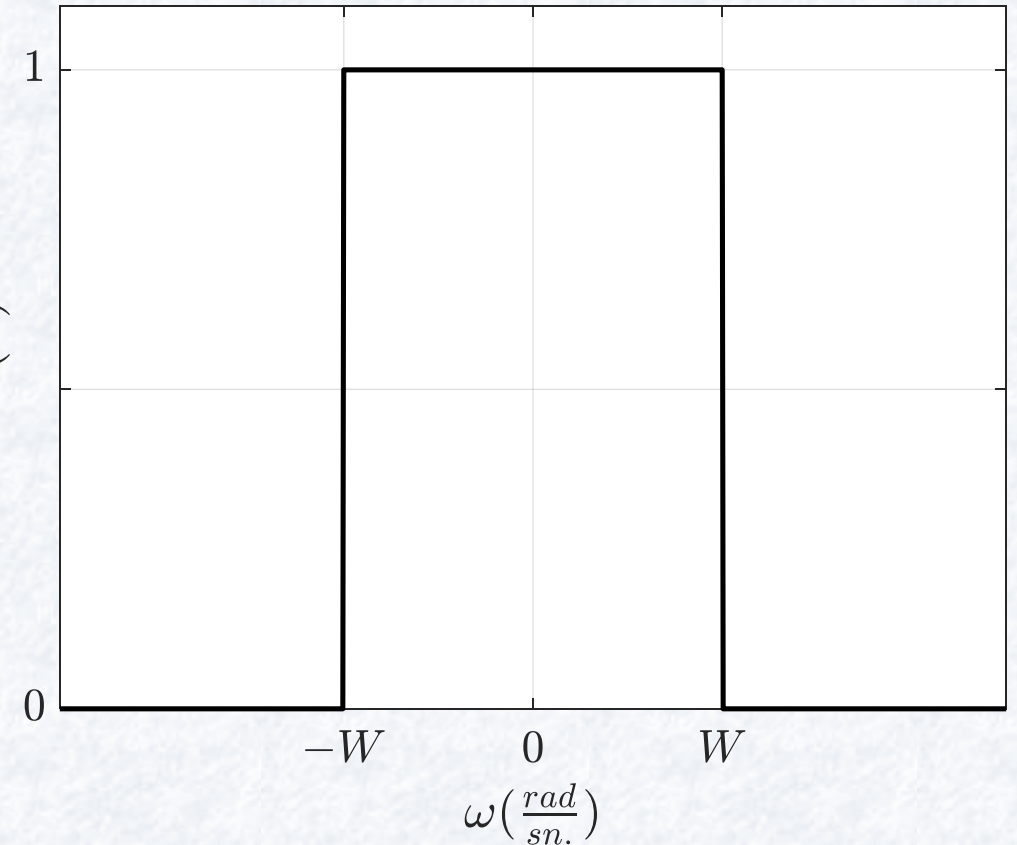
- $X(\omega) = \begin{cases} 1, & |\omega| \leq W \\ 0, & |\omega| > W \end{cases}$  ise  $x(t) = ?$

- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

- $x(t) = \frac{1}{2\pi} \left( \int_{-\infty}^{-W} + \int_{-W}^W + \int_W^{\infty} \right) \underbrace{\quad}_{X(\omega)}$

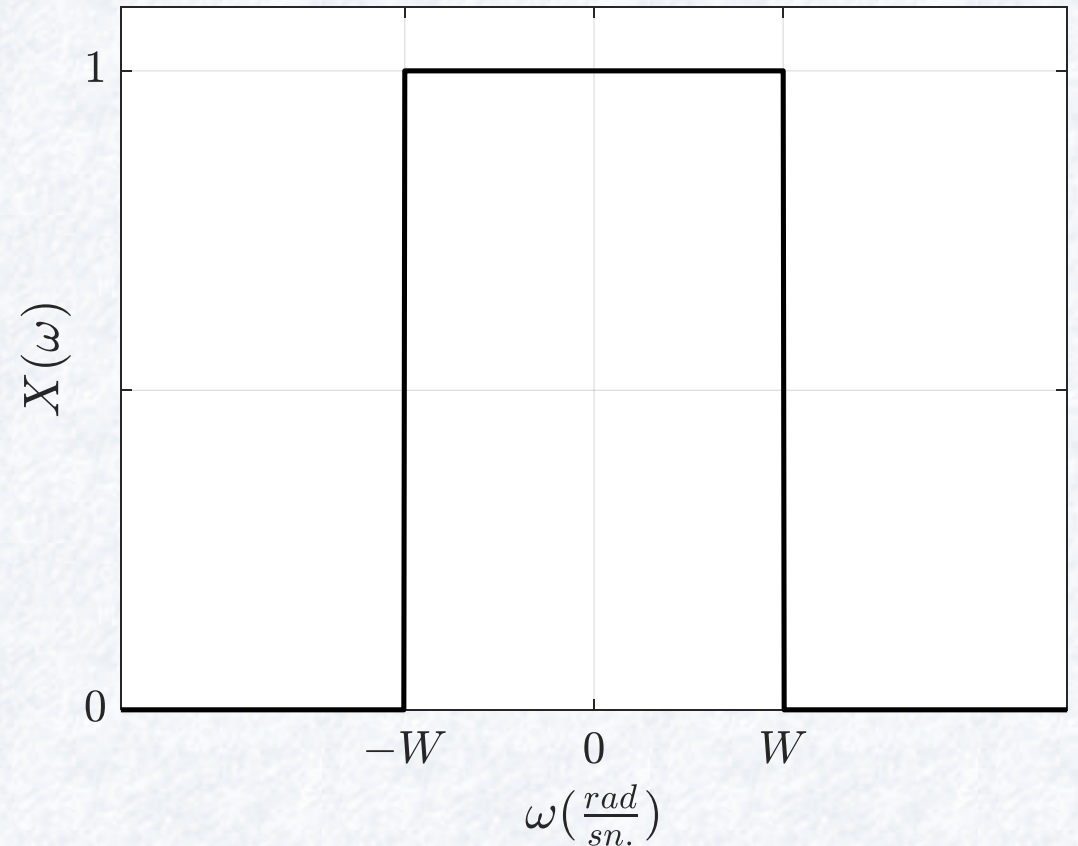
- $x(t) = \frac{1}{2\pi} \int_{-W}^W e^{j\omega t} d\omega$

- $x(t) =$



# Örnek 5

- $X(\omega) = \begin{cases} 1, & |\omega| \leq W \\ 0, & |\omega| > W \end{cases}$  ise  $x(t) = ?$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$
- $x(t) = \frac{1}{2\pi} \left( \int_{-\infty}^{-W} + \int_{-W}^W + \int_W^{\infty} \right)$
- $x(t) = \frac{1}{2\pi} \int_{-W}^W e^{j\omega t} d\omega$
- $x(t) = \frac{1}{2\pi} \frac{1}{jt} e^{j\omega t} \Big|_{-W}^W$
- $x(t) =$



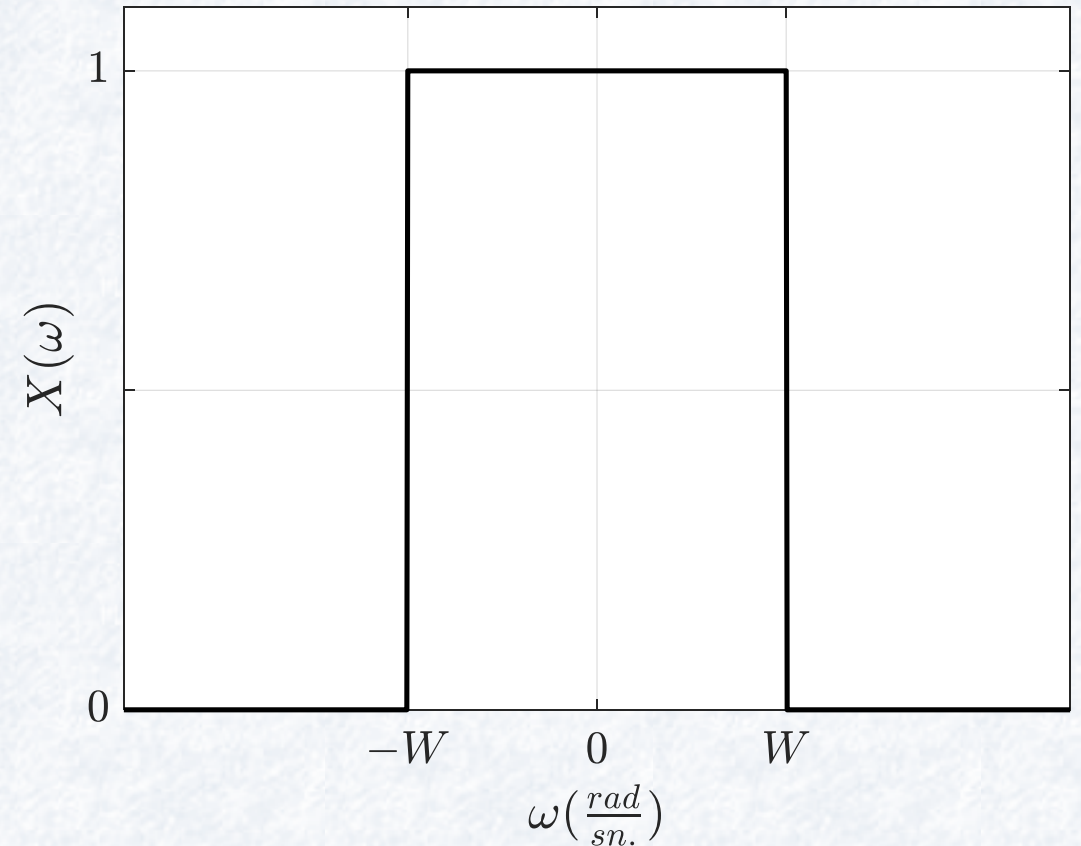
# Örnek 5

- $X(\omega) = \begin{cases} 1, & |\omega| \leq W \\ 0, & |\omega| > W \end{cases}$  ise  $x(t) = ?$

- $x(t) = \frac{1}{2\pi} \frac{1}{jt} e^{j\omega t} \Big|_{-W}^W$

- $x(t) = \frac{1}{2\pi} \frac{1}{jt} (e^{jWt} - e^{-jWt})$

- $x(t) =$



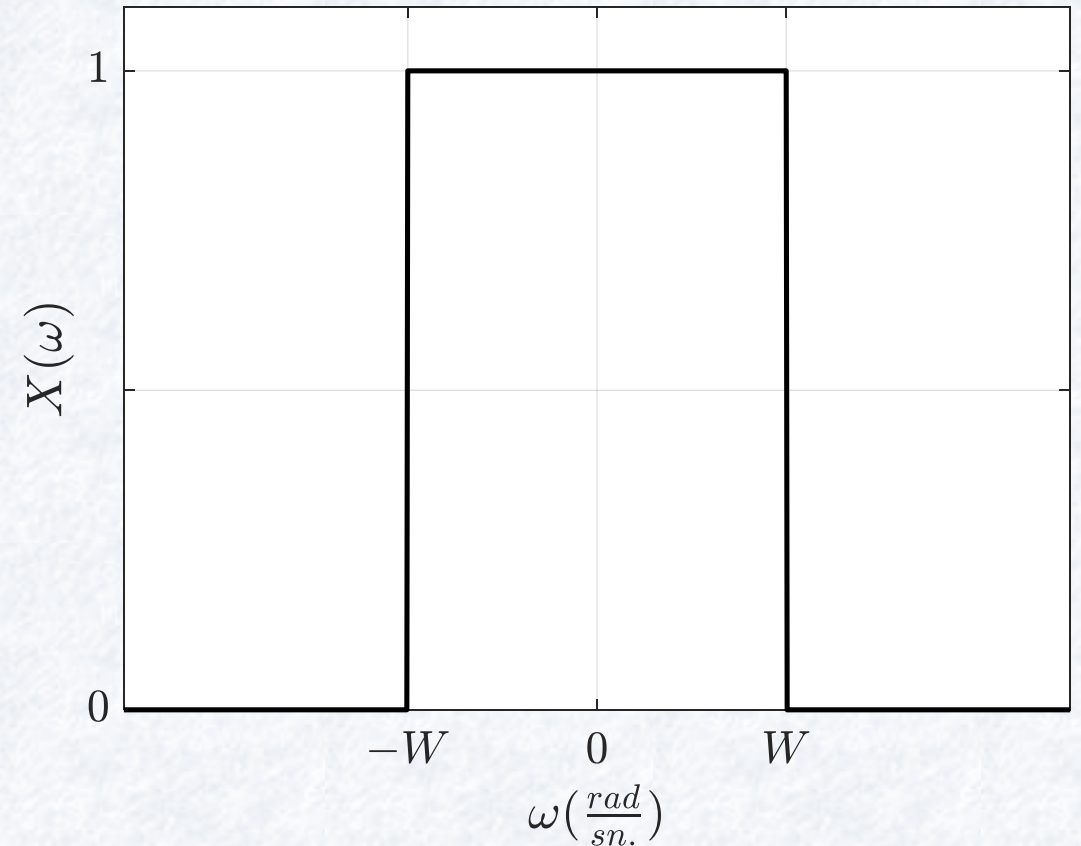
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- $x(t) = \frac{1}{2\pi} \frac{1}{jt} e^{j\omega t} \Big|_{-W}^W$

- $x(t) = \frac{1}{2\pi} \frac{1}{jt} (e^{jWt} - e^{-jWt})$

- $x(t) = \frac{1}{\pi t} \frac{e^{jWt} - e^{-jWt}}{2j} =$





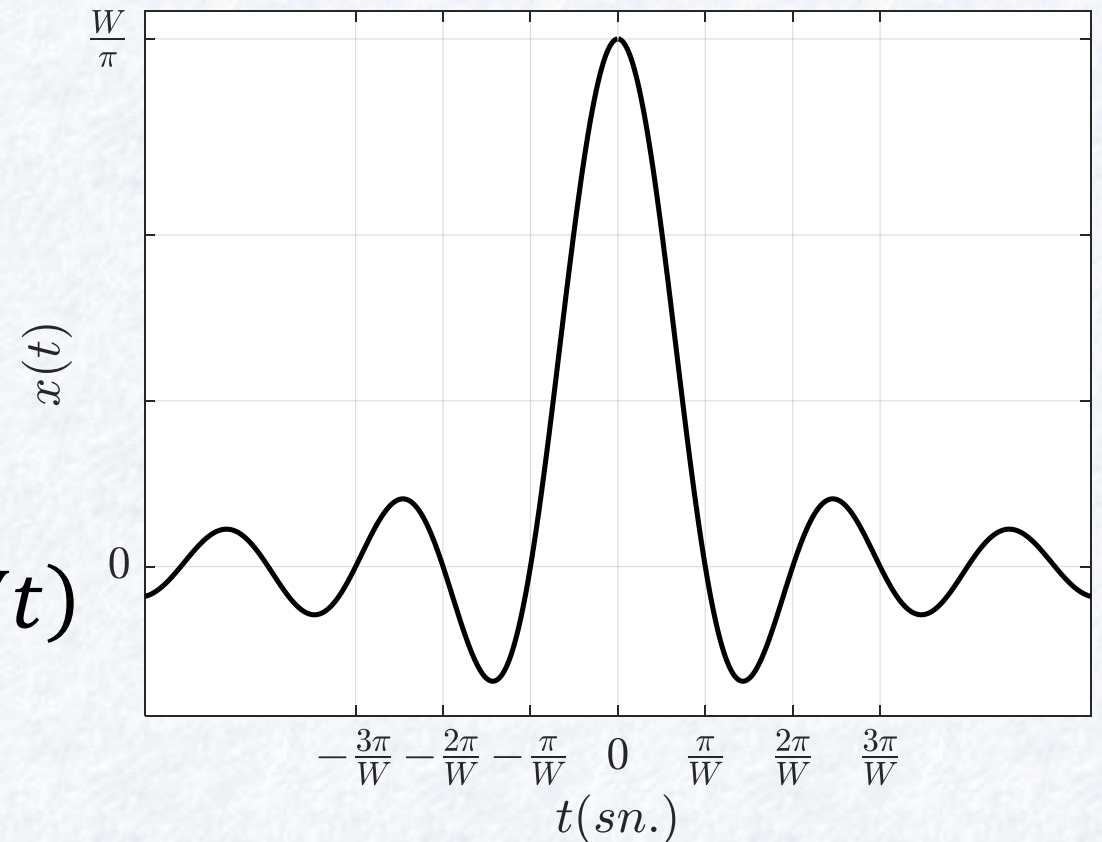
# Örnek 5

- $X(\omega) = \begin{cases} 1, & |\omega| \leq W \\ 0, & |\omega| > W \end{cases}$  ise  $x(t) = ?$

- $x(t) = \frac{1}{2\pi} \frac{1}{jt} e^{j\omega t} \Big|_{-W}^W$

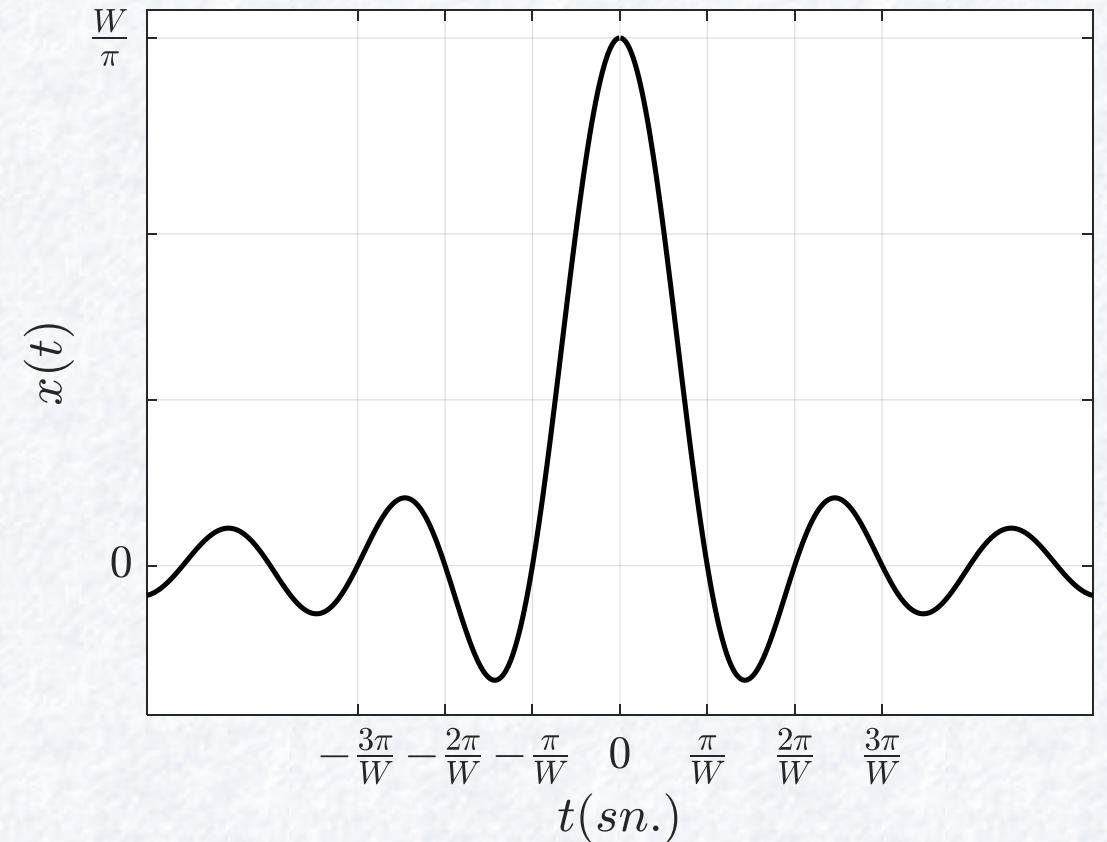
- $x(t) = \frac{1}{2\pi} \frac{1}{jt} (e^{jWt} - e^{-jWt})$

- $x(t) = \frac{1}{\pi t} \frac{e^{jWt} - e^{-jWt}}{2j} = \frac{1}{\pi t} \sin(Wt)$

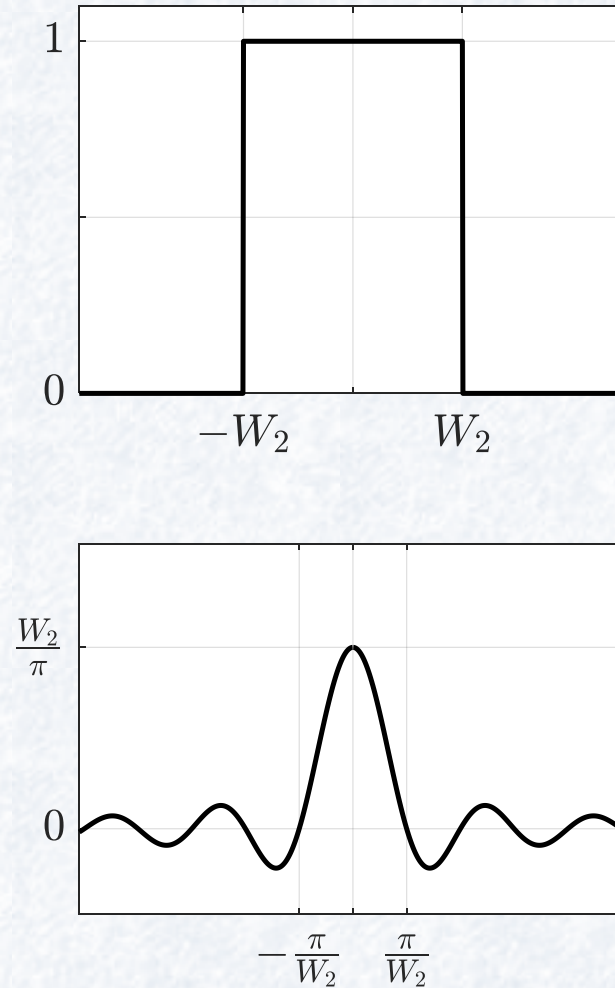


# Örnek 5

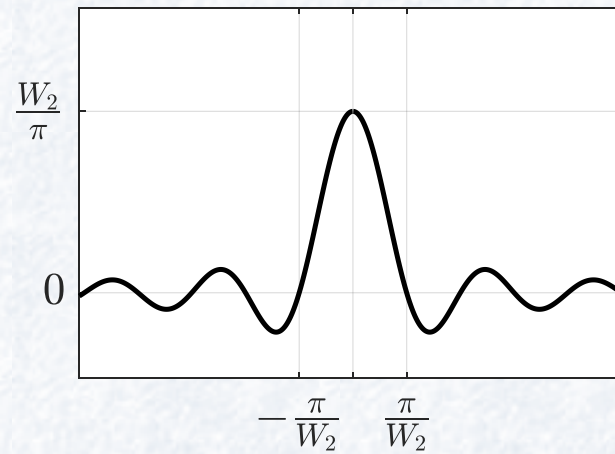
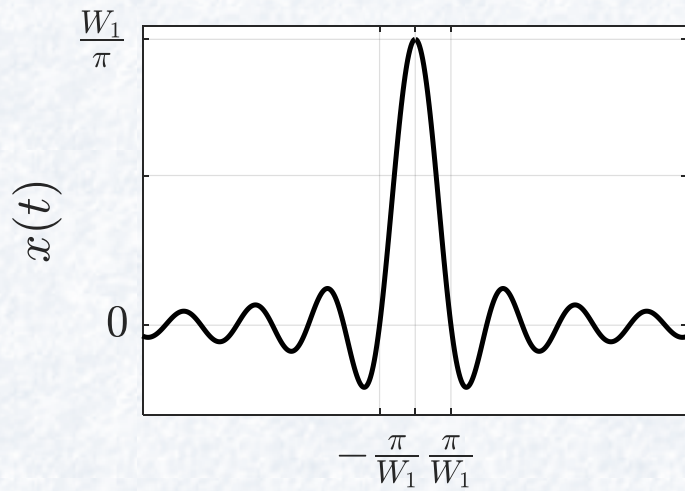
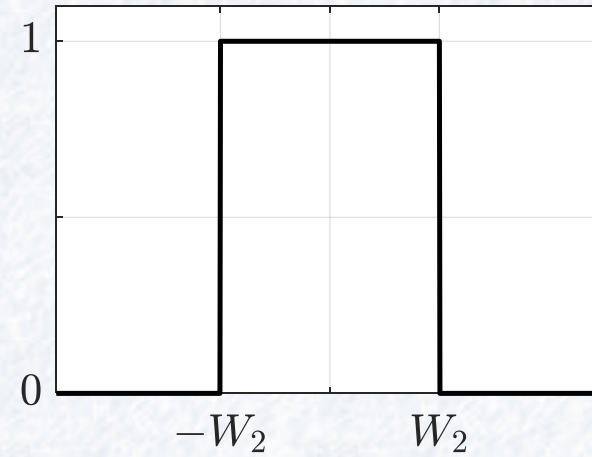
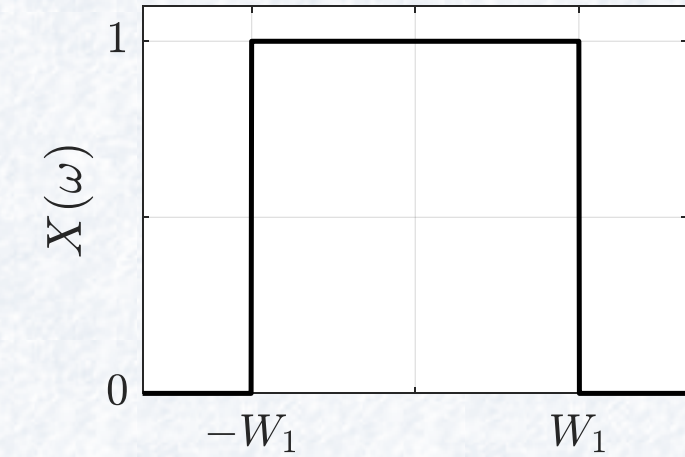
- $X(\omega) = \begin{cases} 1, & |\omega| \leq W \\ 0, & |\omega| > W \end{cases}$  ise  $x(t) = ?$
- $x(t) = \frac{1}{\pi t} \sin(Wt)$
- Frekansta sınırlı
- Zamanda sınırsız



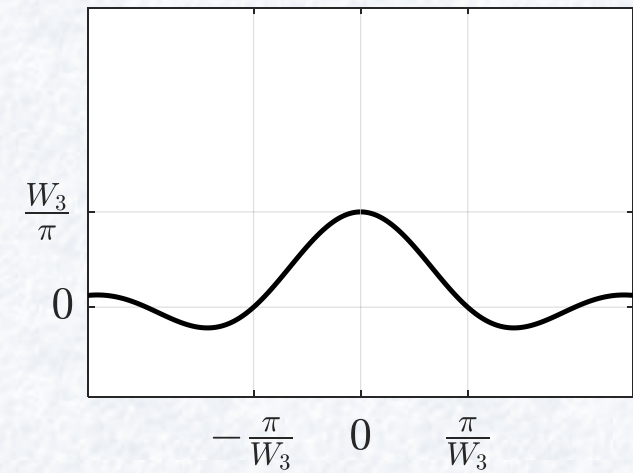
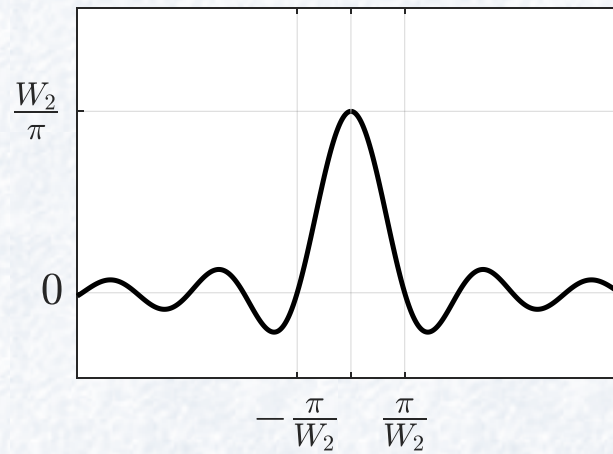
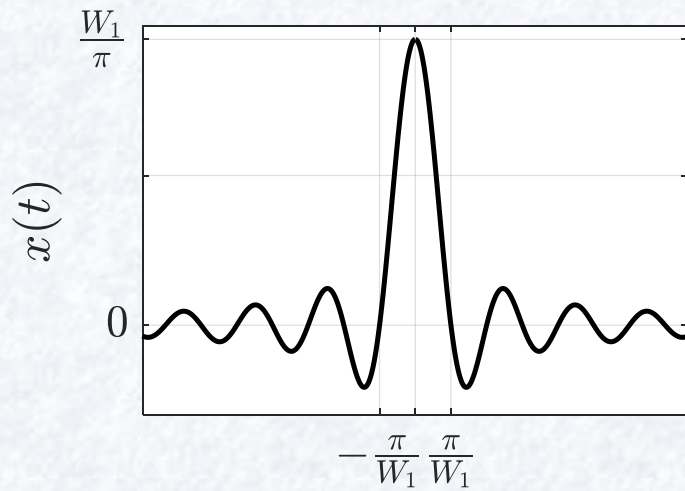
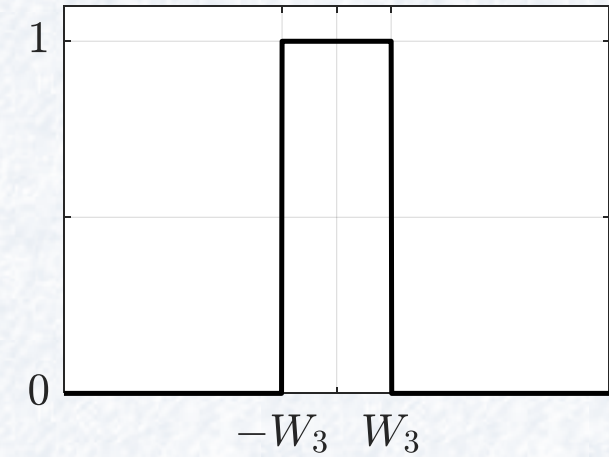
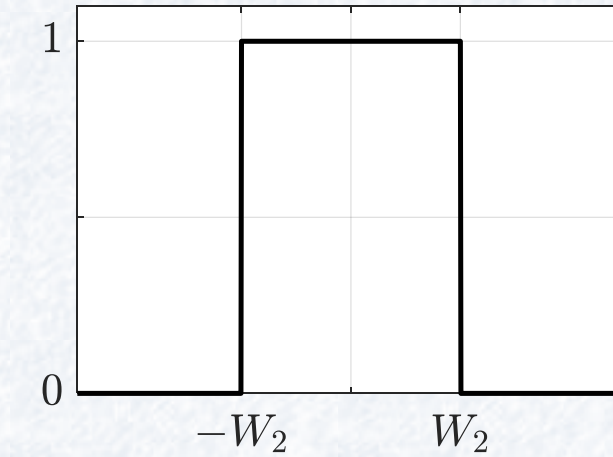
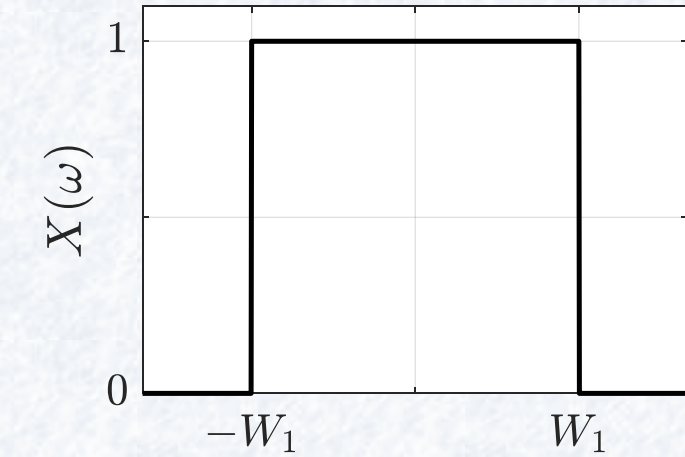
# Örnek 5



# Örnek 5

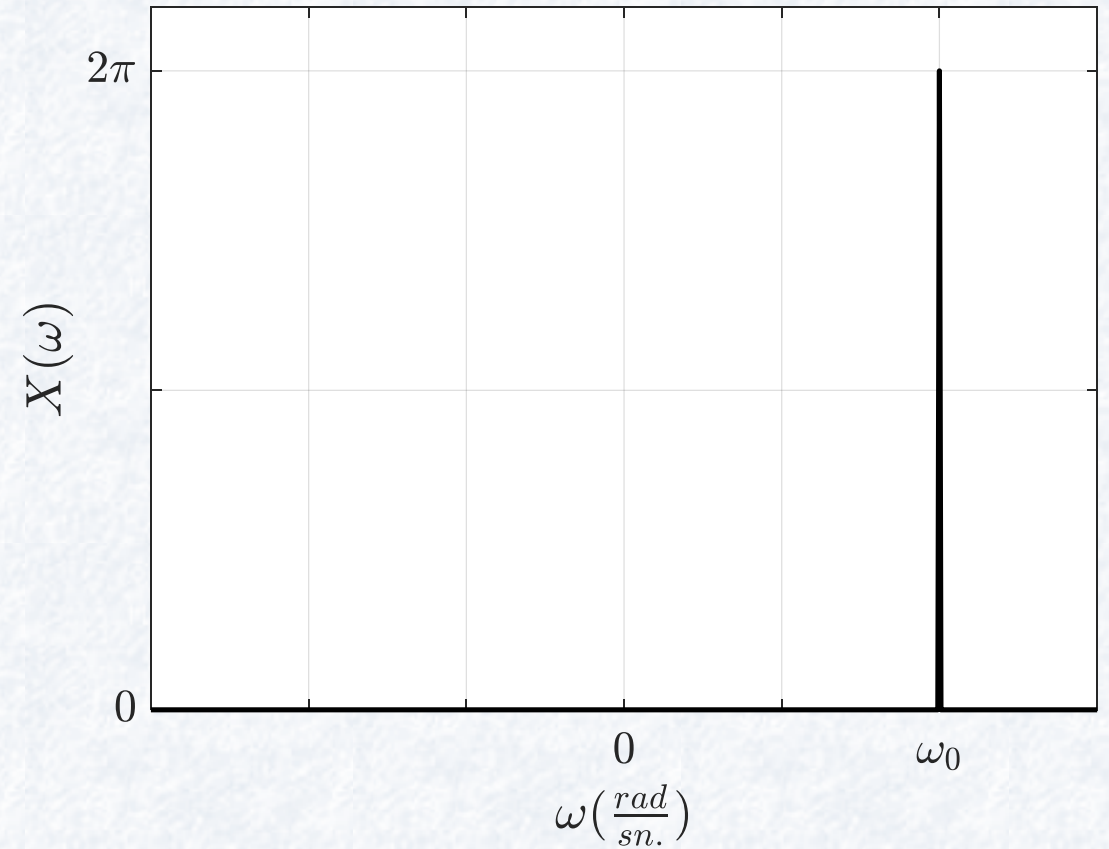


# Örnek 5



# Örnek 6

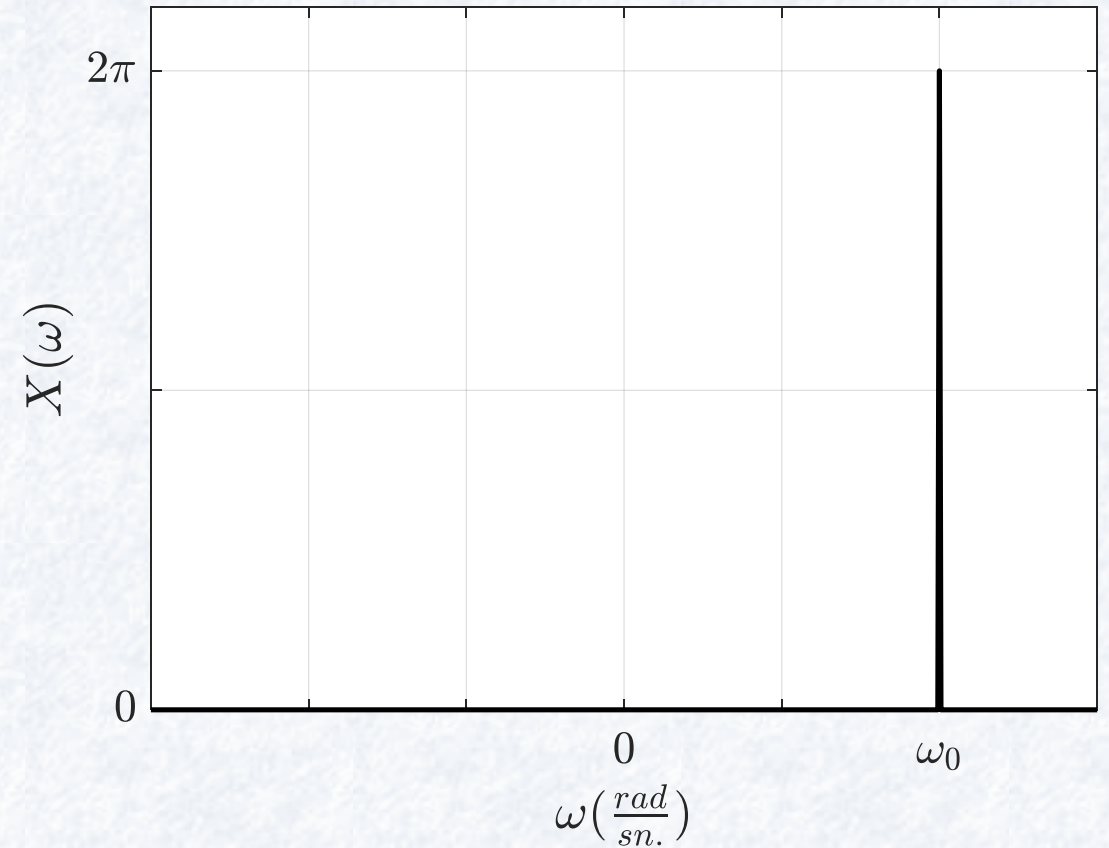
- $X(\omega) = ?$





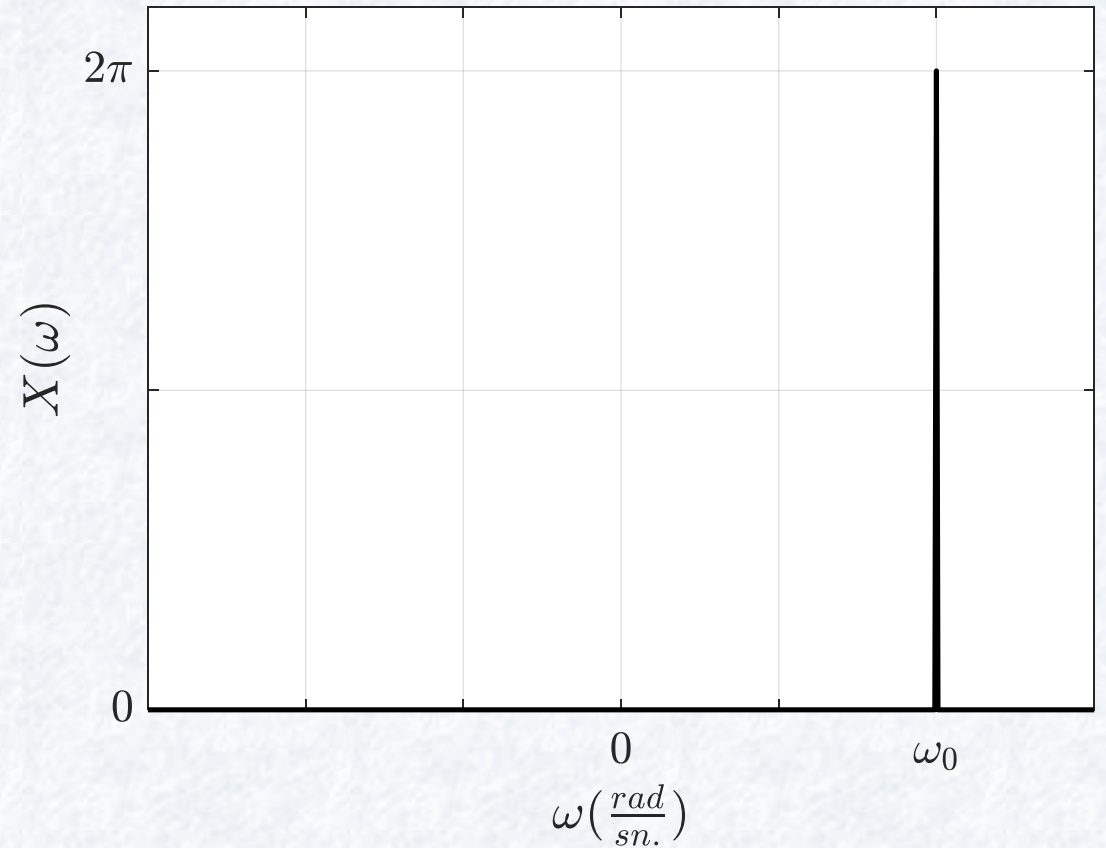
# Örnek 6

- $X(\omega) = 2\pi\delta(\omega - \omega_0)$



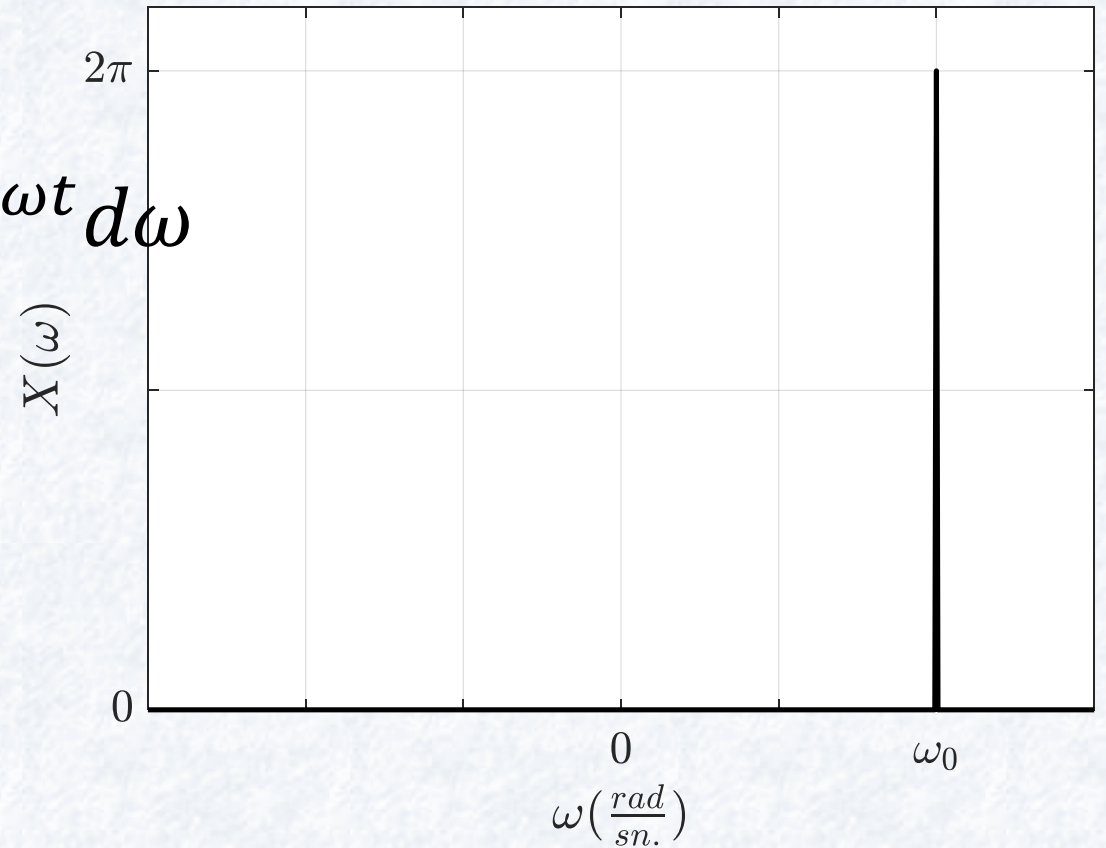
## Örnek 6

- $X(\omega) = 2\pi\delta(\omega - \omega_0)$
- $x(t) =$



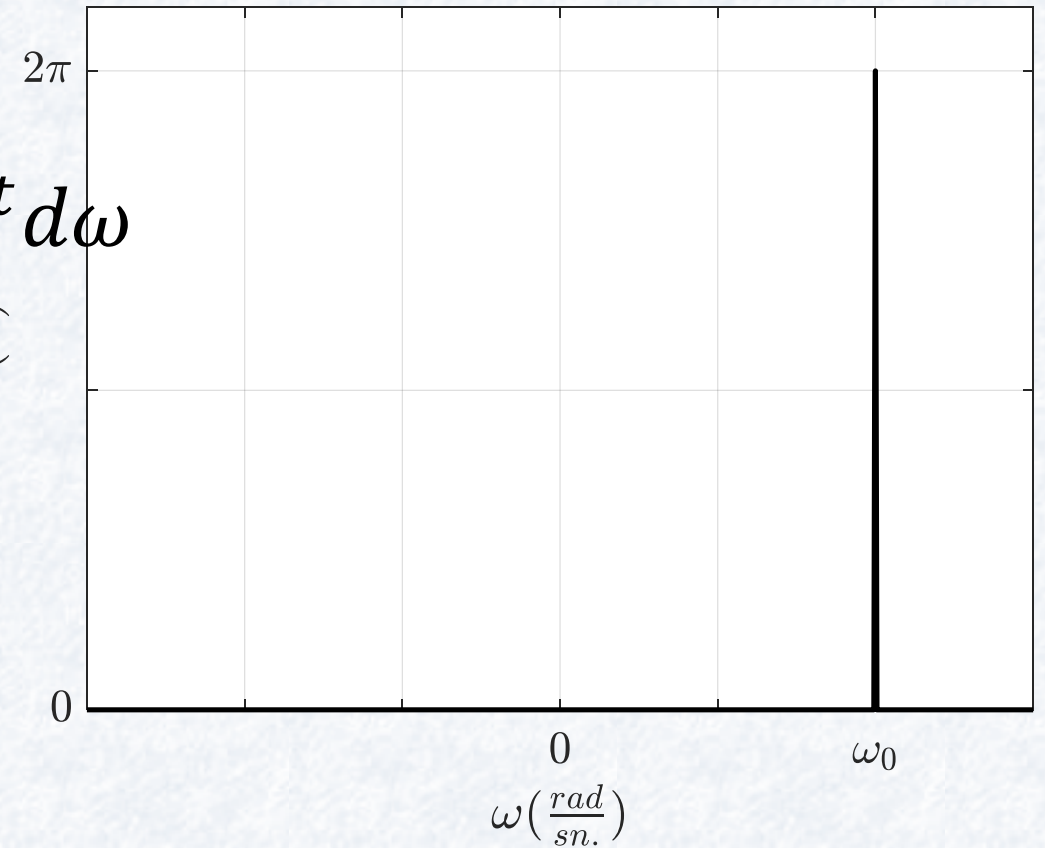
# Örnek 6

- $X(\omega) = 2\pi\delta(\omega - \omega_0)$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi\delta(\omega - \omega_0)e^{j\omega t} d\omega$



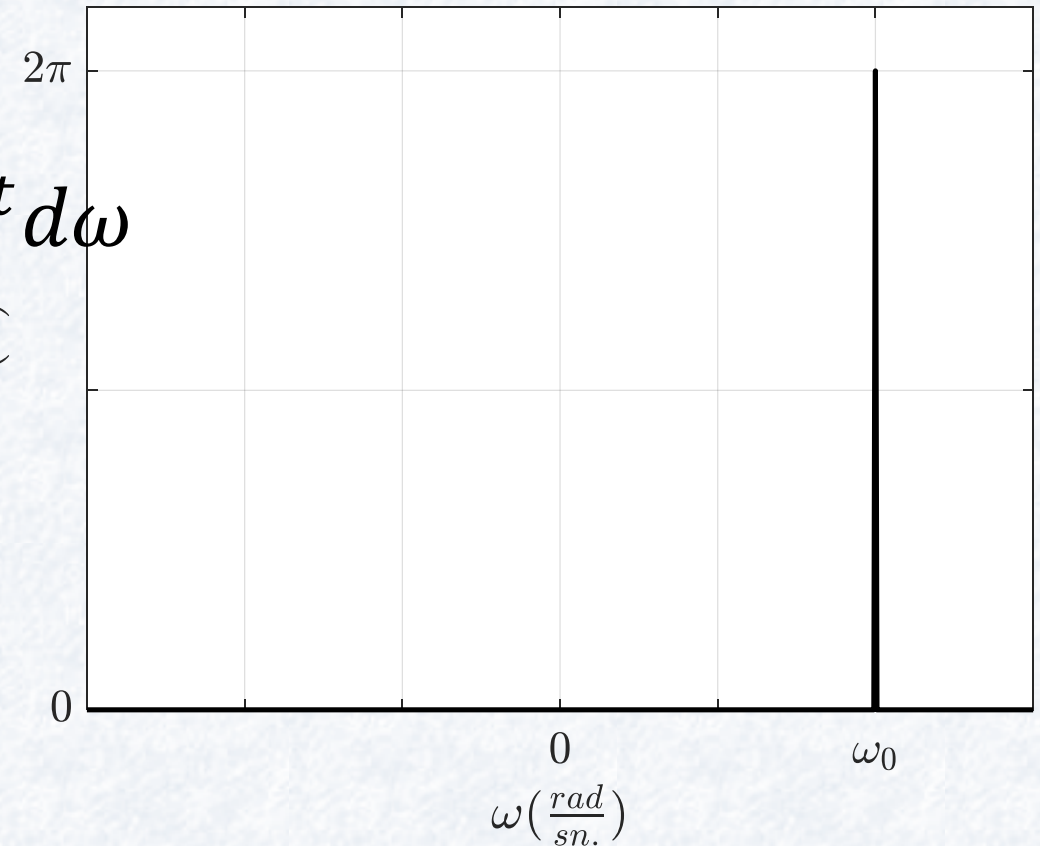
# Örnek 6

- $X(\omega) = 2\pi\delta(\omega - \omega_0)$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi\delta(\omega - \omega_0)e^{j\omega t} d\omega$
- $x(t) = \int_{-\infty}^{\infty} \delta(\omega - \omega_0)e^{j\omega t} d\omega \stackrel{(3)}{X}$
- $x(t) =$



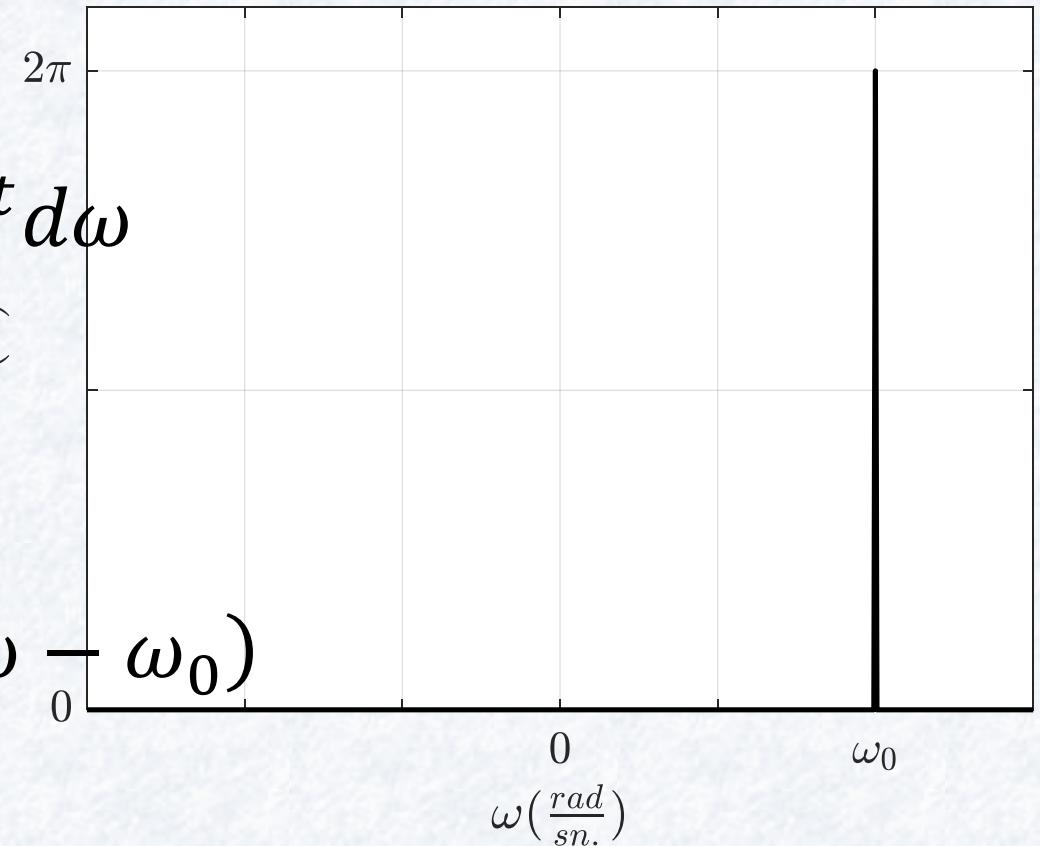
# Örnek 6

- $X(\omega) = 2\pi\delta(\omega - \omega_0)$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi\delta(\omega - \omega_0)e^{j\omega t} d\omega$
- $x(t) = \int_{-\infty}^{\infty} \delta(\omega - \omega_0)e^{j\omega t} d\omega \stackrel{(3)}{X}$
- $x(t) = e^{j\omega_0 t}$



# Örnek 6

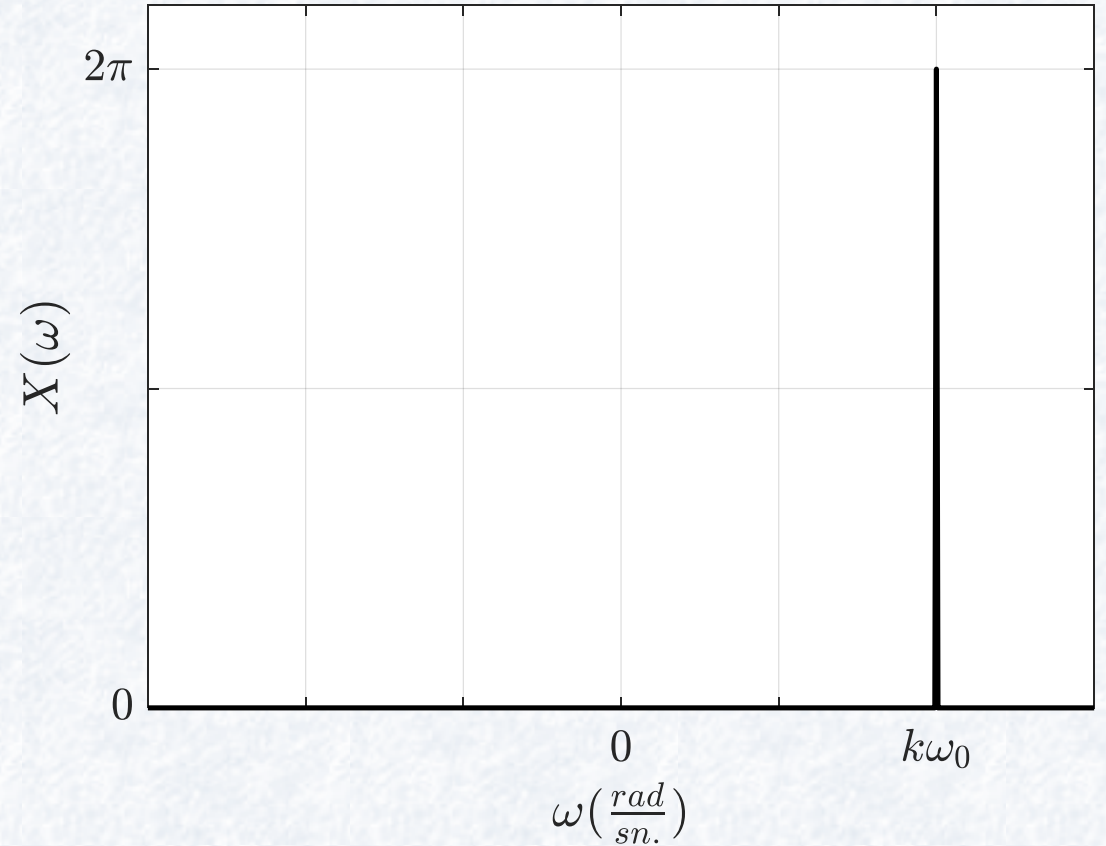
- $X(\omega) = 2\pi\delta(\omega - \omega_0)$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi\delta(\omega - \omega_0)e^{j\omega t} d\omega$
- $x(t) = \int_{-\infty}^{\infty} \delta(\omega - \omega_0)e^{j\omega t} d\omega \stackrel{(3)}{X}$
- $x(t) = e^{j\omega_0 t}$
- $x(t) = e^{j\omega_0 t} \leftrightarrow X(\omega) = 2\pi\delta(\omega - \omega_0)$





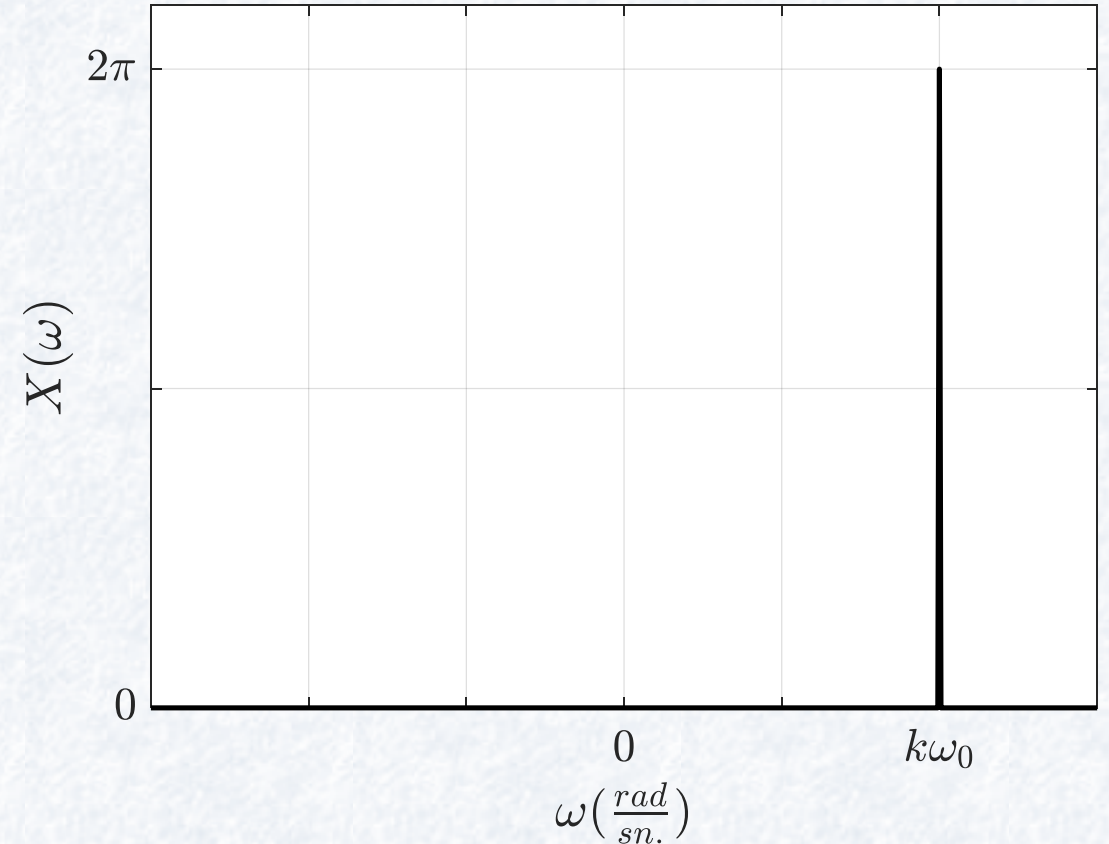
# Örnek 7

- $X(\omega) =$



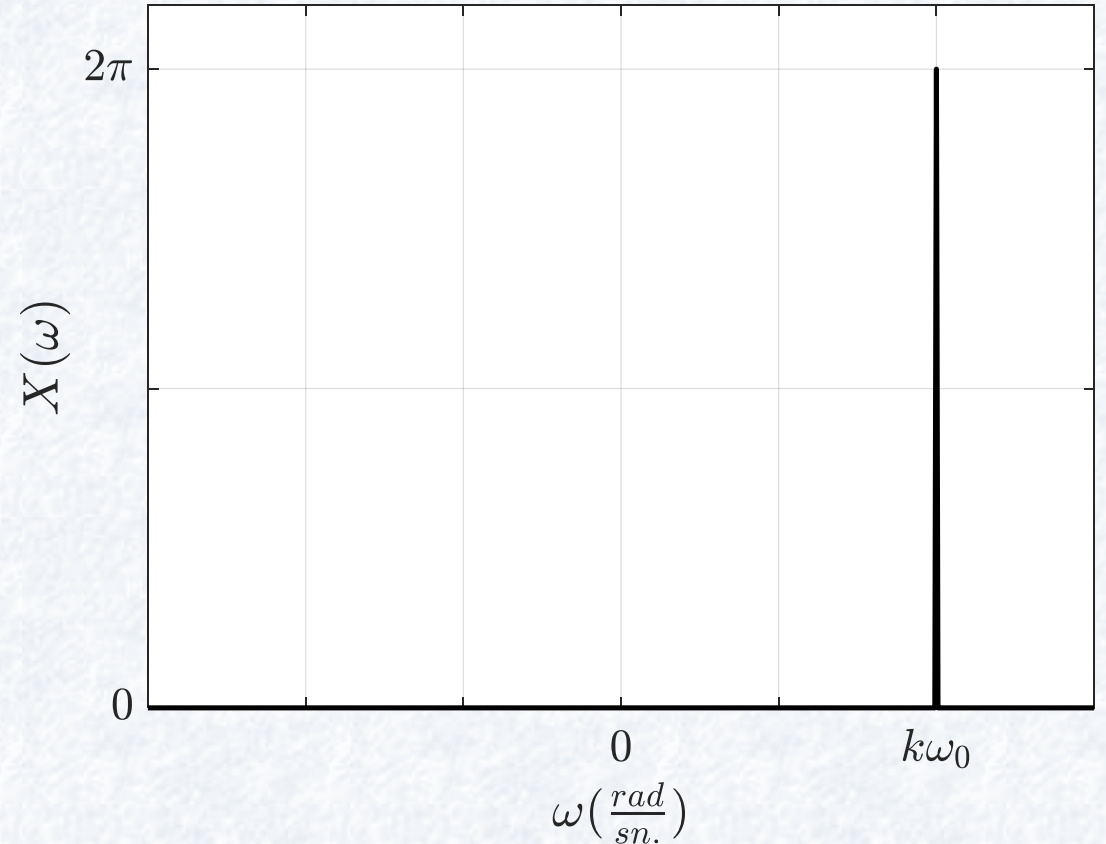
# Örnek 7

- $X(\omega) = 2\pi\delta(\omega - k\omega_0) \leftrightarrow x(t) =$



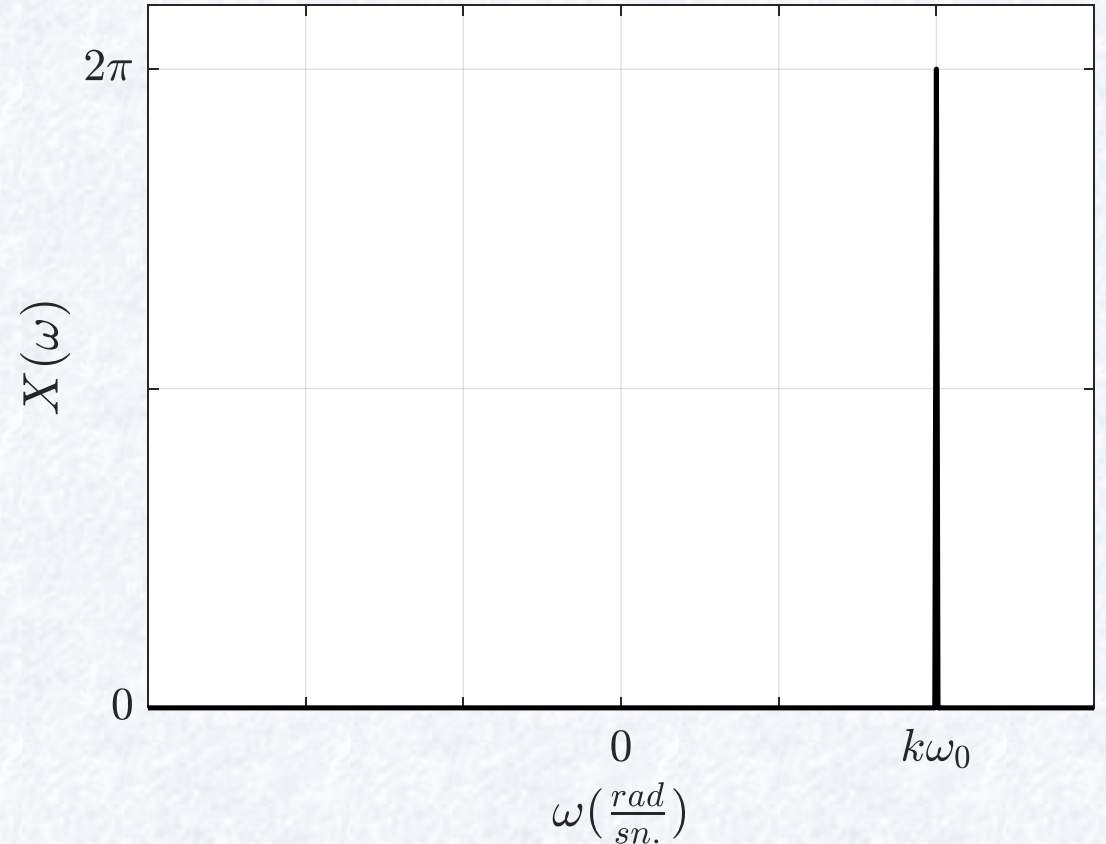
# Örnek 7

- $X(\omega) = 2\pi\delta(\omega - k\omega_0) \leftrightarrow x(t) = e^{jk\omega_0 t}$



# Örnek 7

- $X(\omega) = 2\pi\delta(\omega - k\omega_0) \leftrightarrow x(t) = e^{jk\omega_0 t}$
- Fourier Seri Açılımı



# Fourier Seri Açılımı

- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

# Fourier Seri Açılımı

- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$
- $\mathcal{F}\{x(t)\} = \mathcal{F}\left\{\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right\}$



# Fourier Seri Açılımı

- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$
- $\mathcal{F}\{x(t)\} = \mathcal{F}\left\{\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right\}$
- $\mathcal{F}\{x(t)\} = \sum_{k=-\infty}^{\infty} \mathcal{F}\{a_k e^{jk\omega_0 t}\}$

# Fourier Seri Açılımı

- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$
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- $\mathcal{F}\{x(t)\} = \sum_{k=-\infty}^{\infty} a_k \mathcal{F}\{e^{jk\omega_0 t}\}$

# Fourier Seri Açılımı

- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$
- $\mathcal{F}\{x(t)\} = \mathcal{F}\left\{\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right\}$
- $\mathcal{F}\{x(t)\} = \sum_{k=-\infty}^{\infty} \mathcal{F}\{a_k e^{jk\omega_0 t}\}$
- $\mathcal{F}\{x(t)\} = \sum_{k=-\infty}^{\infty} a_k \mathcal{F}\{e^{jk\omega_0 t}\}$
- $\mathcal{F}\{x(t)\} = \sum_{k=-\infty}^{\infty} a_k 2\pi \delta(\omega - k\omega_0)$
- $X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$

# Fourier Dönüşüm Özellikleri

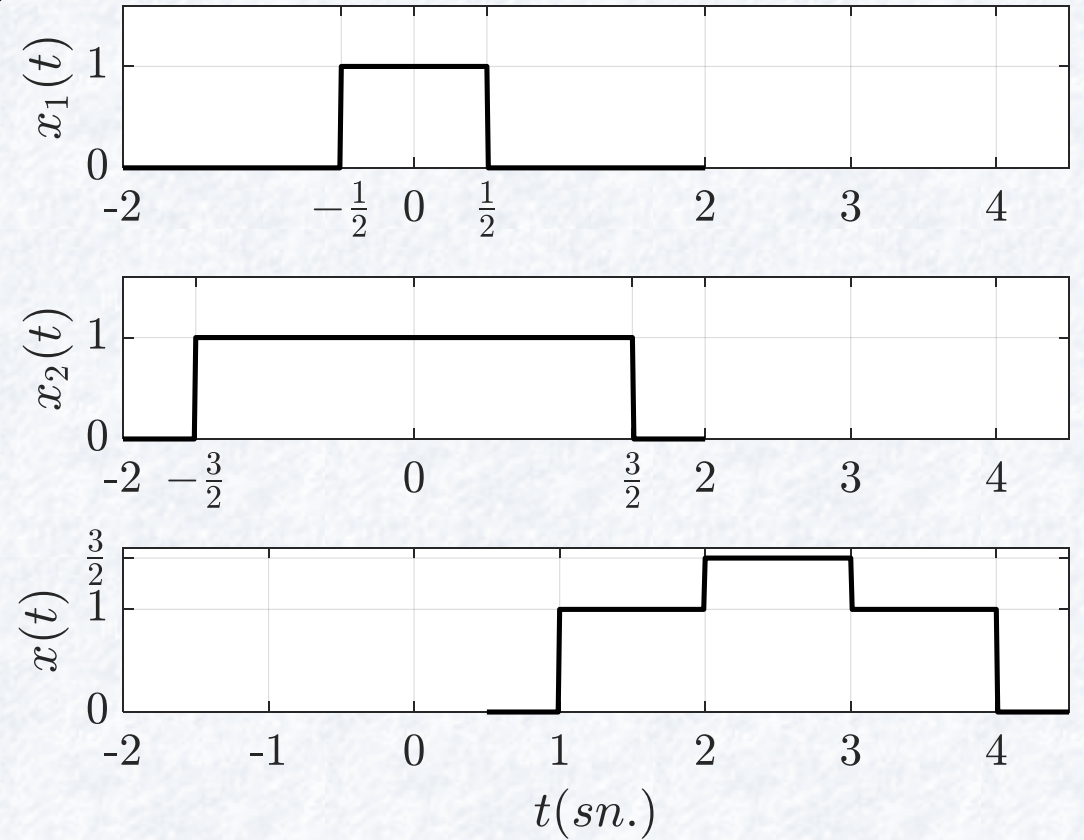
- Doğrusallık
- $x(t) \leftrightarrow X(\omega)$  ve
- $y(t) \leftrightarrow Y(\omega)$  olduğu biliniyorsa
- $z(t) = ax(t) + by(t)$  ise
- $\mathcal{F}\{z(t)\} = a\mathcal{F}\{x(t)\} + b\mathcal{F}\{y(t)\}$
- $Z(\omega) = aX(\omega) + bY(\omega)$

# Fourier Dönüşüm Özellikleri

- Zamanda Öteleme
- $x(t) \leftrightarrow X(\omega)$  olduğu biliniyorsa
- $\mathcal{F}\{x(t - t_0)\} = e^{-j\omega t_0} X(\omega)$

# Örnek 8

- $x(t) = \frac{1}{2}x_1\left(t - \frac{5}{2}\right) + x_2\left(t - \frac{5}{2}\right)$
- $X(\omega) = ?$





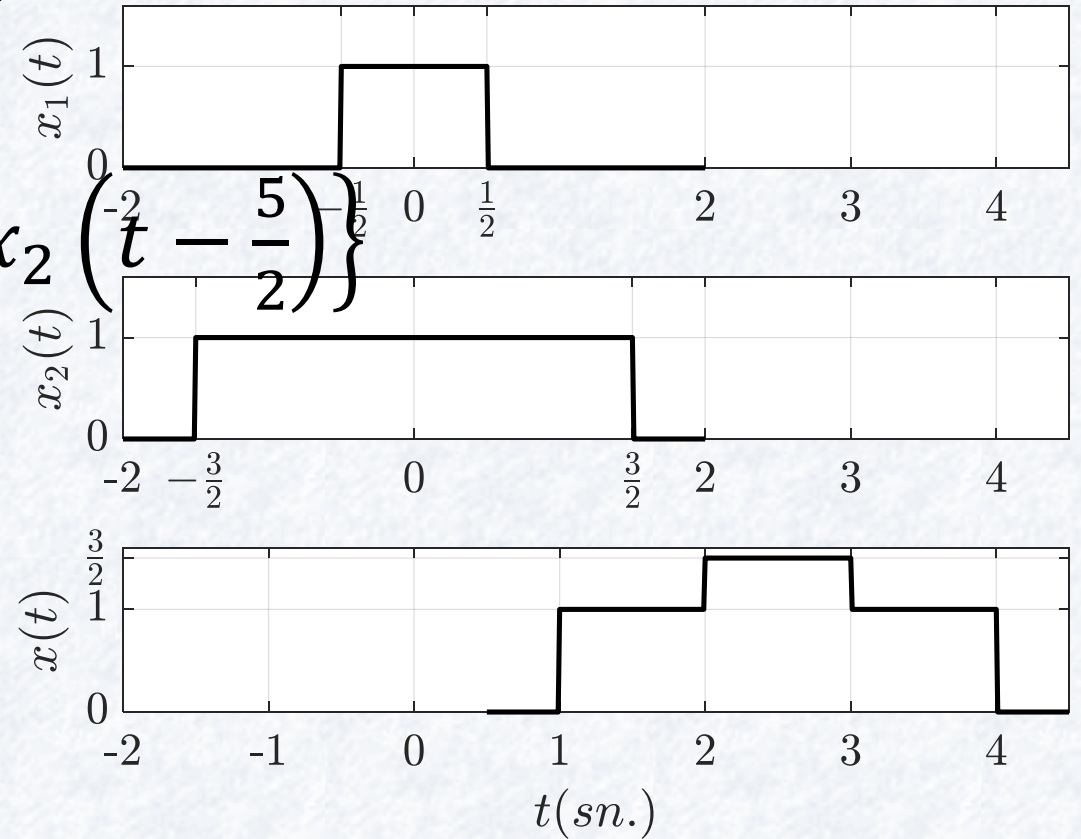
# Örnek 8

- $x(t) = \frac{1}{2}x_1\left(t - \frac{5}{2}\right) + x_2\left(t - \frac{5}{2}\right)$

- $X(\omega) = ?$

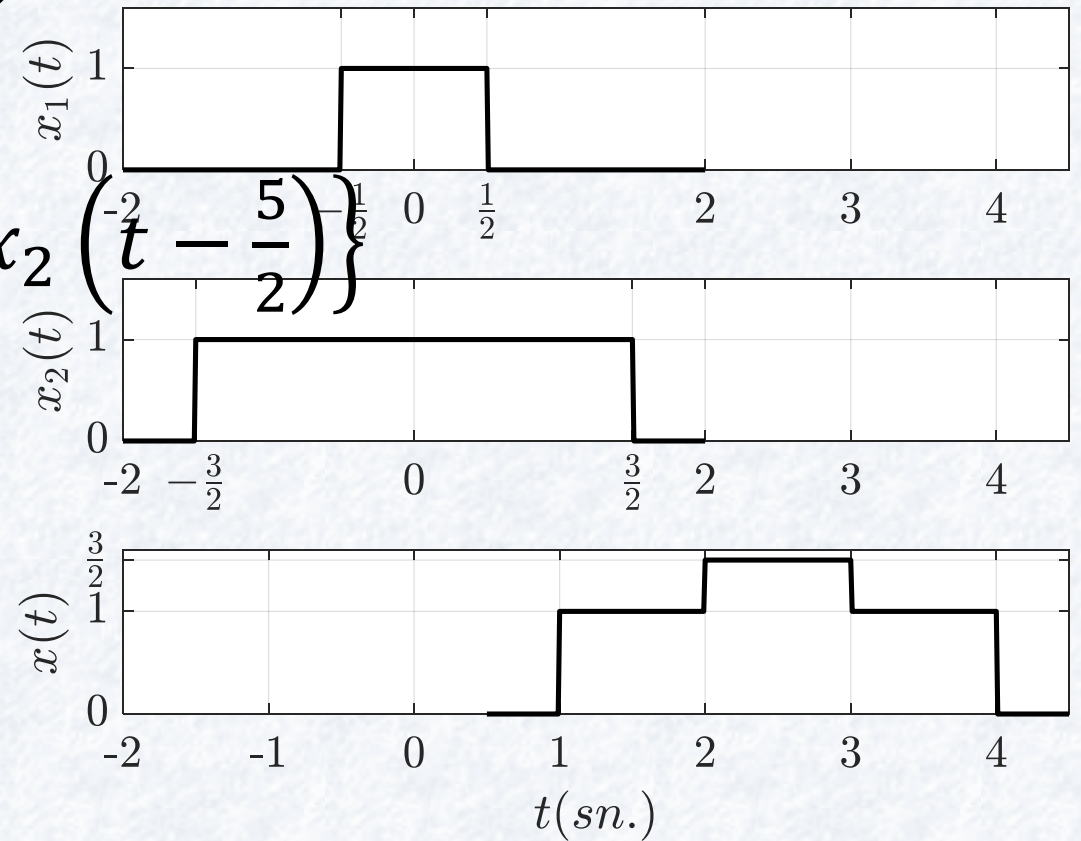
- $X(\omega) = \frac{1}{2}\mathcal{F}\left\{x_1\left(t - \frac{5}{2}\right)\right\} + \mathcal{F}\left\{x_2\left(t - \frac{5}{2}\right)\right\}$

- ♦  $\mathcal{F}\left\{x_1\left(t - \frac{5}{2}\right)\right\} =$



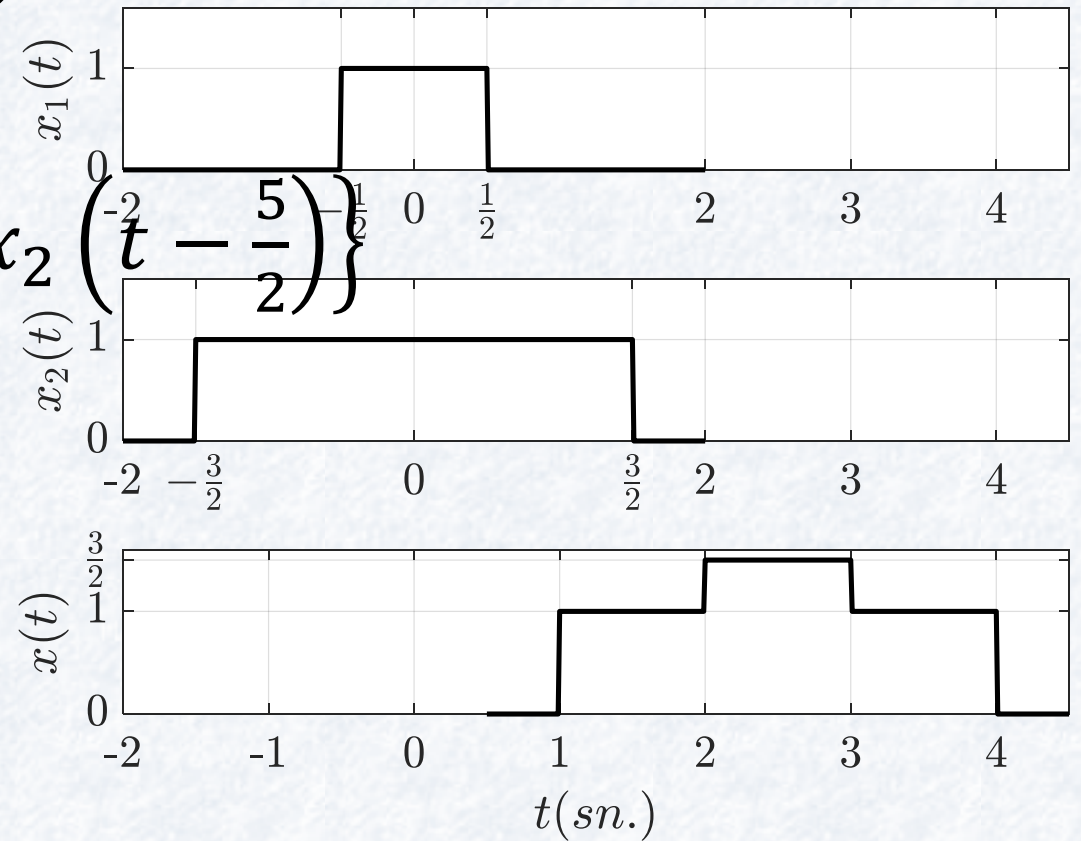
# Örnek 8

- $x(t) = \frac{1}{2}x_1\left(t - \frac{5}{2}\right) + x_2\left(t - \frac{5}{2}\right)$
- $X(\omega) = ?$
- $X(\omega) = \frac{1}{2}\mathcal{F}\left\{x_1\left(t - \frac{5}{2}\right)\right\} + \mathcal{F}\left\{x_2\left(t - \frac{5}{2}\right)\right\}$ 
  - ♦  $\mathcal{F}\left\{x_1\left(t - \frac{5}{2}\right)\right\} = e^{-j\omega\frac{5}{2}}X_1(\omega)$
  - ♦  $\mathcal{F}\left\{x_2\left(t - \frac{5}{2}\right)\right\} =$



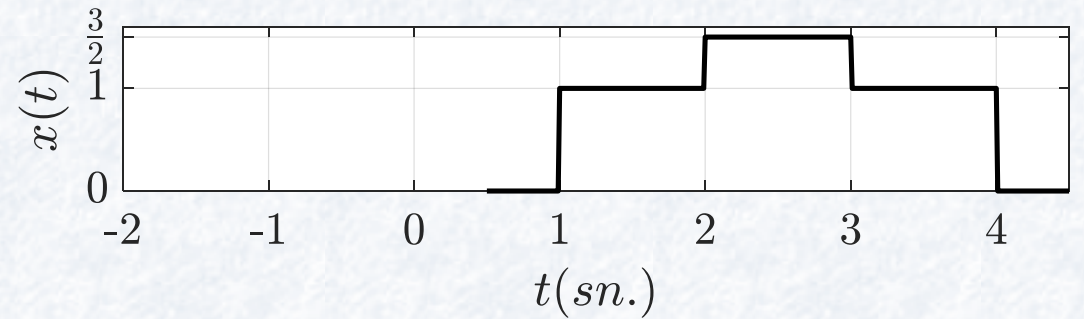
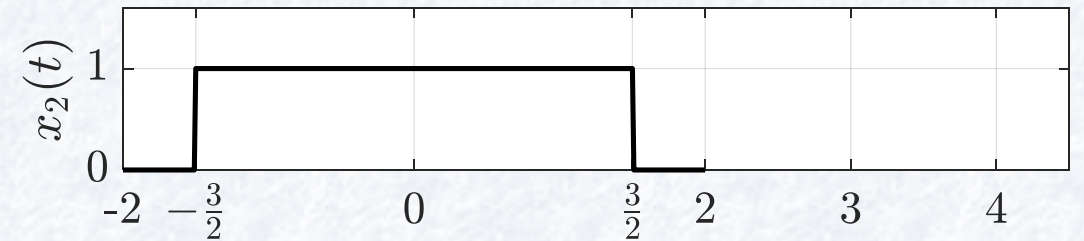
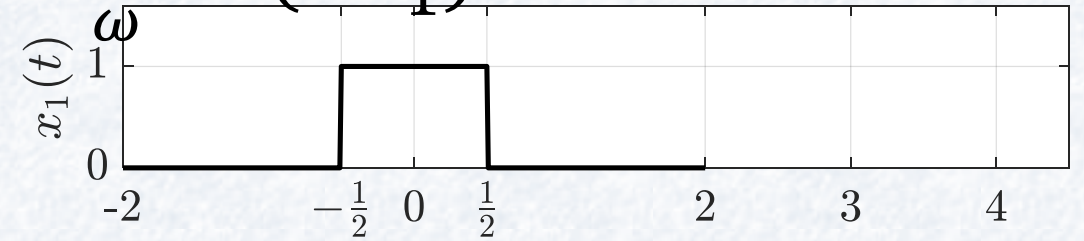
# Örnek 8

- $x(t) = \frac{1}{2}x_1\left(t - \frac{5}{2}\right) + x_2\left(t - \frac{5}{2}\right)$
- $X(\omega) = ?$
- $X(\omega) = \frac{1}{2}\mathcal{F}\left\{x_1\left(t - \frac{5}{2}\right)\right\} + \mathcal{F}\left\{x_2\left(t - \frac{5}{2}\right)\right\}$ 
  - ♦  $\mathcal{F}\left\{x_1\left(t - \frac{5}{2}\right)\right\} = e^{-j\omega\frac{5}{2}}X_1(\omega)$
  - ♦  $\mathcal{F}\left\{x_2\left(t - \frac{5}{2}\right)\right\} = e^{-j\omega\frac{5}{2}}X_2(\omega)$



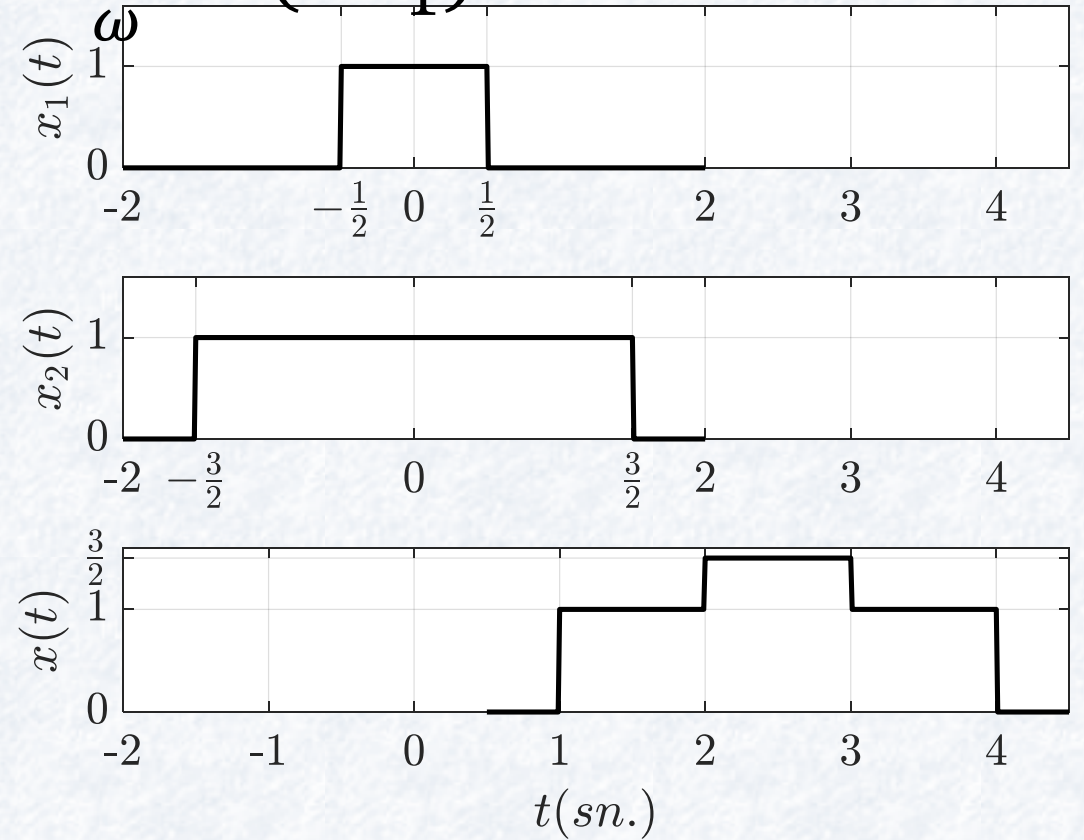
# Örnek 8

- $x(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases} \leftrightarrow X(\omega) = \frac{2}{\omega} \sin(\omega T_1)$
- $X_1(\omega) = ?$



# Örnek 8

- $x(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases} \leftrightarrow X(\omega) = \frac{2}{\omega} \sin(\omega T_1)$
- $X_1(\omega) = \frac{2}{\omega} \sin\left(\omega \frac{1}{2}\right)$
- $X_2(\omega) =$



# Örnek 8

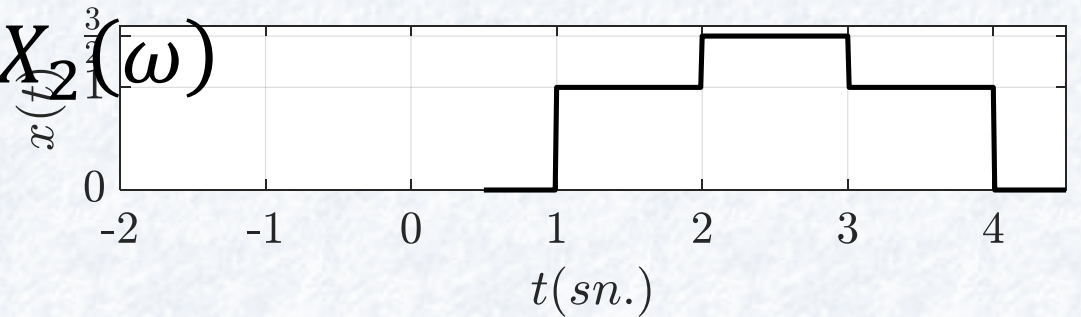
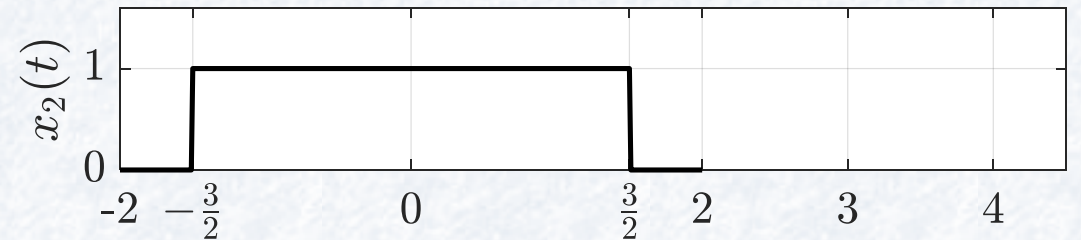
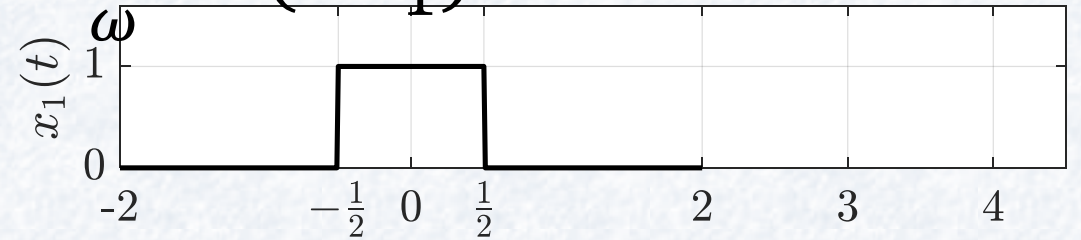
- $x(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases} \leftrightarrow X(\omega) = \frac{2}{\omega} \sin(\omega T_1)$

- $X_1(\omega) = \frac{2}{\omega} \sin\left(\omega \frac{1}{2}\right)$

- $X_2(\omega) = \frac{2}{\omega} \sin\left(\omega \frac{3}{2}\right)$

- $X(\omega) = \frac{1}{2} e^{-j\omega \frac{5}{2}} X_1(\omega) + e^{-j\omega \frac{5}{2}} X_2(\omega)$

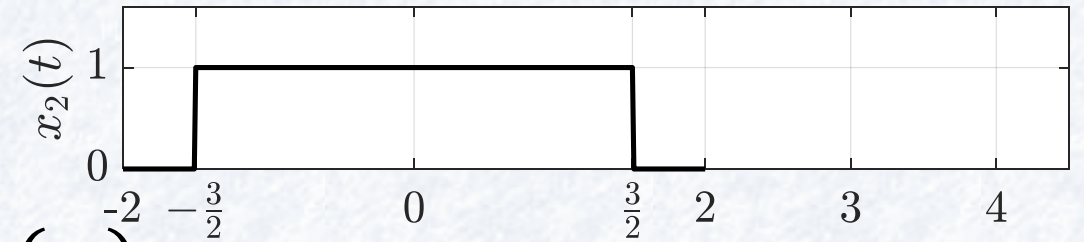
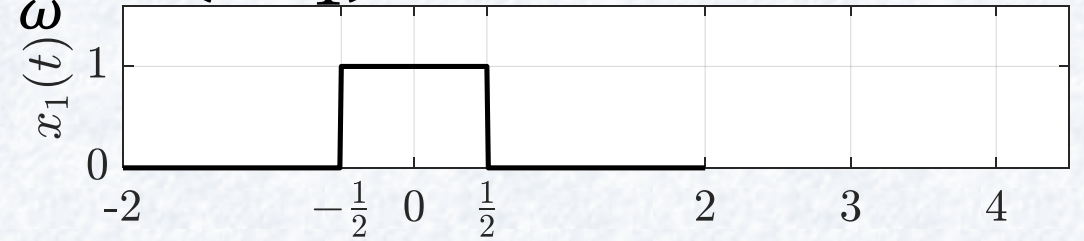
- $X(\omega) =$



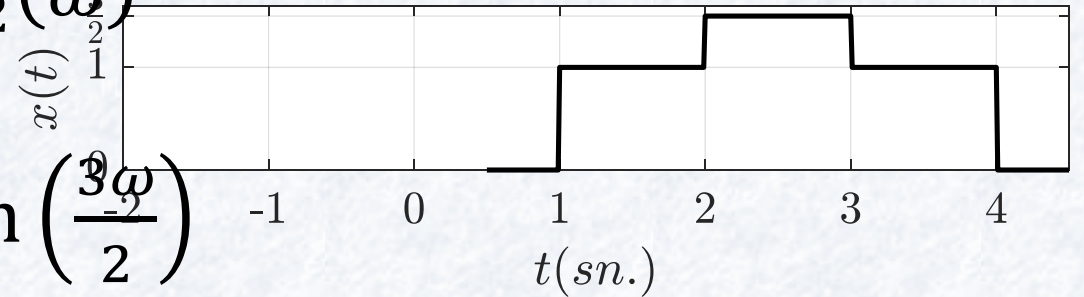


# Örnek 8

- $x(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases} \leftrightarrow X(\omega) = \frac{2}{\omega} \sin(\omega T_1)$



- $X(\omega) = \frac{1}{2} e^{-j\omega \frac{5}{2}} X_1(\omega) + e^{-j\omega \frac{5}{2}} X_2(\omega)$



- $X(\omega) = \frac{e^{-j\omega \frac{5}{2}}}{\omega} \sin\left(\frac{\omega}{2}\right) + \frac{2e^{-j\omega \frac{5}{2}}}{\omega} \sin\left(\frac{3\omega}{2}\right)$

# Fourier Dönüşüm Özellikleri

- Zamanda Ters Çevirme
- $x(t) \leftrightarrow X(\omega)$  olduğu biliniyorsa
- $\mathcal{F}\{x(-t)\} = X(-\omega)$

## Örnek 9

- $x(t) = e^{at}u(-t)$  ise  $X(\omega) = ?$

## Örnek 9

- $x(t) = e^{at}u(-t)$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-at}u(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) =$

## Örnek 9

- $x(t) = e^{at}u(-t)$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-at}u(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{1}{a+j\omega}$
- $x(t) = x_1( \quad )$

## Örnek 9

- $x(t) = e^{at}u(-t)$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-at}u(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{1}{a+j\omega}$
- $x(t) = x_1(-t)$
- $X(\omega) =$



## Örnek 9

- $x(t) = e^{at}u(-t)$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-at}u(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{1}{a+j\omega}$
- $x(t) = x_1(-t)$
- $X(\omega) = X_1(-\omega) = \frac{1}{a-j\omega}$

# Fourier Dönüşüm Özellikleri

- Zamanda Türev ve İntegral
- $x(t) \leftrightarrow X(\omega)$  olduğu biliniyorsa
- $\mathcal{F} \left\{ \frac{\partial x(t)}{\partial t} \right\} = j\omega X(\omega)$

# Fourier Dönüşüm Özellikleri

- Zamanda İntegral
- $x(t) \leftrightarrow X(\omega)$  olduğu biliniyorsa
- $\mathcal{F}\left\{\int x(t)dt\right\} = \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$

# Fourier Dönüşüm Özellikleri

- Zamanda Ölçekleme
- $x(t) \leftrightarrow X(\omega)$  olduğu biliniyorsa
- $\mathcal{F}\{x(at)\} = \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

## Örnek 10

- $x(t) = e^{-2at}u(t)$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-at}u(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{1}{a+j\omega}$
- $x(t) = x_1( \quad )$

## Örnek 10

- $x(t) = e^{-2at}u(t)$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-at}u(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{1}{a+j\omega}$
- $x(t) = x_1(2t)$
- $X(\omega) =$



## Örnek 10

- $x(t) = e^{-2at}u(t)$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-at}u(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{1}{a+j\omega}$
- $x(t) = x_1(2t)$
- $X(\omega) = \frac{1}{2}X_1\left(\frac{\omega}{2}\right) = \frac{1}{2} \frac{1}{a+j\frac{\omega}{2}} = \frac{1}{2a+j\omega}$

# Fourier Dönüşüm Özellikleri

- Frekansta Ölçekleme
- $x(t) \leftrightarrow X(\omega)$  olduğu biliniyorsa
- $\mathcal{F} \left\{ \frac{1}{|a|} x \left( \frac{t}{a} \right) \right\} = X(a\omega)$

# Fourier Dönüşüm Özellikleri

- Frekansta Türev
- $x(t) \leftrightarrow X(\omega)$  olduğu biliniyorsa
- $\mathcal{F}\{tx(t)\} = j \frac{\partial X(\omega)}{\partial \omega}$

# Örnek 11

- $x(t) = te^{-at}u(t)$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-at}u(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{1}{a+j\omega}$
- $x(t) = x_1( \quad )$

# Örnek 11

- $x(t) = te^{-at}u(t)$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-at}u(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{1}{a+j\omega}$
- $x(t) = tx_1(t)$
- $X(\omega) =$

# Örnek 11

- $x(t) = te^{-at}u(t)$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-at}u(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{1}{a+j\omega}$
- $x(t) = tx_1(t)$
- $X(\omega) = j \frac{\partial X_1(\omega)}{\partial \omega} = \frac{1}{(a+j\omega)^2}$



# Fourier Dönüşüm Özellikleri

- Çift Taraflılık
- $x(t) \leftrightarrow X(\omega)$  olduğu biliniyorsa
- $\mathcal{F}\{X(t)\} = 2\pi x(-\omega)$

# Örnek 12

- $x(t) = 1$  ise  $X(\omega) = ?$
- $x_1(t) = \delta(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = 1$
- $x(t) = X_1( \quad )$

# Örnek 12

- $x(t) = 1$  ise  $X(\omega) = ?$
- $x_1(t) = \delta(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = 1$
- $x(t) = X_1(t) = 1$
- $X(\omega) =$

# Örnek 12

- $x(t) = 1$  ise  $X(\omega) = ?$
- $x_1(t) = \delta(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = 1$
- $x(t) = X_1(t) = 1$
- $X(\omega) = 2\pi x_1(-\omega) = 2\pi\delta(-\omega) = 2\pi\delta(\omega)$

# Örnek 13

- $x(t) = \delta(t - a)$  ise  $X(\omega) = ?$
- 1. yol: zamanda öteleme
- $x_1(t) = \delta(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = 1$
- $x(t) = x_1(\quad)$

## Örnek 13

- $x(t) = \delta(t - a)$  ise  $X(\omega) = ?$
- 1. yol: zamanda öteleme
- $x_1(t) = \delta(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = 1$
- $x(t) = x_1(t - a) = \delta(t - a)$
- $X(\omega) =$



# Örnek 13

- $x(t) = \delta(t - a)$  ise  $X(\omega) = ?$
- 1. yol: zamanda öteleme
- $x_1(t) = \delta(t) \xleftrightarrow{\mathcal{F}} X_1(\omega) = 1$
- $x(t) = x_1(t - a) = \delta(t - a)$
- $X(\omega) = e^{-j\omega a} X_1(\omega) = e^{-j\omega a}$

## Örnek 13

- $x(t) = \delta(t - a)$  ise  $X(\omega) = ?$
- 2. yol: çift taraflılık
- $x_1(t) = e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} X_1(\omega) = 2\pi\delta(\omega - \omega_0)$
- $x_2(t) = e^{jat} \xleftrightarrow{\mathcal{F}} X_2(\omega) =$

# Örnek 13

- $x(t) = \delta(t - a)$  ise  $X(\omega) = ?$
- 2. yol: çift taraflılık
- $x_1(t) = e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} X_1(\omega) = 2\pi\delta(\omega - \omega_0)$
- $x_2(t) = e^{jat} \xleftrightarrow{\mathcal{F}} X_2(\omega) = 2\pi\delta(\omega - a)$
- $x(t) = X_2( \quad )$

## Örnek 13

- $x(t) = \delta(t - a)$  ise  $X(\omega) = ?$
- 2. yol: çift taraflılık
- $x_1(t) = e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} X_1(\omega) = 2\pi\delta(\omega - \omega_0)$
- $x_2(t) = e^{jat} \xleftrightarrow{\mathcal{F}} X_2(\omega) = 2\pi\delta(\omega - a)$
- $x(t) = \frac{1}{2\pi} X_2(t) = \delta(t - a)$
- $X(\omega) =$

## Örnek 13

- $x(t) = \delta(t - a)$  ise  $X(\omega) = ?$
- 2. yol: çift taraflılık
- $x_1(t) = e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} X_1(\omega) = 2\pi\delta(\omega - \omega_0)$
- $x_2(t) = e^{jat} \xleftrightarrow{\mathcal{F}} X_2(\omega) = 2\pi\delta(\omega - a)$
- $x(t) = \frac{1}{2\pi} X_2(t) = \delta(t - a)$
- $X(\omega) = \frac{1}{2\pi} 2\pi x_2(-\omega) = e^{-ja\omega}$

# Örnek 14

- $x(t) = \frac{\sin(at)}{\pi t}$  ise  $X(\omega) = ?$
- $x_1(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases} \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{2}{\omega} \sin(\omega T_1)$
- $x_2(t) = \begin{cases} 1, & |t| \leq a \\ 0, & |t| > a \end{cases} \xleftrightarrow{\mathcal{F}} X_2(\omega) =$



# Örnek 14

- $x(t) = \frac{\sin(at)}{\pi t}$  ise  $X(\omega) = ?$
- $x_1(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases} \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{2}{\omega} \sin(\omega T_1)$
- $x_2(t) = \begin{cases} 1, & |t| \leq a \\ 0, & |t| > a \end{cases} \xleftrightarrow{\mathcal{F}} X_2(\omega) = \frac{2}{\omega} \sin(a\omega)$
- $x(t) = X_2( \quad )$

# Örnek 14

- $x(t) = \frac{\sin(at)}{\pi t}$  ise  $X(\omega) = ?$
- $x_1(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases} \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{2}{\omega} \sin(\omega T_1)$
- $x_2(t) = \begin{cases} 1, & |t| \leq a \\ 0, & |t| > a \end{cases} \xleftrightarrow{\mathcal{F}} X_2(\omega) = \frac{2}{\omega} \sin(a\omega)$
- $x(t) = \frac{1}{2\pi} X_2(t) = \frac{1}{2\pi} \frac{2}{t} \sin(at)$
- $X(\omega) =$

# Örnek 14

- $x(t) = \frac{\sin(at)}{\pi t}$  ise  $X(\omega) = ?$
- $x_1(t) = \begin{cases} 1, & |t| \leq T_1 \\ 0, & |t| > T_1 \end{cases} \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{2}{\omega} \sin(\omega T_1)$
- $x_2(t) = \begin{cases} 1, & |t| \leq a \\ 0, & |t| > a \end{cases} \xleftrightarrow{\mathcal{F}} X_2(\omega) = \frac{2}{\omega} \sin(a\omega)$
- $x(t) = \frac{1}{2\pi} X_2(t) = \frac{1}{2\pi} \frac{2}{t} \sin(at)$
- $X(\omega) = \frac{1}{2\pi} 2\pi x_2(-\omega) = x_2(-\omega) = \begin{cases} 1, & |-\omega| \leq a \\ 0, & |-\omega| > a \end{cases} = \begin{cases} 1, & |\omega| \leq a \\ 0, & |\omega| > a \end{cases}$

## Örnek 15

- $x(t) = \frac{1}{a^2+t^2}$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-a|t|} \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{2a}{a^2+\omega^2}$
- $x(t) = X_1( \quad )$

# Örnek 15

- $x(t) = \frac{1}{a^2+t^2}$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-a|t|} \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{2a}{a^2+\omega^2}$
- $x(t) = \frac{1}{2a} X_1(t) = \frac{1}{2a} \frac{2}{t} \frac{2a}{a^2+t^2} = \frac{1}{a^2+t^2}$
- $X(\omega) =$

# Örnek 15

- $x(t) = \frac{1}{a^2+t^2}$  ise  $X(\omega) = ?$
- $x_1(t) = e^{-a|t|} \xleftrightarrow{\mathcal{F}} X_1(\omega) = \frac{2a}{a^2+\omega^2}$
- $x(t) = \frac{1}{2a} X_1(t) = \frac{1}{2a} \frac{2}{t} \frac{2a}{a^2+t^2} = \frac{1}{a^2+t^2}$
- $X(\omega) = \frac{1}{2a} 2\pi x_1(-\omega) = \frac{\pi}{a} x_1(-\omega) = \frac{\pi}{a} e^{-a|-\omega|} = \frac{\pi}{a} e^{-a|\omega|}$



# Fourier Dönüşüm Özellikleri

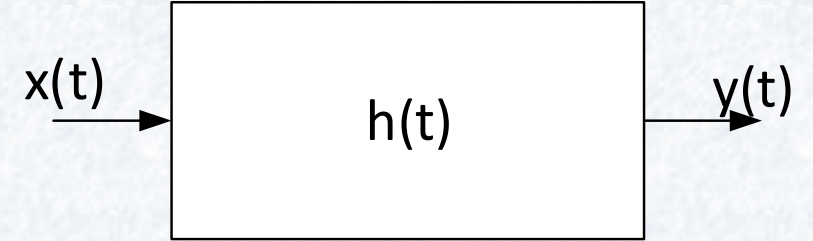
- **Konvolüsyon**
- $x(t) \leftrightarrow X(\omega)$  ve
- $h(t) \leftrightarrow H(\omega)$  olduğu biliniyorsa
- $y(t) =$

# Fourier Dönüşüm Özellikleri

- **Konvolüsyon**
- $x(t) \leftrightarrow X(\omega)$  ve
- $h(t) \leftrightarrow H(\omega)$  olduğu biliniyorsa
- $y(t) = x(t) * h(t)$
- $\mathcal{F}\{y(t)\} = \mathcal{F}\{x(t) * h(t)\}$
- $Y(\omega) = X(\omega)H(\omega)$
- $H(\omega) = \frac{Y(\omega)}{X(\omega)}$  Sistemin frekans domeninde transfer fonksiyonu

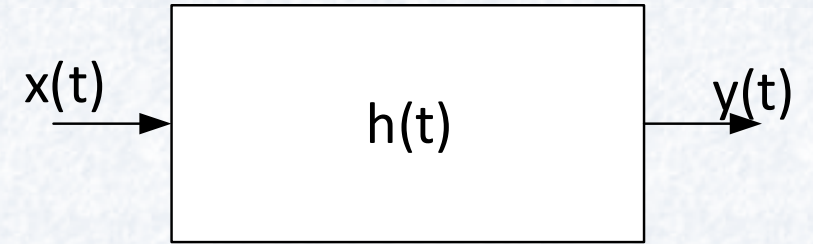
# Örnek 16

- $h(t) = \delta(t - t_0)$  ise  $y(t) = ?$



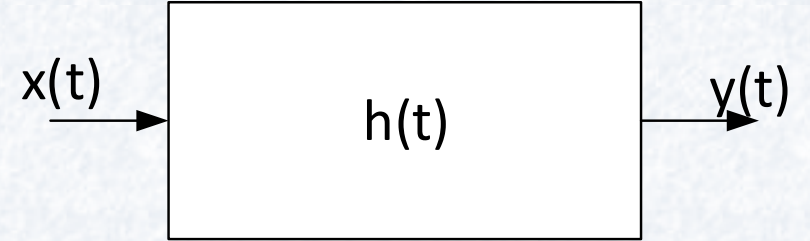
# Örnek 16

- $h(t) = \delta(t - t_0)$  ise  $y(t) = ?$
- $H(\omega) =$



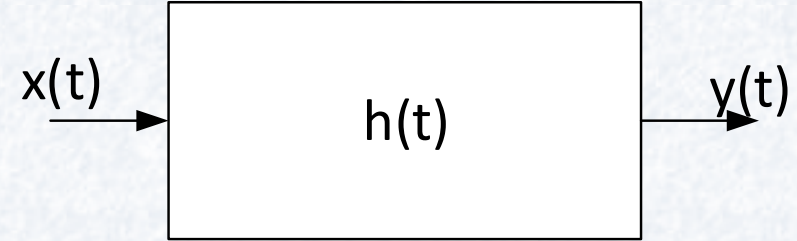
# Örnek 16

- $h(t) = \delta(t - t_0)$  ise  $y(t) = ?$
- $H(\omega) = e^{-j\omega t_0}$
- $Y(\omega) =$



# Örnek 16

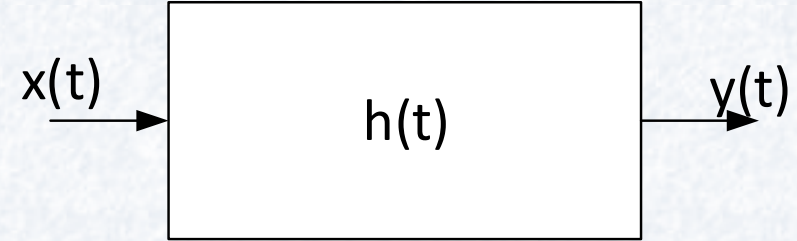
- $h(t) = \delta(t - t_0)$  ise  $y(t) = ?$
- $H(\omega) = e^{-j\omega t_0}$
- $Y(\omega) = H(\omega)X(\omega)$





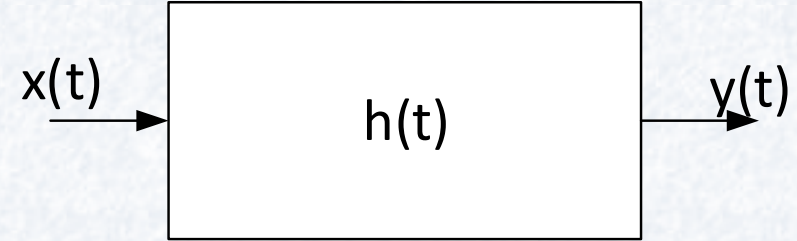
# Örnek 16

- $h(t) = \delta(t - t_0)$  ise  $y(t) = ?$
- $H(\omega) = e^{-j\omega t_0}$
- $Y(\omega) = H(\omega)X(\omega)$
- $Y(\omega) = e^{-j\omega t_0}X(\omega)$
- $y(t) =$



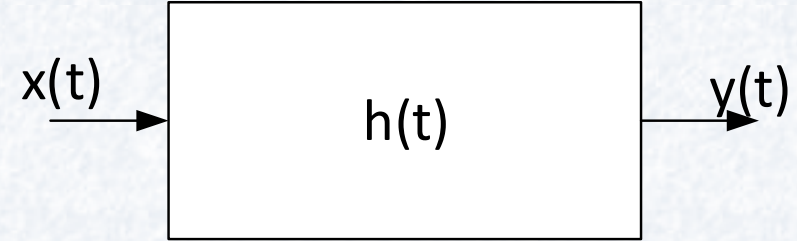
# Örnek 16

- $h(t) = \delta(t - t_0)$  ise  $y(t) = ?$
- $H(\omega) = e^{-j\omega t_0}$
- $Y(\omega) = H(\omega)X(\omega)$
- $Y(\omega) = e^{-j\omega t_0}X(\omega)$ 
  - ♦ *Zamanda öteleme*
- $y(t) =$



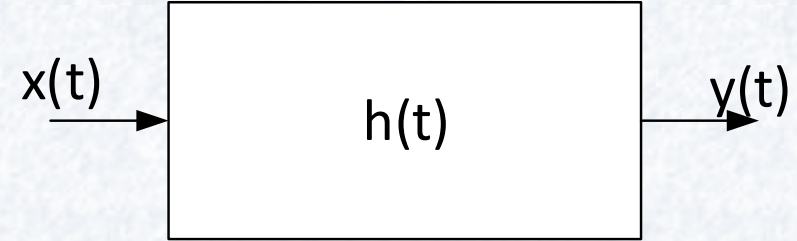
# Örnek 16

- $h(t) = \delta(t - t_0)$  ise  $y(t) = ?$
- $H(\omega) = e^{-j\omega t_0}$
- $Y(\omega) = H(\omega)X(\omega)$
- $Y(\omega) = e^{-j\omega t_0}X(\omega)$ 
  - ♦ *Zamanda öteleme*
- $y(t) = x(t - t_0)$



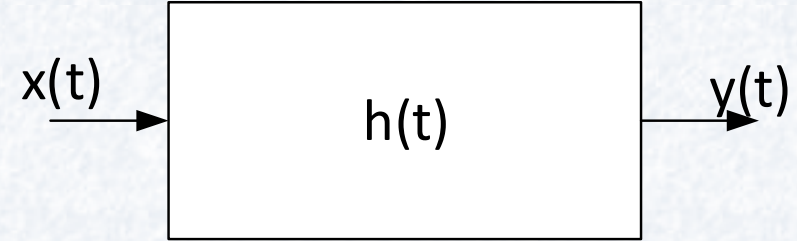
# Örnek 17

- $h(t) = e^{-at}u(t)$  ve
- $x(t) = e^{-bt}u(t)$  ise  $y(t) = ?$
- $H(\omega) =$
- $X(\omega) =$



# Örnek 17

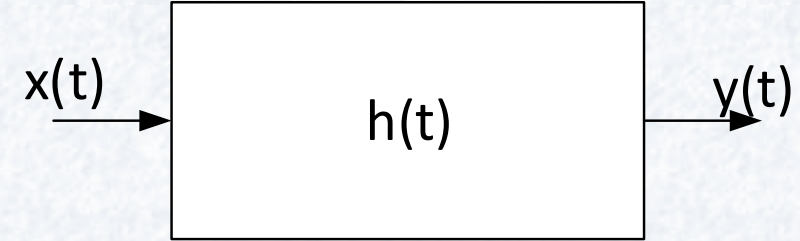
- $h(t) = e^{-at}u(t)$  ve
- $x(t) = e^{-bt}u(t)$  ise  $y(t) = ?$
- $H(\omega) = \frac{1}{a+j\omega}$
- $X(\omega) = \frac{1}{b+j\omega}$
- $Y(\omega) = H(\omega)X(\omega) = \frac{1}{a+j\omega} \frac{1}{b+j\omega}$



$$\bullet \quad Y(\omega) = \begin{cases} \frac{1}{b-a} \left( \frac{1}{a+j\omega} - \frac{1}{b+j\omega} \right), & a \neq b \\ \frac{1}{(a+j\omega)^2}, & a = b \end{cases} \Leftrightarrow y(t) = \begin{cases} \boxed{\phantom{0}}, & a \neq b \\ \boxed{\phantom{0}}, & a = b \end{cases}$$

# Örnek 17

- $h(t) = e^{-at}u(t)$  ve
- $x(t) = e^{-bt}u(t)$  ise  $y(t) = ?$



- $H(\omega) = \frac{1}{a+j\omega}$

- $X(\omega) = \frac{1}{b+j\omega}$

- $Y(\omega) = H(\omega)X(\omega) = \frac{1}{a+j\omega} \frac{1}{b+j\omega}$

- $Y(\omega) = \begin{cases} \frac{1}{b-a} \left( \frac{1}{a+j\omega} - \frac{1}{b+j\omega} \right), & a \neq b \\ \frac{1}{(a+j\omega)^2}, & a = b \end{cases} \Leftrightarrow y(t) = \begin{cases} \frac{1}{b-a} \left( e^{-at}u(t) - e^{-bt}u(t) \right), & a \neq b \\ te^{-at}u(t), & a = b \end{cases}$



# Fourier Dönüşüm Özellikleri

- **Çarpma**
- $p(t) \leftrightarrow P(\omega)$  ve
- $r(t) \leftrightarrow R(\omega)$  olduğu biliniyorsa
- $s(t) = p(t)r(t)$
- $\mathcal{F}\{s(t)\} = \mathcal{F}\{p(t)r(t)\}$
- $$S(\omega) = \frac{1}{2\pi} \left( P(\omega) * R(\omega) \right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(\theta) S(\omega - \theta) d\theta$$

## Örnek 18

- $x(t) = \cos\left(\frac{\pi}{4}t\right) \sin\left(\frac{2\pi}{3}t\right)$  ise  $X(\omega) = ?$

## Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$
- $X(\omega) =$

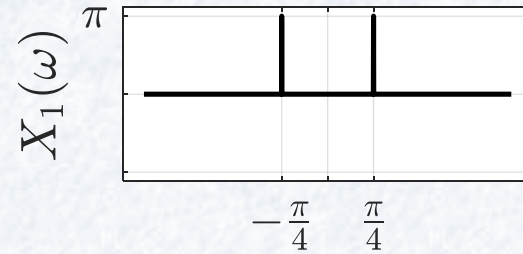
## Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$
- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$
- $X_1(\omega) =$

# Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$



- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$

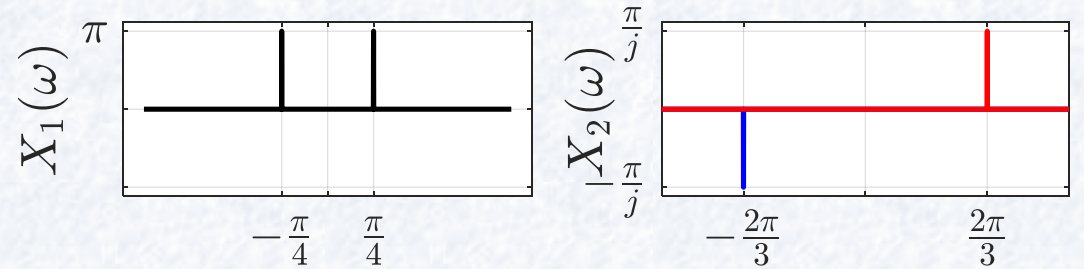
- $X_2(\omega) =$

- $X(\omega) =$

# Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$



- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X_2(\omega) = \frac{\pi}{j} \left( \delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right) \right)$

- $X(\omega) =$



# Örnek 18

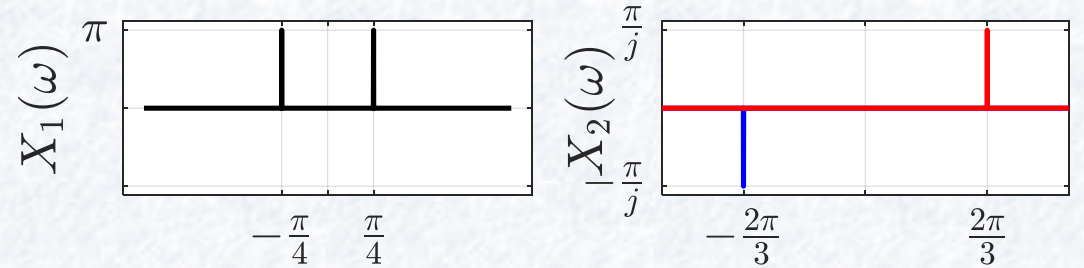
- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$

- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X_2(\omega) = \frac{\pi}{j} \left( \delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right) \right)$

- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \left( \delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right) \right) \right)$



# Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$

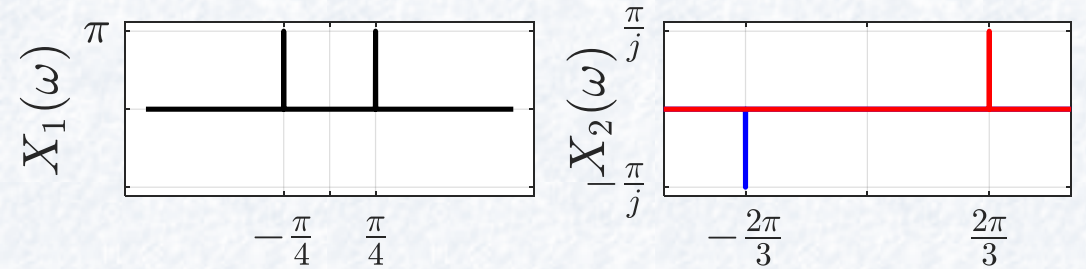
- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X_2(\omega) = \frac{\pi}{j} \left( \delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right) \right)$

- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \left( \delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right) \right) \right)$

- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \delta\left(\omega - \frac{2\pi}{3}\right) - X_1(\omega) * \frac{\pi}{j} \delta\left(\omega + \frac{2\pi}{3}\right) \right)$

- $X(\omega) =$



# Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$

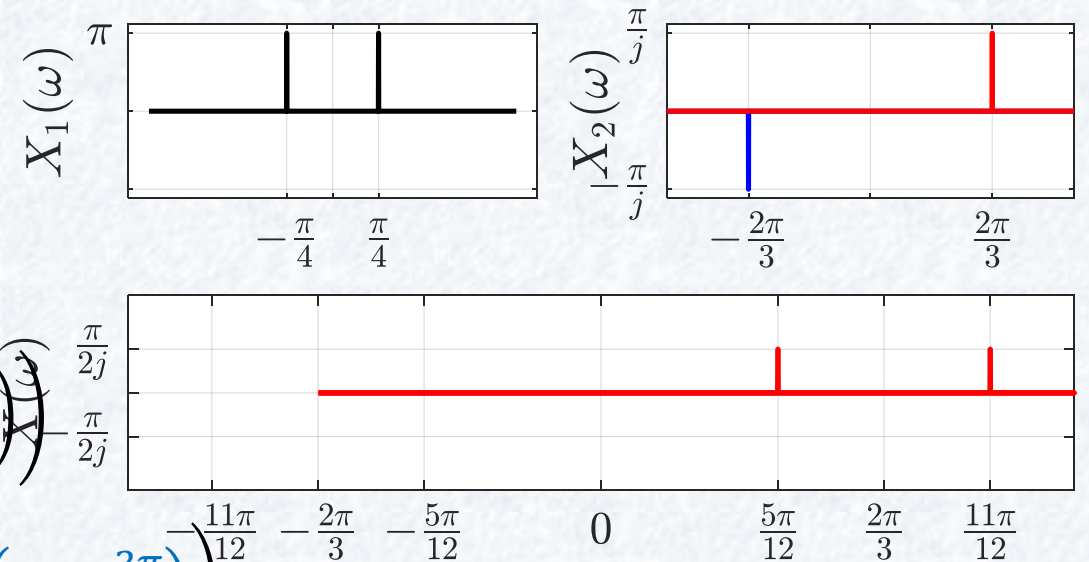
- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X_2(\omega) = \frac{\pi}{j} \left( \delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right) \right)$

- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \left( \delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right) \right) \right)$

- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \delta\left(\omega - \frac{2\pi}{3}\right) - X_1(\omega) * \frac{\pi}{j} \delta\left(\omega + \frac{2\pi}{3}\right) \right)$

- $X(\omega) = \frac{1}{2j} X_1\left(\omega - \frac{2\pi}{3}\right) - \frac{1}{2j} X_1\left(\omega + \frac{2\pi}{3}\right)$



# Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$

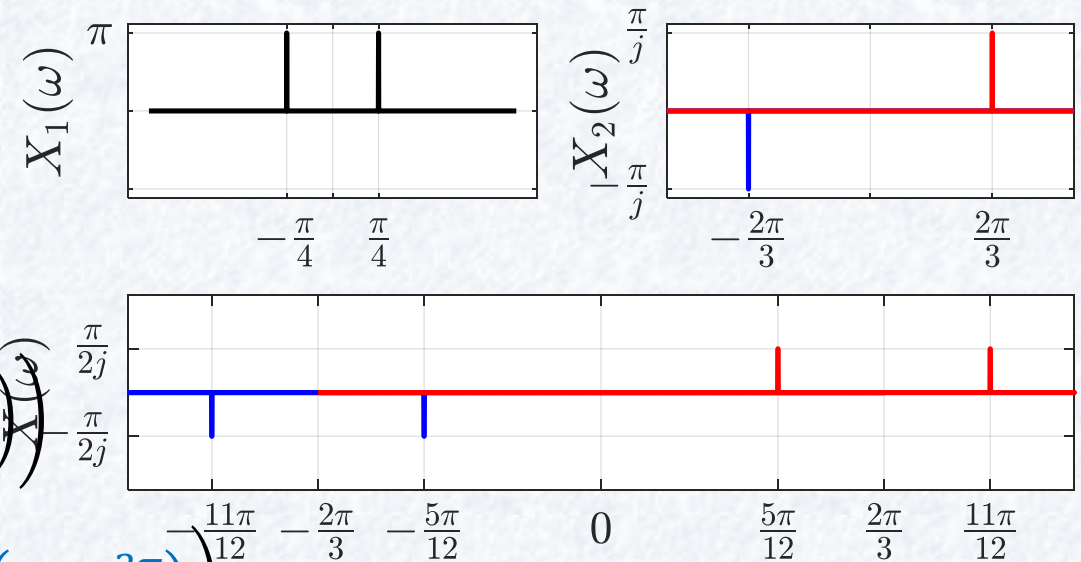
- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X_2(\omega) = \frac{\pi}{j} \left( \delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right) \right)$

- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \left( \delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right) \right) \right)$

- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \delta\left(\omega - \frac{2\pi}{3}\right) - X_1(\omega) * \frac{\pi}{j} \delta\left(\omega + \frac{2\pi}{3}\right) \right)$

- $X(\omega) = \frac{1}{2j} X_1\left(\omega - \frac{2\pi}{3}\right) - \frac{1}{2j} X_1\left(\omega + \frac{2\pi}{3}\right)$



# Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$

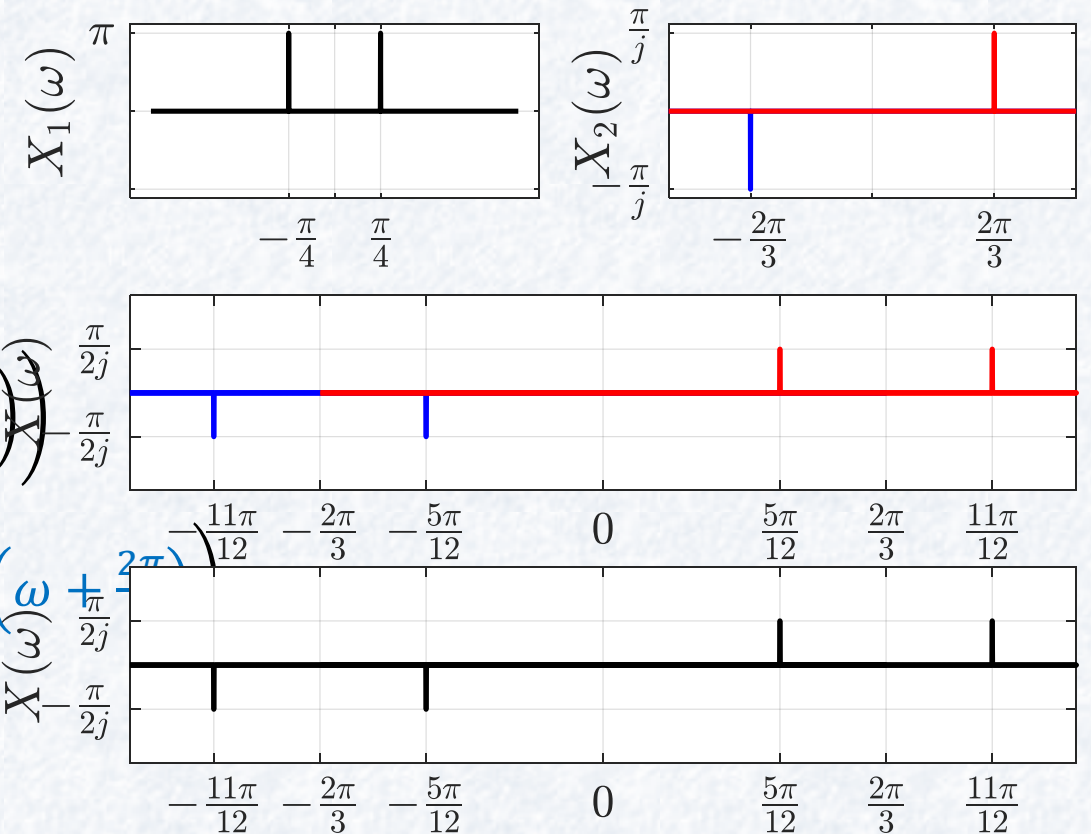
- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X_2(\omega) = \frac{\pi}{j} \left( \delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right) \right)$

- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \left( \delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right) \right) \right)$

- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \delta\left(\omega - \frac{2\pi}{3}\right) - X_1(\omega) * \frac{\pi}{j} \delta\left(\omega + \frac{2\pi}{3}\right) \right)$

- $X(\omega) = \frac{1}{2j} X_1\left(\omega - \frac{2\pi}{3}\right) - \frac{1}{2j} X_1\left(\omega + \frac{2\pi}{3}\right)$



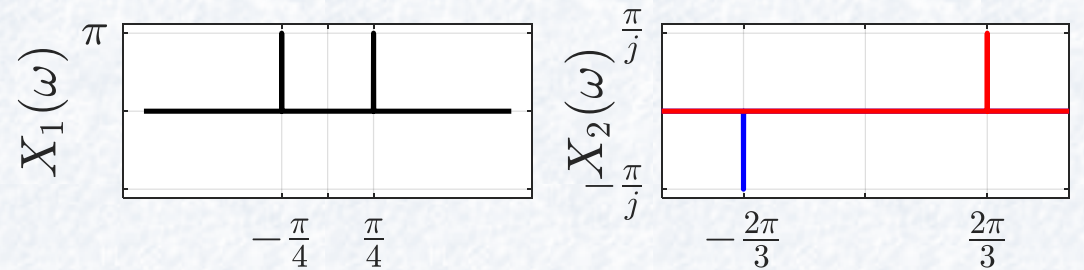


# Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

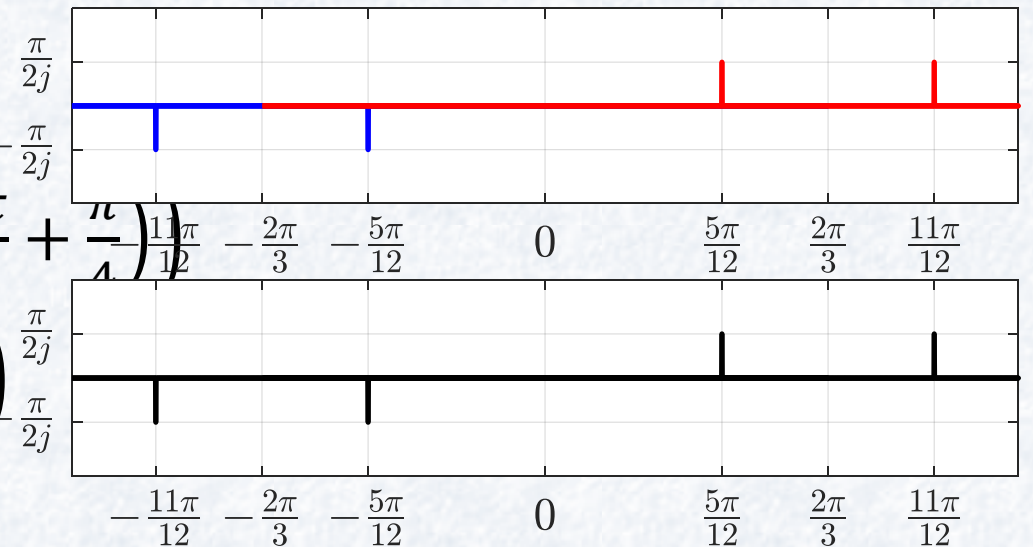
- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$

- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$



- $X(\omega) = \frac{1}{2j} X_1\left(\omega - \frac{2\pi}{3}\right) - \frac{1}{2j} X_1\left(\omega + \frac{2\pi}{3}\right)$

- $X(\omega) = \frac{1}{2j} \pi \left( \delta\left(\omega - \frac{2\pi}{3} - \frac{\pi}{4}\right) + \delta\left(\omega - \frac{2\pi}{3} + \frac{\pi}{4}\right) \right) - \frac{1}{2j} \pi \left( \delta\left(\omega + \frac{2\pi}{3} - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{2\pi}{3} + \frac{\pi}{4}\right) \right)$



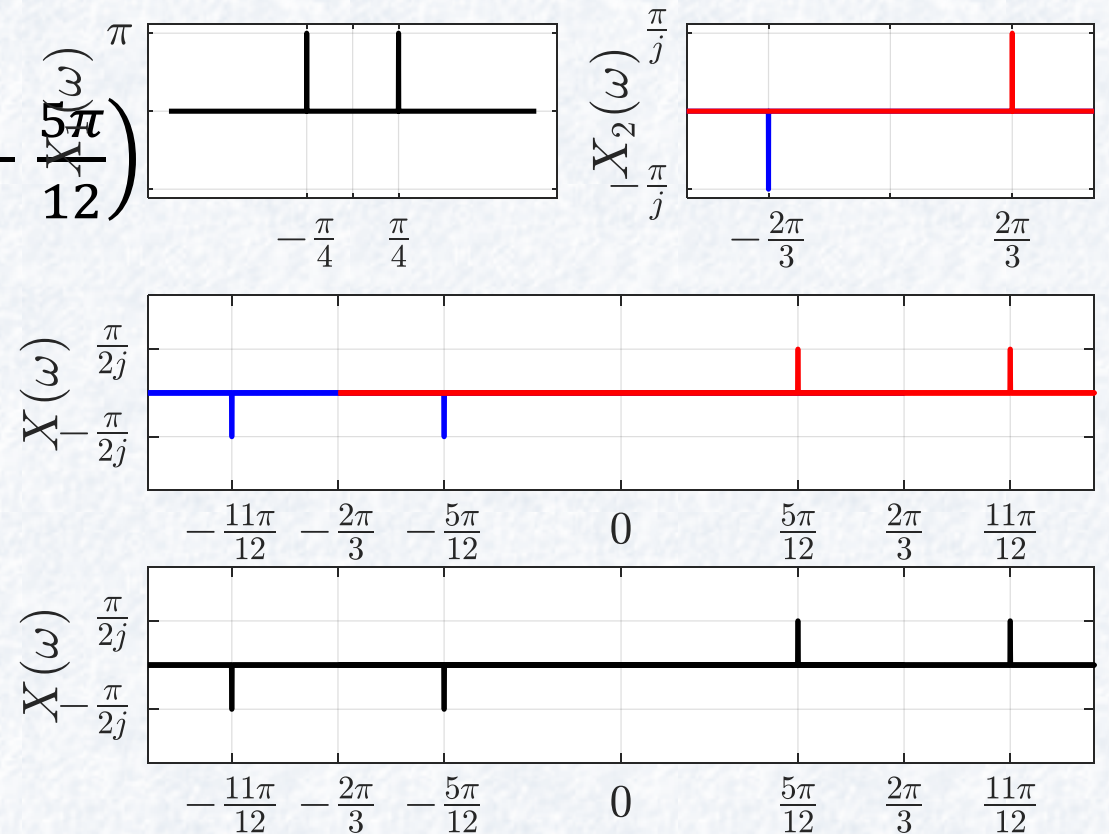


# Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{\pi}{2j} \delta\left(\omega - \frac{11\pi}{12}\right) + \frac{\pi}{2j} \delta\left(\omega - \frac{5\pi}{12}\right) - \frac{\pi}{2j} \delta\left(\omega + \frac{5\pi}{12}\right) - \frac{\pi}{2j} \delta\left(\omega + \frac{11\pi}{12}\right)$

- $x(t) =$

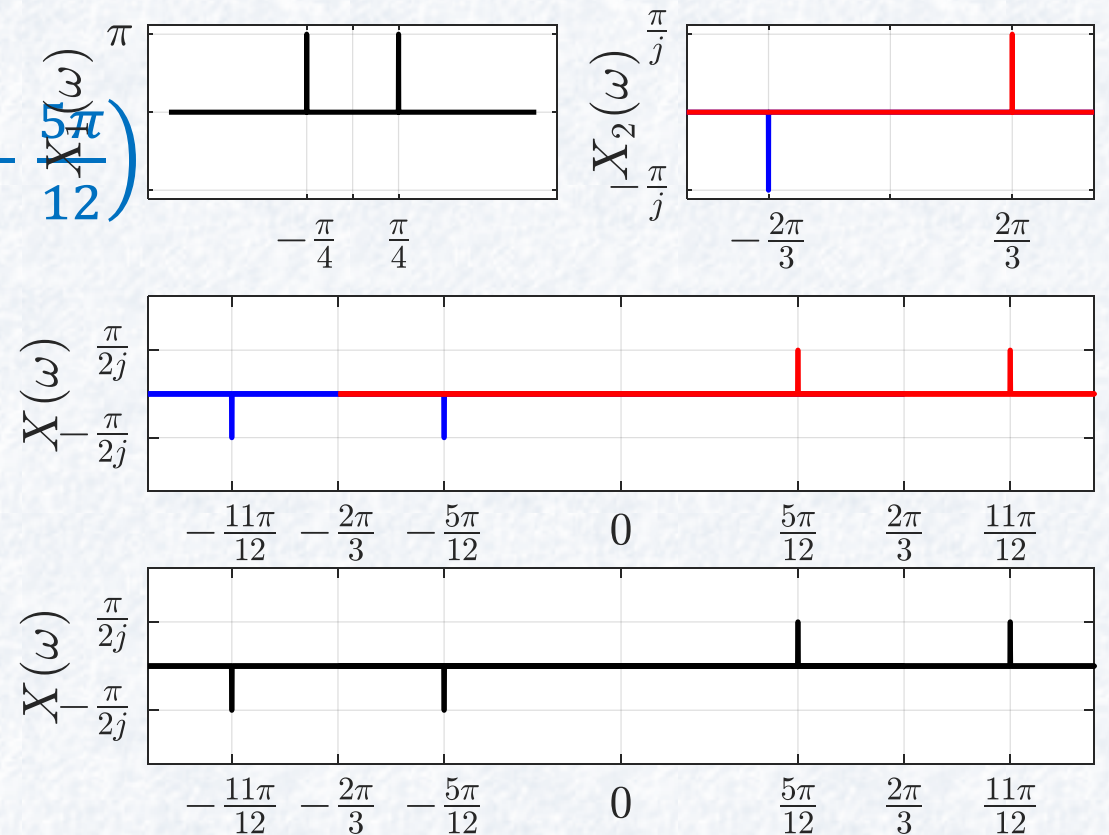


# Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{\pi}{2j} \delta\left(\omega - \frac{11\pi}{12}\right) + \frac{\pi}{2j} \delta\left(\omega - \frac{5\pi}{12}\right) - \frac{\pi}{2j} \delta\left(\omega + \frac{5\pi}{12}\right) - \frac{\pi}{2j} \delta\left(\omega + \frac{11\pi}{12}\right)$

- $x(t) = \frac{1}{2} \sin\left(\frac{11\pi}{12}t\right) +$

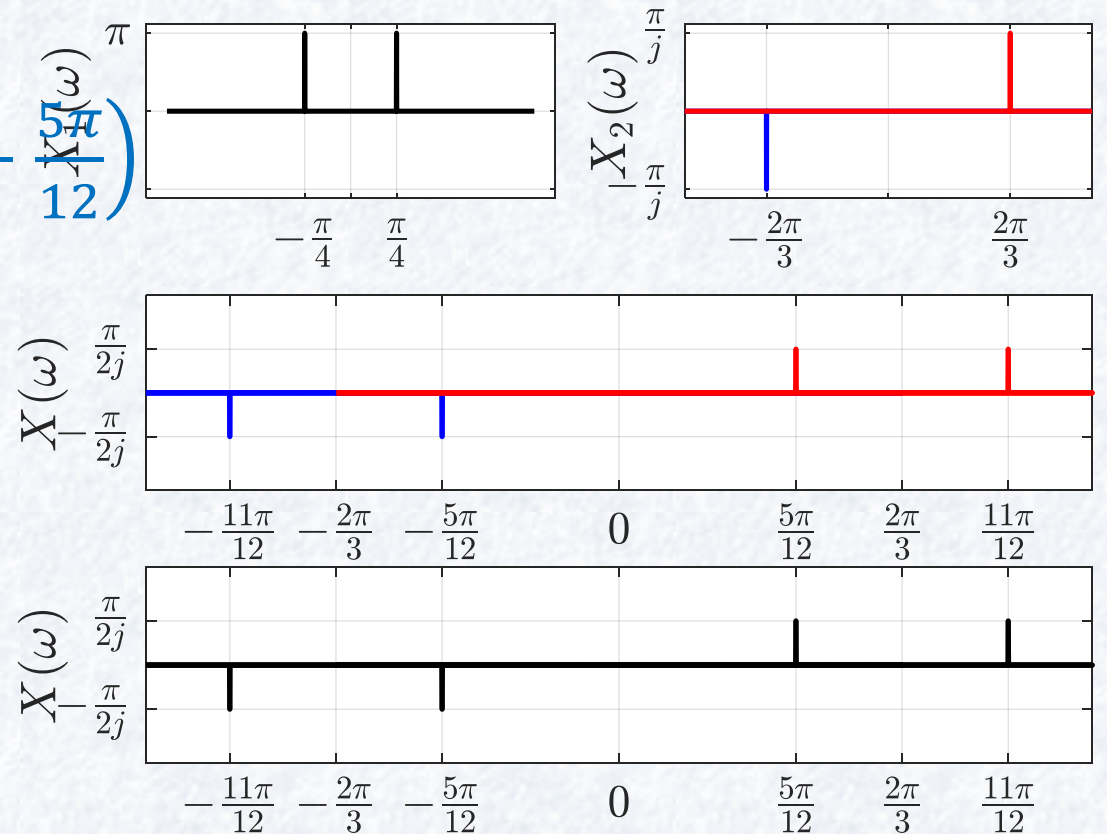


# Örnek 18

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{2\pi}{3}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{\pi}{2j} \delta\left(\omega - \frac{11\pi}{12}\right) + \frac{\pi}{2j} \delta\left(\omega - \frac{5\pi}{12}\right) - \frac{\pi}{2j} \delta\left(\omega + \frac{5\pi}{12}\right) - \frac{\pi}{2j} \delta\left(\omega + \frac{11\pi}{12}\right)$

- $x(t) = \frac{1}{2} \sin\left(\frac{11\pi}{12}t\right) + \frac{1}{2} \sin\left(\frac{5\pi}{12}t\right)$



## Örnek 19

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{\pi}{4}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

## Örnek 19

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{\pi}{4}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$
- $X(\omega) =$

## Örnek 19

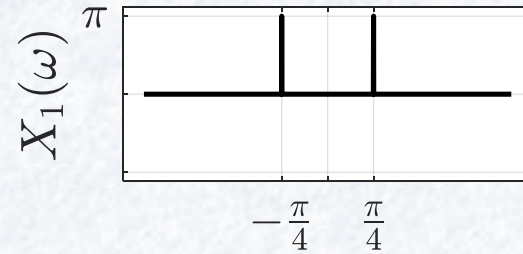
- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{\pi}{4}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$
- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$
- $X_1(\omega) =$



# Örnek 19

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{\pi}{4}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$



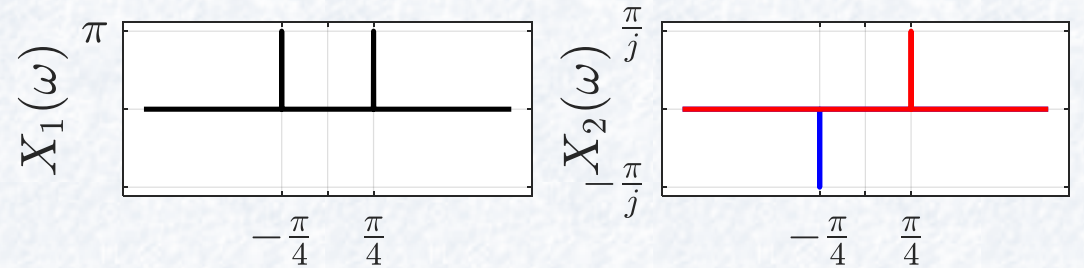
- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X_2(\omega) =$

# Örnek 19

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{\pi}{4}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$



- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X_2(\omega) = \frac{\pi}{j} \left( \delta\left(\omega - \frac{\pi}{4}\right) - \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X(\omega) =$

# Örnek 19

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{\pi}{4}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$

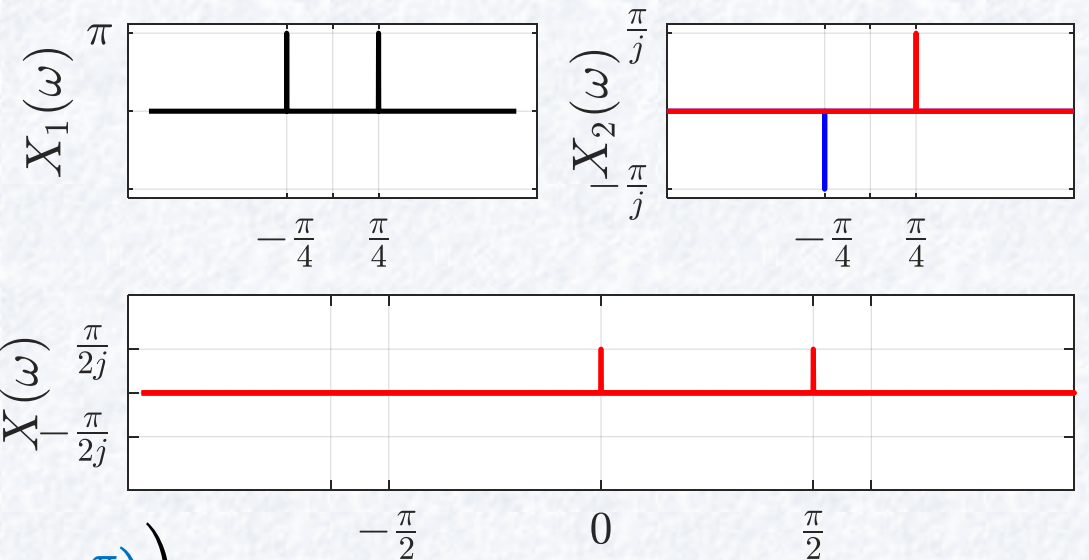
- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X_2(\omega) = \frac{\pi}{j} \left( \delta\left(\omega - \frac{\pi}{4}\right) - \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \left( \delta\left(\omega - \frac{\pi}{4}\right) - \delta\left(\omega + \frac{\pi}{4}\right) \right) \right)$

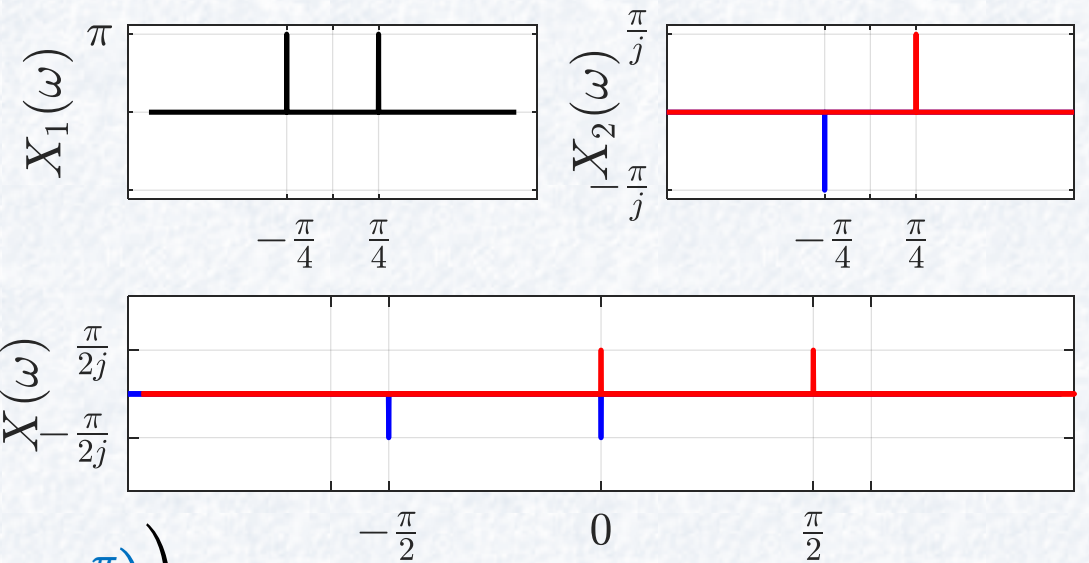
- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \delta\left(\omega - \frac{\pi}{4}\right) - X_1(\omega) * \frac{\pi}{j} \delta\left(\omega + \frac{\pi}{4}\right) \right)$

- $X(\omega) = \frac{1}{2j} X_1\left(\omega - \frac{\pi}{4}\right)$



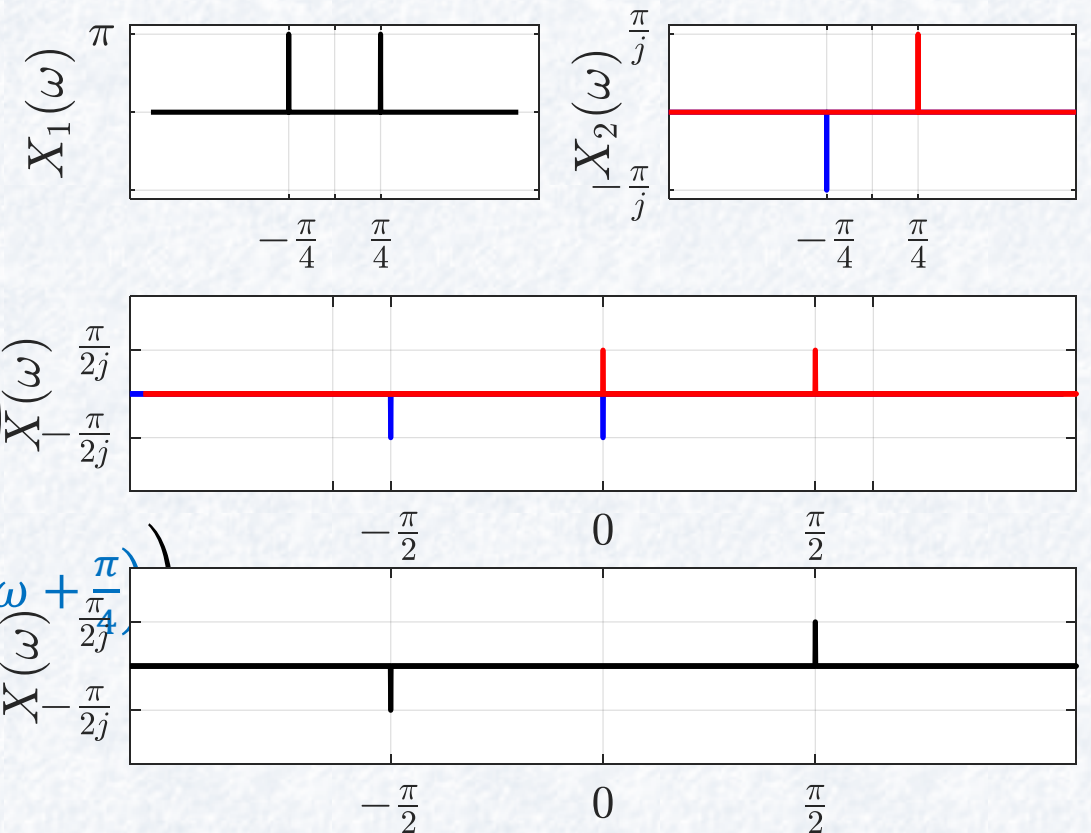
# Örnek 19

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{\pi}{4}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$
- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$
- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$
- $X_2(\omega) = \frac{\pi}{j} \left( \delta\left(\omega - \frac{\pi}{4}\right) - \delta\left(\omega + \frac{\pi}{4}\right) \right)$
- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \left( \delta\left(\omega - \frac{\pi}{4}\right) - \delta\left(\omega + \frac{\pi}{4}\right) \right) \right)$
- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \delta\left(\omega - \frac{\pi}{4}\right) - X_1(\omega) * \frac{\pi}{j} \delta\left(\omega + \frac{\pi}{4}\right) \right)$
- $X(\omega) = \frac{1}{2j} X_1\left(\omega - \frac{\pi}{4}\right) - \frac{1}{2j} X_1\left(\omega + \frac{\pi}{4}\right)$



# Örnek 19

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{\pi}{4}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$
- $X(\omega) = \frac{1}{2\pi} (X_1(\omega) * X_2(\omega))$
- $X_1(\omega) = \pi \left( \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) \right)$
- $X_2(\omega) = \frac{\pi}{j} \left( \delta\left(\omega - \frac{\pi}{4}\right) - \delta\left(\omega + \frac{\pi}{4}\right) \right)$
- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \left( \delta\left(\omega - \frac{\pi}{4}\right) - \delta\left(\omega + \frac{\pi}{4}\right) \right) \right)$
- $X(\omega) = \frac{1}{2\pi} \left( X_1(\omega) * \frac{\pi}{j} \delta\left(\omega - \frac{\pi}{4}\right) - X_1(\omega) * \frac{\pi}{j} \delta\left(\omega + \frac{\pi}{4}\right) \right)$
- $X(\omega) = \frac{1}{2j} X_1\left(\omega - \frac{\pi}{4}\right) - \frac{1}{2j} X_1\left(\omega + \frac{\pi}{4}\right)$



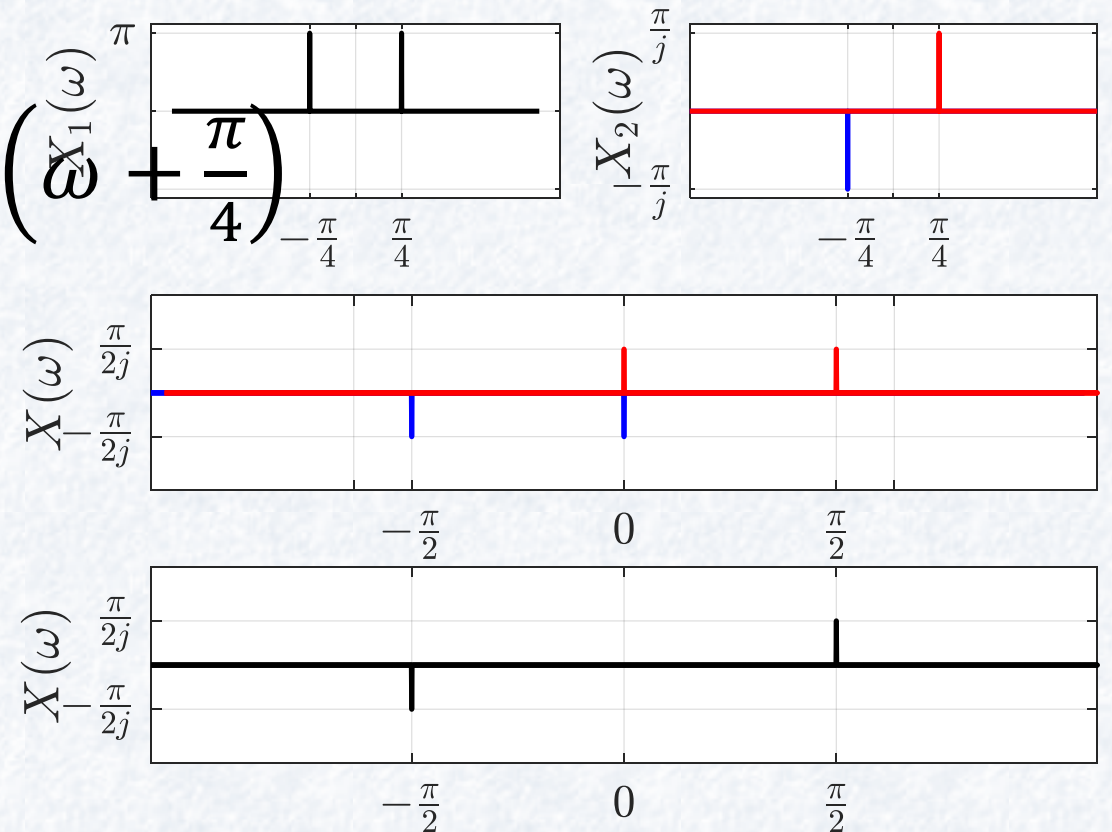


# Örnek 19

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{\pi}{4}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2j} X_1\left(\omega - \frac{\pi}{4}\right) - \frac{1}{2j} X_1\left(\omega + \frac{\pi}{4}\right)$

- $x(t) =$





# Örnek 19

- $x(t) = \underbrace{\cos\left(\frac{\pi}{4}t\right)}_{x_1(t)} \underbrace{\sin\left(\frac{\pi}{4}t\right)}_{x_2(t)}$  ise  $X(\omega) = ?$

- $X(\omega) = \frac{1}{2j} X_1\left(\omega - \frac{\pi}{4}\right) - \frac{1}{2j} X_1\left(\omega + \frac{\pi}{4}\right)$

- $x(t) = \frac{1}{2} \sin\left(\frac{\pi}{2}t\right)$

