



# BSM307

## İşaretler ve Sistemler

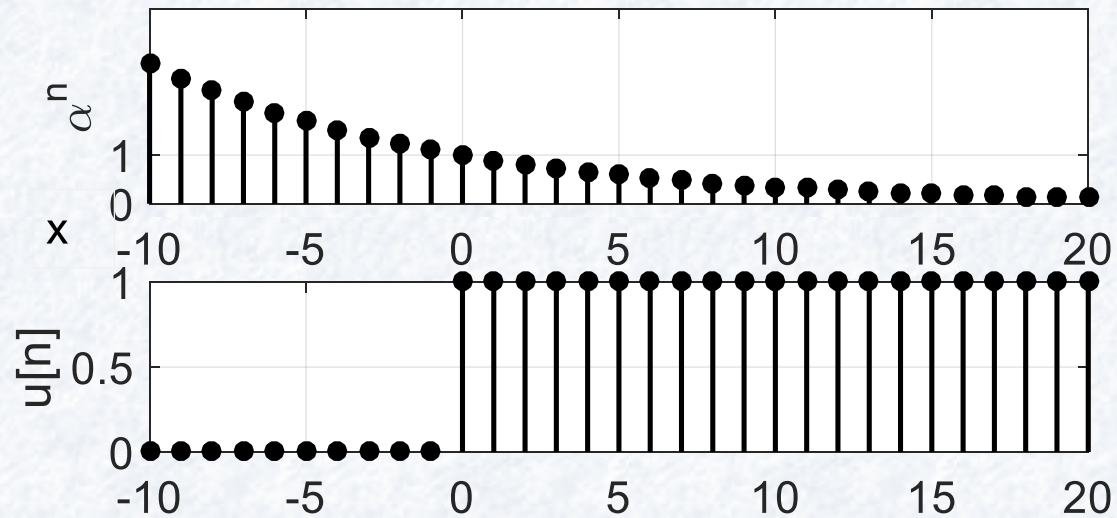
Dr. Seçkin Arı

Konvolüsyon

- Temel Sistem Özellikleri
- Doğrusal Zamanla Değişmez Sistemler
- Birim Darbe Cevabı

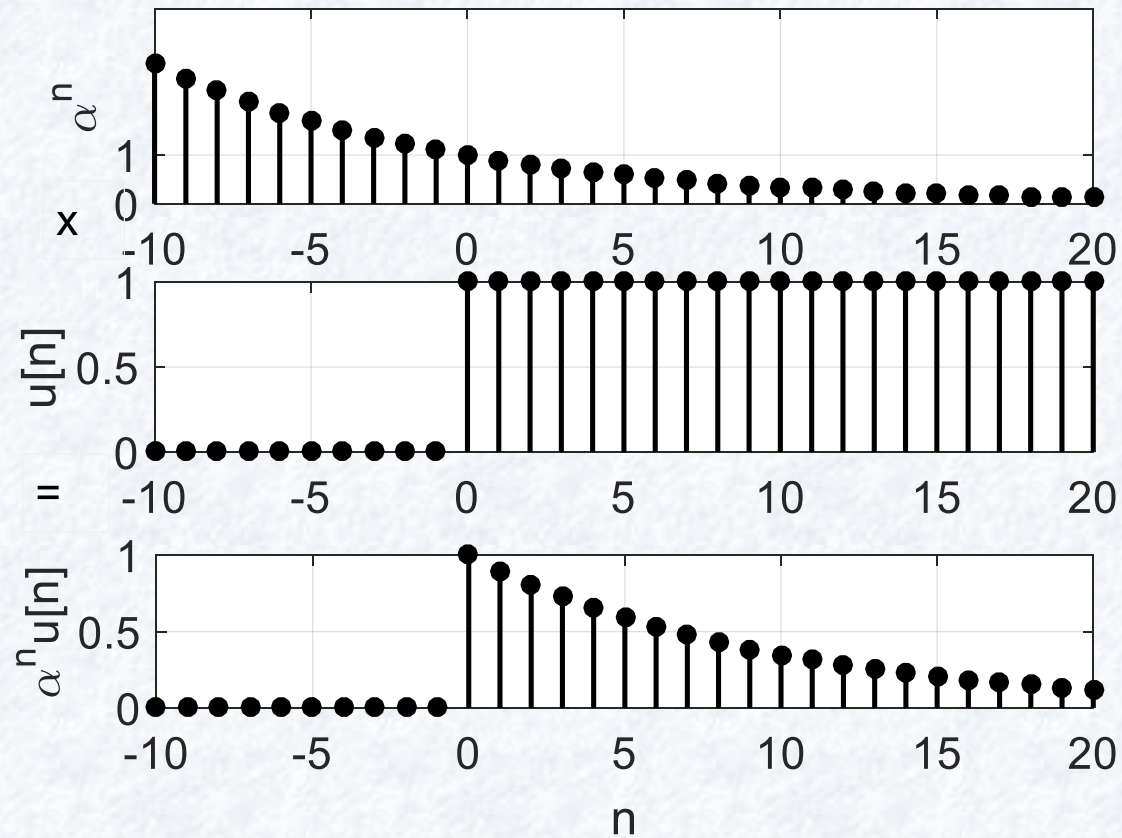
# Örnek 1

- $x[n] = \alpha^n u[n]$



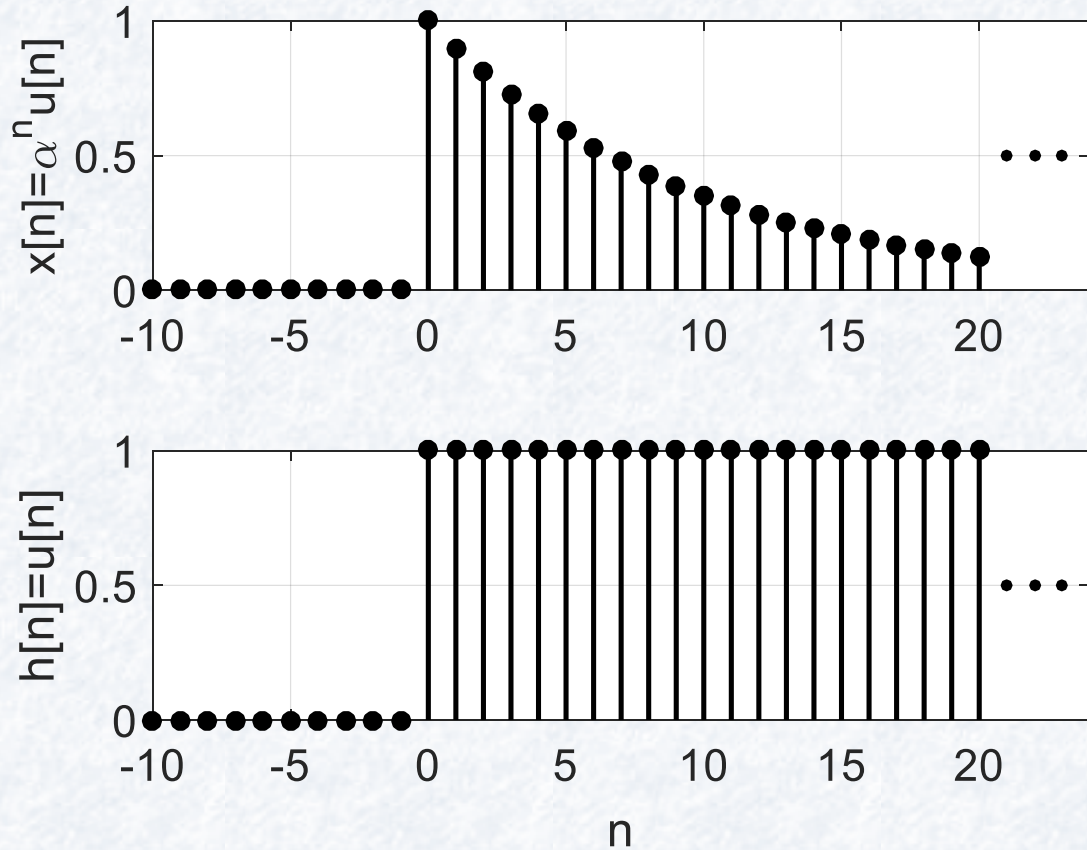
# Örnek 1

- $x[n] = \alpha^n u[n]$



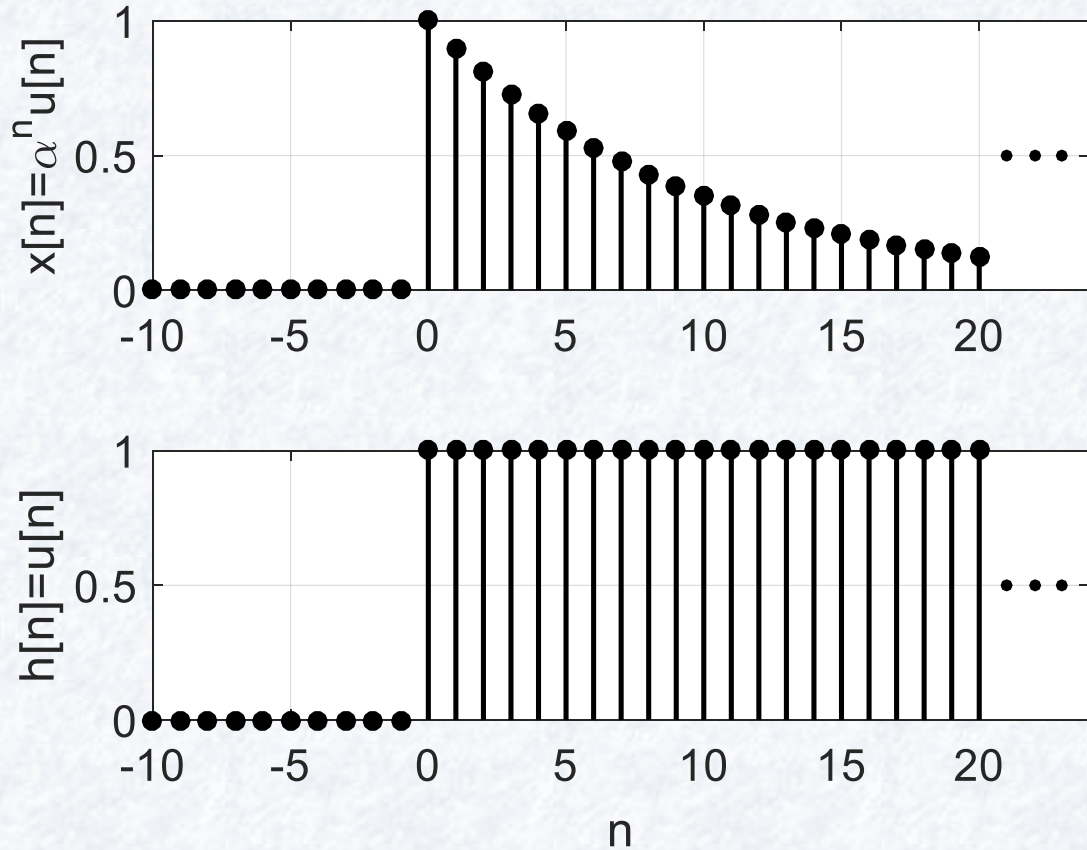
# Örnek 1

- $x[n] = \alpha^n u[n]$
- $h[n] = u[n]$
- $y[n] = ?$



# Örnek 1

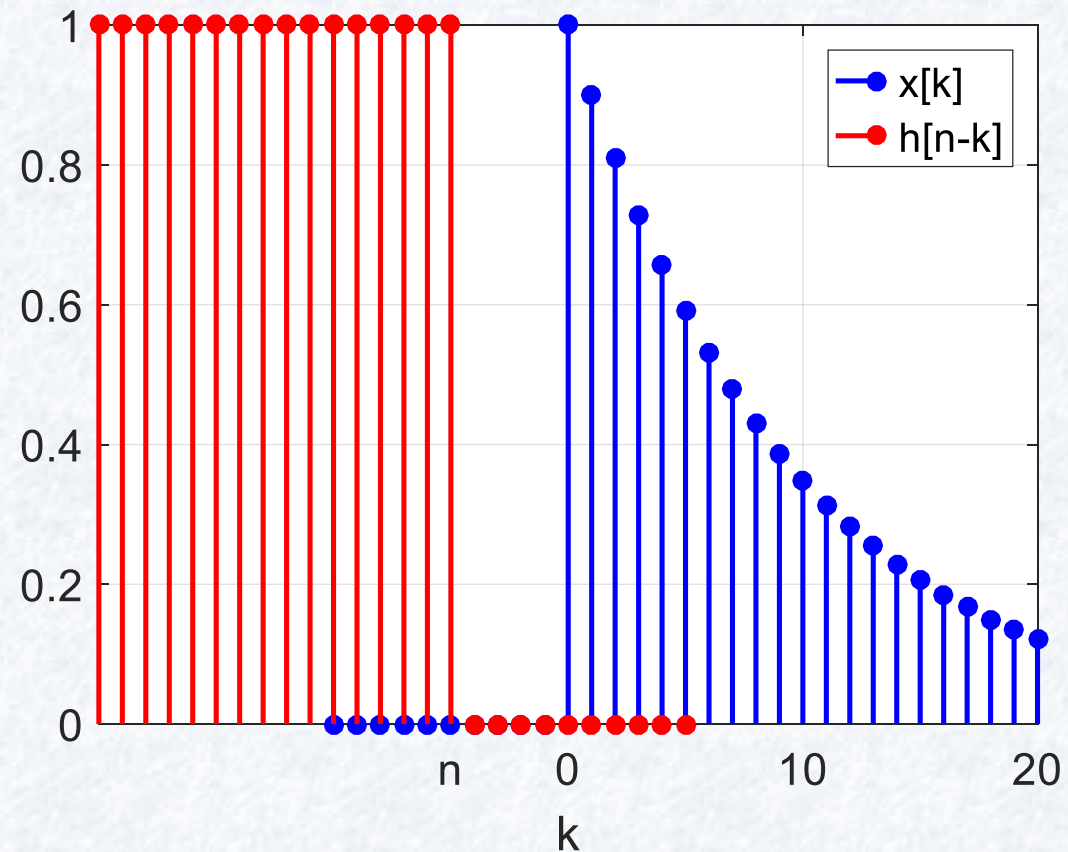
- $x[n] = \alpha^n u[n]$
- $h[n] = u[n]$
- $y[n] = x[n] * h[n]$





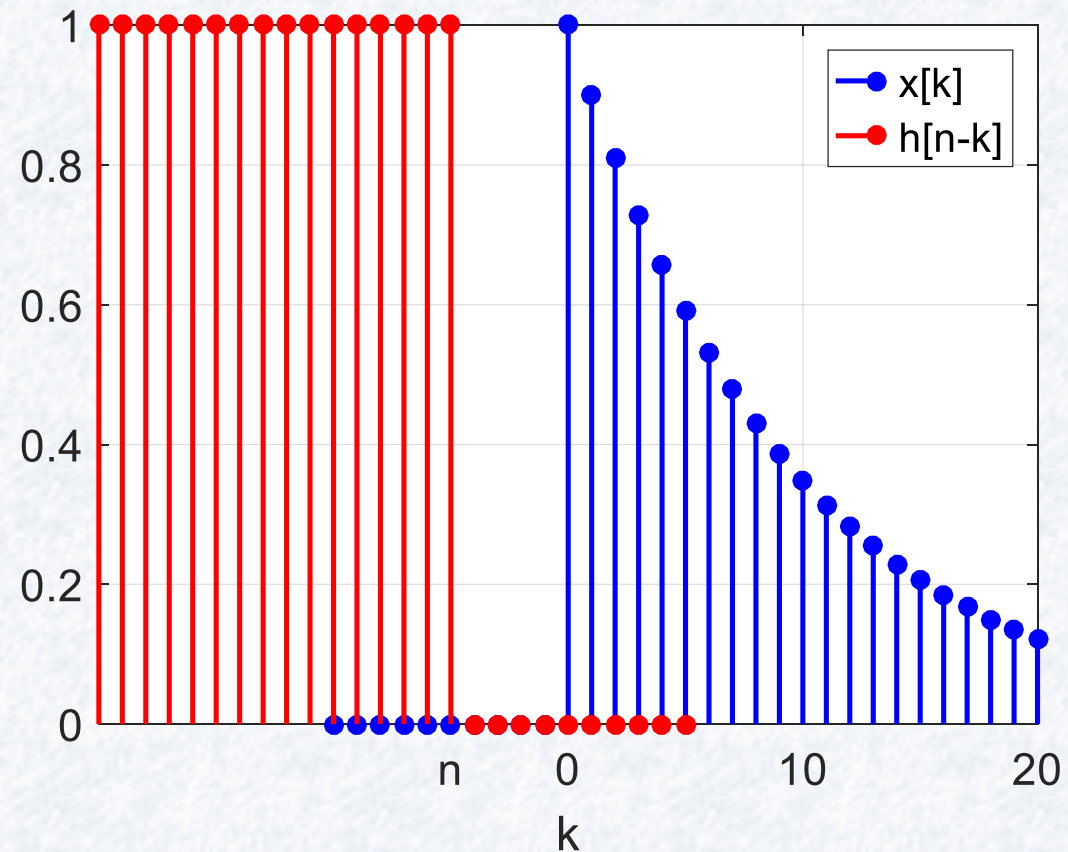
# Örnek 1

- $x[n] = \alpha^n u[n]$
- $h[n] = u[n]$
- $y[n] = x[n] * h[n]$
- $n < 0$  iken



# Örnek 1

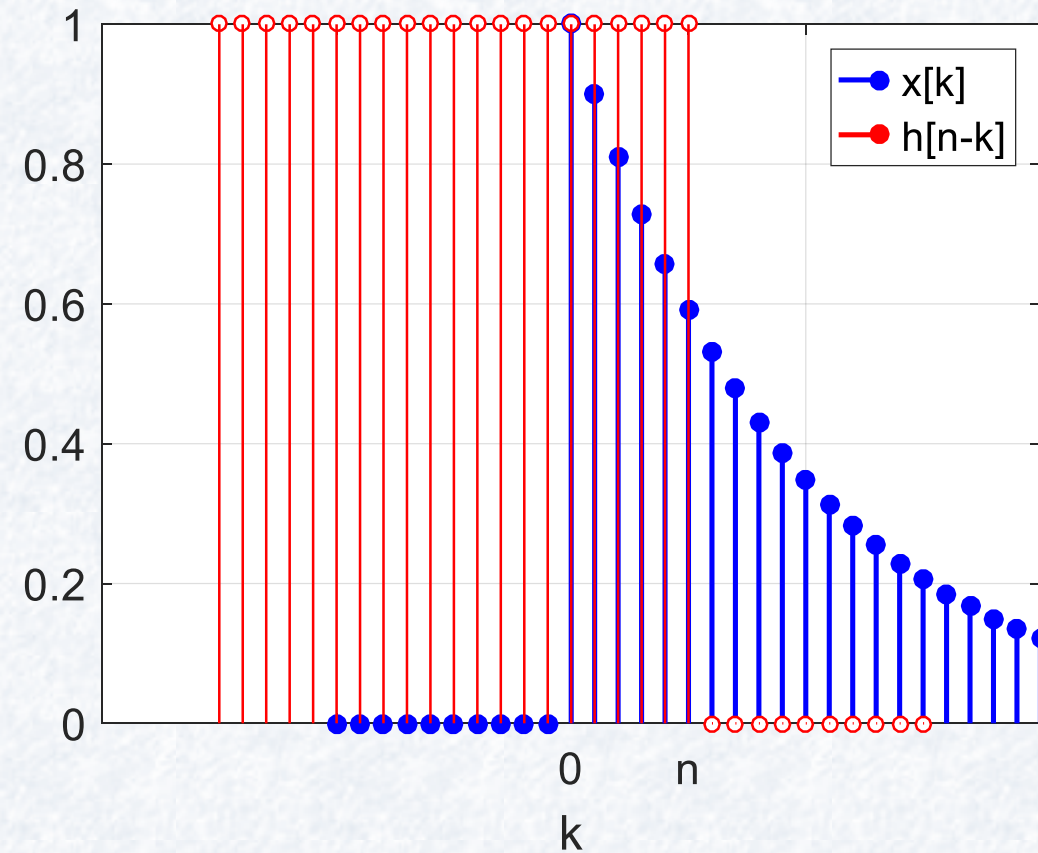
- $x[n] = \alpha^n u[n]$
- $h[n] = u[n]$
- $y[n] = x[n] * h[n]$
- $n < 0$  iken
  - ♦ Çakışma yok
- $y[n] = 0$





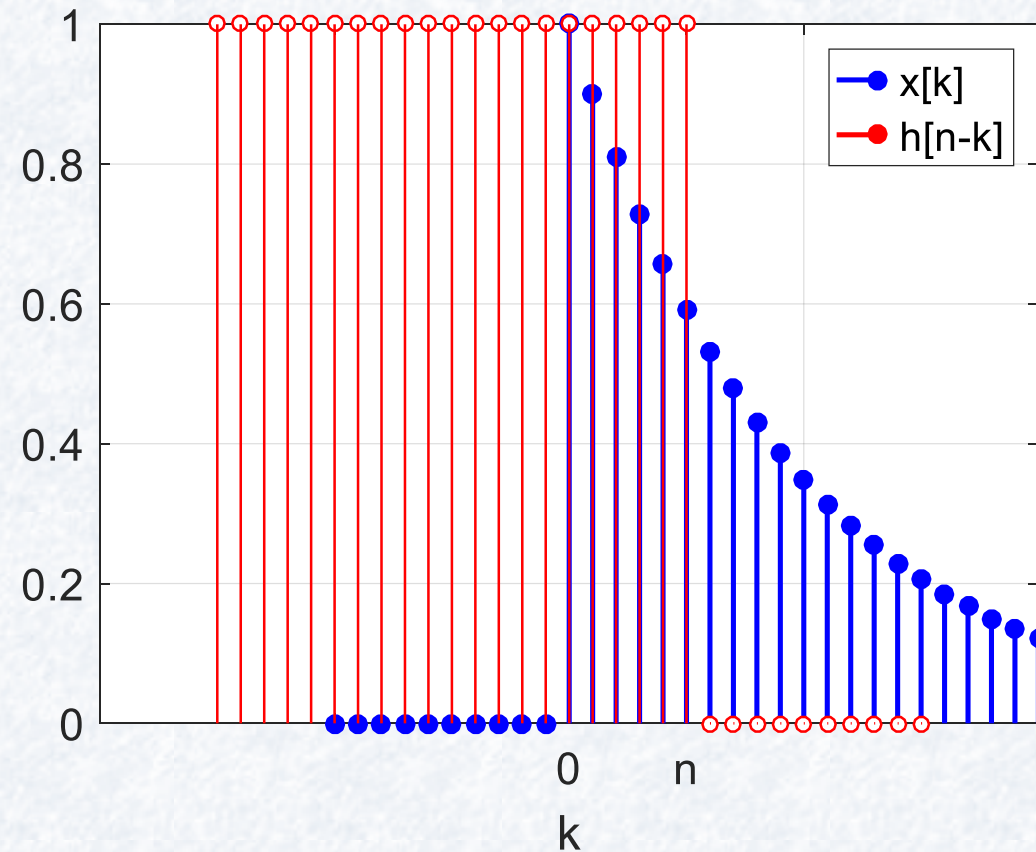
# Örnek 1

- $x[n] = \alpha^n u[n]$
- $h[n] = u[n]$
- $y[n] = x[n] * h[n]$
- $n \geq 0$  iken



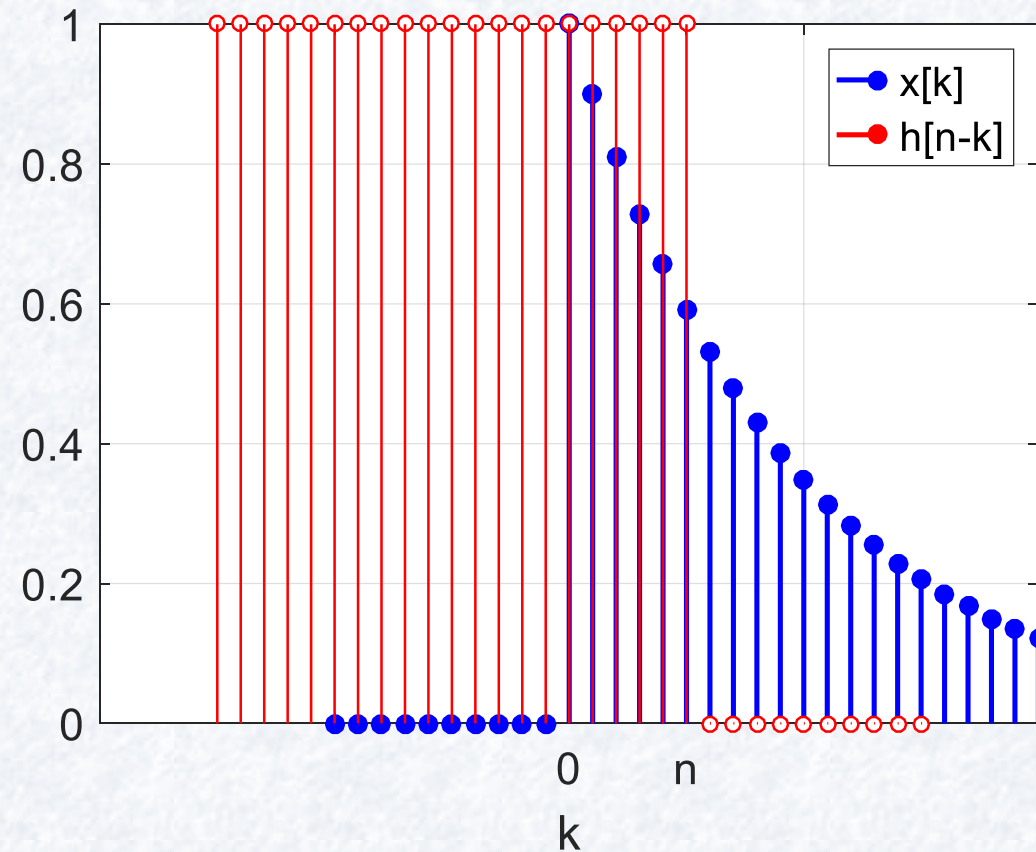
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- $x[n] = \alpha^n u[n]$
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- $n \geq 0$  iken
  - ♦ Çakışma 0-n arası
- $y[n] = \sum_{k=0}^n \alpha^k \cdot 1$
- $y[n] =$



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- $x[n] = \alpha^n u[n]$
- $h[n] = u[n]$
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- $y[n] = \frac{1 - \alpha^{n+1}}{1 - \alpha}$



# Örnek 1

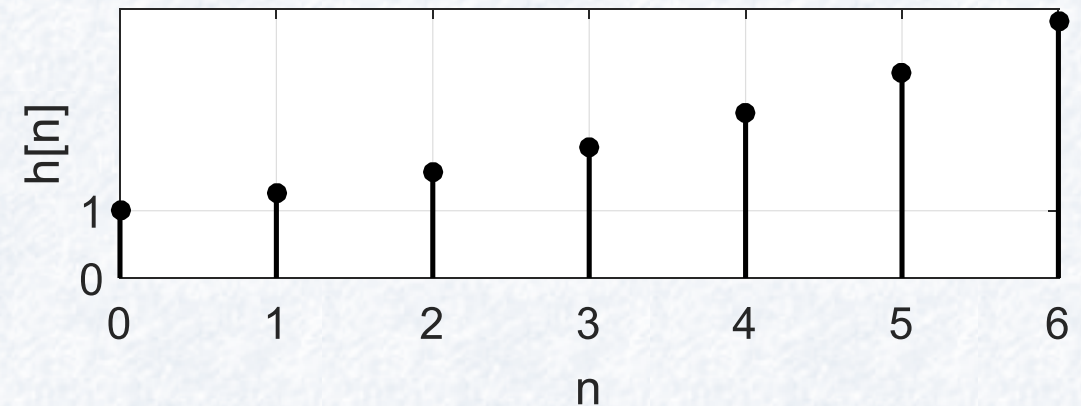
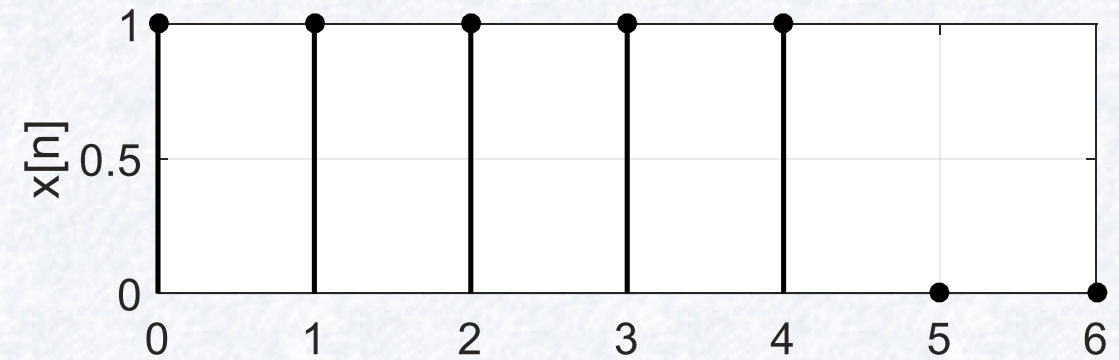
- $x[n] = \alpha^n u[n]$
- $h[n] = u[n]$
- $n < 0$  iken  $y[n] = 0$
- $n \geq 0$  iken  $y[n] = \frac{1 - \alpha^{n+1}}{1 - \alpha}$

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- $x[n] = \alpha^n u[n]$
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- $n < 0$  iken  $y[n] = 0$
- $n \geq 0$  iken  $y[n] = \frac{1-\alpha^{n+1}}{1-\alpha}$
- $y[n] = \frac{1-\alpha^{n+1}}{1-\alpha} u[n]$

# Örnek 2

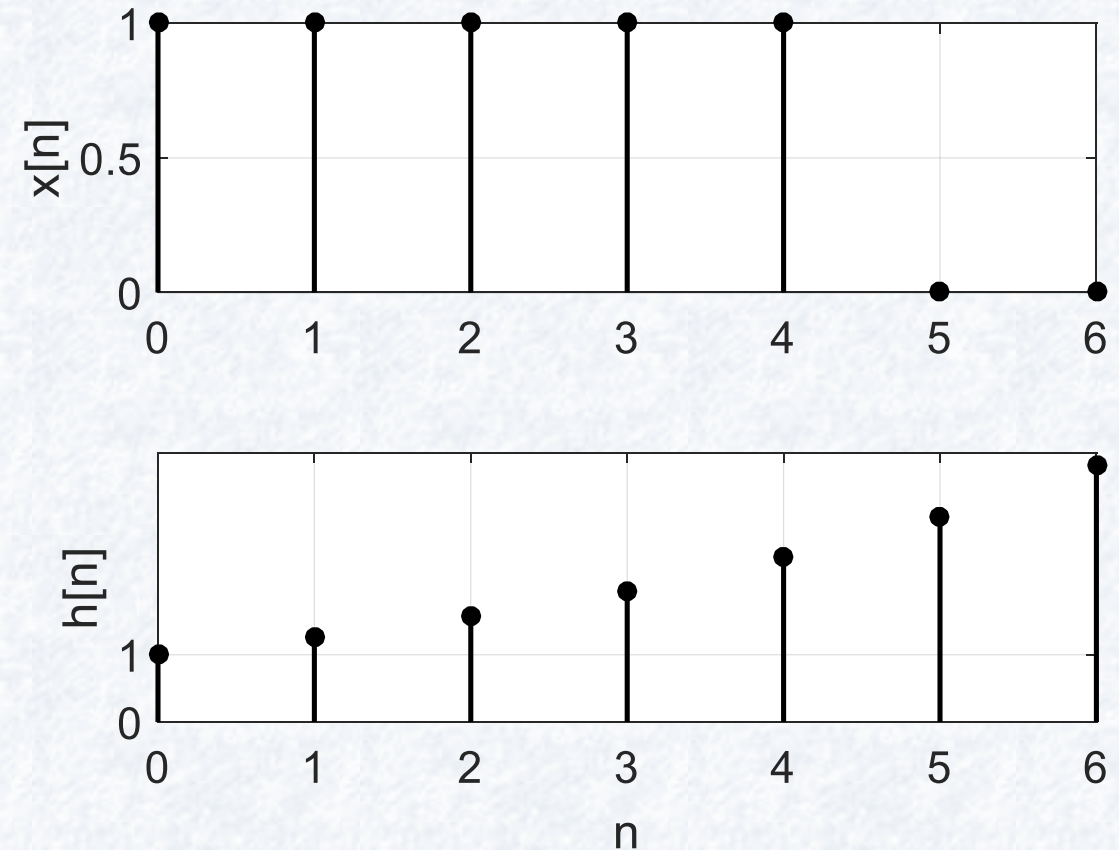
- $x[n] = \begin{cases} 1, & 0 \leq n \leq 4 \\ 0, & \text{diğer} \end{cases}$
- $h[n] = \begin{cases} \alpha^n, & 0 \leq n \leq 6 \\ 0, & \text{diğer} \end{cases}$
- $y[n] = ?$





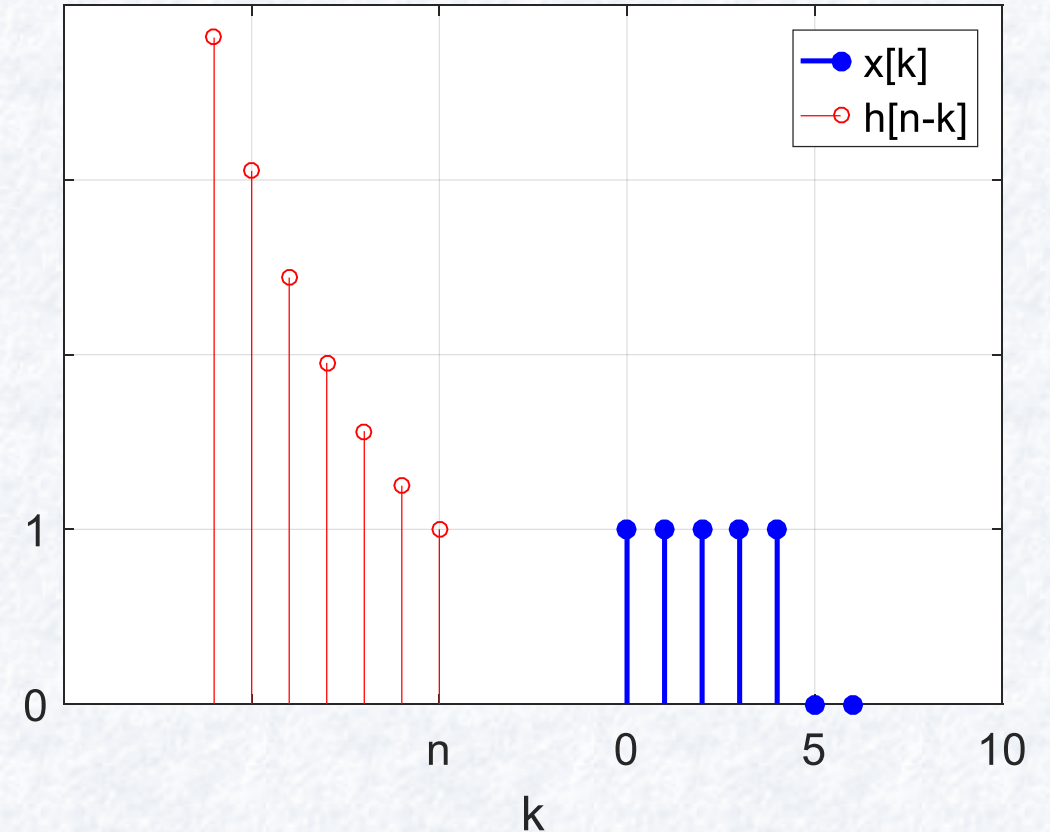
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- $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$
- $n < 0$  iken



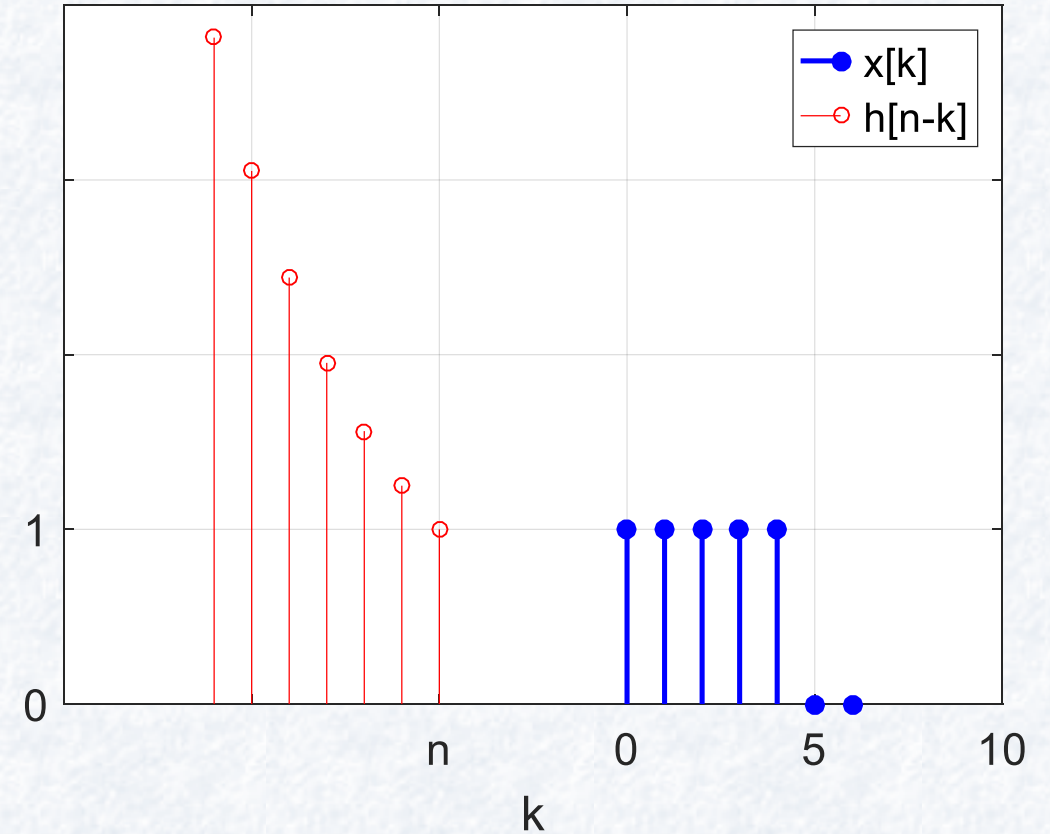
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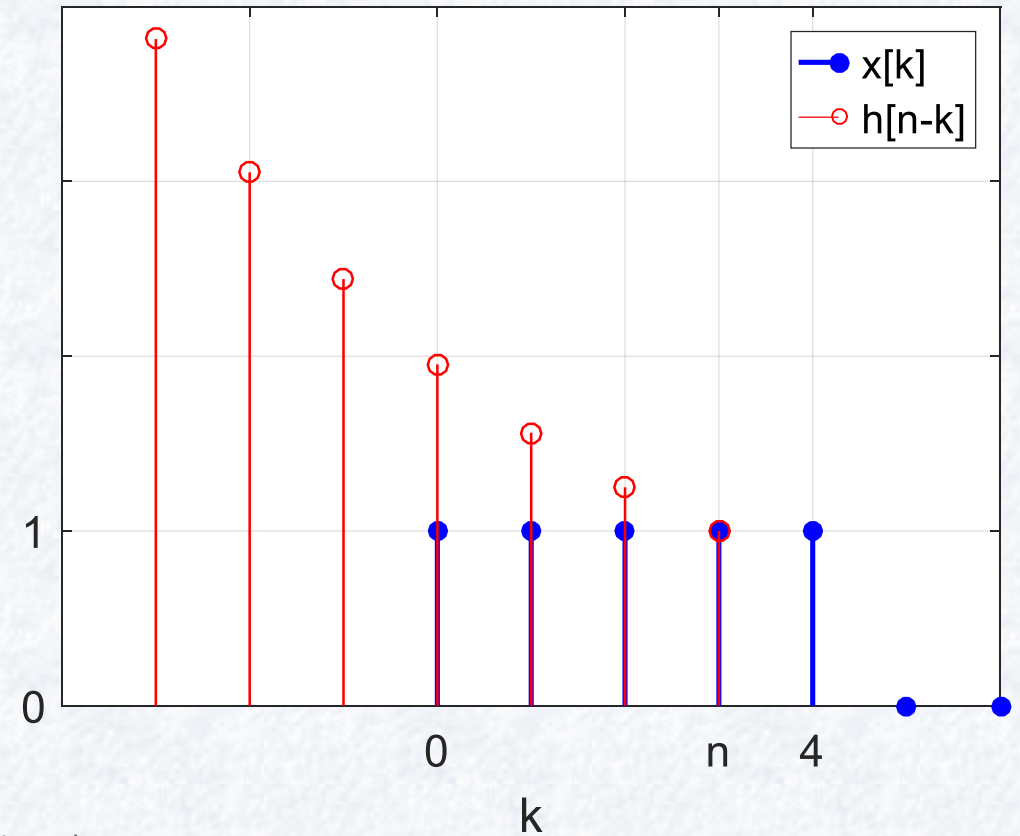
# Örnek 2

- $0 \leq n \leq 4$  iken



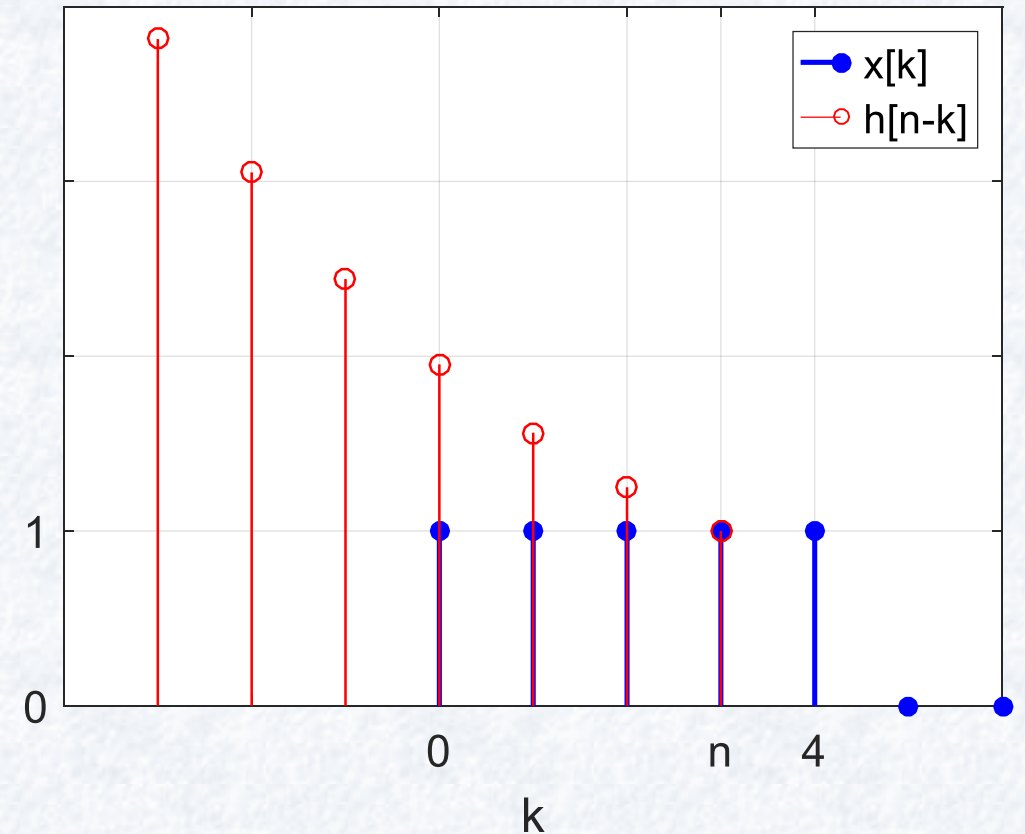
# Örnek 2

- $0 \leq n \leq 4$  iken
  - ♦ Çakışma,



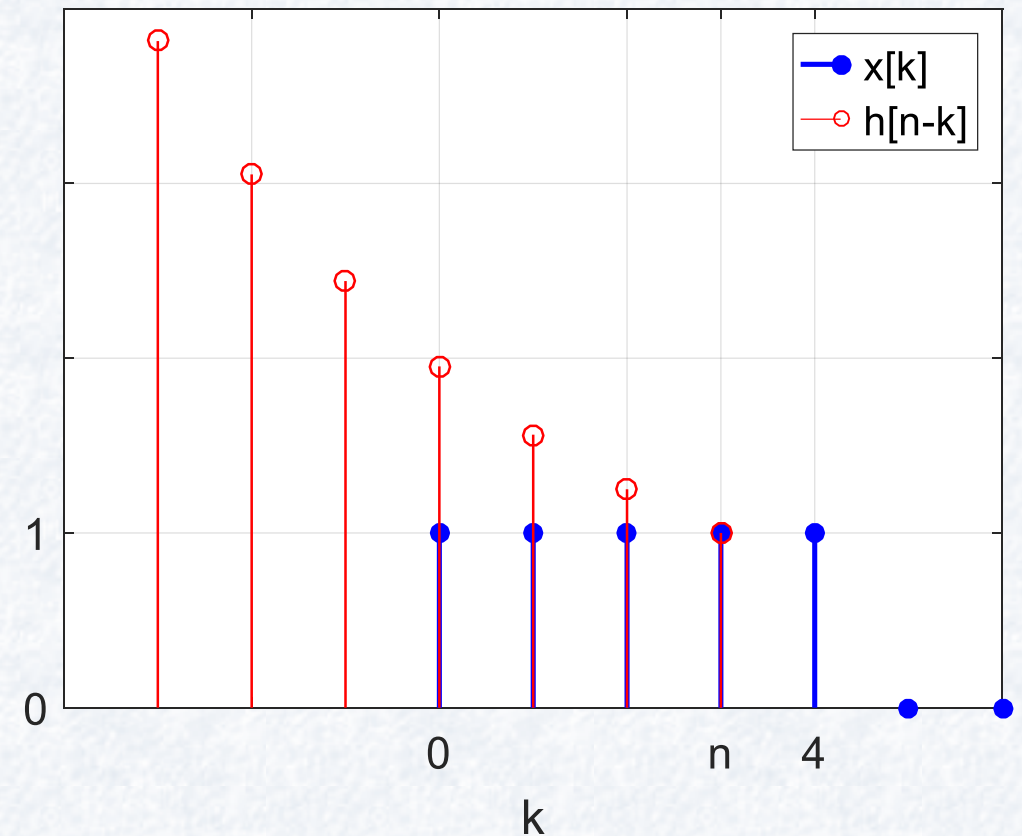
# Örnek 2

- $0 \leq n \leq 4$  iken
  - ♦ Çakışma, 0-n arası
- $y[n] = \sum_{k=0}^n x[k]h[n-k]$
- $y[n] = \sum_{k=0}^n 1 \cdot \alpha^{n-k}$



# Örnek 2

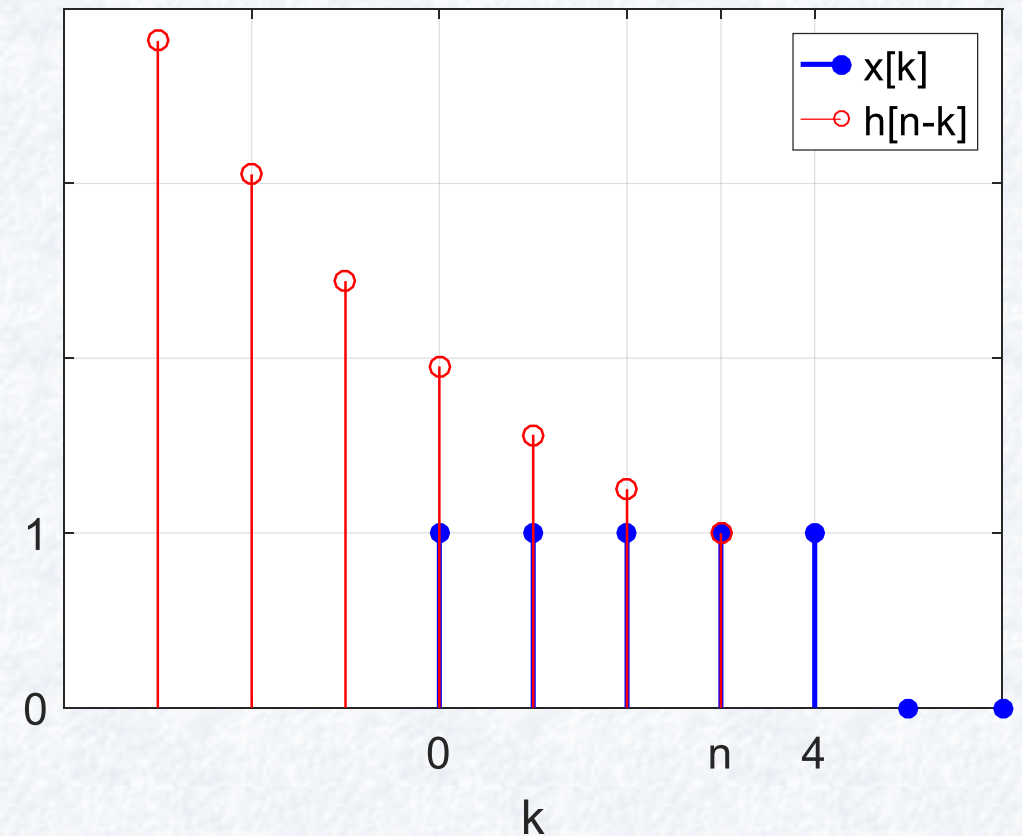
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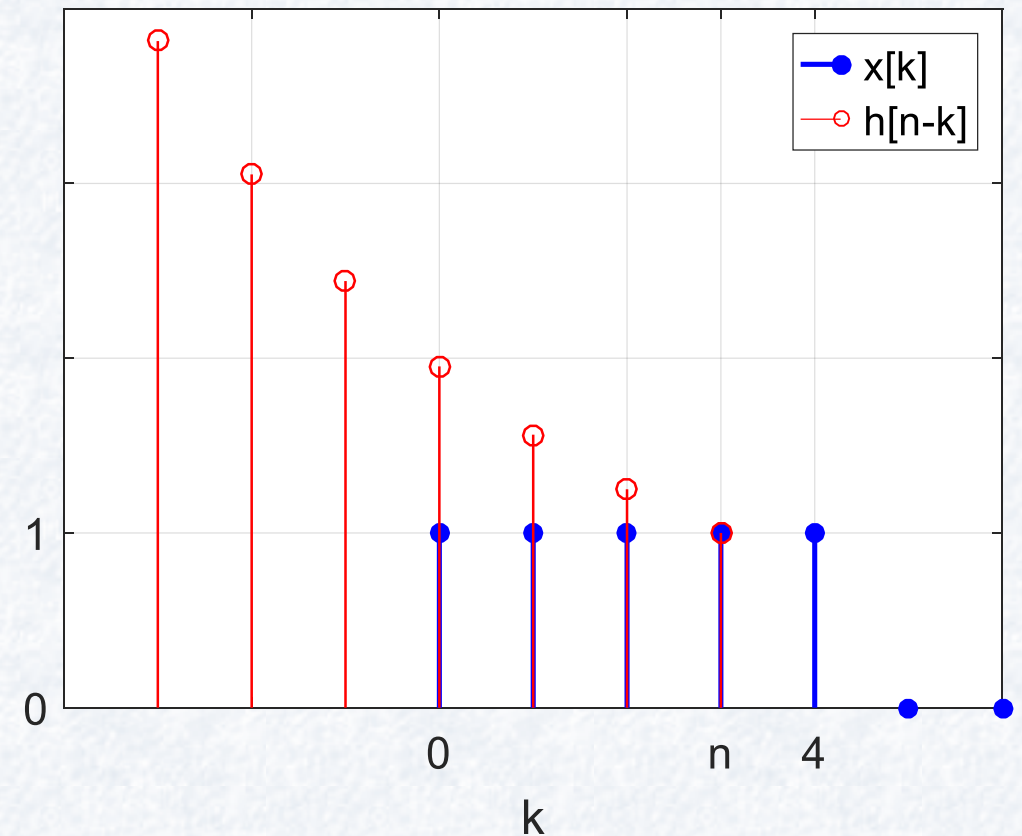
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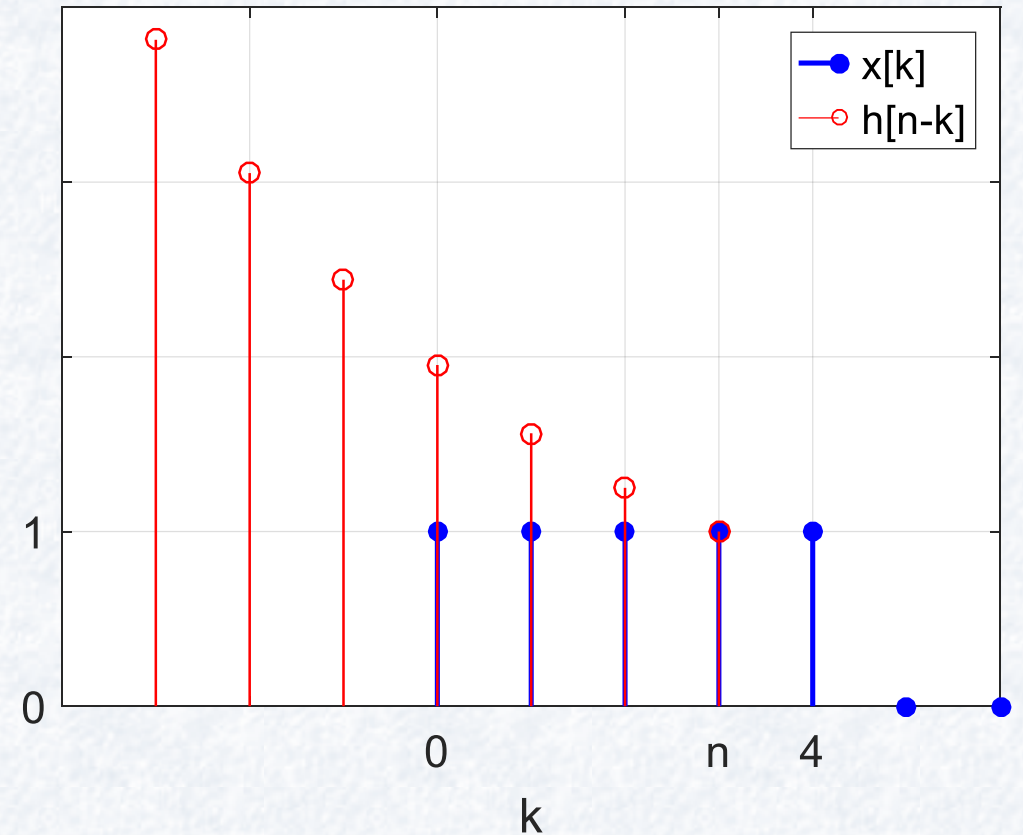
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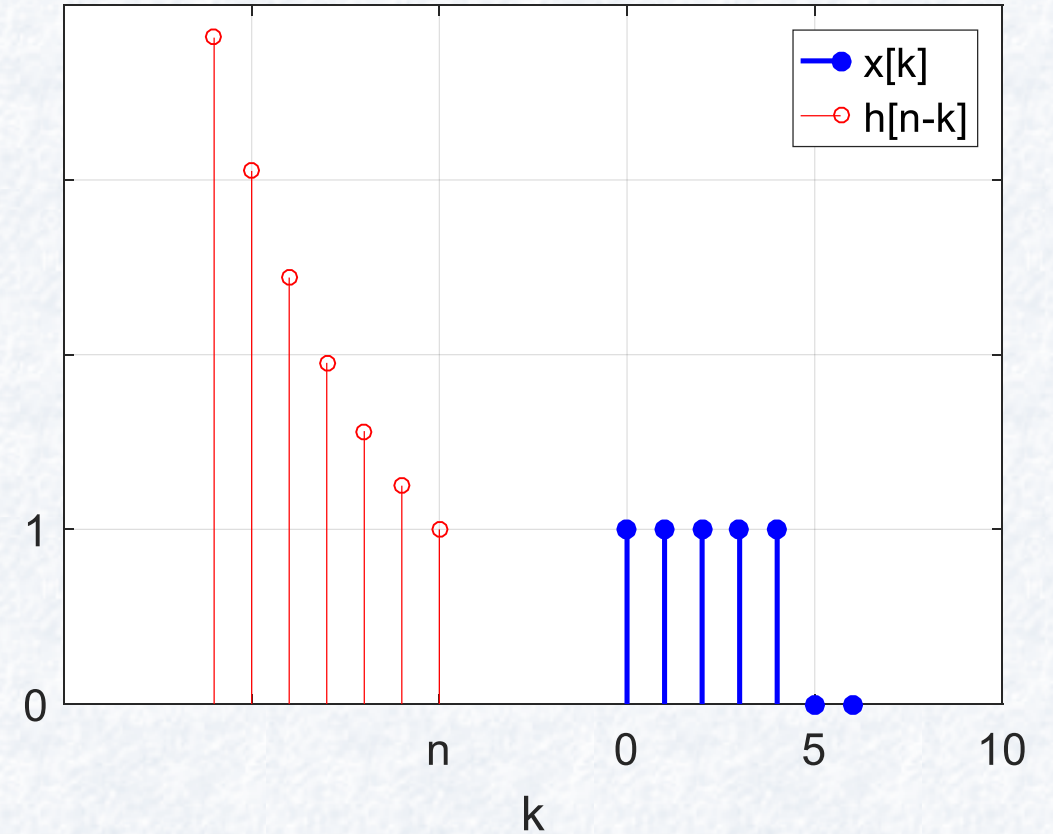
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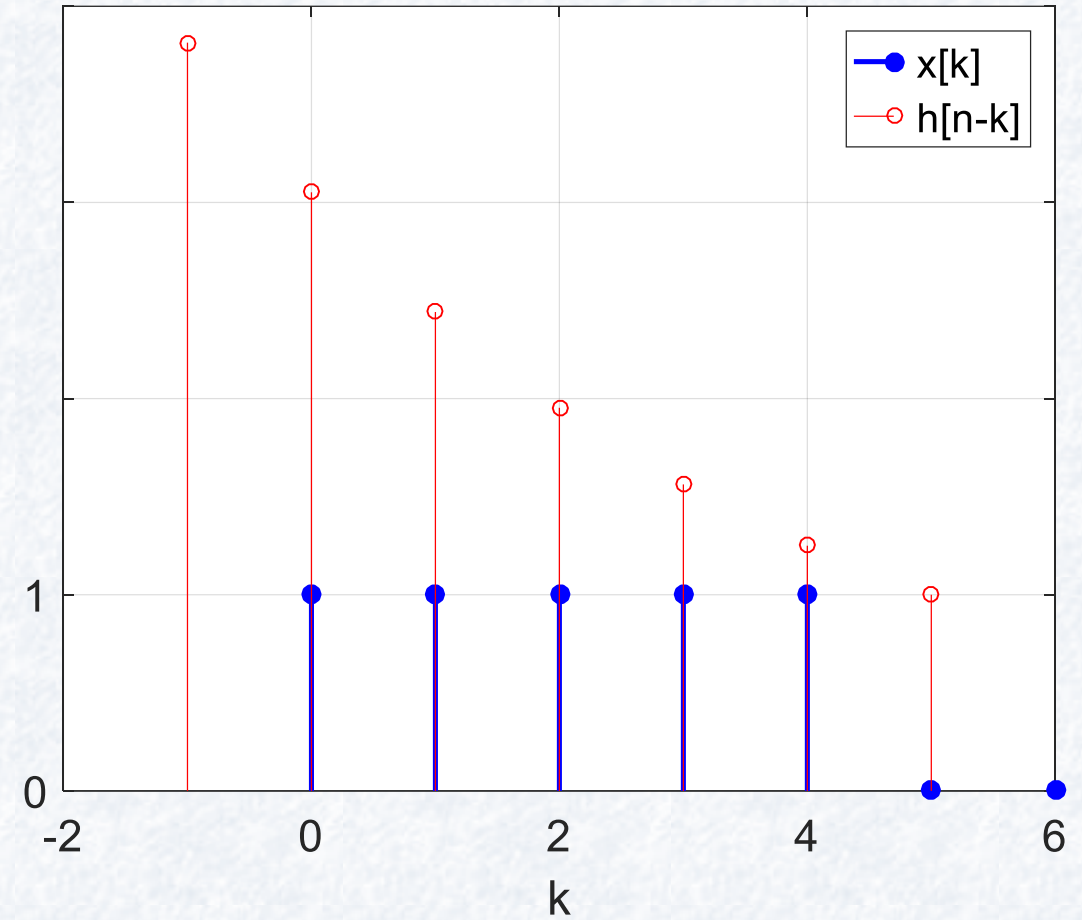
# Örnek 2

- $4 < n \leq 6$  iken



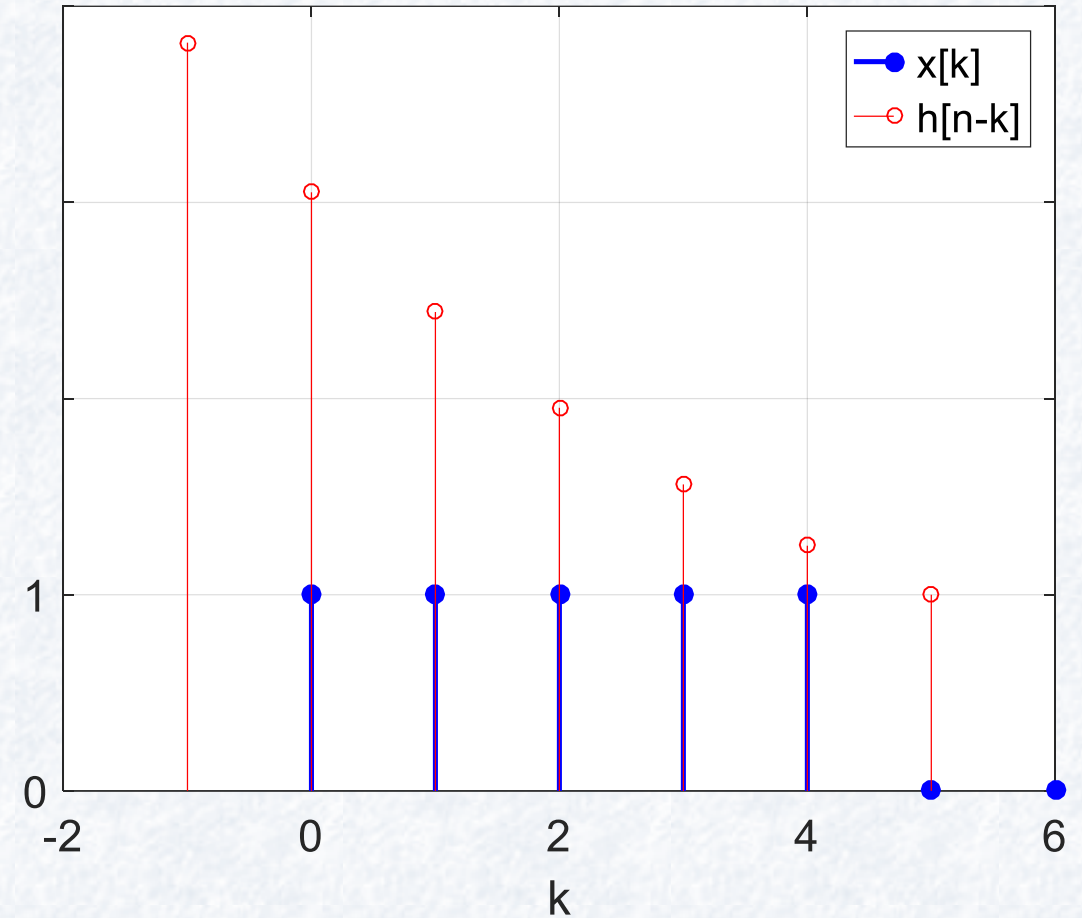
# Örnek 2

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  - ♦ Çakışma,



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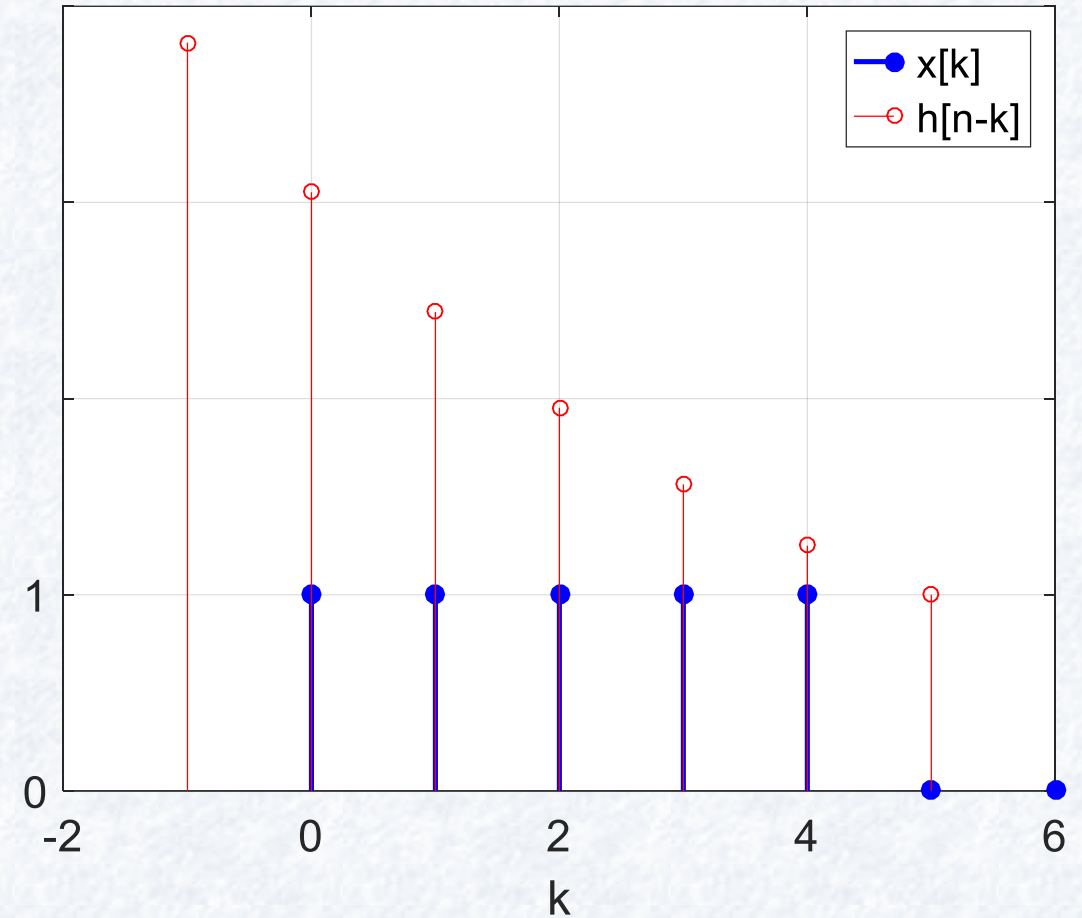
- $4 < n \leq 6$  iken
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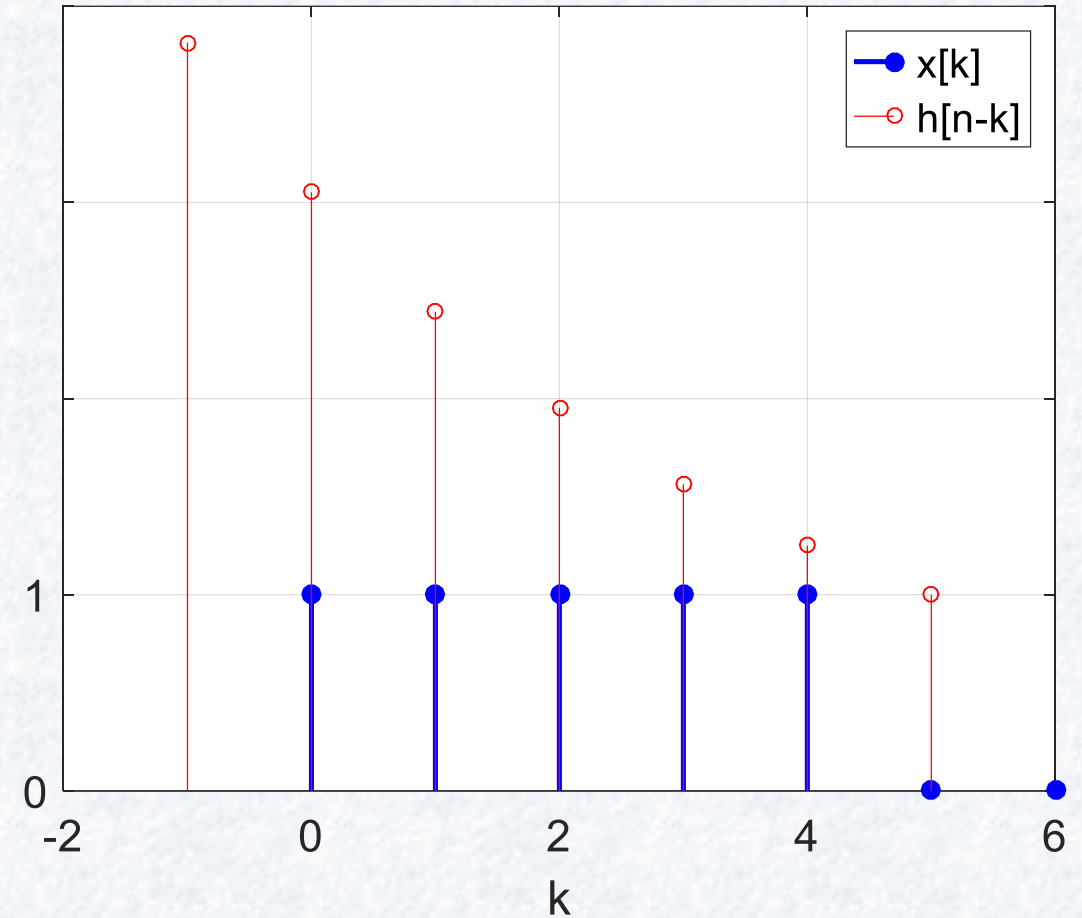
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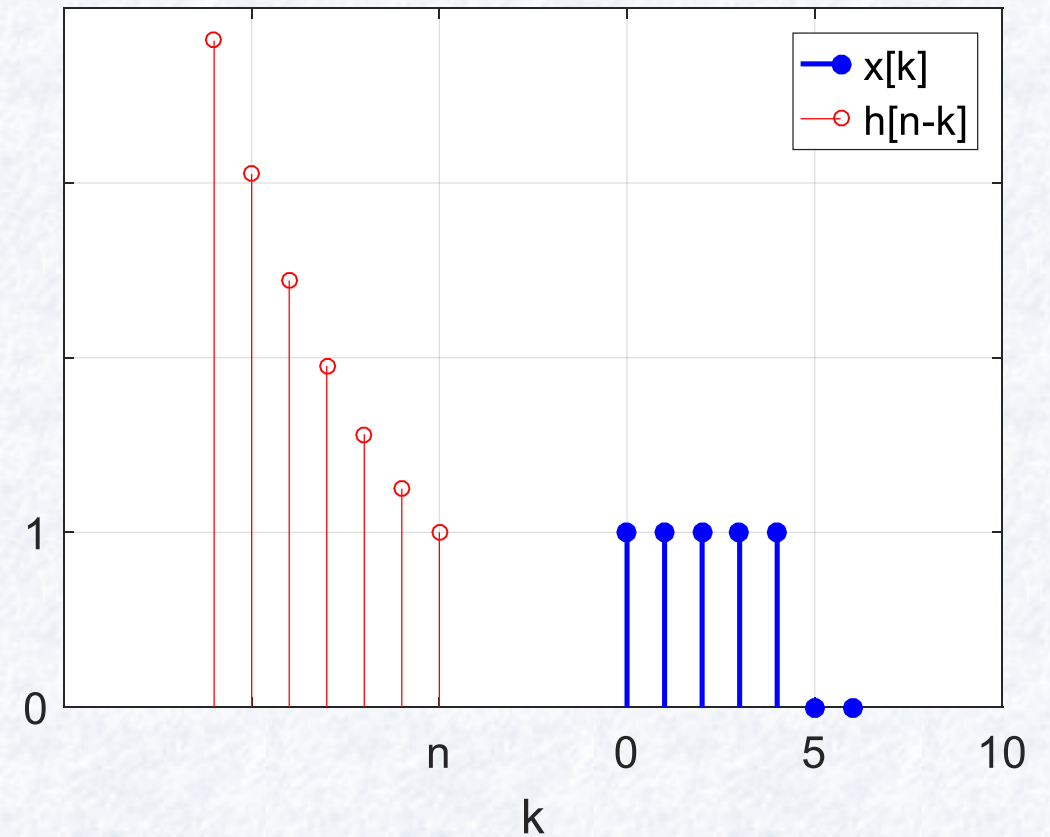
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- $y[n] = \alpha^n \frac{1 - \left(\frac{1}{\alpha}\right)^5}{1 - \frac{1}{\alpha}}$



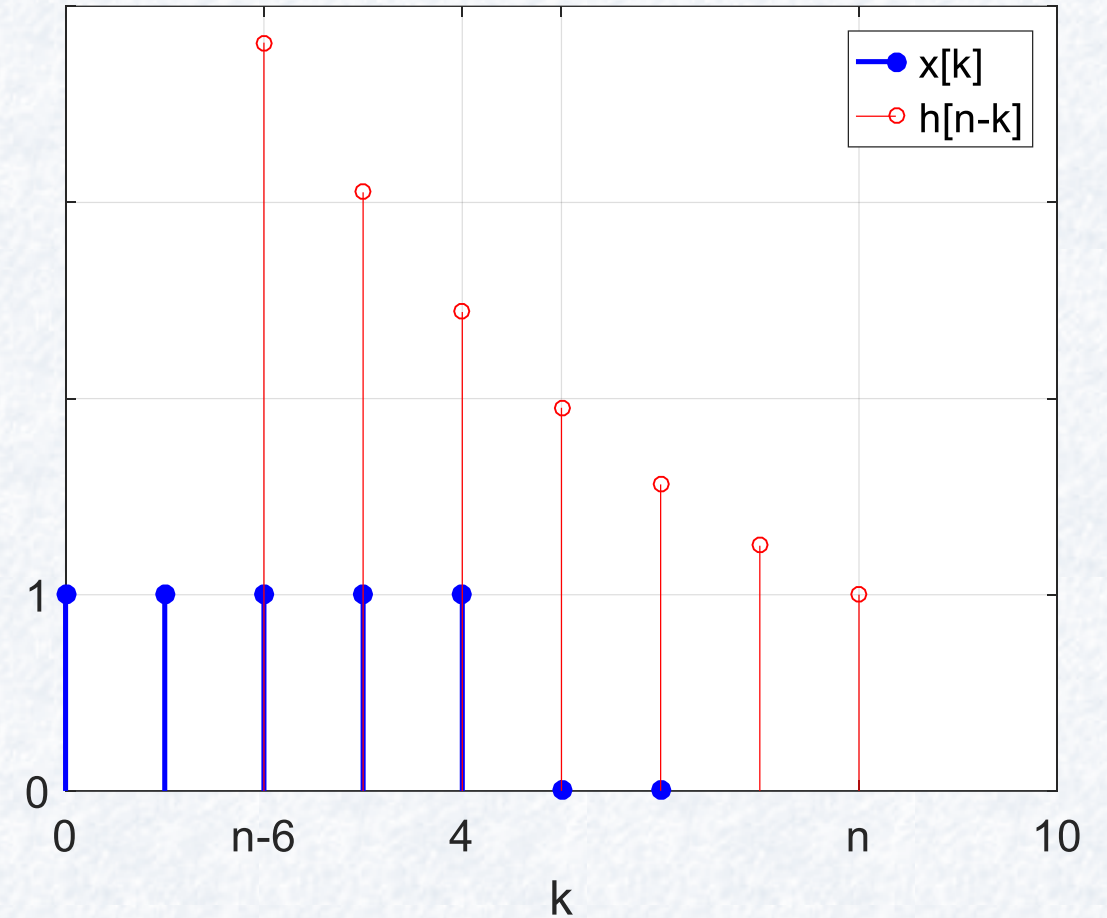
## Örnek 2

- $6 < n \leq 10$  iken



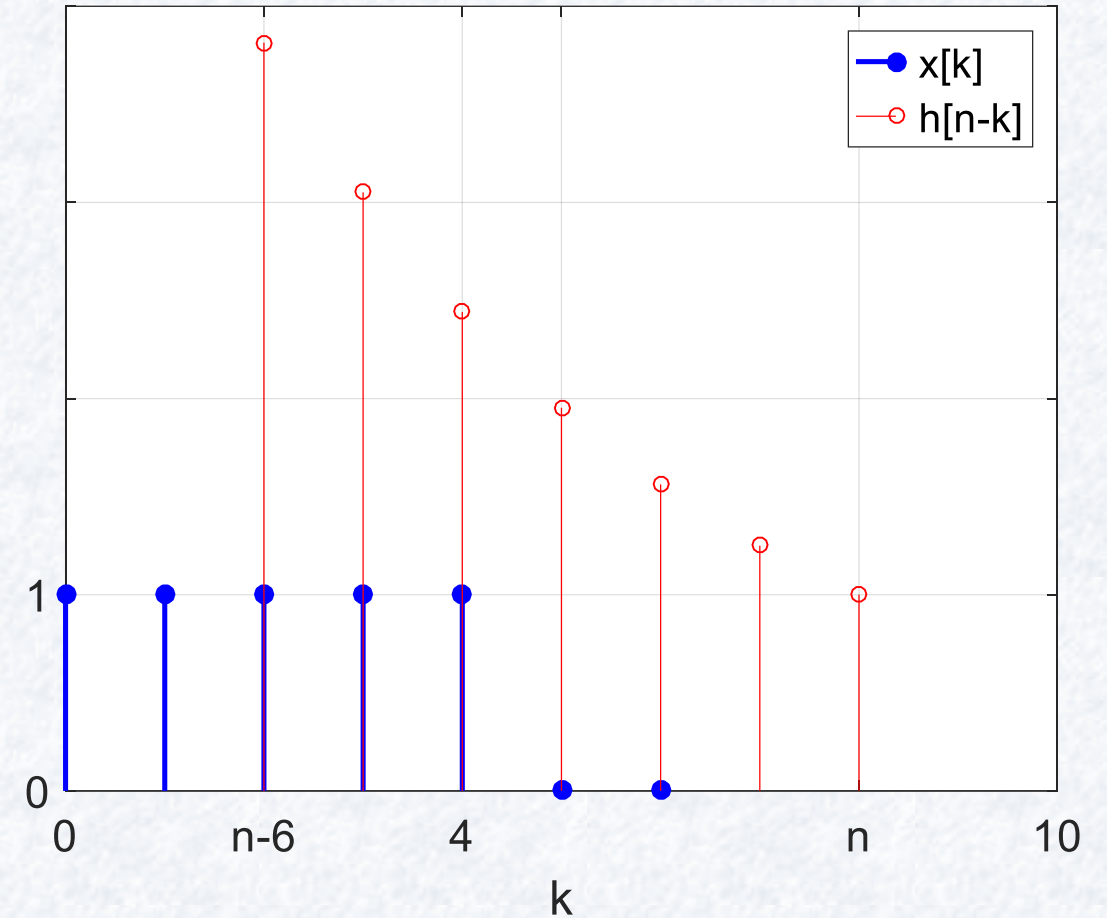
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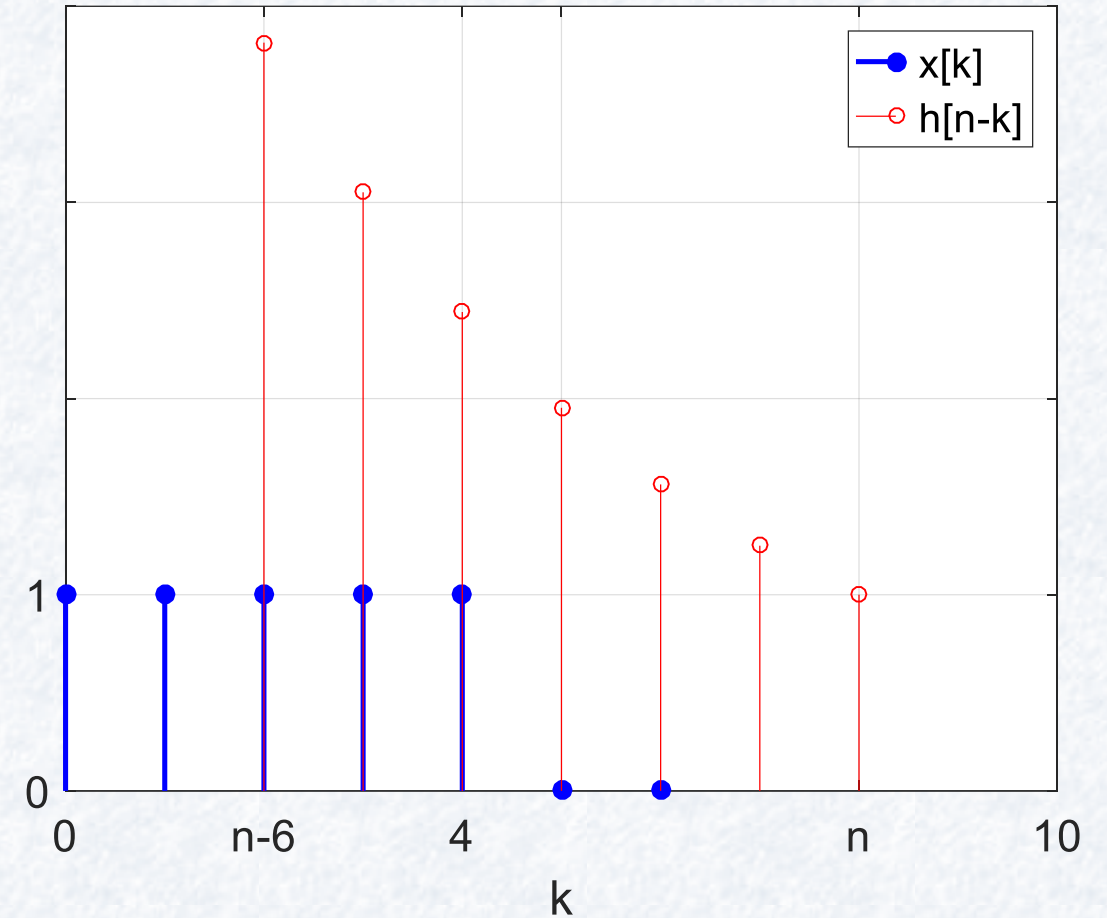
# Örnek 2

- $6 < n \leq 10$  iken
  - ♦ Çakışma,  $n-6 - 4$  arası



## Örnek 2

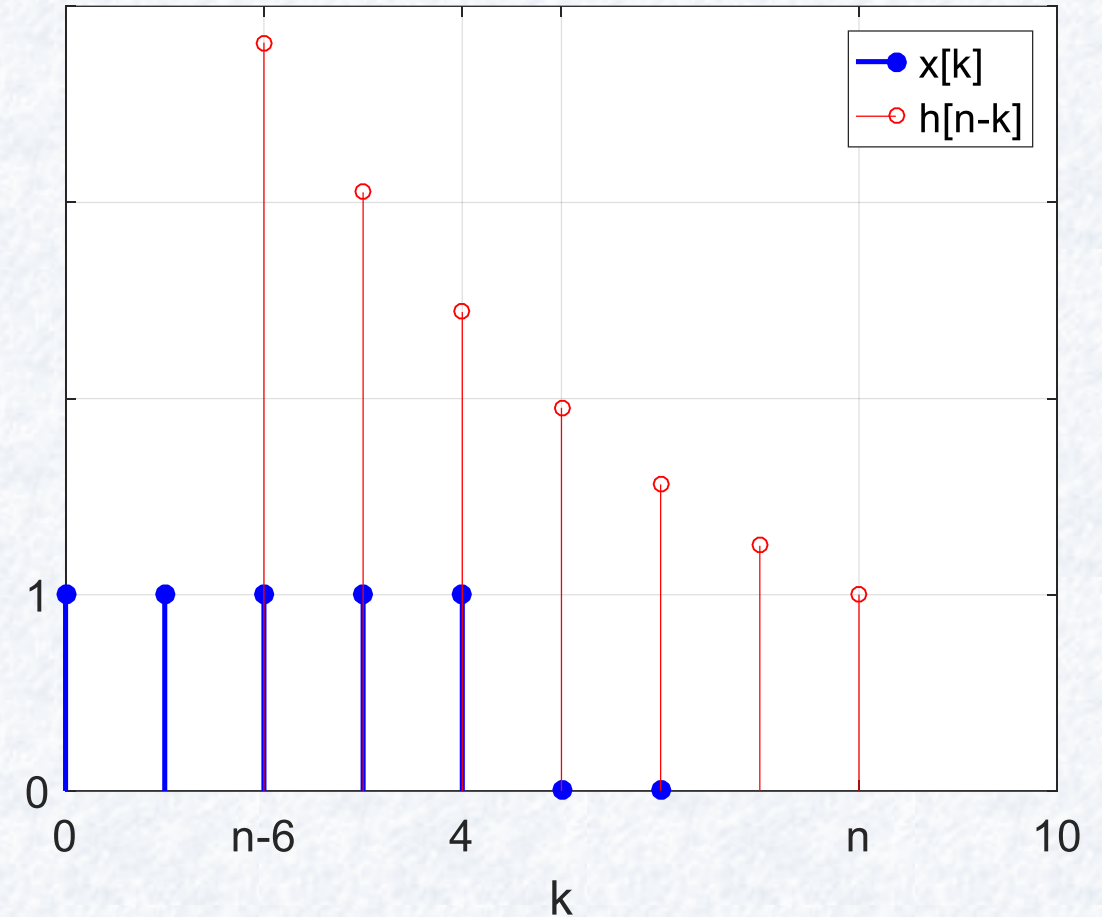
- $6 < n \leq 10$  iken
  - ♦ Çakışma,  $n-6 - 4$  arası
- $y[n] = \sum_{k=n-6}^4 x[k]h[n-k]$





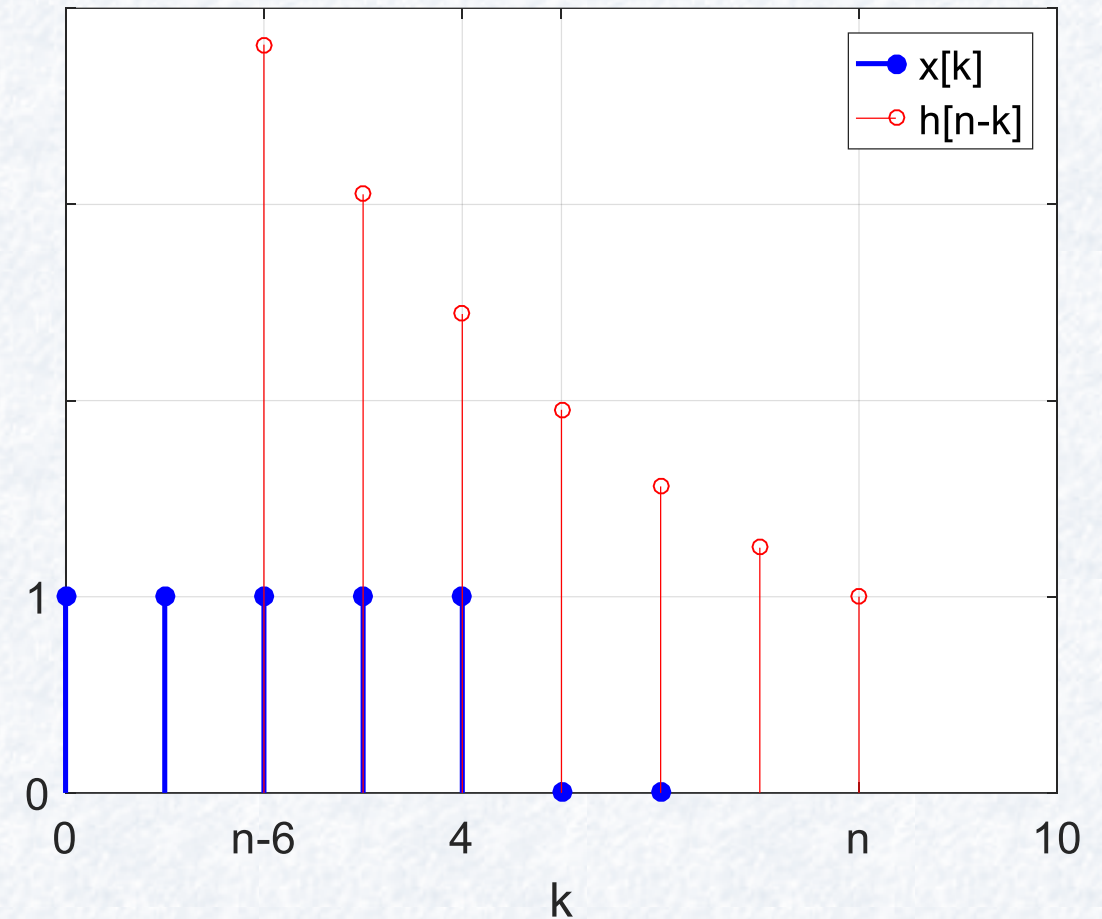
# Örnek 2

- $6 < n \leq 10$  iken
  - ♦ Çakışma,  $n-6 - 4$  arası
- $y[n] = \sum_{k=n-6}^4 x[k]h[n-k]$
- $y[n] = \alpha^n \sum_{k=n-6}^4 \alpha^{-k}$ 
  - ♦  $l = k - n + 6$



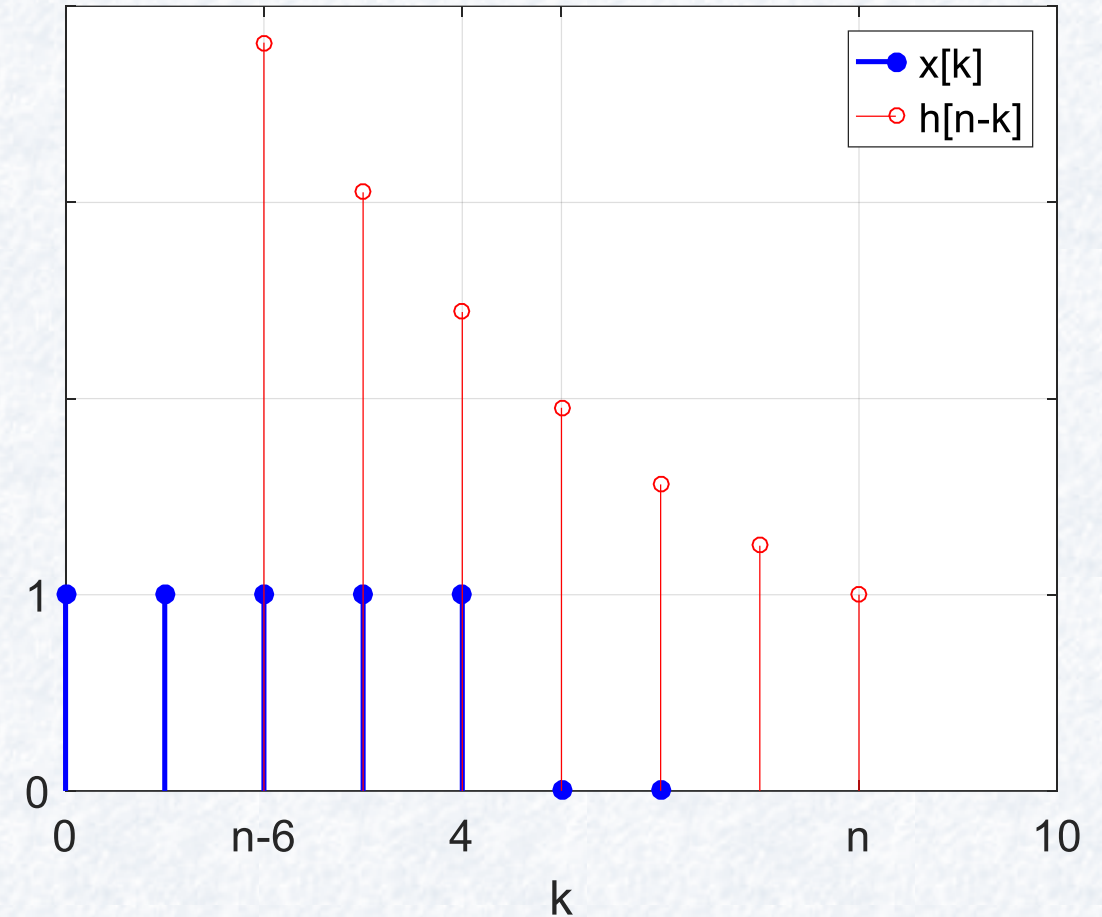
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  - ♦  $l = k - n + 6$
- $y[n] = \alpha^n \sum_{l=0}^{10-n} \alpha^{-l-n+6}$



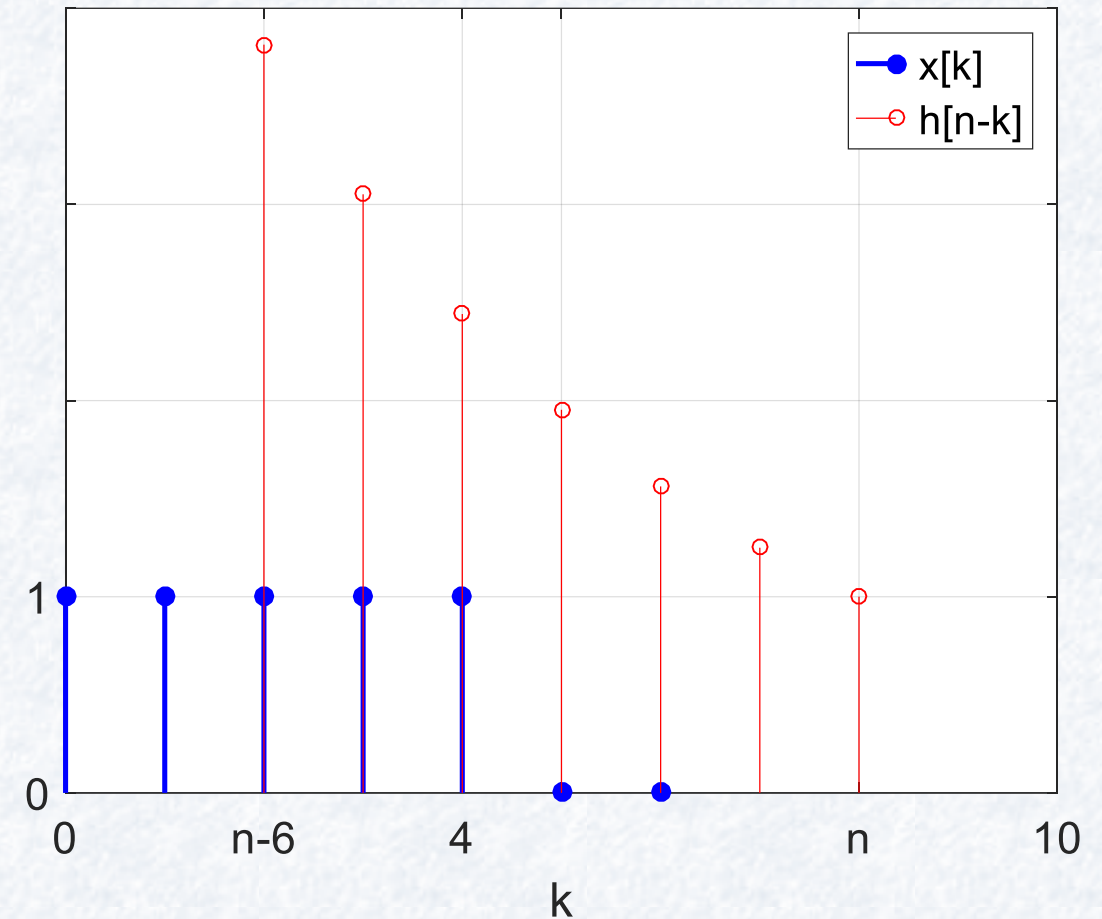
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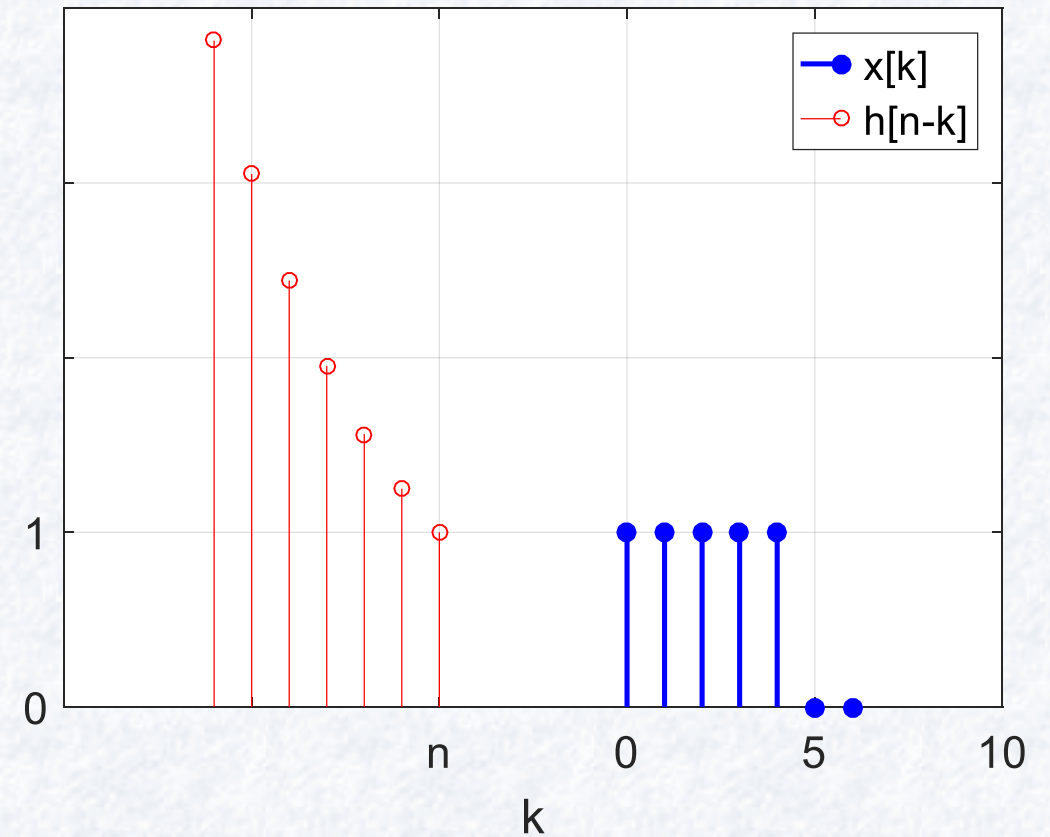
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- $y[n] = \alpha^n \alpha^{-n+6} \sum_{l=0}^{10-n} \alpha^{-l}$
- $y[n] = \alpha^6 \frac{1 - \left(\frac{1}{\alpha}\right)^{11-n}}{1 - \frac{1}{\alpha}}$



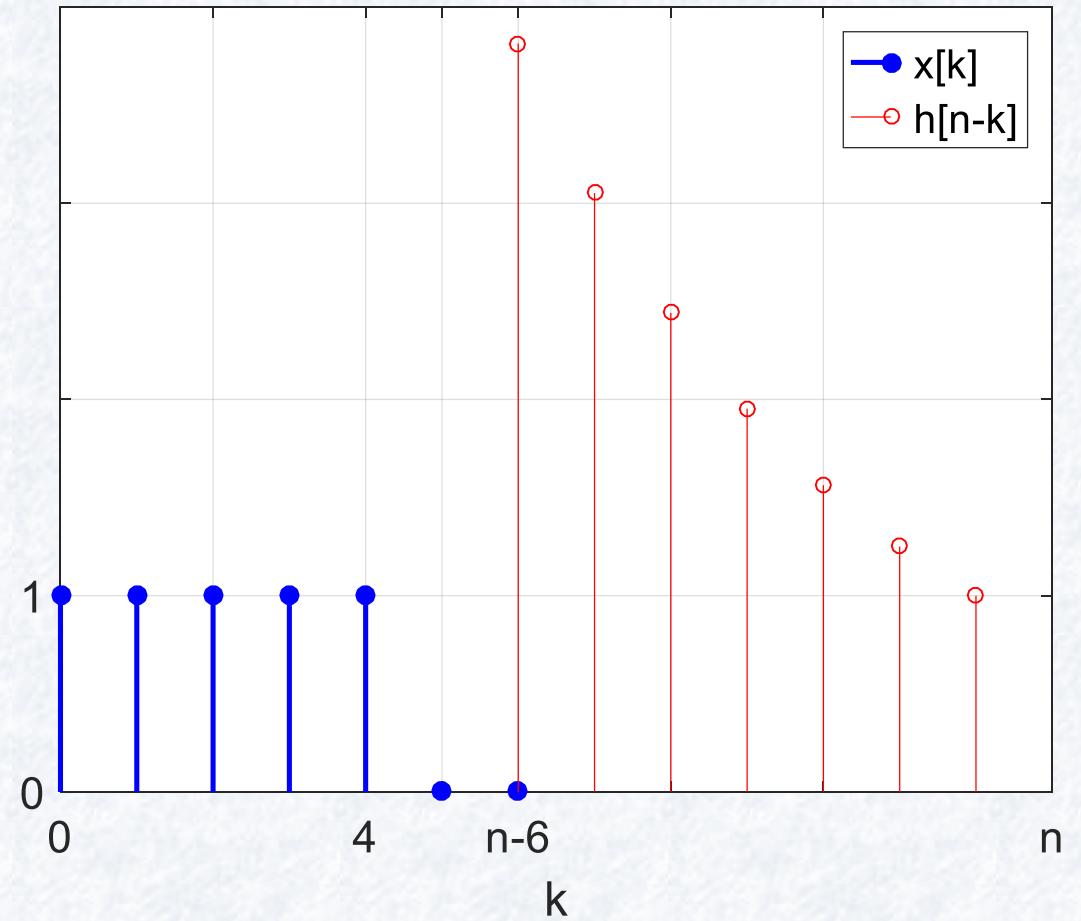
# Örnek 2

- $10 < n$  iken



# Örnek 2

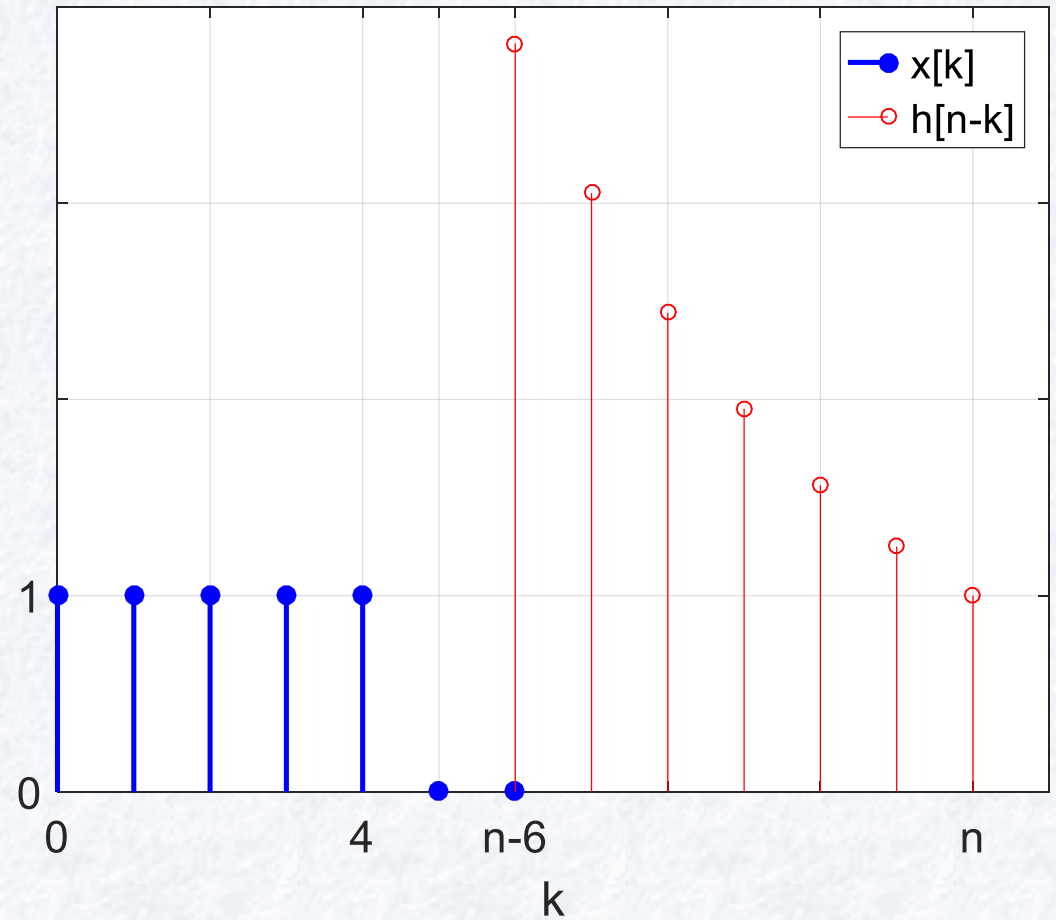
- $10 < n$  iken
  - ♦ Çakışma yok.





# Örnek 2

- $10 < n$  iken
  - ♦ Çakışma yok.
- $y[n] = 0$

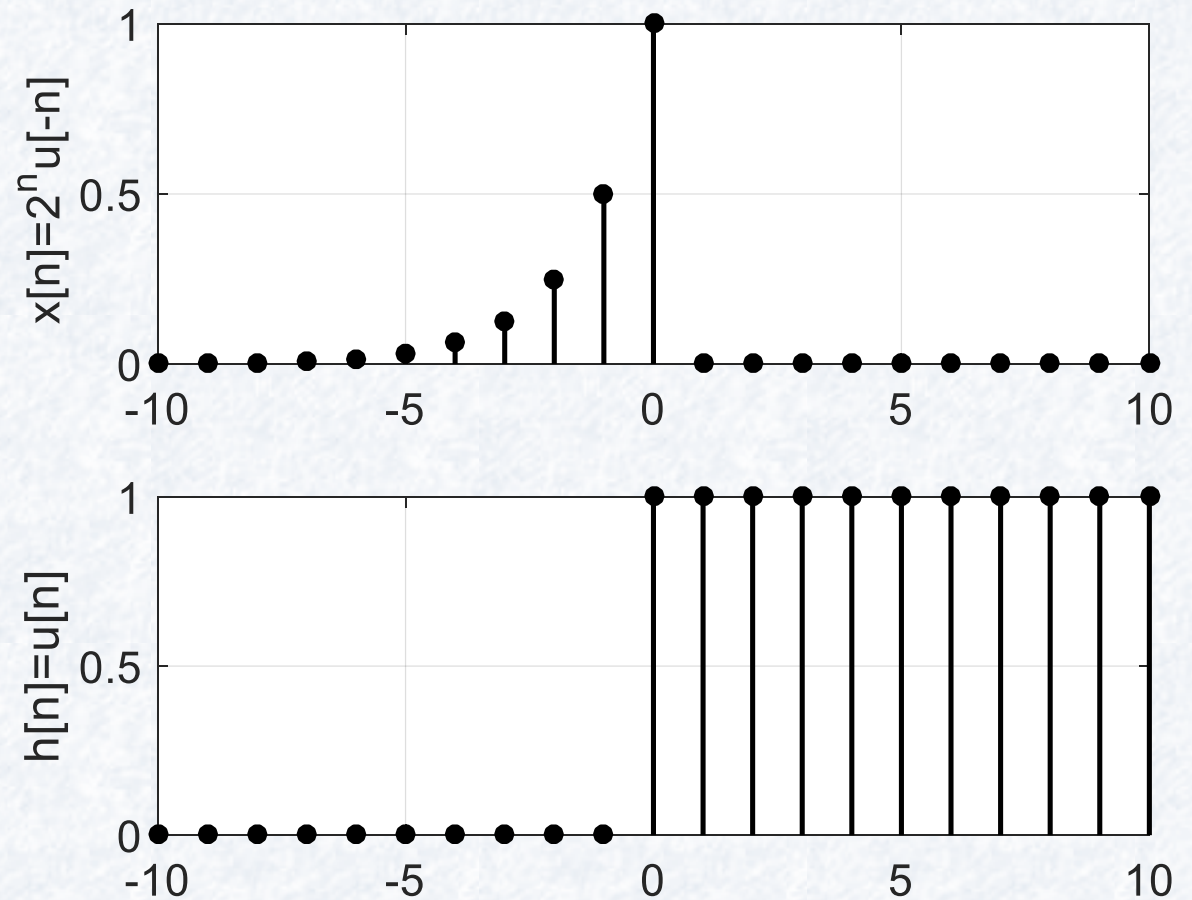


## Örnek 2

- $n < 0$  iken  $y[n] = 0$
- $0 \leq n \leq 4$  iken  $y[n] = \alpha^n \frac{1 - \left(\frac{1}{\alpha}\right)^{n+1}}{1 - \frac{1}{\alpha}}$
- $4 < n \leq 6$  iken  $y[n] = \alpha^n \frac{1 - \left(\frac{1}{\alpha}\right)^5}{1 - \frac{1}{\alpha}}$
- $6 < n \leq 10$  iken  $y[n] = \alpha^6 \frac{1 - \left(\frac{1}{\alpha}\right)^{11-n}}{1 - \frac{1}{\alpha}}$
- $10 < n$  iken  $y[n] = 0$

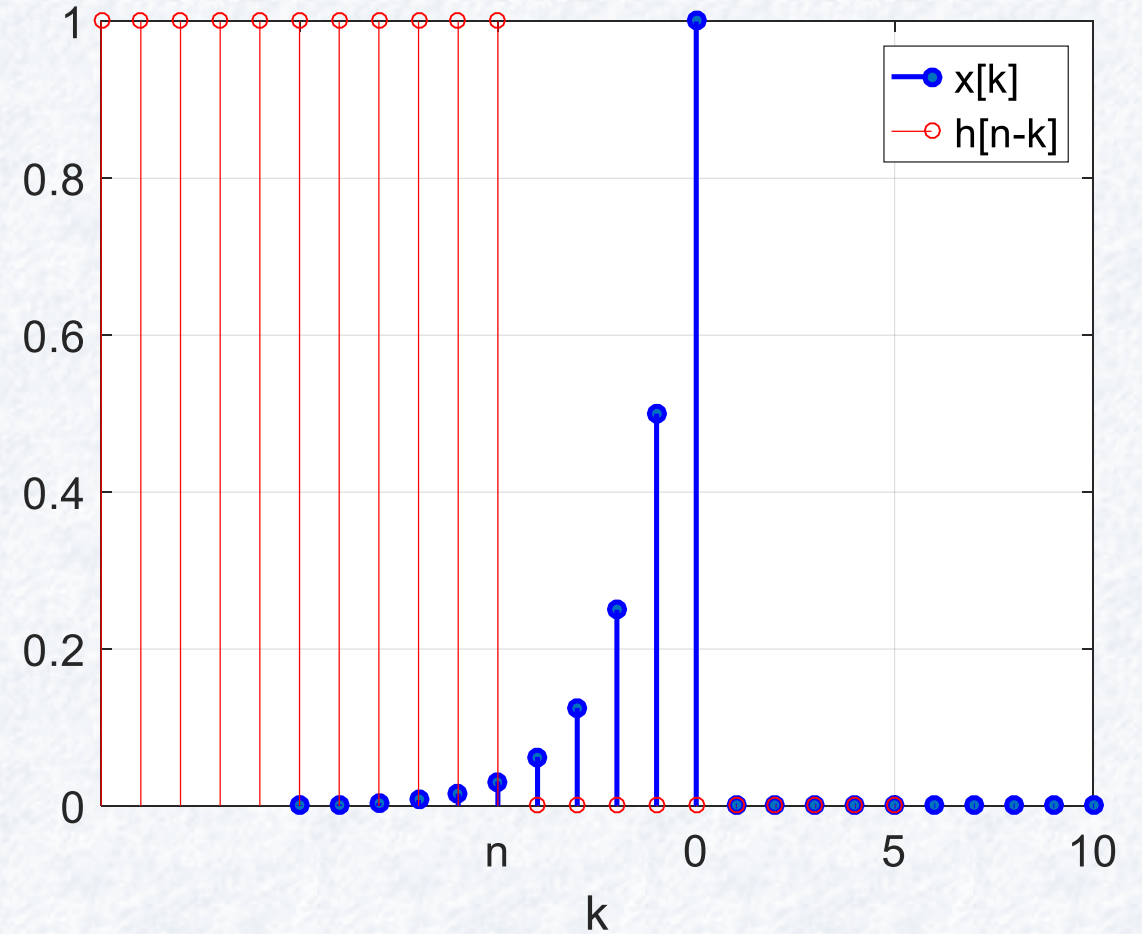
# Örnek 3

- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
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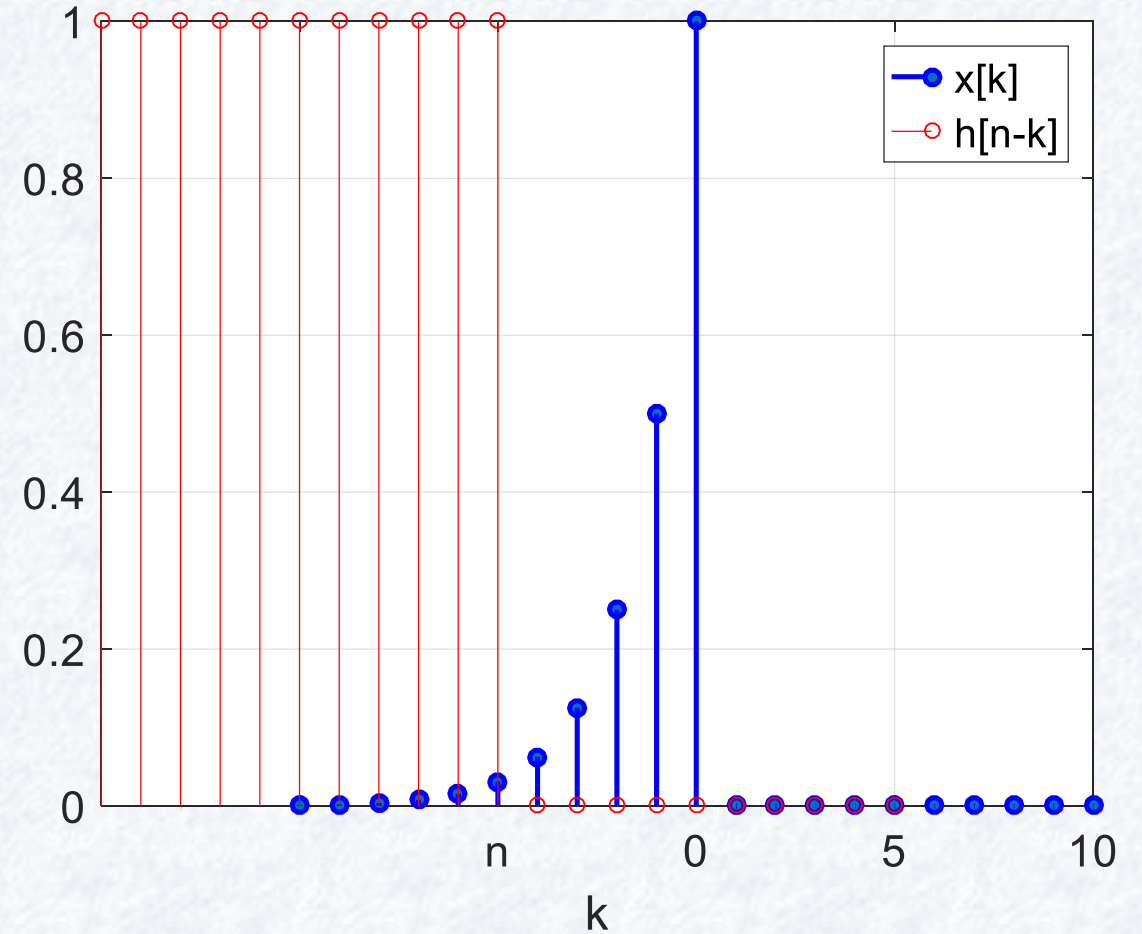
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- $x[n] = 2^n u[-n]$
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  - ♦ Çakışma,



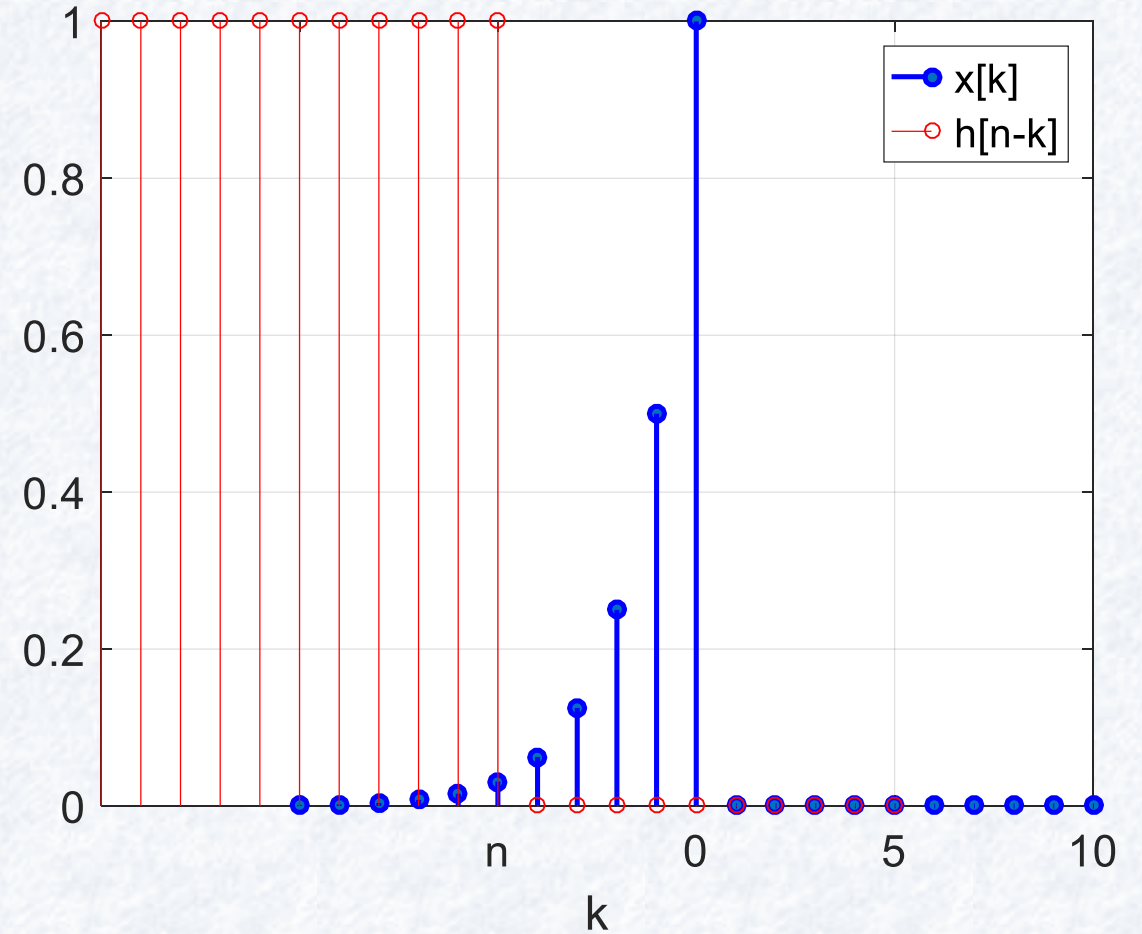
# Örnek 3

- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
- $n < 0$  iken
  - ♦ Çakışma,  $-\infty - n$  arası



# Örnek 3

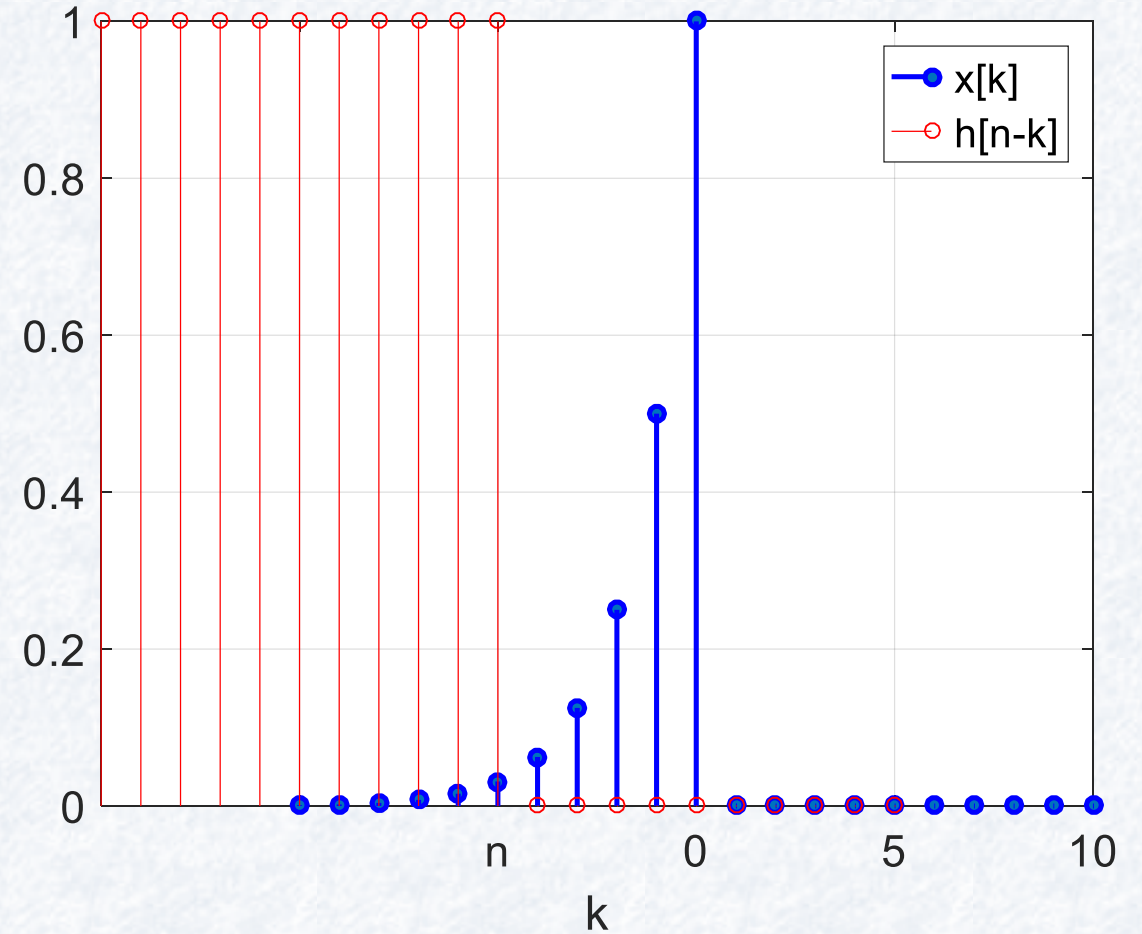
- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
- $n < 0$  iken
  - ♦ Çakışma,  $-\infty - n$  arası
- $y[n] = \sum_{-\infty}^n x[k]h[n-k]$





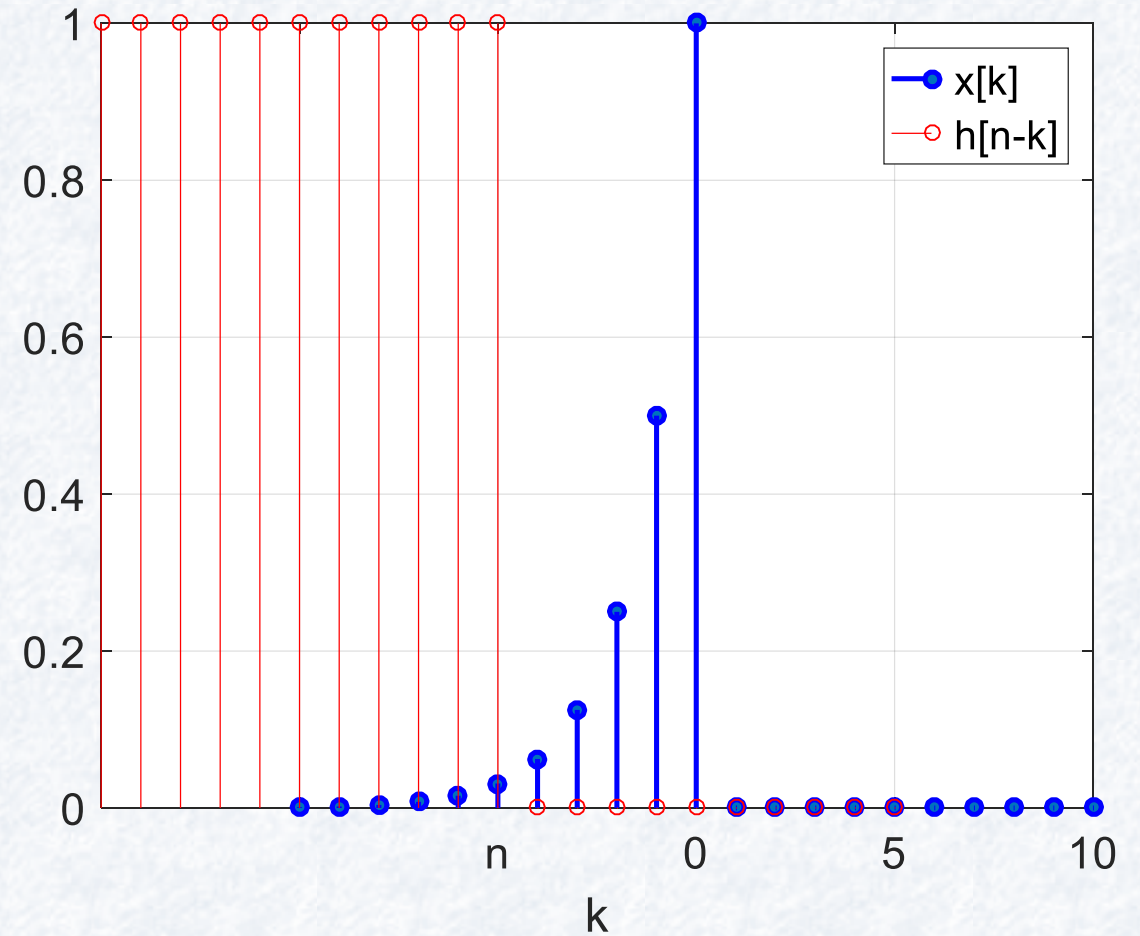
# Örnek 3

- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
- $n < 0$  iken
  - ♦ Çakışma,  $-\infty - n$  arası
- $y[n] = \sum_{k=-\infty}^n x[k]h[n-k]$
- $y[n] = \sum_{k=-\infty}^n 2^k 1$



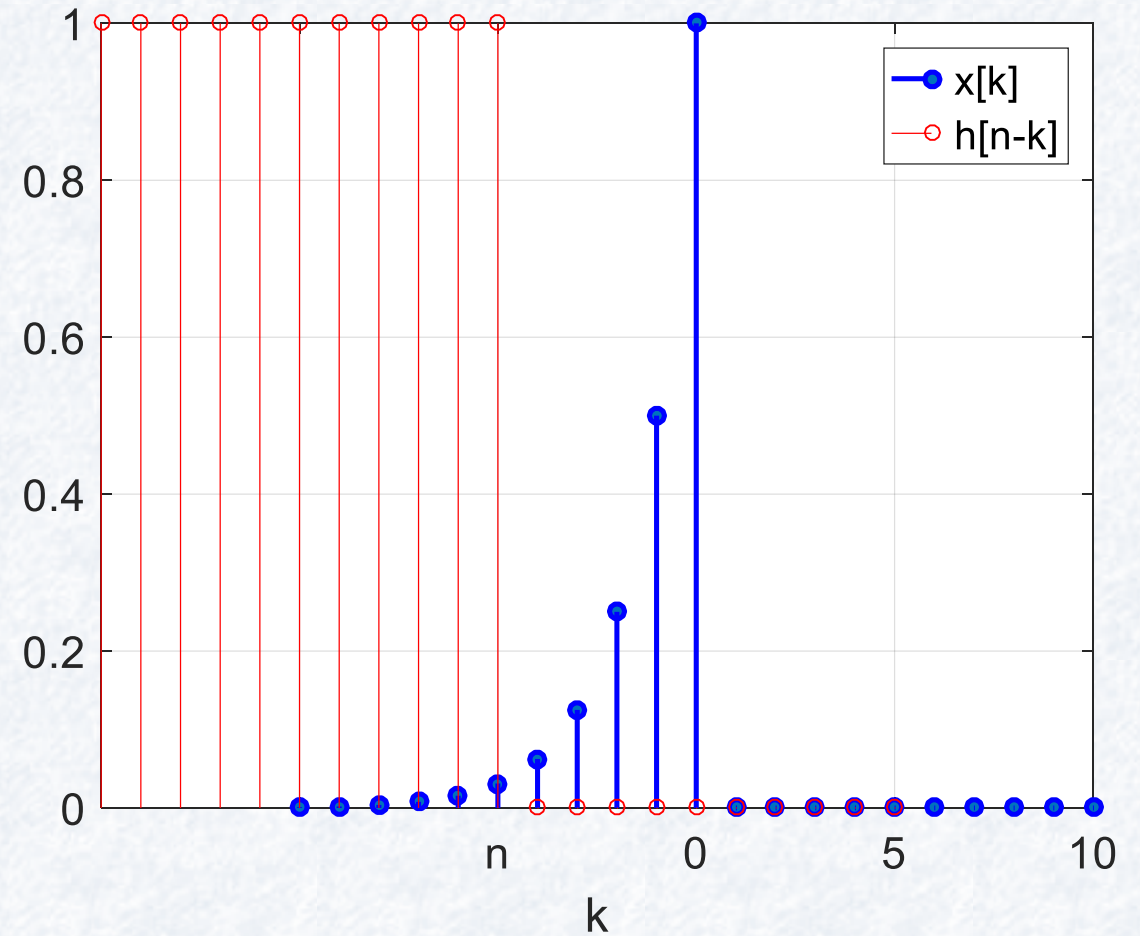
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  - ♦  $l = -k + n$



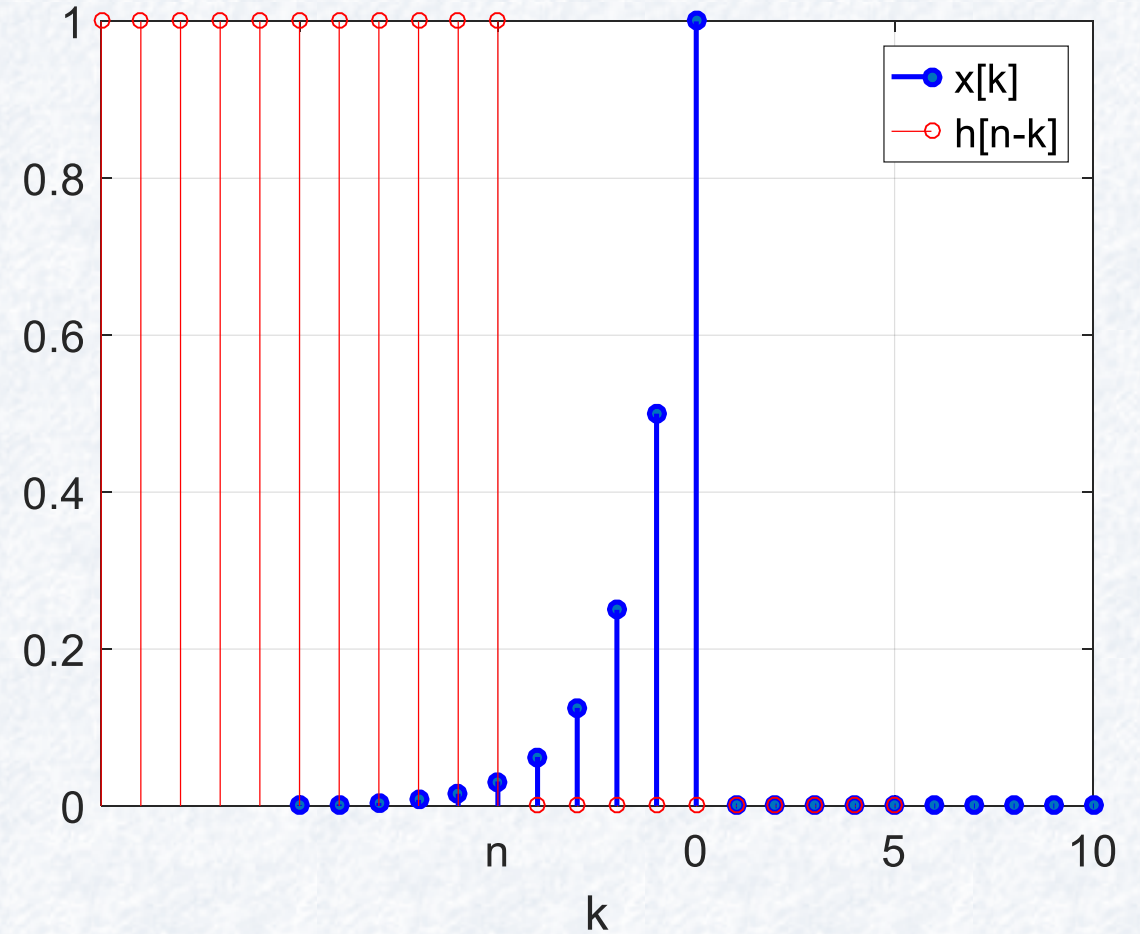
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- $y[n] = \sum_{k=-\infty}^n 2^k \mathbf{1}$ 
  - ♦  $l = -k + n$
- $y[n] = \sum_{l=\infty}^0 2^{n-l} \mathbf{1}$



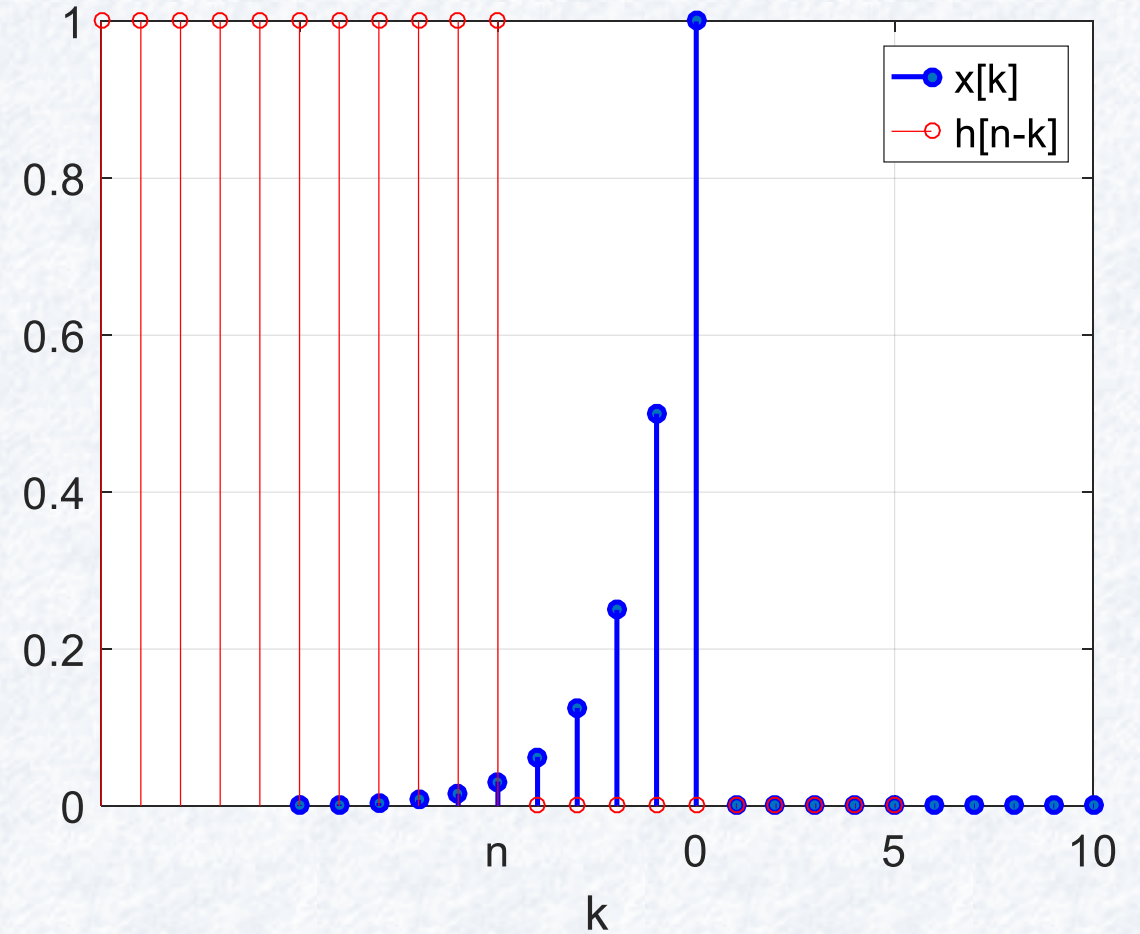
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- $y[n] = \sum_{k=-\infty}^n 2^k \mathbf{1}$ 
  - ♦  $l = -k + n$
- $y[n] = \sum_{l=-\infty}^0 2^{n-l} \mathbf{1}$
- $y[n] = \sum_{l=0}^{\infty} 2^n 2^{-l} \mathbf{1}$



# Örnek 3

- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
- $n < 0$  iken
  - ♦ Çakışma,  $-\infty - n$  arası
- $y[n] = \sum_{k=-\infty}^n x[k]h[n-k]$
- $y[n] = \sum_{k=-\infty}^n 2^k \mathbf{1}$ 
  - ♦  $l = -k + n$
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- $y[n] = \sum_{l=0}^{\infty} 2^n 2^{-l} \mathbf{1}$
- $y[n] = 2^n \sum_{l=0}^{\infty} \left(\frac{1}{2}\right)^l$



# Örnek 3

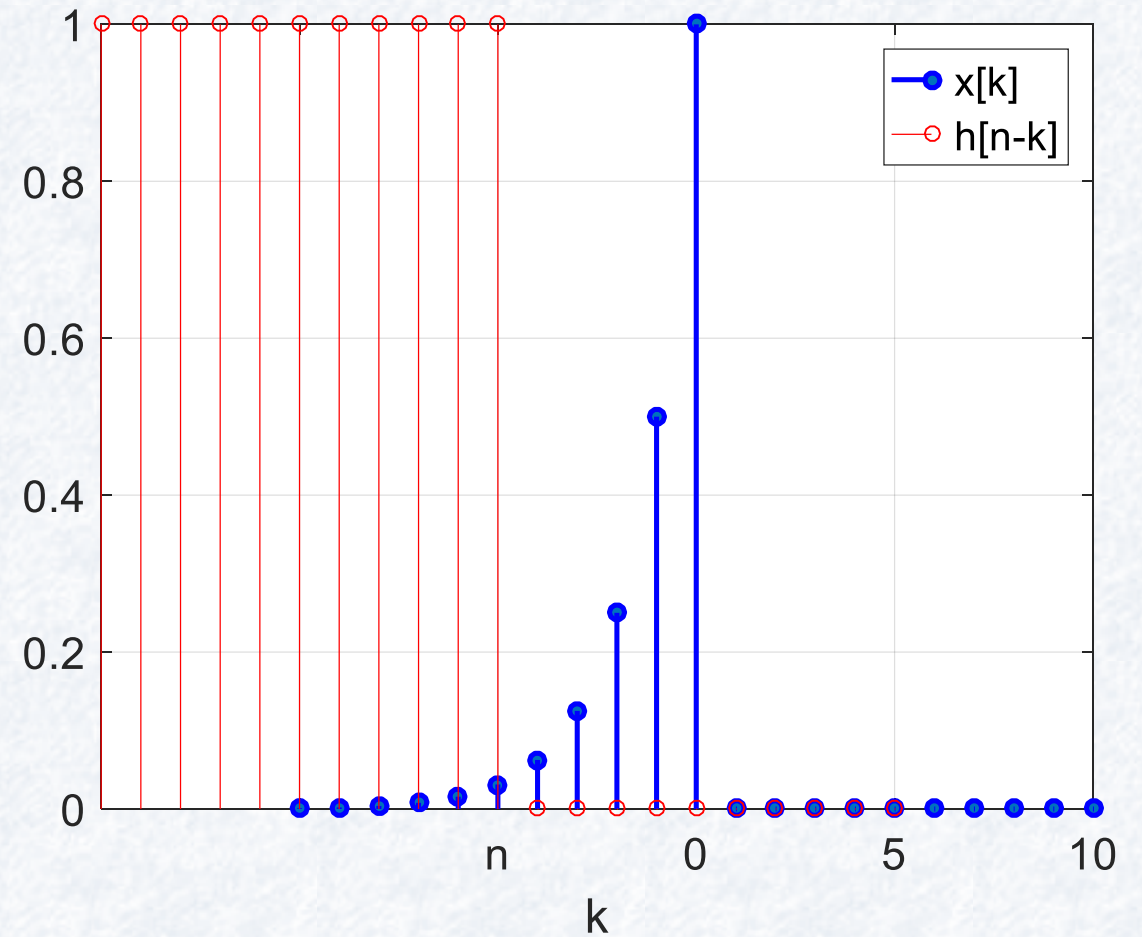
- $x[n] = 2^n u[-n]$

- $h[n] = u[n]$

- $n < 0$  iken

- $y[n] = 2^n \sum_{l=0}^{\infty} \left(\frac{1}{2}\right)^l$

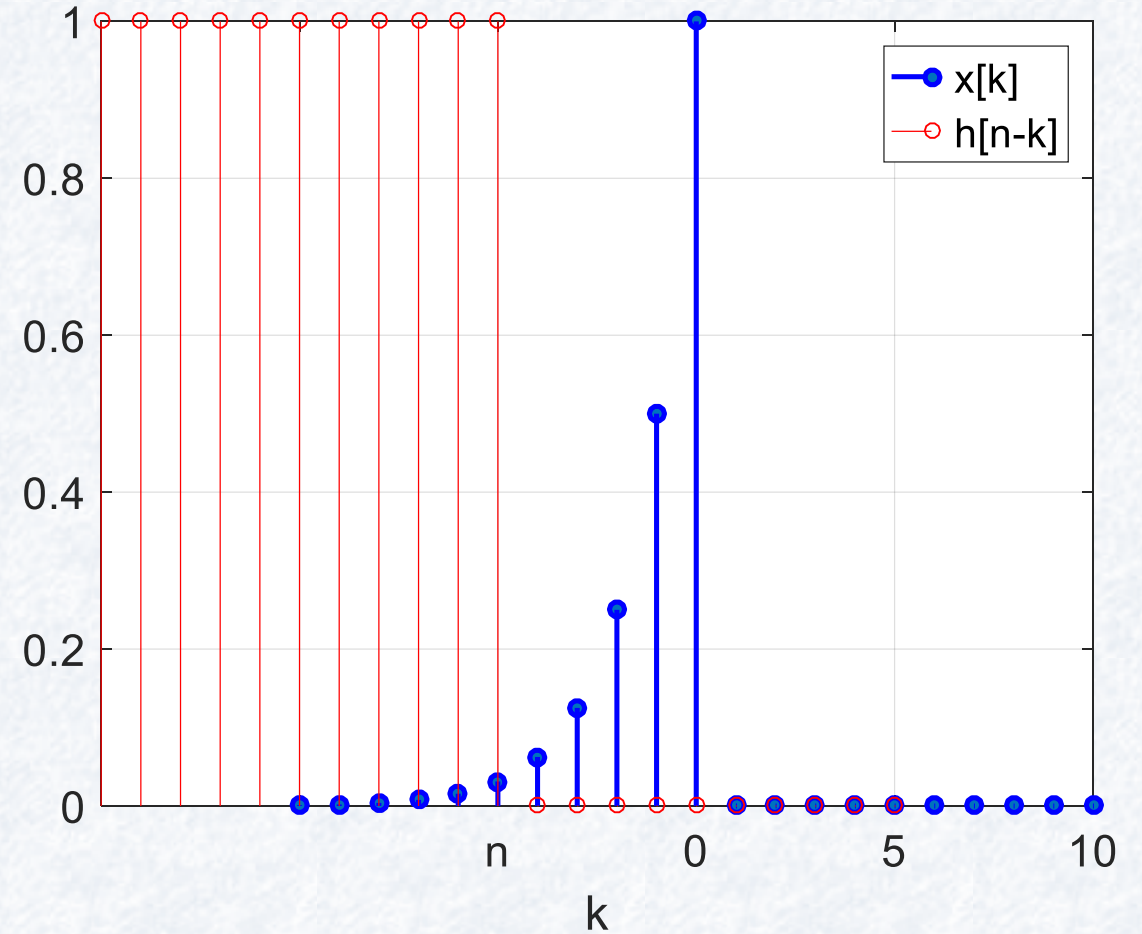
$$\diamond \sum_{l=0}^{\infty} a^l = \begin{cases} \frac{1}{1-a}, & a < 1 \\ \infty, & a \geq 1 \end{cases}$$





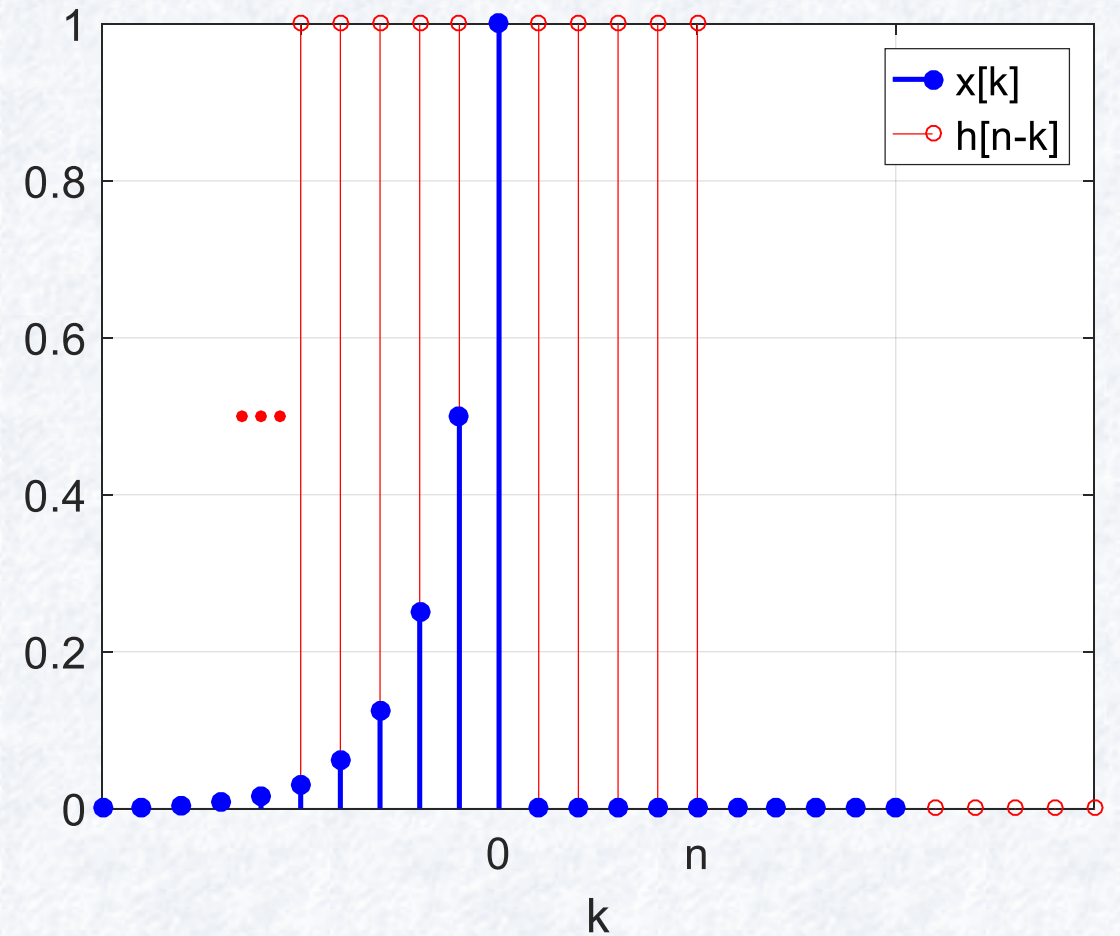
# Örnek 3

- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
- $n < 0$  iken
- $y[n] = 2^n \sum_{l=0}^{\infty} \left(\frac{1}{2}\right)^l$ 
  - ♦  $\sum_{l=0}^{\infty} a^l = \begin{cases} \frac{1}{1-a}, & a < 1 \\ \infty, & a \geq 1 \end{cases}$
- $y[n] = 2^n \frac{1}{1-\frac{1}{2}} = 2^{n+1}$



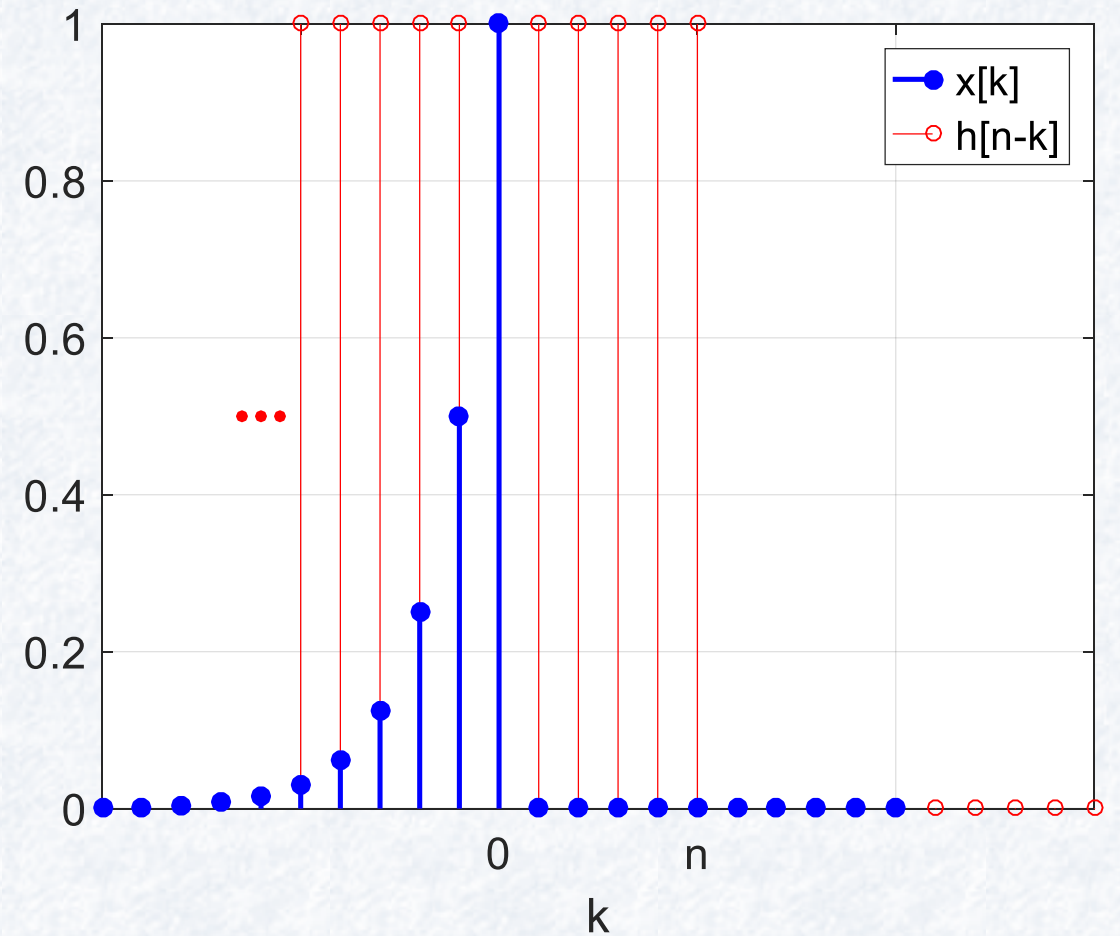
# Örnek 3

- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
- $n \geq 0$  iken
  - ♦ Çakışma,



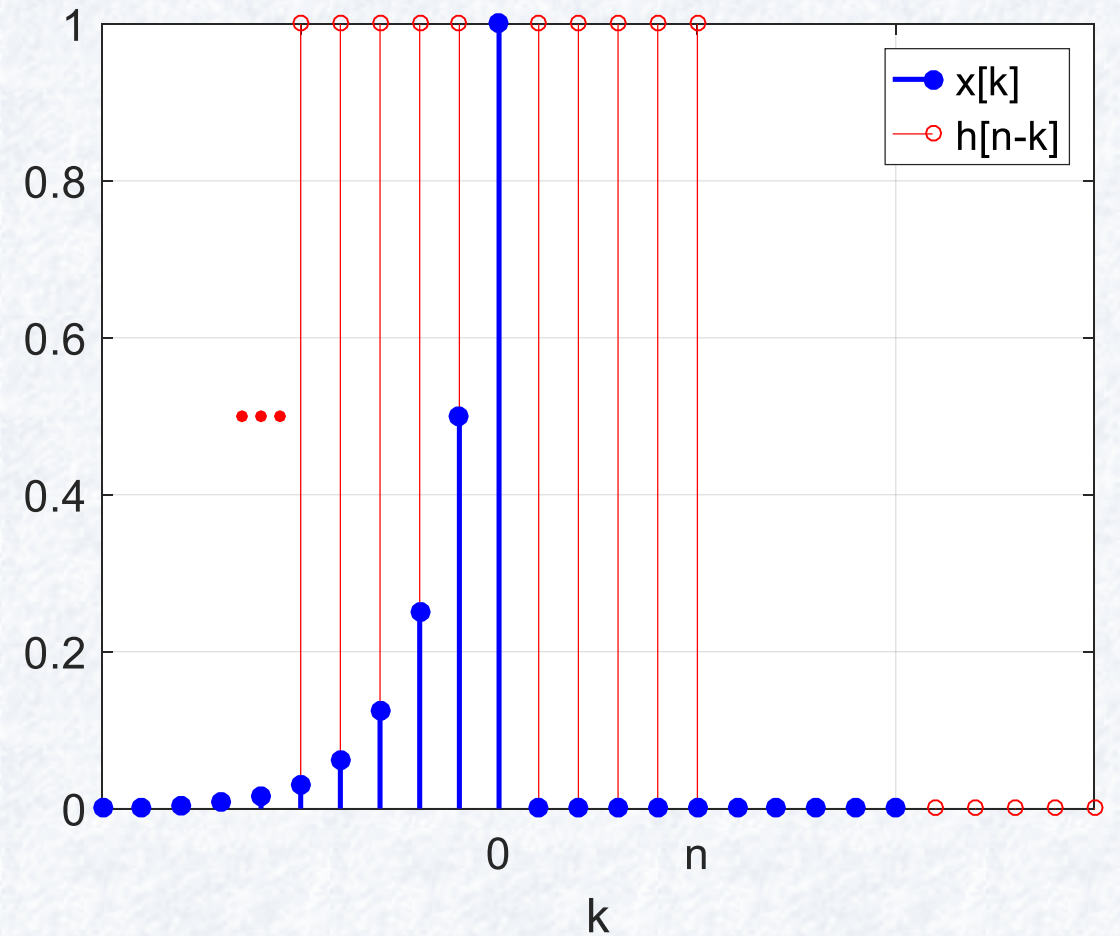
# Örnek 3

- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
- $n \geq 0$  iken
  - ♦ Çakışma,  $-\infty - 0$  arası



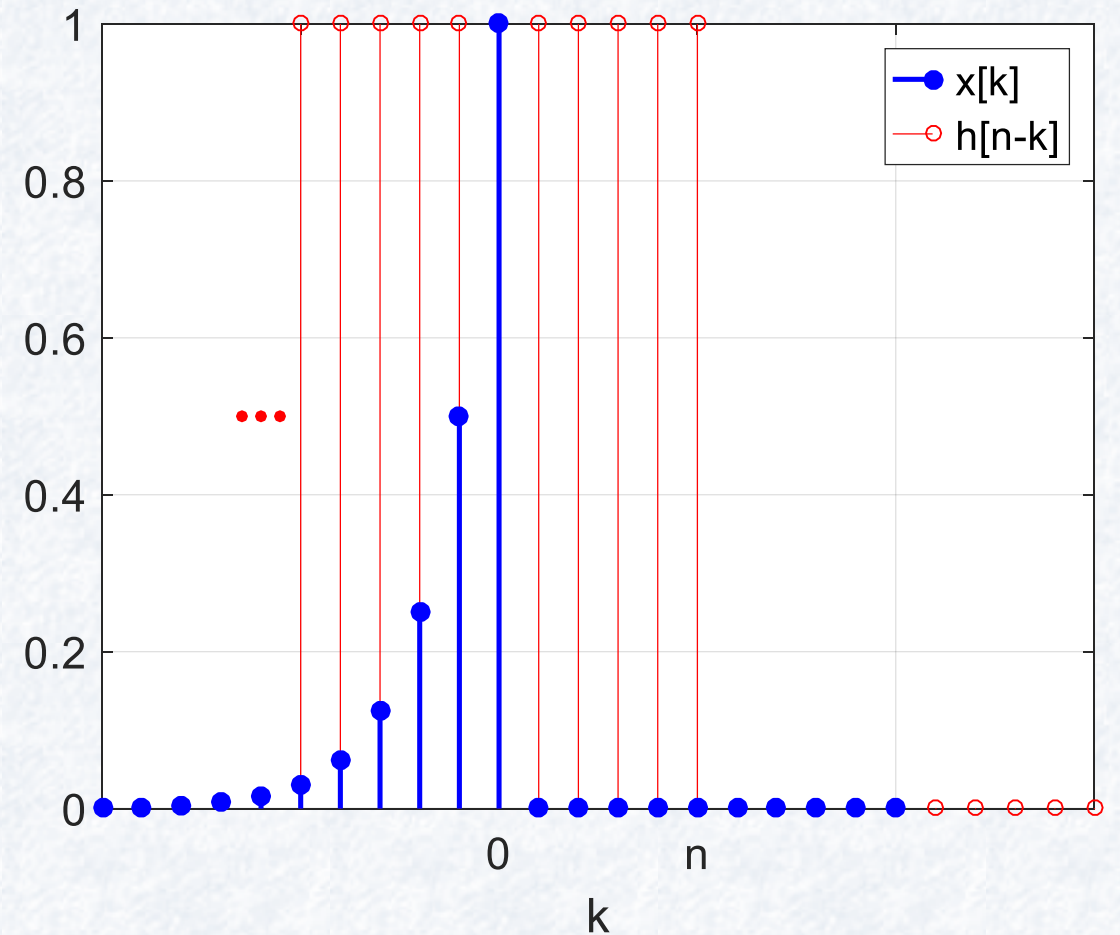
# Örnek 3

- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
- $n \geq 0$  iken
  - ♦ Çakışma,  $-\infty - 0$  arası
- $y[n] = \sum_{k=-\infty}^0 x[k]h[n-k]$



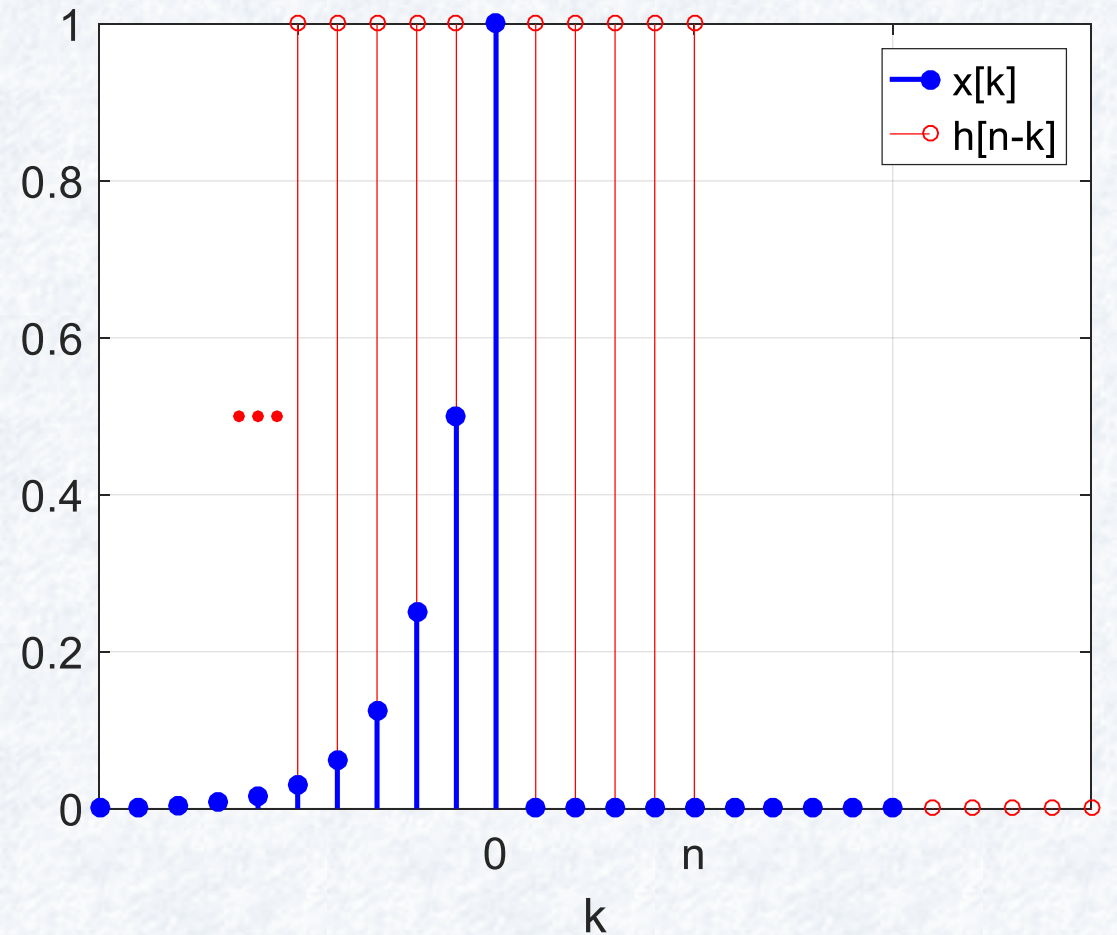
# Örnek 3

- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
- $n \geq 0$  iken
  - ♦ Çakışma,  $-\infty - 0$  arası
- $y[n] = \sum_{k=-\infty}^0 x[k]h[n-k]$
- $y[n] = \sum_{k=-\infty}^0 2^k 1$



# Örnek 3

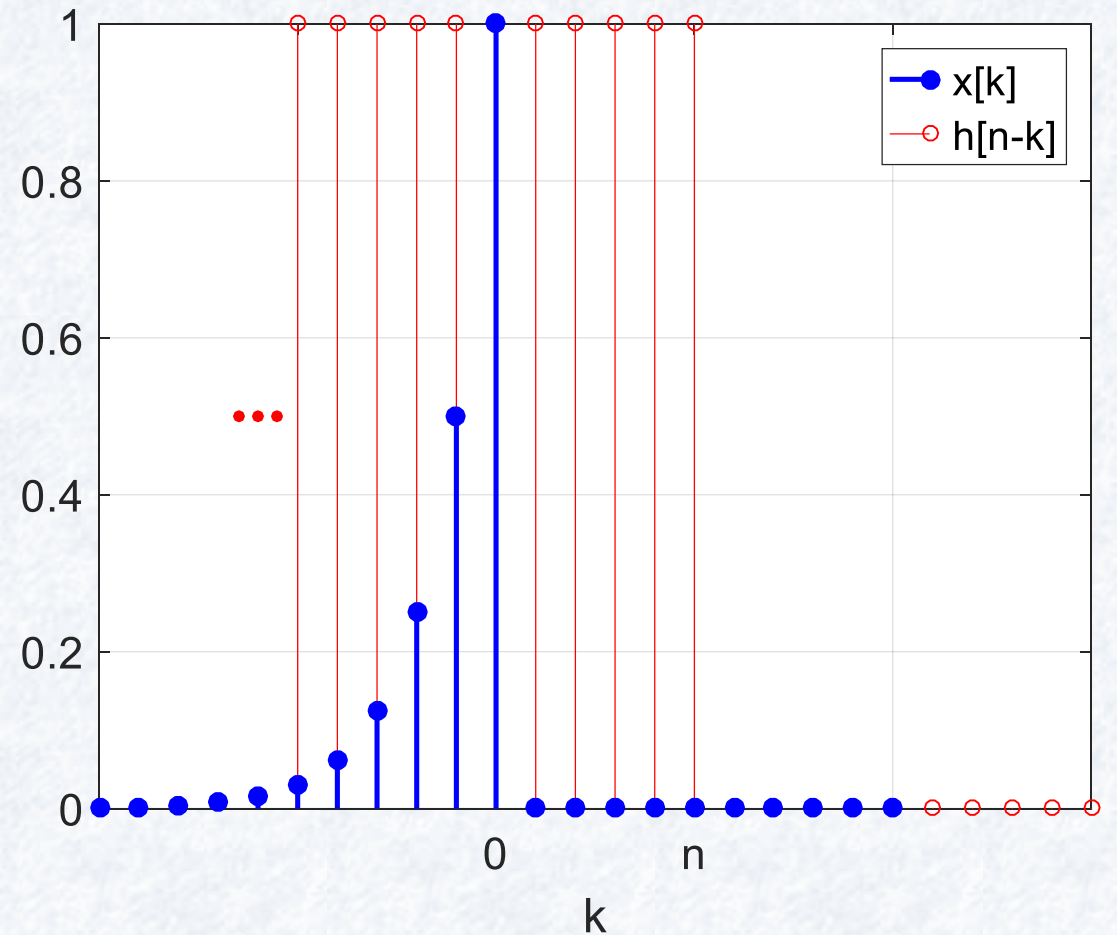
- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
- $n \geq 0$  iken
  - ♦ Çakışma,  $-\infty - 0$  arası
- $y[n] = \sum_{k=-\infty}^0 x[k]h[n-k]$
- $y[n] = \sum_{k=-\infty}^0 2^k \mathbf{1}$ 
  - ♦  $l = -k$





# Örnek 3

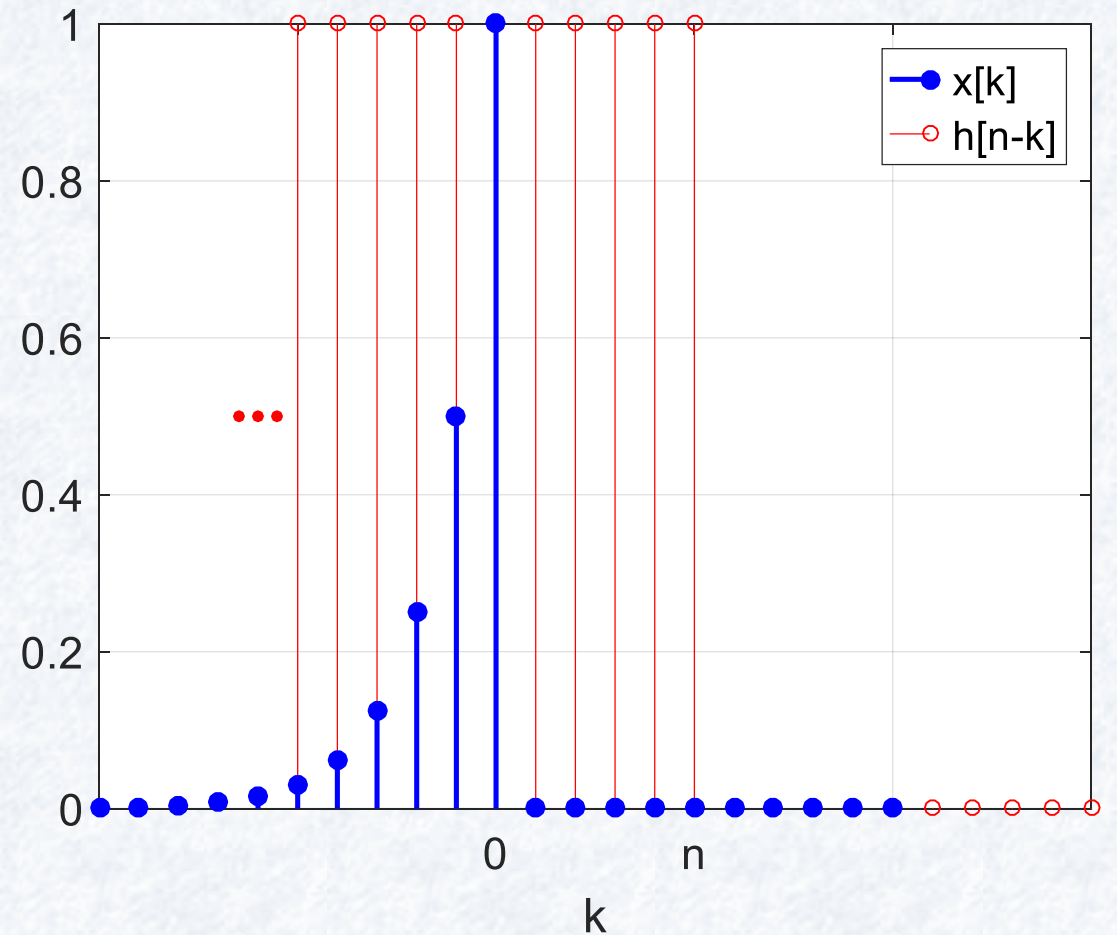
- $x[n] = 2^n u[-n]$
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  - ♦  $l = -k$
- $y[n] = \sum_{l=\infty}^0 2^{-l} \mathbf{1}$





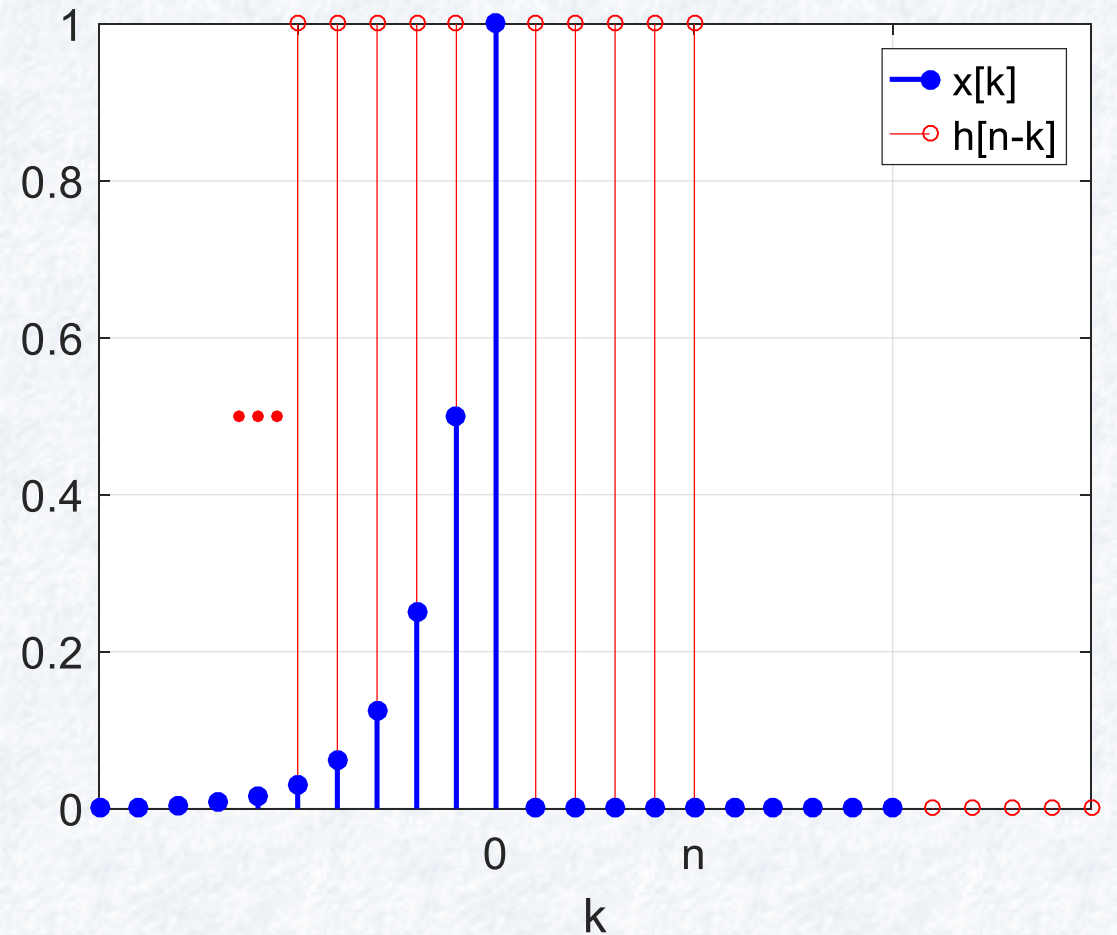
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- $y[n] = \sum_{l=0}^{\infty} \left(\frac{1}{2}\right)^l$



# Örnek 3

- $x[n] = 2^n u[-n]$
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- $y[n] = \sum_{k=-\infty}^0 2^k \mathbf{1}$ 
  - ♦  $l = -k$
- $y[n] = \sum_{l=-\infty}^0 2^{-l} \mathbf{1}$
- $y[n] = \sum_{l=0}^{\infty} \left(\frac{1}{2}\right)^l = \frac{1}{1-\frac{1}{2}} = 2$



## Örnek 3

- $x[n] = 2^n u[-n]$
- $h[n] = u[n]$
- $n < 0$  iken  $y[n] = 2^n \frac{1}{1 - \frac{1}{2}} = 2^{n+1}$
- $n \geq 0$  iken  $y[n] = \sum_{l=0}^{\infty} \left(\frac{1}{2}\right)^l = \frac{1}{1 - \frac{1}{2}} = 2$

## Örnek 3

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- $h[n] = u[n]$
- $n < 0$  iken  $y[n] = 2^n \frac{1}{1 - \frac{1}{2}} = 2^{n+1}$
- $n \geq 0$  iken  $y[n] = \sum_{l=0}^{\infty} \left(\frac{1}{2}\right)^l = \frac{1}{1 - \frac{1}{2}} = 2$
- $y[n] = 2^{n+1} u[-n - 1] + 2u[n]$

# Konvolüsyon Özellikleri

- Değişme Özelliği
  - ♦  $x[n] * h[n] = h[n] * x[n]$

# Konvolüsyon Özellikleri

- Değişme Özelliği

- ◆  $x[n] * h[n] = h[n] * x[n]$

- Dağılma Özelliği

- ◆  $x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$



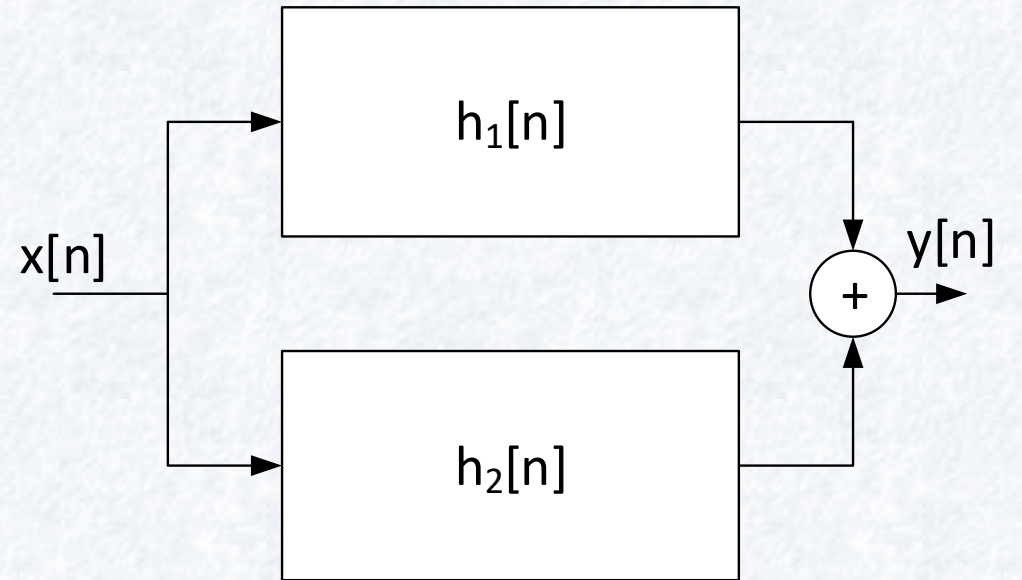
# Konvolüsyon Özellikleri

- Değişme Özelliği
  - ♦  $x[n] * h[n] = h[n] * x[n]$
- Dağılma Özelliği
  - ♦  $x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$
- Birleşme Özelliği
  - ♦  $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$



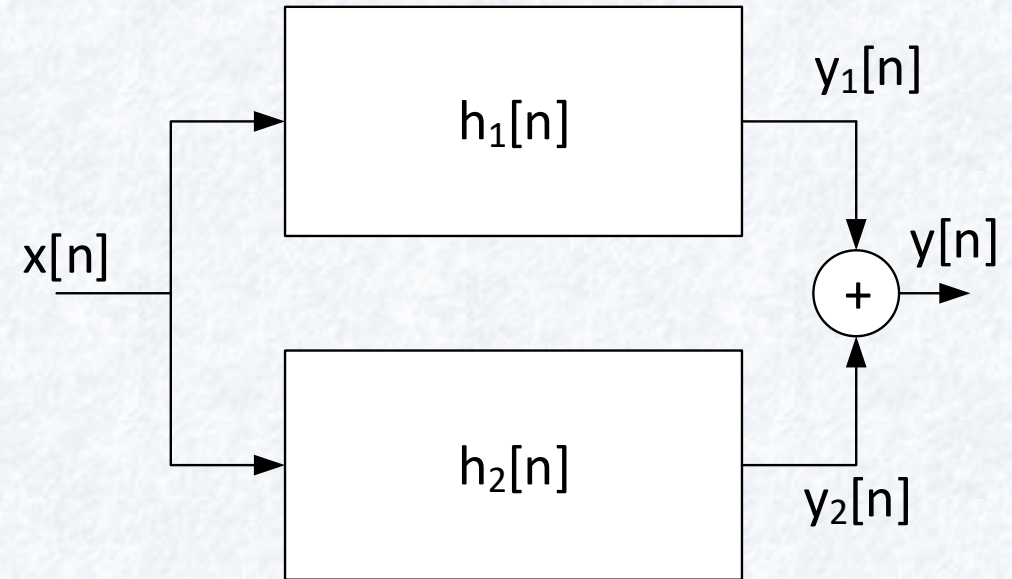
# Konvolüsyon Özellikleri

- Dağılma Özelliği
- $y[n] = ?$



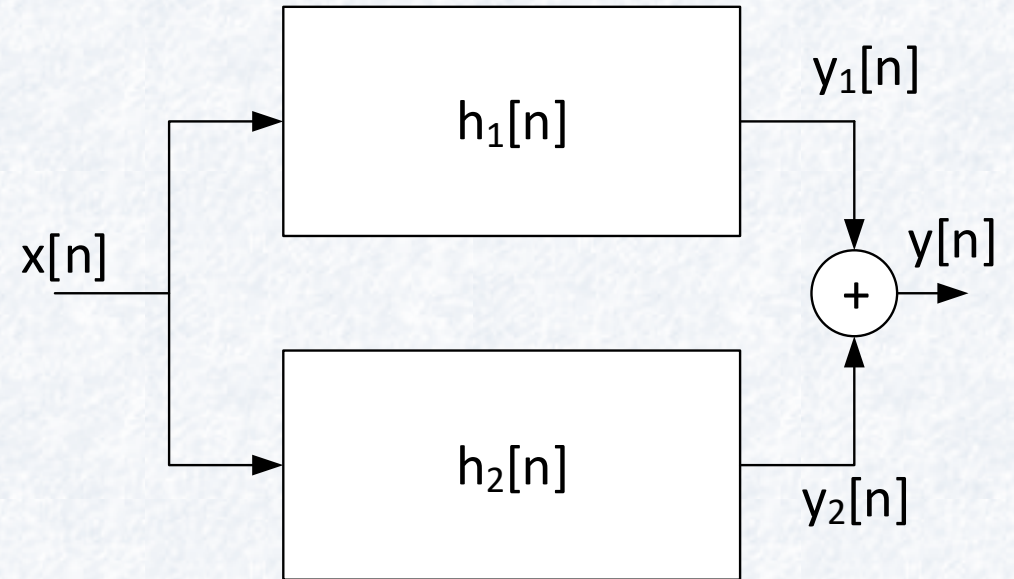
# Konvolüsyon Özellikleri

- Dağılma Özelliği
- $y[n] = ?$
- $y[n] = y_1[n] + y_2[n]$
- $y_1[n] = ?$
- $y_2[n] = ?$



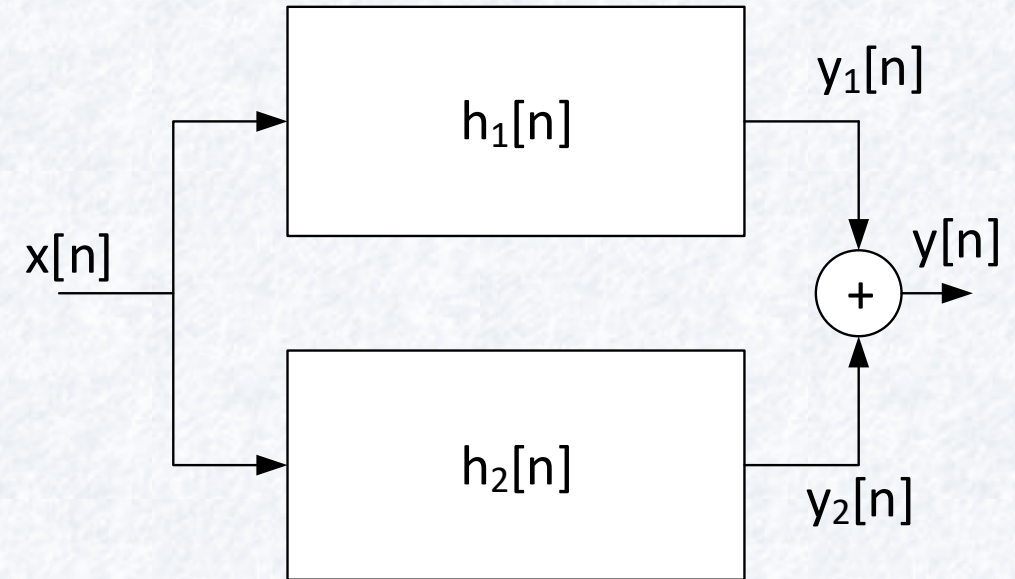
# Konvolüsyon Özellikleri

- Dağılma Özelliği
- $y[n] = ?$
- $y[n] = y_1[n] + y_2[n]$
- $y_1[n] = x[n] * h_1[n]$
- $y_2[n] = x[n] * h_2[n]$



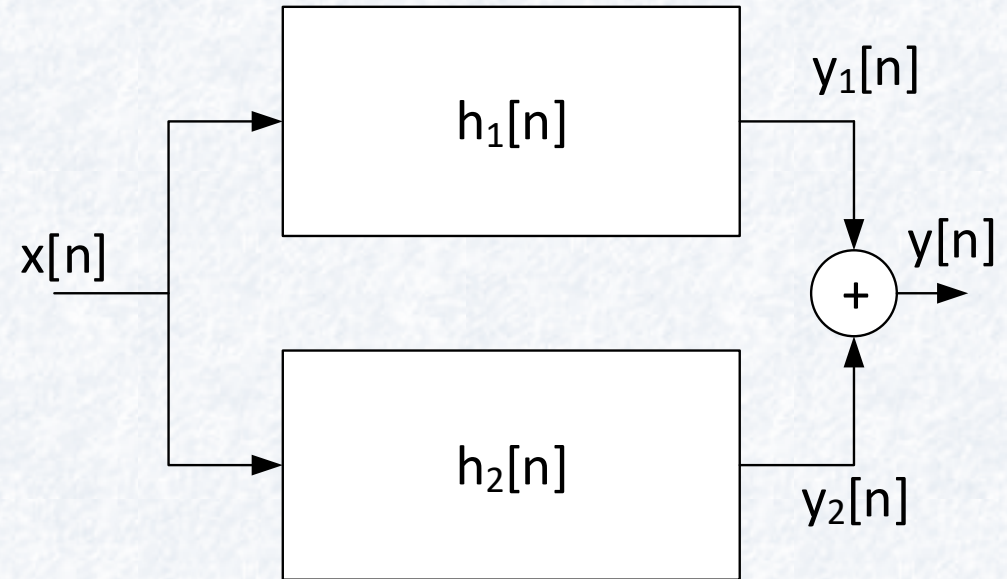
# Konvolüsyon Özellikleri

- Dağılma Özelliği
- $y[n] = ?$
- $y[n] = y_1[n] + y_2[n]$
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- $y_2[n] = x[n] * h_2[n]$
- $y[n] = x[n] * h_1[n] + x[n] * h_2[n]$



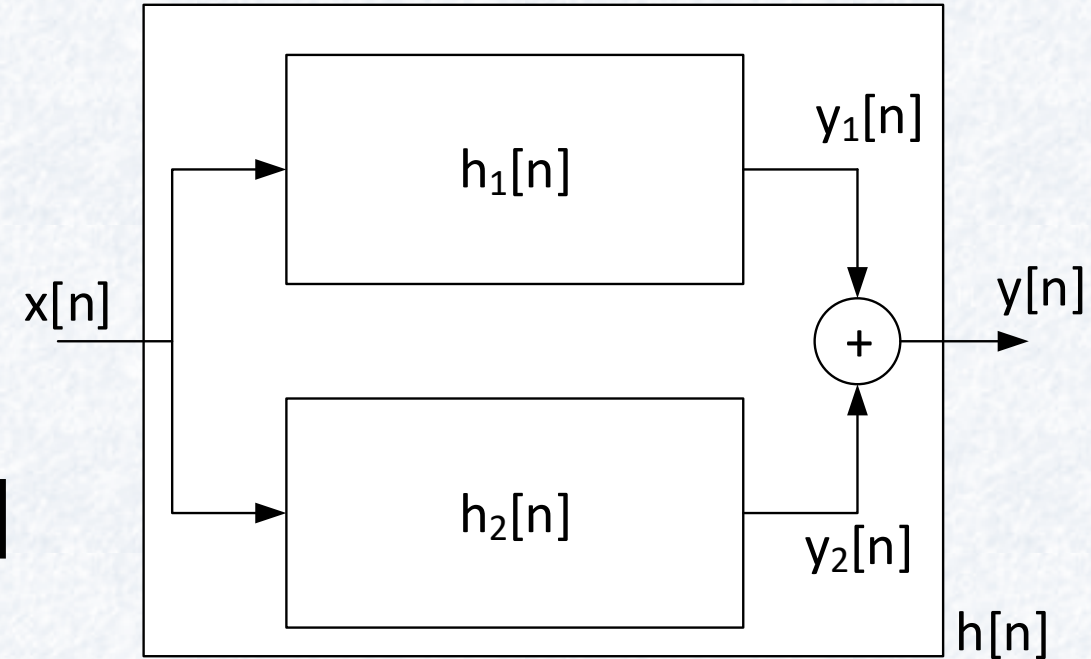
# Konvolüsyon Özellikleri

- Dağılma Özelliği
- $y[n] = ?$
- $y[n] = y_1[n] + y_2[n]$
- $y_1[n] = x[n] * h_1[n]$
- $y_2[n] = x[n] * h_2[n]$
- $y[n] = x[n] * h_1[n] + x[n] * h_2[n]$



# Konvolüsyon Özellikleri

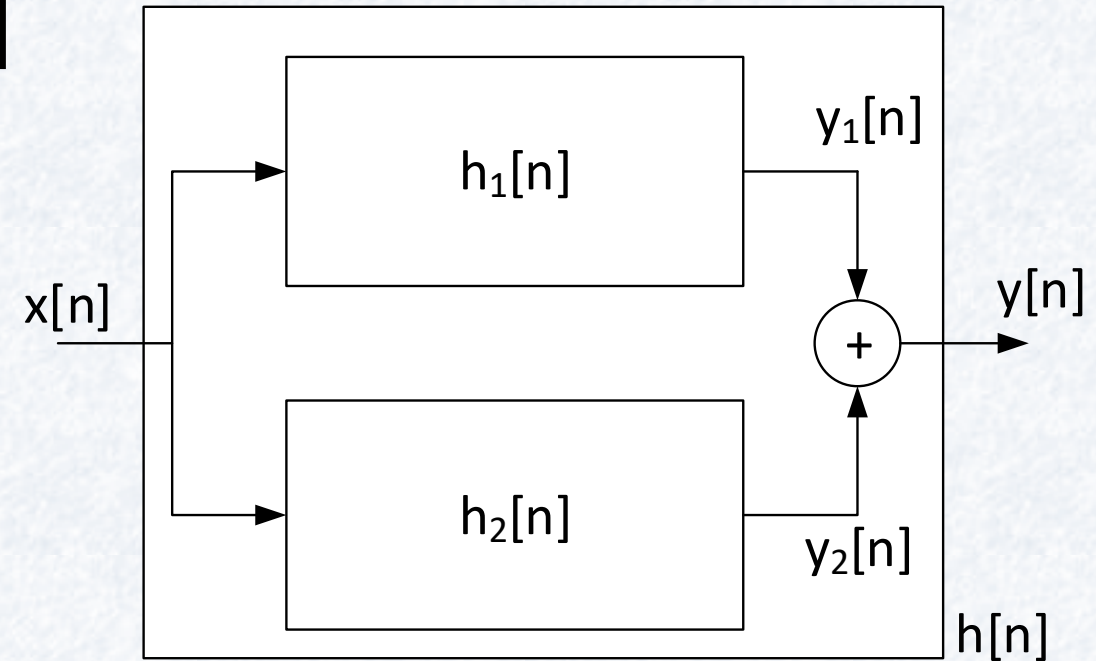
- Dağılma Özelliği
- $y[n] = ?$
- $y[n] = y_1[n] + y_2[n]$
- $y_1[n] = x[n] * h_1[n]$
- $y_2[n] = x[n] * h_2[n]$
- $y[n] = x[n] * h_1[n] + x[n] * h_2[n]$
- $y[n] = x[n] * (h_1[n] + h_2[n])$
- $h[n] = ?$





# Konvolüsyon Özellikleri

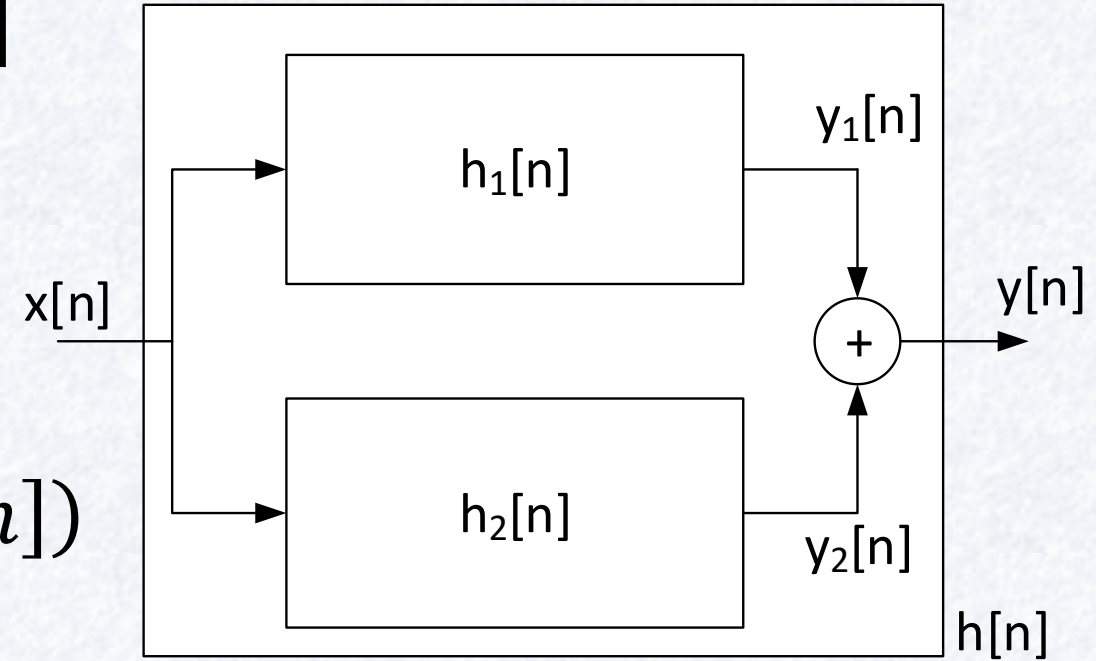
- Dağılma Özelliği
- $y[n] = x[n] * h_1[n] + x[n] * h_2[n]$
- $y[n] = x[n] * (h_1[n] + h_2[n])$
- $h[n] = ?$
- $y[n] = x[n] * h[n]$





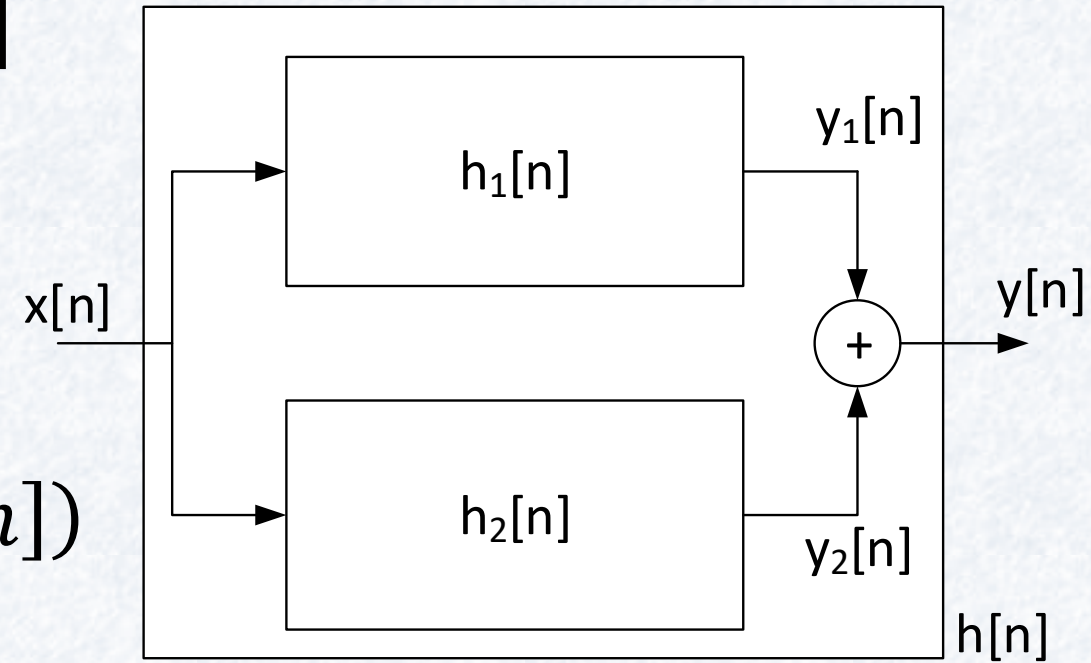
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- Dağılma Özelliği
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- $y[n] = x[n] * h[n]$
- $x[n] * h[n] = x[n] * (h_1[n] + h_2[n])$



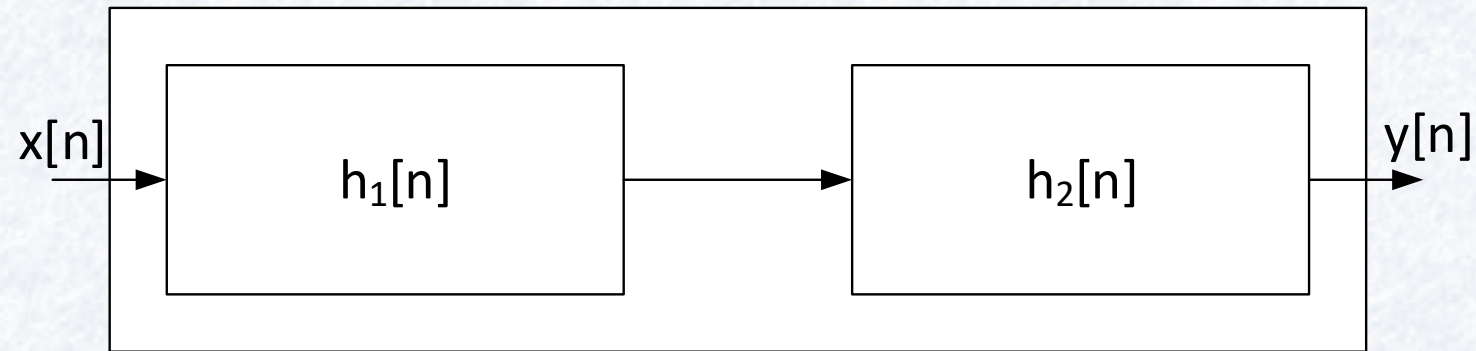
# Konvolüsyon Özellikleri

- Dağılma Özelliği
- $y[n] = x[n] * h_1[n] + x[n] * h_2[n]$
- $y[n] = x[n] * (h_1[n] + h_2[n])$
- $h[n] = ?$
- $y[n] = x[n] * h[n]$
- $x[n] * h[n] = x[n] * (h_1[n] + h_2[n])$
- $h[n] = (h_1[n] + h_2[n])$



# Konvolüsyon Özellikleri

- Birleşme Özelliği
  - ♦  $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$
- $y[n] = ?$



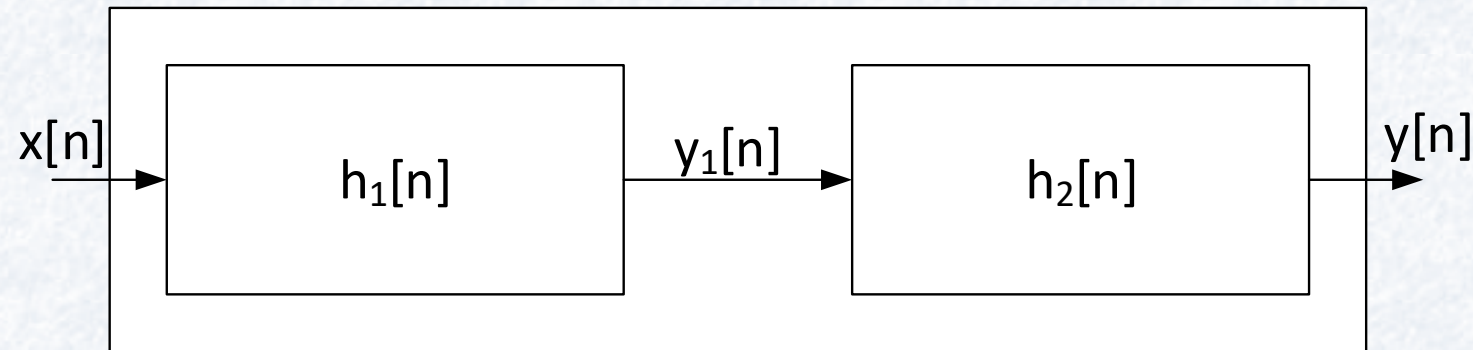
# Konvolüsyon Özellikleri

- Birleşme Özelliği

- ♦  $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$

- $y[n] = y_1[n] * h_2[n]$

- $y_1[n] = ?$



# Konvolüsyon Özellikleri

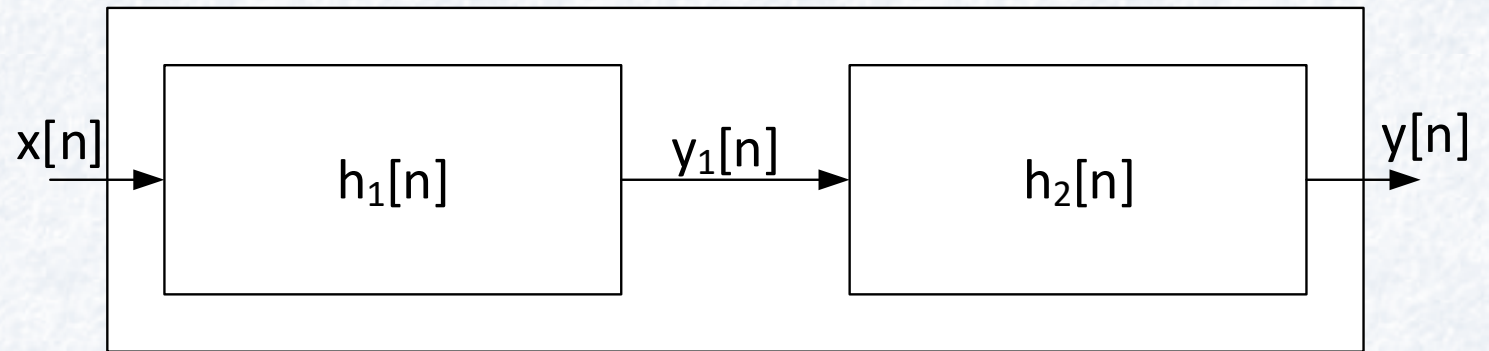
- Birleşme Özelliği

- ◆  $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$

- $y[n] = y_1[n] * h_2[n]$

- $y_1[n] = x[n] * h_1[n]$

- $y[n] = ?$



# Konvolüsyon Özellikleri

- Birleşme Özelliği

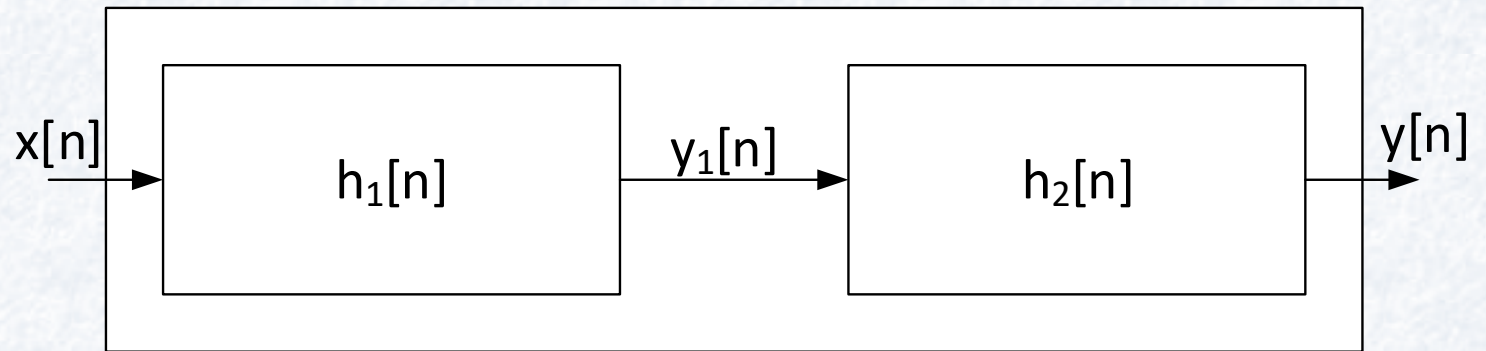
- ♦  $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$

- $y[n] = y_1[n] * h_2[n]$

- $y_1[n] = x[n] * h_1[n]$

- $y[n] = x[n] * h_1[n] * h_2[n]$

- $h[n] = ?$





# Konvolüsyon Özellikleri

- Birleşme Özelliği

- ♦  $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$

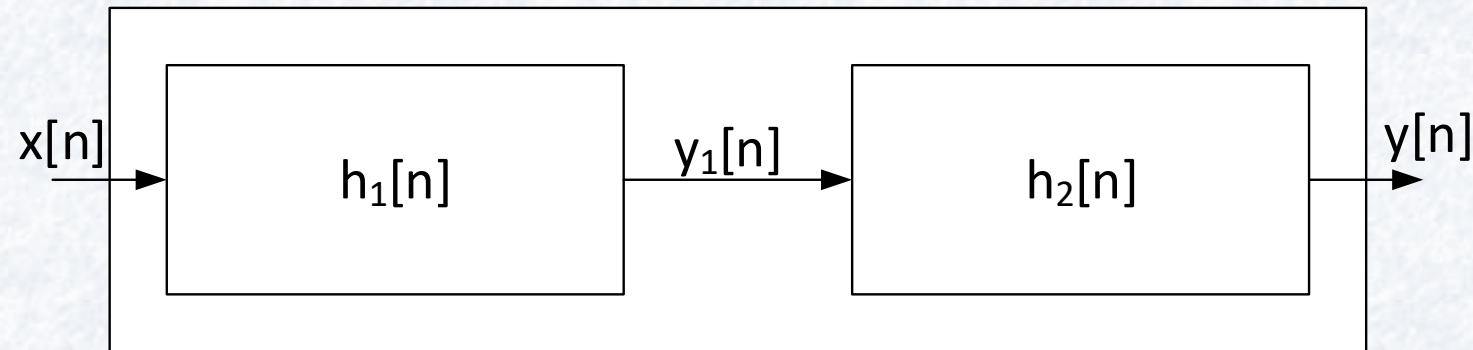
- $y[n] = y_1[n] * h_2[n]$

- $y_1[n] = x[n] * h_1[n]$

- $y[n] = x[n] * h_1[n] * h_2[n]$

- $h[n] = ?$

- $y[n] = x[n] * h[n]$





# Konvolüsyon Özellikleri

- Birleşme Özelliği

- ♦  $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$

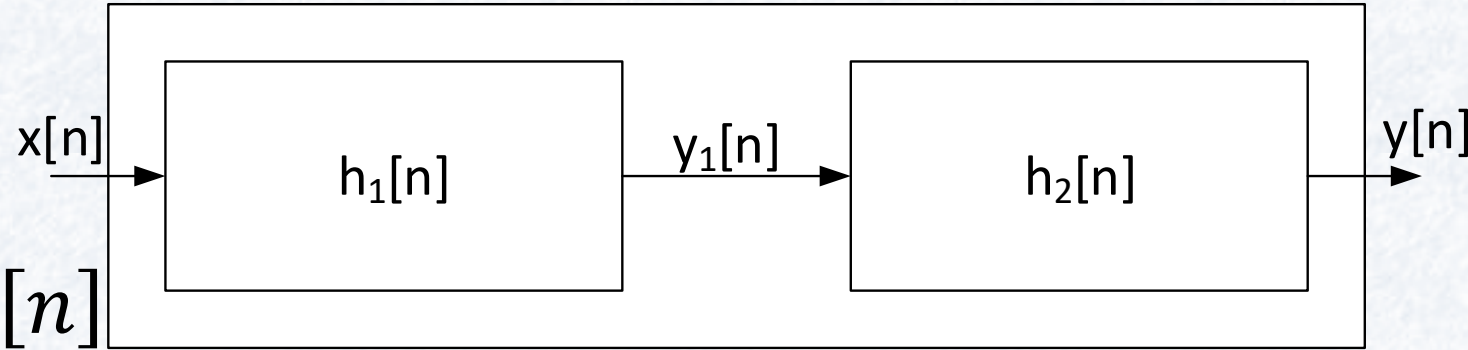
- $y[n] = y_1[n] * h_2[n]$

- $y_1[n] = x[n] * h_1[n]$

- $y[n] = x[n] * h_1[n] * h_2[n]$

- $h[n] = ?$

- $y[n] = x[n] * h[n] = x[n] * h_1[n] * h_2[n]$



# Konvolüsyon Özellikleri

- Birleşme Özelliği

- ♦  $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$

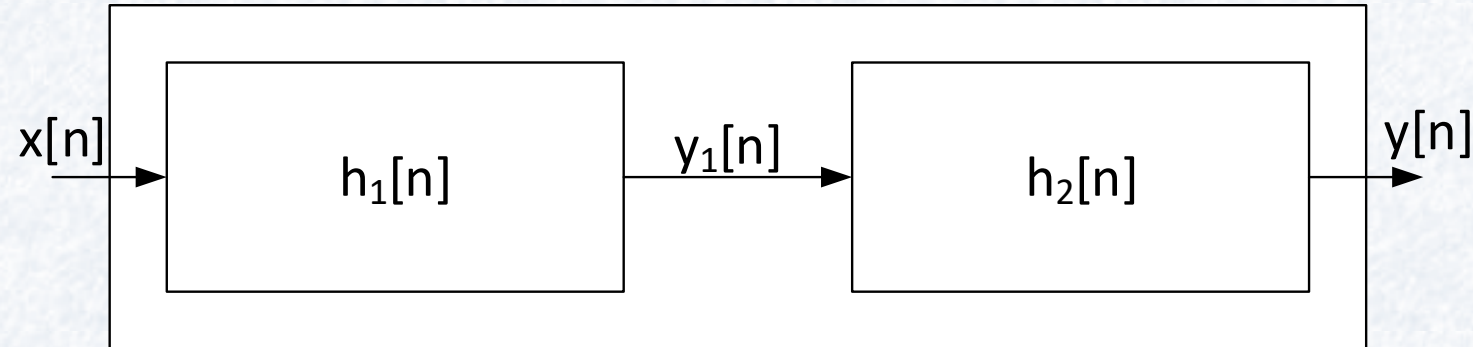
- $y[n] = y_1[n] * h_2[n]$

- $y_1[n] = x[n] * h_1[n]$

- $y[n] = x[n] * h_1[n] * h_2[n]$

- $y[n] = x[n] * h[n] = x[n] * h_1[n] * h_2[n]$

- $h[n] = h_1[n] * h_2[n]$



# Sistem Özellikleri

- Hafızalılık
- Hafızasız
  - ♦ Sistem çıkışının, giriş işaretinin zamanın sadece o andaki bilgisine bağlı olması
- Hafızalı
  - ♦ Sistem çıkışının, giriş işaretinin ötelenmiş hallerine bağlı olması
- $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$
- $y[n] = \cdots + h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1] + \cdots$

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- Hafızalılık
- Hafızasız
  - ♦ Sistem çıkışının, giriş işaretinin zamanın sadece o andaki bilgisine bağlı olması
- $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$
- $y[n] = \cdots + h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1] + \cdots$ 
  - ♦ Hafızasız:  $y[n]$ , sadece  $x[n]$ 'ye bağlı olması

# Sistem Özellikleri

- **Hafızalılık**

- $y[n] = \cdots + h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1] + \cdots$

- ♦ Hafızasız:  $y[n]$ , sadece  $x[n]$ 'ye bağlı olması

- $y[n] = \underbrace{\cdots + h[-1]x[n+1]}_0 + h[0]x[n] + \underbrace{h[1]x[n-1] + \cdots}_0$

- $n \neq 0$  iken  $h[n] = 0$  olursa Hafızasız.



# Sistem Özellikleri

- **Hafızalılık**
- $y[n] = \cdots + h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1] + \cdots$ 
  - ♦ Hafızasız:  $y[n]$ , sadece  $x[n]$ 'ye bağlı olması
- $n \neq 0$  iken  $h[n] = 0$  olursa Hafızasız.

# Sistem Özellikleri

- **Hafızalılık**
- $y[n] = \cdots + h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1] + \cdots$ 
  - ♦ Hafızasız:  $y[n]$ , sadece  $x[n]$ 'ye bağlı olması
- $\forall n \neq 0$  iken  $h[n] = 0$  olursa Hafızasız.
  - ♦  $h[n] = A\delta[n]$



# Sistem Özellikleri

- Hafızalılık

- $y[n] = \dots + h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1] + \dots$

- ♦ Hafızasız:  $y[n]$ , sadece  $x[n]$ 'ye bağlı olması

- $\forall n \neq 0$  iken  $h[n] = 0$  olursa Hafızasız.

- ♦  $h[n] = A\delta[n]$

- $\exists n \neq 0$  iken  $h[n] \neq 0$  olursa Hafızalı.

- ♦  $h[n] \neq A\delta[n]$

## Örnek 4

- $h[n] = a^n u[n]$ , Hafızalı mıdır?

## Örnek 4

- $h[n] = a^n u[n]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n] = ?$

## Örnek 4

- $h[n] = a^n u[n]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n] \neq 0$

## Örnek 4

- $h[n] = a^n u[n]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n] \neq 0$ 
  - ♦  $n = 1$  iken  $h[n] = a$
  - ♦  $n = 2$  iken  $h[n] = a^2$
  - ♦  $\vdots$

# Örnek 4

- $h[n] = a^n u[n]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n] \neq 0$ 
  - ♦  $n = 1$  iken  $h[n] = a$
  - ♦  $n = 2$  iken  $h[n] = a^2$
  - ♦  $\vdots$
- Hafızalı



## Örnek 5

- $h[n] = \delta[n - n_0]$ , Hafızalı mıdır?

## Örnek 5

- $h[n] = \delta[n - n_0]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n] = ?$

## Örnek 5

- $h[n] = \delta[n - n_0]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n]$
- $h[n] = \delta[n - n_0] = \begin{cases} 0, & n \neq n_0 \\ 1, & n = n_0 \end{cases}$

# Örnek 5

- $h[n] = \delta[n - n_0]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n]$
- $h[n] = \delta[n - n_0] = \begin{cases} 0, & n \neq n_0 \\ 1, & n = n_0 \end{cases}$ 
  - ♦  $n_0 = 0$  ise

# Örnek 5

- $h[n] = \delta[n - n_0]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n]$
- $h[n] = \delta[n - n_0] = \begin{cases} 0, & n \neq n_0 \\ 1, & n = n_0 \end{cases}$ 
  - ♦  $n_0 = 0$  ise Hafızasız
  - ♦  $n_0 \neq 0$  ise

# Örnek 5

- $h[n] = \delta[n - n_0]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n]$
- $h[n] = \delta[n - n_0] = \begin{cases} 0, & n \neq n_0 \\ 1, & n = n_0 \end{cases}$ 
  - ♦  $n_0 = 0$  ise Hafızasız
  - ♦  $n_0 \neq 0$  ise Hafızalı



## Örnek 6

- $h[n] = u[n]$ , Hafızalı mıdır?

## Örnek 6

- $h[n] = u[n]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n] = ?$

## Örnek 6

- $h[n] = u[n]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n] \neq 0$

## Örnek 6

- $h[n] = u[n]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n] \neq 0$ 
  - ♦  $n = 1$  iken  $h[n] = 1$
  - ♦  $n = 2$  iken  $h[n] = 1$
  - ♦  $\vdots$

## Örnek 6

- $h[n] = u[n]$ , Hafızalı mıdır?
- $n \neq 0$  iken  $h[n] \neq 0$ 
  - ♦  $n = 1$  iken  $h[n] = 1$
  - ♦  $n = 2$  iken  $h[n] = 1$
  - ♦  $\vdots$
- Hafızalı

# Sistem Özellikleri

- Nedensellik
- $y[n] = \cdots + h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1] + \cdots$ 
  - ♦ Nedensel:  $y[n]$ , sadece



- Nedensellik

- $y[n] = \cdots + h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1] + \cdots$

- ♦ Nedensel:  $y[n]$ , sadece  $x[n]$  ve/veya  $x[n-k]$  'ya bağlı olması

- $y[n] = \underbrace{\cdots + h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1] + \cdots}_0$

- $\forall n < 0$  iken  $h[n] = 0$  ise Nedensel.

- Nedensellik

- $y[n] = \dots + h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1] + \dots$

- ♦ Nedensel:  $y[n]$ , sadece  $x[n]$  ve/veya  $x[n-k]$  'ya bağlı olması

- $y[n] = \underbrace{\dots + h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1] + \dots}_0$

- $\forall n < 0$  iken  $h[n] = 0$  ise Nedensel.

- $\exists n < 0$  iken  $h[n] \neq 0$  ise Nedensel değil.

# Örnek 7

- $h[n] = a^n u[n]$ , Nedensel midir?
  - ♦ Hafızalı

# Örnek 7

- $h[n] = a^n u[n]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = ?$

# Örnek 7

- $h[n] = a^n u[n]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = 0$

# Örnek 7

- $h[n] = a^n u[n]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = 0$ 
  - ♦  $n < 0$  iken  $u[n] = 0$



# Örnek 7

- $h[n] = a^n u[n]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = 0$ 
  - ♦  $n < 0$  iken  $u[n] = 0$
- Nedensel

## Örnek 8

- $h[n] = \delta[n - n_0]$ , Nedensel midir?
  - ♦ Hafızalı

## Örnek 8

- $h[n] = \delta[n - n_0]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = ?$

## Örnek 8

- $h[n] = \delta[n - n_0]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = ?$
- $$\delta[n - n_0] = \begin{cases} 0, & n \neq n_0 \\ 1, & n = n_0 \end{cases}$$

# Örnek 8

- $h[n] = \delta[n - n_0]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = ?$
- $\delta[n - n_0] = \begin{cases} 0, & n \neq n_0 \\ 1, & n = n_0 \end{cases}$
- $n = n_0 < 0$  iken  $h[n] = 1$ 
  - ♦ Nedensel değil.

# Örnek 8

- $h[n] = \delta[n - n_0]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = ?$
- $\delta[n - n_0] = \begin{cases} 0, & n \neq n_0 \\ 1, & n = n_0 \end{cases}$
- $n = n_0 < 0$  iken  $h[n] = 1$ 
  - ♦ Nedensel değil.
- $n = n_0 \geq 0$  iken  $h[n] = 1$ 
  - ♦ Nedensel.



## Örnek 9

- $h[n] = u[n]$ , Nedensel midir?
  - ♦ Hafızalı

# Örnek 9

- $h[n] = u[n]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = ?$

# Örnek 9

- $h[n] = u[n]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = 0$

# Örnek 9

- $h[n] = u[n]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = 0$ 
  - ♦  $n < 0$  iken  $u[n] = 0$

# Örnek 9

- $h[n] = u[n]$ , Nedensel midir?
  - ♦ Hafızalı
- $n < 0$  iken  $h[n] = 0$ 
  - ♦  $n < 0$  iken  $u[n] = 0$
- Nedensel

# Sistem Özellikleri

- Kararlılık
- $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$  ise kararlı.



- **Kararlılık**
- $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$  ise Kararlı.
- $\sum_{n=-\infty}^{\infty} |h[n]| \rightarrow \infty$  ise Kararsız.

# Örnek 10

- $h[n] = a^n u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel

# Örnek 10

- $h[n] = a^n u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$

# Örnek 10

- $h[n] = a^n u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=-\infty}^{\infty} a^n u[n] = ?$

# Örnek 10

- $h[n] = a^n u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=0}^{\infty} a^n = \begin{cases} \infty, & a \geq 1 \end{cases}$

# Örnek 10

- $h[n] = a^n u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=0}^{\infty} a^n = \begin{cases} \infty, & a \geq 1 \\ \frac{1}{1-a} & a < 1 \end{cases}$



# Örnek 10

- $h[n] = a^n u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=0}^{\infty} a^n = \begin{cases} \infty, & a \geq 1 \\ \frac{1}{1-a} & a < 1 \end{cases}$
- $a \geq 1$  iken Kararsız

# Örnek 10

- $h[n] = a^n u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=0}^{\infty} a^n = \begin{cases} \infty, & a \geq 1 \\ \frac{1}{1-a} & a < 1 \end{cases}$
- $a \geq 1$  iken Kararsız
- $a < 1$  iken Kararlı

# Örnek 11

- $h[n] = \delta[n - n_0]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel

# Örnek 11

- $h[n] = \delta[n - n_0]$ , Kararlı mıdır?
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  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$

# Örnek 11

- $h[n] = \delta[n - n_0]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=-\infty}^{\infty} \delta[n - n_0] =$

# Örnek 11

- $h[n] = \delta[n - n_0]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=-\infty}^{\infty} \delta[n - n_0] = 1$



# Örnek 11

- $h[n] = \delta[n - n_0]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=-\infty}^{\infty} \delta[n - n_0] = 1 < \infty$
- Kararlı

# Örnek 12

- $h[n] = u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel

# Örnek 12

- $h[n] = u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$

# Örnek 12

- $h[n] = u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=-\infty}^{\infty} u[n] =$

# Örnek 12

- $h[n] = u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=-\infty}^{\infty} u[n] = \sum_{n=0}^{\infty} 1$

# Örnek 12

- $h[n] = u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=-\infty}^{\infty} u[n] = \sum_{n=0}^{\infty} 1 = \infty$



# Örnek 12

- $h[n] = u[n]$ , Kararlı mıdır?
  - ♦ Hafızalı
  - ♦ Nedensel
- $\sum_{n=-\infty}^{\infty} |h[n]| = ?$
- $\sum_{n=-\infty}^{\infty} u[n] = \sum_{n=0}^{\infty} 1 = \infty$
- Kararsız