IE453 - Energy Systems Planning

Homework 4 - Report

Group 15

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Q1.

d: Demand points, d = 1,...,7

f: Facilities, f = 1, 2, 3

Parameters:

D_{df} : Manhattan distance between demand point d and facility f

C_f : Cost of opening the facility f

U : Unit cost of distance

Decision Variables:

a_{df} : 1 if demand point d is covered by facility f, 0 o/w

x_f : 1 if facility f is opened

Mathematical Model:

$$\min \quad \sum_f x_f * C_f + \sum_{d,f} a_{df} * D_{df} * U$$

$$\text{s.t.} \quad \sum_f x_f \geq 1$$

$$\sum_f a_{df} = 1 \qquad \forall d$$

$$x_f \geq a_{df} \qquad \forall d$$

Xpress code:

```
model "FLP"
 uses "mmxprs"

 declarations
  ND = 7
  NF = 3
  UNIT_COST = 1
  DEMANDS = 1..ND
  FACILITIES = 1..NF

  DISTANCE: array(DEMANDS, FACILITIES) of integer
  COST_f: array(FACILITIES) of integer

  a_df: array(DEMANDS, FACILITIES) of mpvar
  x_f: array(FACILITIES) of mpvar

 end-declarations

 initializations from 'project.data'
  DISTANCE COST_f
 end-initializations

 ! Objective: minimize cost
 TotalCost:= sum(f in FACILITIES) COST_f(f) * x_f(f) + sum(d in DEMANDS, f in FACILITIES) a_df(d, f) * DISTANCE(d, f) * UNIT_COST

 ! CONSTRAINTS !
 sum(f in FACILITIES) x_f(f) >= 1

 forall(d in DEMANDS)
  sum(f in FACILITIES) a_df(d, f) = 1

forall(f in FACILITIES)
 forall(d in DEMANDS)
  x_f(f) >= a_df(d, f)

 ! Setting decision variables as binary
 forall(d in DEMANDS, f in FACILITIES) a_df(d, f) is_binary
 forall(f in FACILITIES) x_f(f) is_binary

! Solve the problem
 minimize(TotalCost)

end-model
```

In the first iteration we set the facility opening costs as 30 for all of them and the model used only one facility and it was facility number 2 with a total cost of 63 units.

In the second iteration we increased the cost of facility number 2 by 5 units. This time the model only used facility number 1 and the total cost was 67 units.

In the third iteration we increased the cost of facility number 1 by 5 units. Costs of facilities were 35, 35, and 30. The model only used facility number 2 and the total cost was 68. At this stage, we realized that although the number 3 facility is 5 units cheaper than the others, its distance to the demand points might be considerably higher than the others.

After a few iterations, we found that the facility number 3 should be 10 units cheaper than the facility number 2 and 7 units cheaper than the facility number 1 in order to be opened. Also, the facility number 1 should be 4 units cheaper than the facility number 2 in order to be opened.

When we ran the model with the costs of 2, 5, 3 respectively, we saw that the facilities number 1 and 3 were opened. When we made the costs 2, 4, 3, all the facilities were used.

Q2.

a)

Step 1:

S(1) =	10
S(2) =	12
S(3) =	5
S(4) =	10
S(5) =	9
S(6) =	8
S(7) =	7
S(8) =	8
S(9) =	6
S(10) =	14
S(11) =	9
S(12) =	11
S(13) =	9
S(14) =	14 ***
S(15) =	5

Locate the first facility at node 14

Step 2:

S(1) =	9 ***
S(3) =	5
S(5) =	3
S(6) =	3
S(7) =	2
S(8) =	6
S(9) =	4
S(11) =	6
S(13) =	9 ***
S(15) =	5

Locate the first facility at node 1 or 13
Let's choose 13

Step 3:

S(3) =	3 ***
S(5) =	3 ***
S(6) =	3 ***
S(7) =	1
S(9) =	3 ***

Let's choose the node 3

Step 4:

S(5) =	3
S(6) =	3
S(7) =	1
S(9) =	3

Let's choose the node 6

Step 5:

S(7) =	1

Locate the facility at node 5

Minimum number of transformers is 5 and they are located at T14, T13, T3, T6, T5.

b) Size of the largest set covered is 14, approximation factor is
H(14) = 1 + 1/2 + 1/3 + … + 1/14 = 3.25

c) $$min \sum_j x_j$$

$$s.t. \quad \sum_i a_{1j} * x_j \geq 1$$

$$\sum_i a_{2j} * x_j \geq 1$$

$$\sum_i a_{4j} * x_j \geq 1$$

$$\sum_i a_{5j} * x_j \geq 1$$

$$\sum_i a_{6j} * x_j \geq 1$$

$$\sum_i a_{7j} * x_j \geq 1$$

$$\sum_i a_{8j} * x_j \geq 1$$

$$\sum_i a_{9j} * x_j \geq 1$$

$$\sum_i a_{10j} * x_j \geq 1$$

$$\sum_i a_{11j} * x_j \geq 1$$

$$\sum_i a_{12j} * x_j \geq 1$$

$$\sum_i a_{13j} * x_j \geq 1$$

$$\sum_j a_{14j} * x_j \geq 1$$

$$\sum_j a_{15j} * x_j \geq 1$$

$$\sum_j a_{18j} * x_j \geq 1$$

$$\sum_j a_{19j} * x_j \geq 1$$

$$\sum_j a_{20j} * x_j \geq 1$$

$$\sum_j a_{21j} * x_j \geq 1$$

$$\sum_j a_{22j} * x_j \geq 1$$

$$\sum_j a_{23j} * x_j \geq 1$$

$$\sum_j a_{24j} * x_j \geq 1$$

$$\sum_j a_{25j} * x_j \geq 1$$

$$\sum_j a_{26j} * x_j \geq 1$$

$$\sum_j a_{27j} * x_j \geq 1$$

$$\sum_j a_{28j} * x_j \geq 1$$

$$\sum_j a_{29j} * x_j \geq 1$$

$$\sum_j a_{30j} * x_j \geq 1$$

Since households H3, H16 and H17 can only be covered by facility T3, we eliminate their coverage constraints in our model. We had 27 constraints left after elimination redundant constraints.

d) We solved the model in Xpress solver and the code is as follows:

```
model "SCP"
 uses "mmxprs"

 declarations
  ND = 30
  NF = 15
  DEMANDS = 1..ND
  FACILITIES = 1..NF

  COVERAGE: array(FACILITIES, DEMANDS) of integer

  x_f: array(FACILITIES) of mpvar

 end-declarations

 initializations from '2.data'
  COVERAGE
 end-initializations

 ! Objective: minimize cost
 TotalCost:= sum(f in FACILITIES) x_f(f)

 ! CONSTRAINTS !
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 1) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 2) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 4) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 5) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 6) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 7) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 8) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 9) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 10) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 11) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 12) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 13) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 14) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 15) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 18) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 19) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 20) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 21) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 22) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 23) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 24) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 25) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 26) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 27) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 28) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 29) >= 1
 sum(f in FACILITIES) x_f(f) * COVERAGE(f, 30) >= 1


 ! Setting decision variables as binary
 forall(f in FACILITIES) x_f(f) is_binary

! Solve the problem
 minimize(TotalCost)
end- model
```

The model found an optimal solution with 3 facilities which are T1, T10, T12. Since we eliminated household H3, H16 and H17 model did not cover them. We also included T3 to cover these households. At the end we had an optimal solution with facilities located at T1, T3, T10 and T12.

e) We checked the optimality and found out that every household was covered with at least one facility and the largest set covered was 14 the same as we found with greedy heuristic in part a. Although the approximation factor is the same as that we found in part a, minimum number of facilities was higher when we used greedy heuristic. Since our objective was minimizing the cost greedy heuristic could not find the optimal solution.