Интерполационен полином на Лагранж

Генериране на данни (съставяне на табличната функция)

In[1]:=
```
xt = Table[3 + i * 0.1, {i, -5, 5}]
```

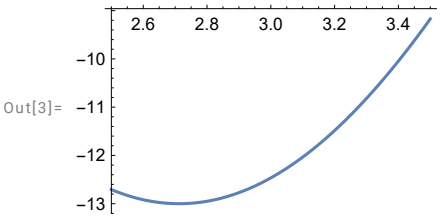
Out[1]=
```
{2.5, 2.6, 2.7, 2.8, 2.9, 3., 3.1, 3.2, 3.3, 3.4, 3.5}
```

In[2]:=
```
f[x_] := 13 Sin[x + 2]
```

за да сравняваме резултатите визуализираме функцията

In[3]:=
```
Plot[f[x], {x, 2.5, 3.5}]
```

Out[3]=



In[4]:=
```
yt = f[xt]
```

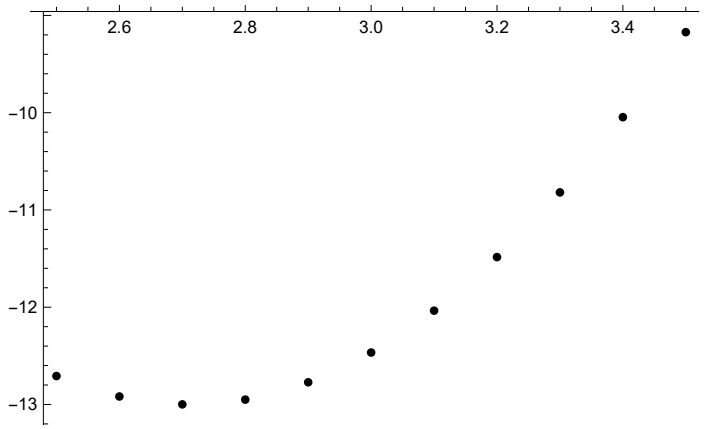
Out[4]=
```
{-12.7079, -12.918, -12.999, -12.9501, -12.7719,
 -12.466, -12.0356, -11.4849, -10.8195, -10.0459, -9.17202}
```

In[17]:=
```
n = Length[xt]
points = Table[{xt[[i]], yt[[i]]}, {i, 1, n}]
grp = ListPlot[points, PlotStyle → Black]
```

Out[17]=
```
11
```

Out[18]=
```
{{2.5, -12.7079}, {2.6, -12.918}, {2.7, -12.999},
 {2.8, -12.9501}, {2.9, -12.7719}, {3., -12.466}, {3.1, -12.0356},
 {3.2, -11.4849}, {3.3, -10.8195}, {3.4, -10.0459}, {3.5, -9.17202}}
```

Out[19]=



Интерполация в т. z = 2.97

Линейна интерполация

In[8]:=
```
L1[x_] := -12.7719 * (x - 3)/(2.9 - 3) - 12.466 * (x - 2.9)/(3 - 2.9)
Expand[L1[x]]
```

Out[9]=
```
-21.643 + 3.059 x
```

Проверка на интерполационните условия

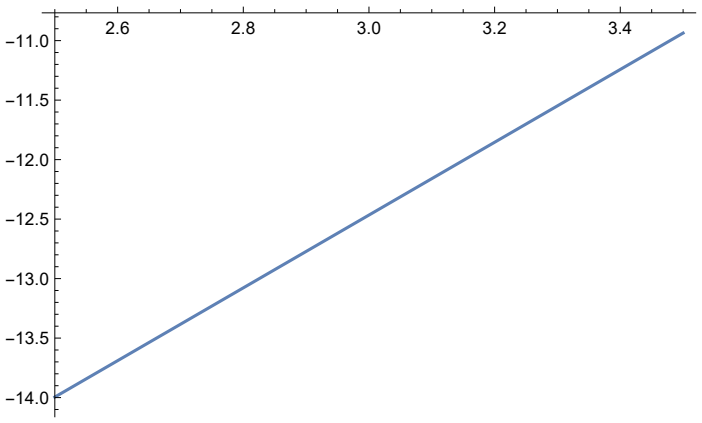
In[10]:=
```
L1[2.9]
L1[3]
```

Out[10]=
```
-12.7719
```

Out[11]=
```
-12.466
```

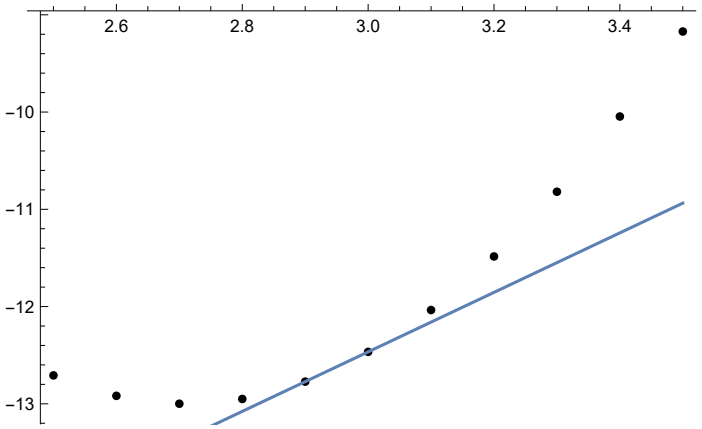
In[15]:=
```
grL1 = Plot[L1[x], {x, 2.5, 3.5}]
```

Out[15]=



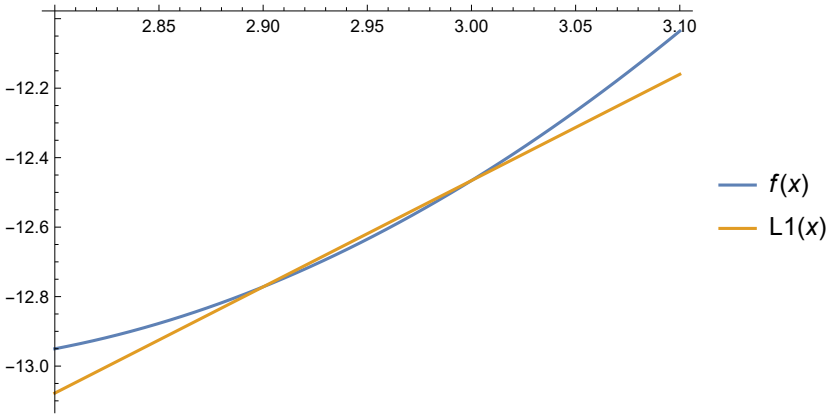
In[20]:=
```
Show[grp, grL1]
```

Out[20]=



In[22]:=
```
Plot[{f[x], L1[x]}, {x, 2.8, 3.1}, PlotLegends → "Expressions"]
```

Out[22]=



Пресмятане приближена стойност на функцията в z = 2.97

In[23]:=
```
L1[2.97]
```

Out[23]=

−12.5578

Оценка на грешката

истинска грешка

In[24]:=
```
Abs[f[2.97] - L1[2.97]]
```

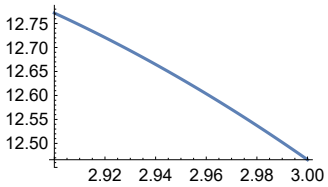
Out[24]=
```
0.0132479
```

теоретична грешка

намираме M2

In[25]:=
```
Plot[Abs[f''[x]], {x, 2.9, 3}]
```

Out[25]=



от графиката се вижда, че

In[26]:=
```
M2 = Abs[f''[2.9]]
```

Out[26]=
```
12.7719
```

In[27]:=
```
R1[x_] := M2/2! * Abs[(x - 2.9) (x - 3)]
```

In[28]:=
```
R1[2.97]
```

Out[28]=
```
0.0134105
```

за сравнение истинската грешка е 0.0132479

Квадратична интерполация

In[29]:=
```
L2[x_] := -12.7719 * (x - 3) (x - 3.1)/((2.9 - 3) (2.9 - 3.1)) -
  12.466 * (x - 2.9) (x - 3.1)/((3 - 2.9) (3 - 3.1)) - 12.0356 * (x - 2.9) (x - 3.)/((3.1 - 2.9) (3.1 - 3.))
Expand[L2[x]]
```

Out[30]=
```
32.5145 - 33.6685 x + 6.225 x^2
```

Проверка на интерполационните условия

In[31]:=
```
L2[2.9]
L2[3]
L2[3.1]
```

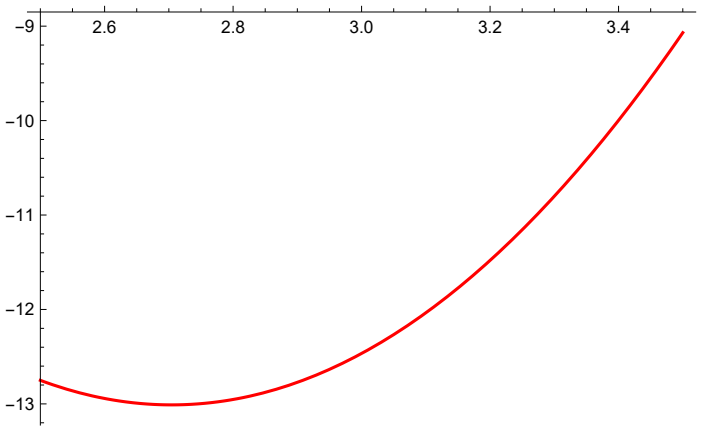
Out[31]=
```
-12.7719
```

Out[32]=
```
-12.466
```

Out[33]=
```
-12.0356
```

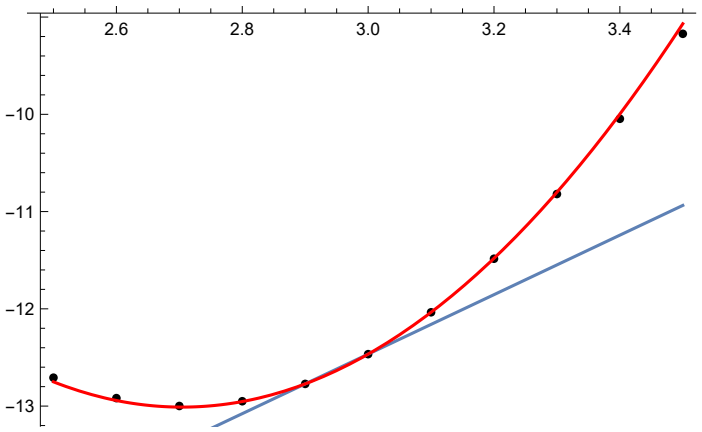
In[35]:=
```
grL2 = Plot[L2[x], {x, 2.5, 3.5}, PlotStyle → Red]
```

Out[35]=



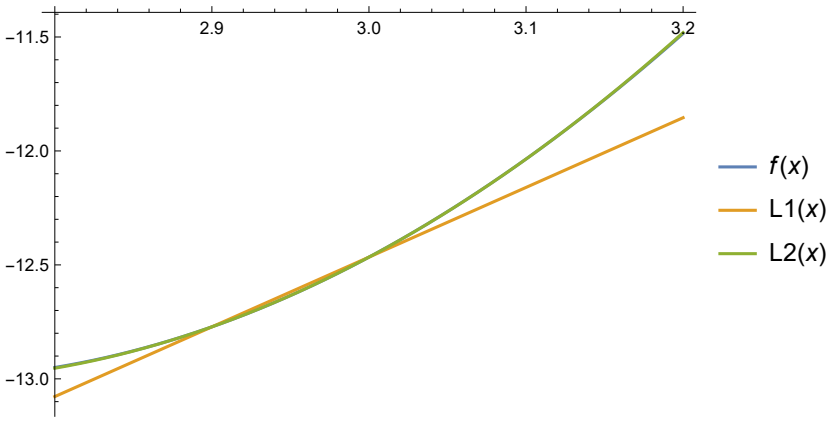
In[36]:=
```
Show[grp, grL1, grL2]
```

Out[36]=



In[37]:=
```
Plot[{f[x], L1[x], L2[x]}, {x, 2.8, 3.2}, PlotLegends → "Expressions"]
```

Out[37]=



Пресмятане приближена стойност на функцията в z = 2.97

In[38]:=
```
L2[2.97]
```

Out[38]=

−12.5708

Оценка на грешката

истинска грешка

In[39]:=
```
Abs[f[2.97] - L2[2.97]]
```

Out[39]=

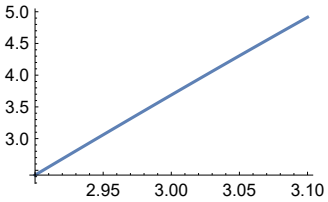
0.000175443

теоретична грешка

намираме M3

In[40]:=
```
Plot[Abs[f'''[x]], {x, 2.9, 3.1}]
```

Out[40]=



от графиката се вижда, че

In[41]:=
```
M3 = Abs[f'''[3.1]]
```

Out[41]=

4.91371

In[42]:=
```
R2[x_] := M3/3! * Abs[(x - 2.9) (x - 3) (x - 3.1)]
```

In[43]:=
```
R2[2.97]
```

Out[43]=

0.000223574

за сравнение истинската грешка е 0.000175443

Кубична интерполация

Естраполация в т. z = 3.56 (**близка** до дадените точки)

Естраполация в т. z = 10 (**далечна** от дадените точки)

In[44]:=
```
L2[10]
```
Out[44]=
```
318.329
```

In[45]:=
```
R2[10]
```
Out[45]=
```
280.843
```