DESIGN via ROOT LOCUS

In this chapter you will learn the following :

❖How to use the root locus to design cascade compansators to improve the steady-state error

❖How to use the root locus to design cascade compansators to improve the transient response

❖How to use the root locus to design cascade compansators to improve both the transient response and steady-state error

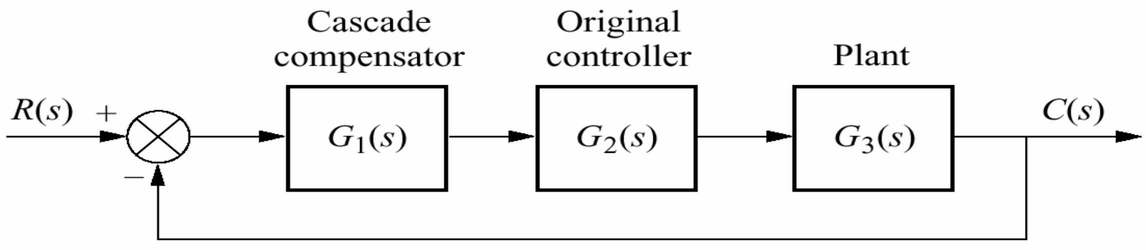
❖How to use the root locus to design feedback compansators to improve the transent response

❖How to realize the designed compensators physically

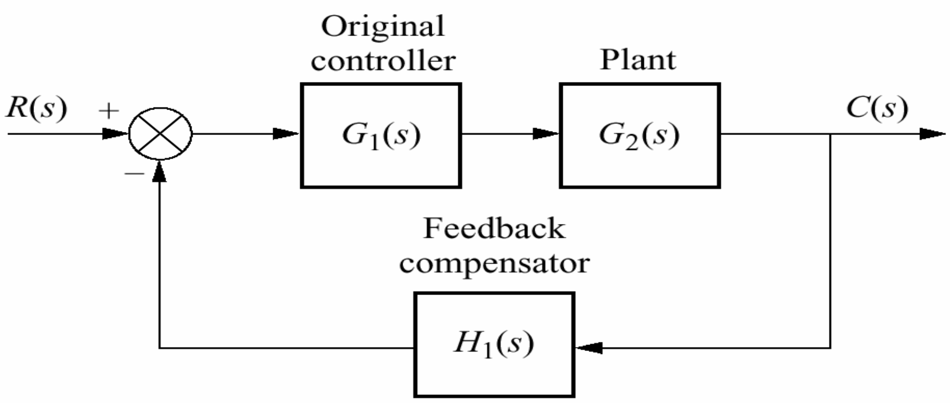
The root locus allows us to choose the proper loop gain to meet a transient response specification. As the gain varied, we move through different region of response. But we are limited to responses that exists along to root locus.

We ***compansate*** the system with additional poles and zeros, so that the compansated system has a root locus that goes through the desired pole location for some value of gain. Compansators are also used to improve the steady state error characteristics. Generally, the transient response is improved with the addition of differetiation, and the stesdy state error is improved with the addition of integration in the forward path.

Two configration of compansation are covered in this lecture : Cascade compansation and feedback compansation as modeled in the following figures. In the cascade compansation, the compansating network, $G_1(s)$, is placed at the low-power end of thew forward path in cascade with the plant. If feedback compansation is used, the compansator, $H_1(s)$ is placed in the feedback path. Both methods change the open loop poles and zeros, thereby creating a new root locus that goes through the desired closed loop pole location.



(a)



(b)

Compansators that pure integration for improving steady state error or pure differentiation for improving the transient response are defined as ***ideal compansators***. Ideal compansators must be implemented with active networks, which, in the case of electrical networks, require the use of active amplifiers and possible additional power sources. An advantage of ideal compansators is that steady state error is reduced to zero. Electromechanical ideal compansators, such as tachometers, are often used to improve transient response.

Other design tevhniques that preclude the use of active devices for compansation can be adopted. These compansators, which can be implemented with passive elements such as resistors and capacitors, do not use pure integration and differentiation and are not ideal compansators. Advantages of passive networks are that they are less expensive and do not require additional power sources for their operations. Their disadvantage is that the steady state error is not driven to zero in cases where ideal compansators yield zero error.

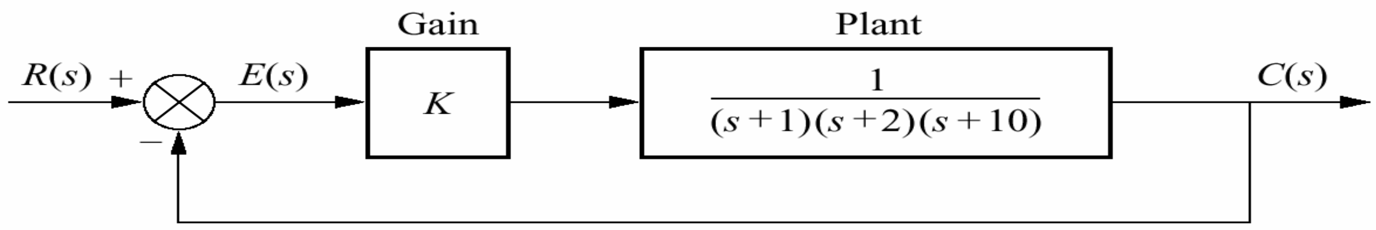
Thus the choice between an active or passive compansator revolves around the cost, weight, desired performance, transfer function. We first discuss cascade compansator design using the ideal compansation nad follow with cascade compansation using compansators that are not implemented with pure integration and differentiation.

Improving the Steady-State error via Cascade Compensation

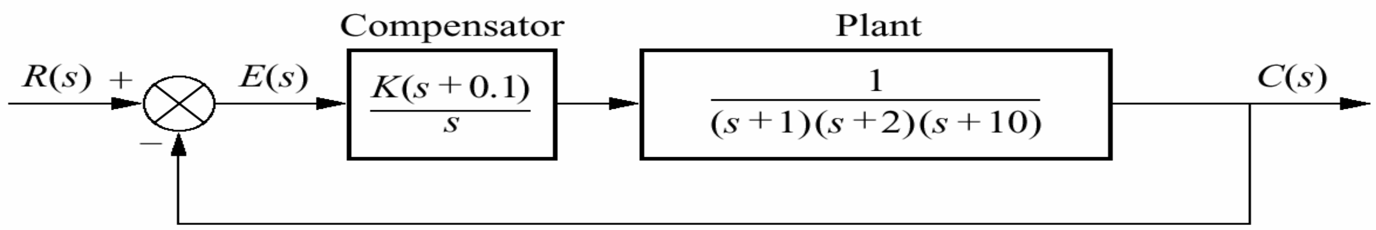
We will discuss two ways to improve the steady state error using cascade compansation

1) Ideal Integral Compansation (PI) : Steady-state error can be improved by placing an open loop pole at the origin. A compansator with a pole at the origin and a zero close to the pole is called ideal compansator. In the following example, we demonstrate the effect of ideal integral compansation. An open loop pole will drive the steady state error to zero. An open loop zero will be placed very close to the open loop pole at the origin so that the original closed loop poles on the original root locus still remain at approximately the same points on the compansated root locus.

Example : Given the system of figure(a), operating a damping ratio of 0.174, show that the addition of the integral compansator shown in figure(b) reduces the steady state error to zero for a step input without appreciably affecting transient response.

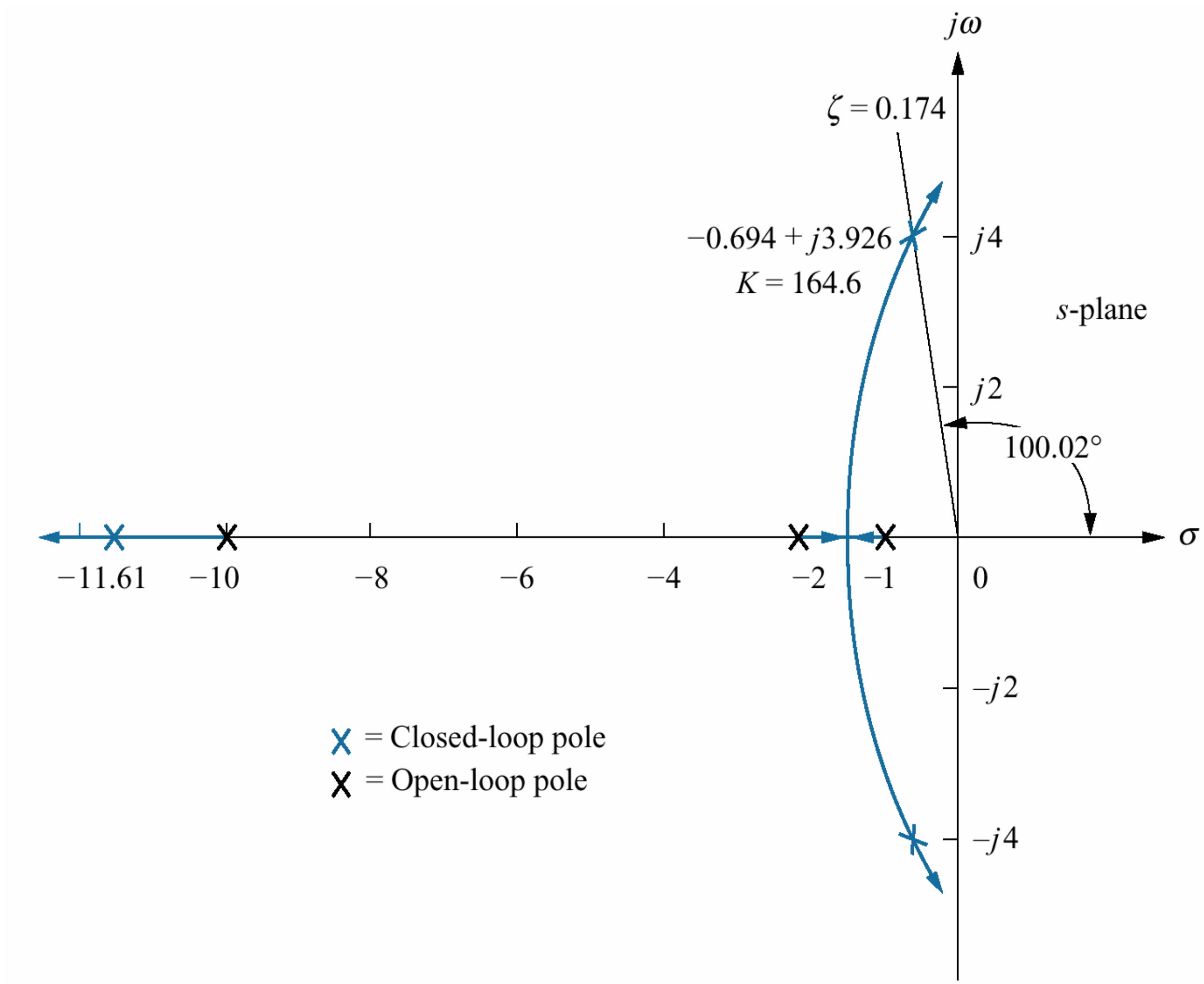


(a)

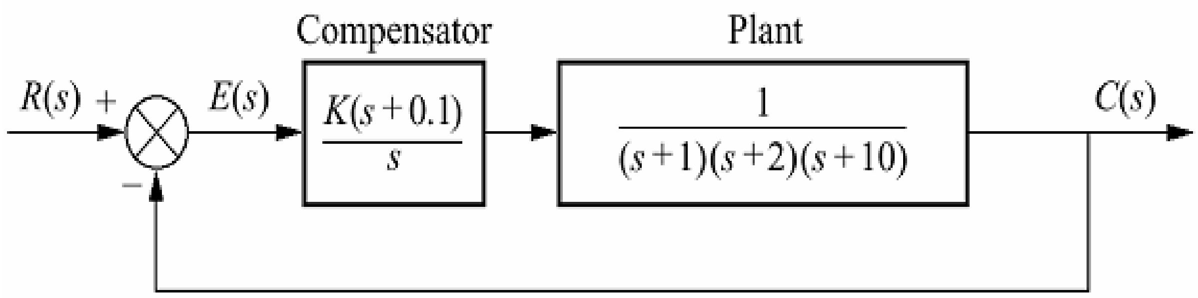


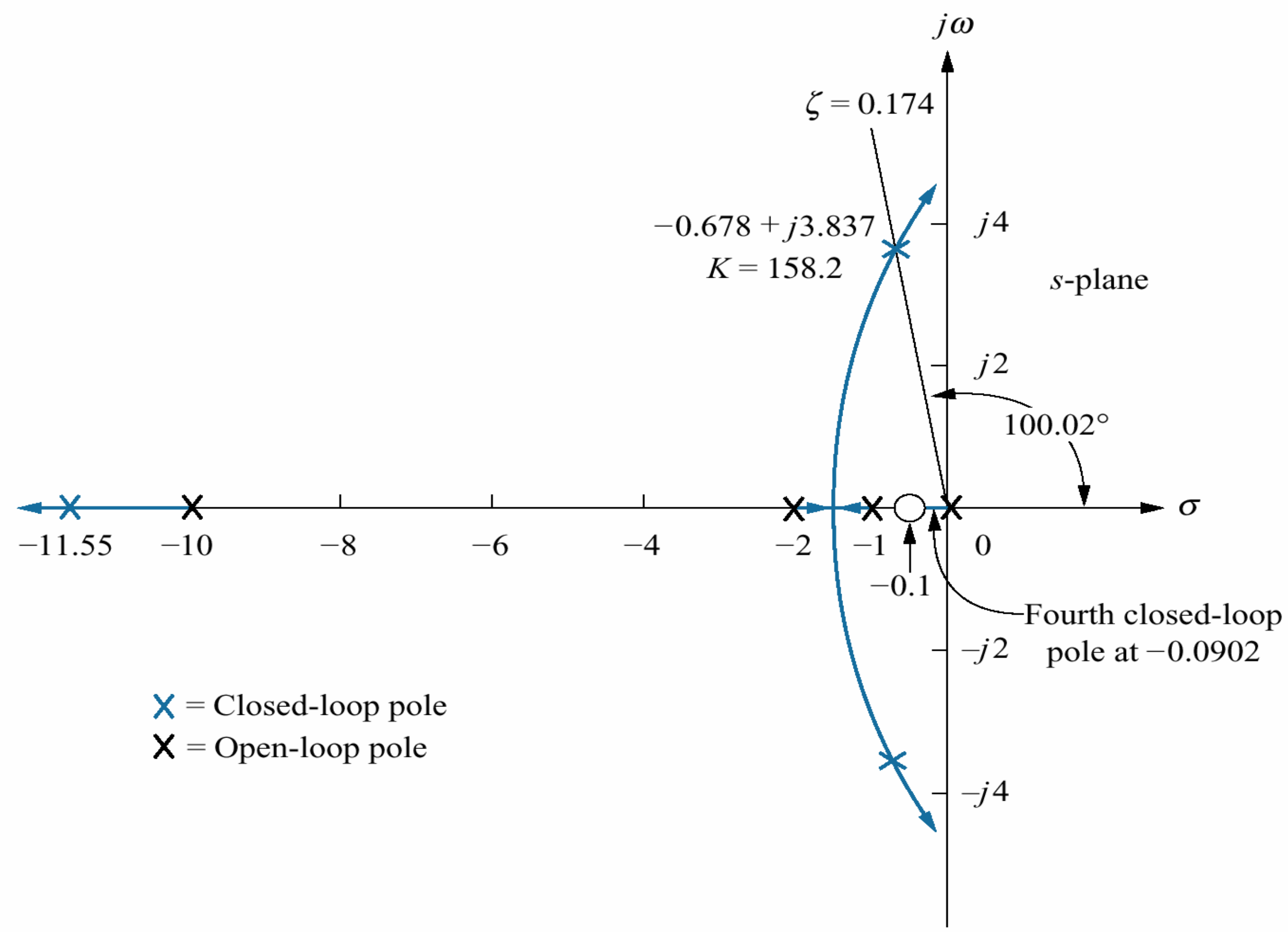
(b)

Solution : We first analyze the uncompansated system and determine the location of dominant second order poles. Next we evaluate the uncompansated steady state error for a unit step input. The root locus of uncompansated system is shown in following figure.

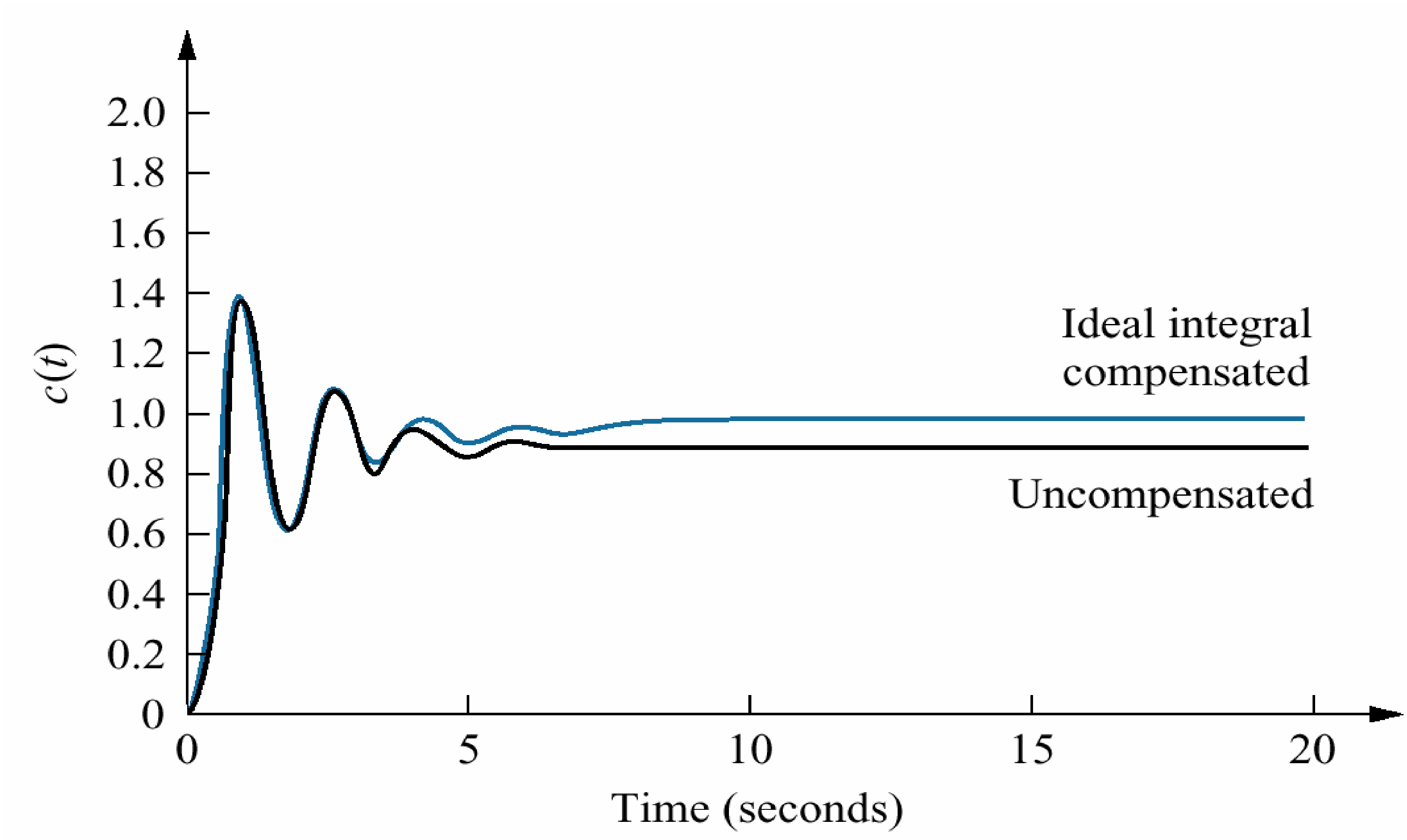


Adding an integral compansator with a zero at 0.1 as shown in the figure, we obtain the root locus as shown in other figure.

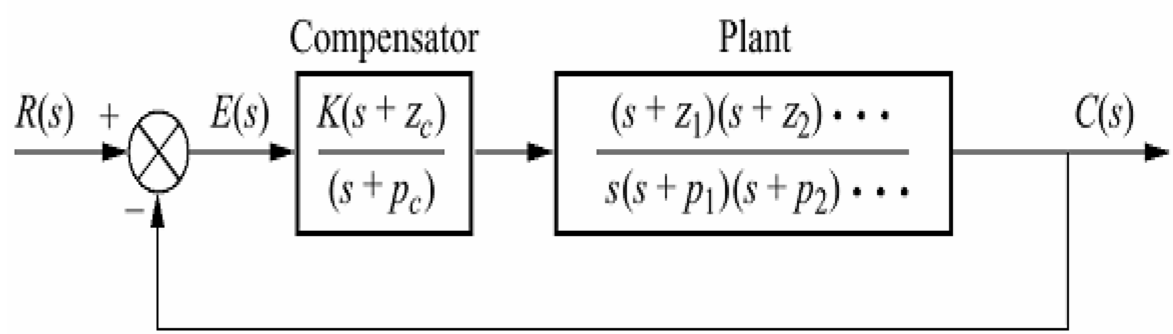




Following figure compares the uncompansated response with ideal integral compansated response. The step response of ideal integral compansated system approaches unity in steady state, while uncompansated system approaches 0.892. **Thus the ideal integral compansated system responds with zero steady state error. But the transient responses for two cases are similar.**



2) Lag Compansation : Ideal integral compansator requires an active integrator. But as we will see later, circuit configration for the lag compansator can be obtained with passive networks and thus do not require the active amplifiers and possible additional power sources. Hovewer, lag compansators can not drive the steady-state error to zero. But very small steady state error can be obtained. The configration and transfer function of lag compansators shown in following figure.

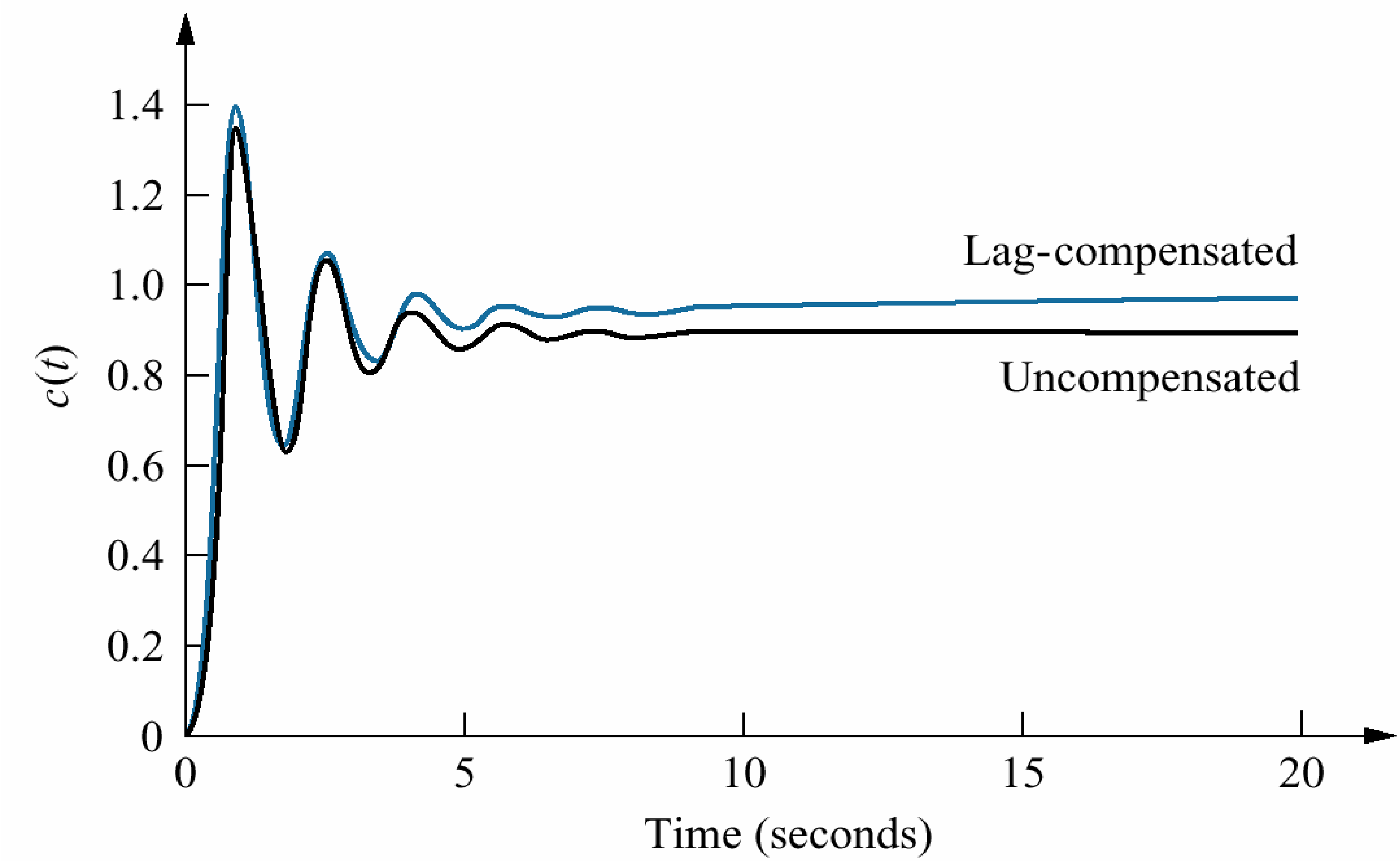


Example : Compansate the system of previous example to improve the steady –state error by a factor of 10 if the system is operating with a damping ratio of 0.174.

Solution : Remember that steady state error of uncompansated system is 0.108 with a gain K=8.23. From the problem statement, e(∞)=0.108/10=0.0108. If we want to design a lag compansator, the following algorithm can be used to select K, z_c and p_c conventionally. Using the formulation for steady state error which we have learned in the lecture “Steady-State Errors”, we can find the value of K as follow:

$$e(\infty)=\frac{1}{1+K}\Rightarrow K=\frac{1-e(\infty)}{e(\infty)}=\frac{1-0.0108}{0.0108}=91.59 \qquad \frac{z_c}{p_c}=\frac{K_{new}}{K_{old}}=\frac{91.59}{8.23}=11.13$$

Arbitrarily select p_c=0.01 and z_c=11.13p_c≈0.111 . Let us compare the compansated system with uncompansated system as shown in following figure. Steady-state error of the uncompansated system is 0.108 while the compansated system has steady state error of 0.011.

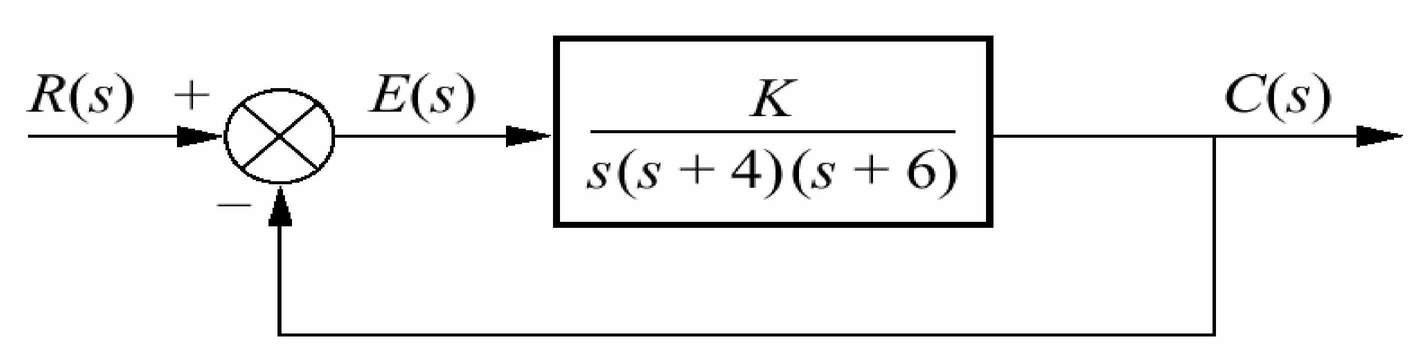


Improving Transient Response via Cascade Compensation

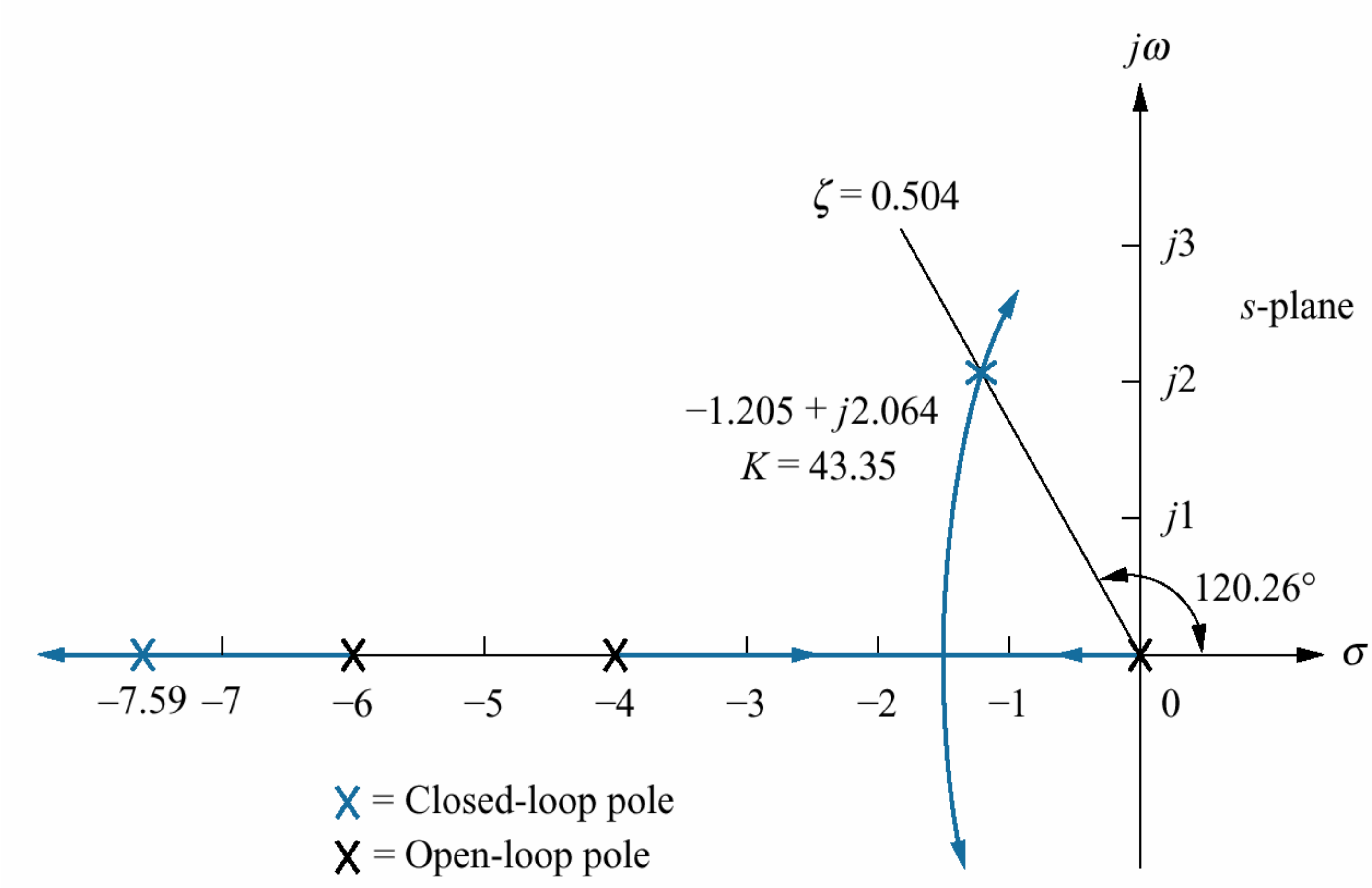
We will discuss two ways to improve transient response using cascade compensation.

1)Ideal Derivative Compensation (PD) : The transient response of a system can be selected by choosing an appropriate closed loop pole location on the s-plane. If this point is on the root locus, then a simple gain adjustment is all that is required in order to meet the transient response specification. A compensator whose transfer function $G_c(s) = s + z_c$, the sum of a differentiator and a pure gain, is called an *ideal derivative*, or *PD controller*.

Example : Given the system of following figure, design an ideal derivative compensator to yield a 16% overshoot, with a threefold reduction in settling time.



Solution : Let us first evaluate the performance of uncompansated system with 16% overshoot. Since 16% overshoot is eqivalent to $\zeta=0.504$, we search along that damping ratio line for an odd multiple of 180° and find that the dominant, second order pair of poles is at $-1.205 \pm j2.064$. Thus the settling time of the uncompansated system is $T_s = 4/(\zeta\omega_n) = 3.320$. The root locus of uncompansated system is shown in following figure.

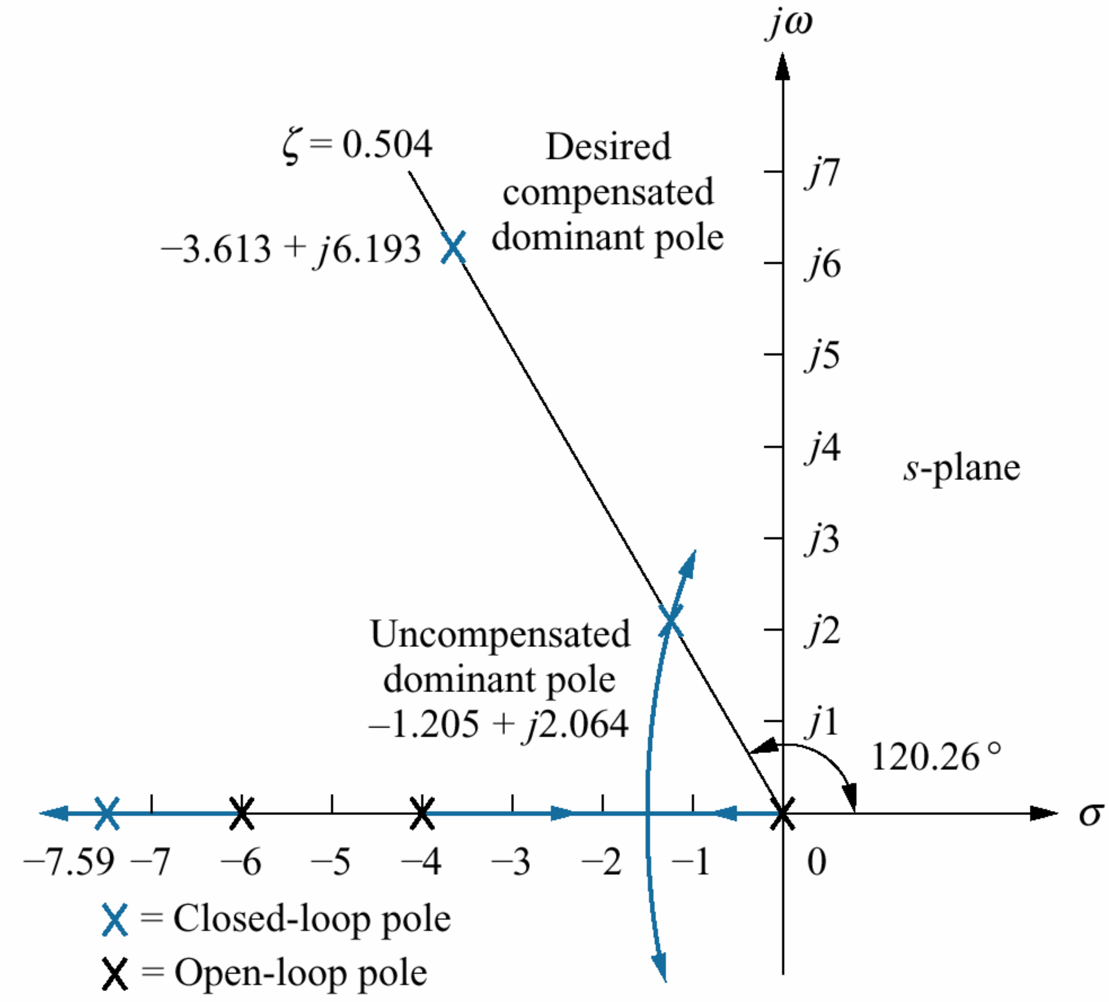


Root locus for Uncompensated system

Now we proceed to compansate the system. First we find the location of the compansated system's dominant poles. In order to have a threefold reduction in the settling time, the compansated system' settling time will be 1.107 . Therefore, real part of the compansated system dominant pole is

$$\sigma = \frac{4}{T_s} = \frac{4}{1.107} = 3.613$$

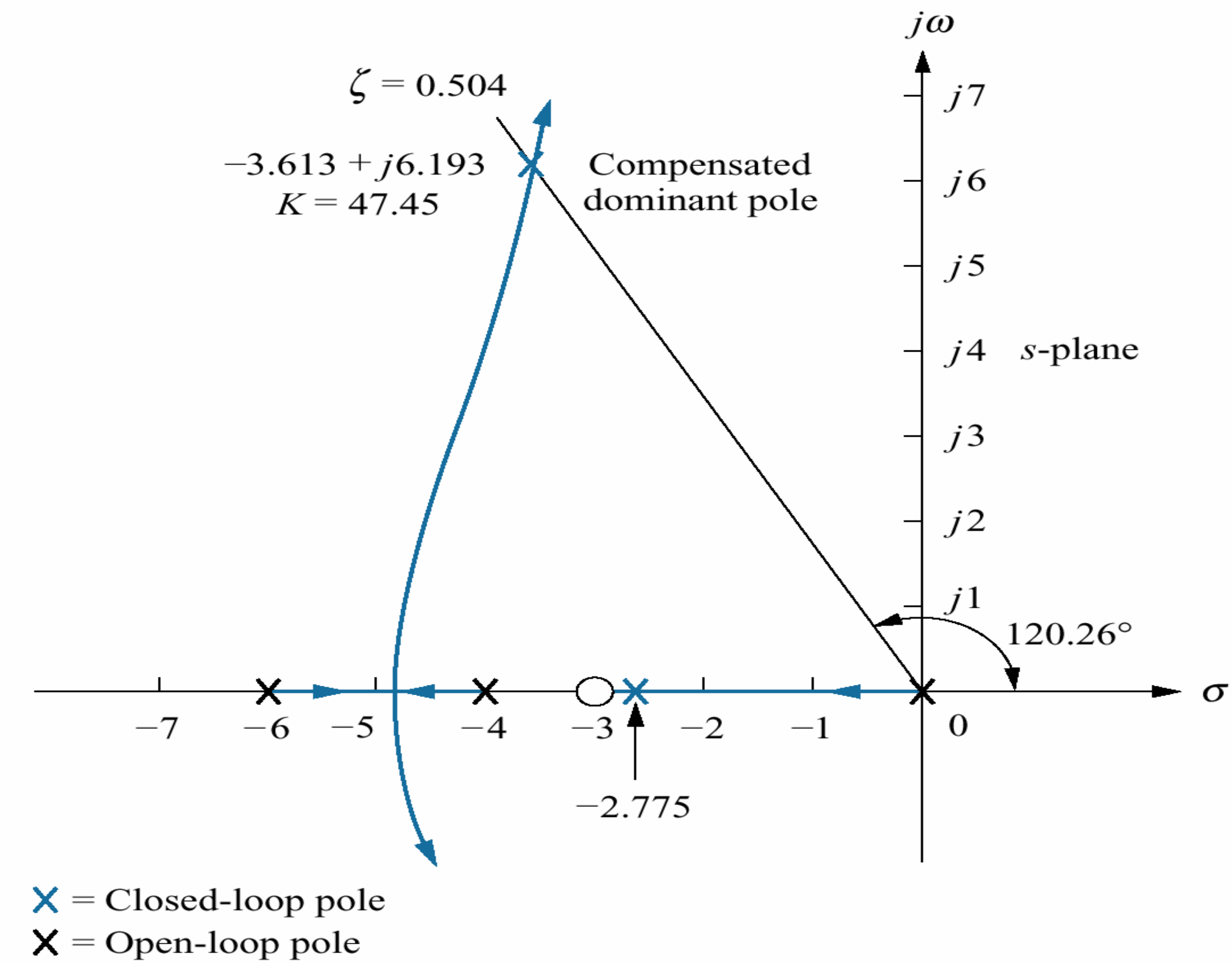
Following figure shows the designed dominant second order pole with a real part equal to -3.613 .



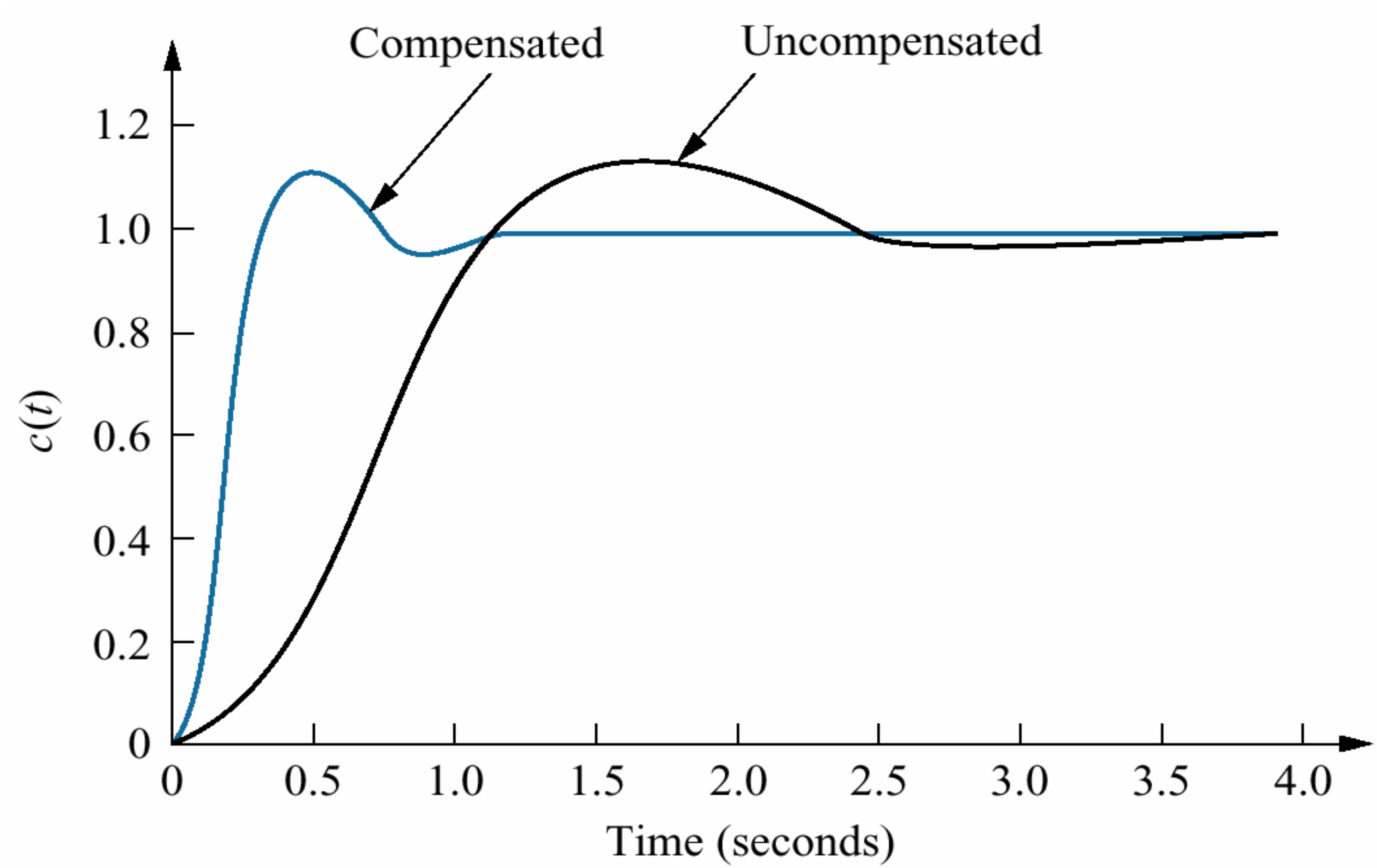
Therefore imaginary part of compansated system second order dominant pole is

$$\omega_d = 3.613\tan(180^\circ - 120.26^\circ) = 6.193$$

Next we design the location of compansator zero. Using a computer code, such as the code extended to book's cd, input the uncompansated system poles and zeros and find the optimum value for zero. For this example, σ=3. Following figure shows the root locus of compansated system.



Following figure shows the step response and comparison result of compansated and uncompansated system

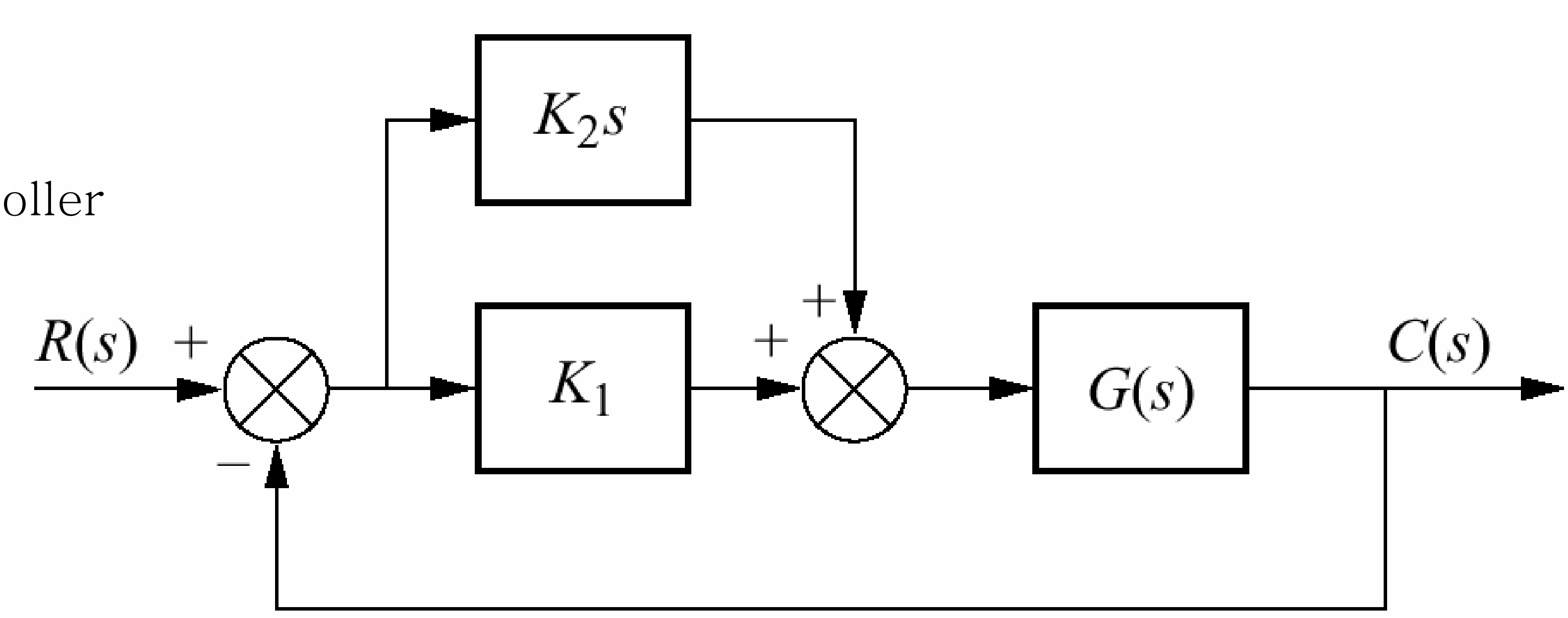


Block Diagram and Transfer Function of PD Controller : Once we decide on the location of the compansating zero, how do we implement the ideal derivative, or PD controller? The ideal integral compansatorthat improved steady-state error was implemented with a proportional-plus-integral (PI) controller. The ideal derivative compansator used to improve transient response is implemented with a proportional-plus-derivative (PD) controller. For example, in the following figure the transfer function of controller is

$$G_c(s) = K_2 s + K_1 = K_2\left(s + \frac{K_1}{K_2}\right)$$

Hence, K_1/K_2 is chosen to equal the negative of the compansator zero, and K_2 is chosen to contribute to the reqired loop gain value.

PD Controller



2) Lead Compensation : If we want to use passive elements instead of active ones, we must use the lead compansation to improve transient response. If we use the lead compansation, noise due to differentiation is reduced. Let us look at the concept behind lead compansation. If we select a desired dominant second order pole, the sum of the angles from the uncompansated system's poles and zeros to the design point can be found. The difference between 180° and the sum of angles must be angular contribution required of the compansator. For example, look at the figure below. We see that

$$\theta_2-\theta_1-\theta_3-\theta_4+\theta_5=(2k+1)180°$$

where $\theta_2-\theta_1=\theta_c$ is the angular contribution of the lead compansator. From the figure, we see that θ_c is the angle of a ray extending from the design point and intersectingthe real axis at the pole value and zero value of the compansator.

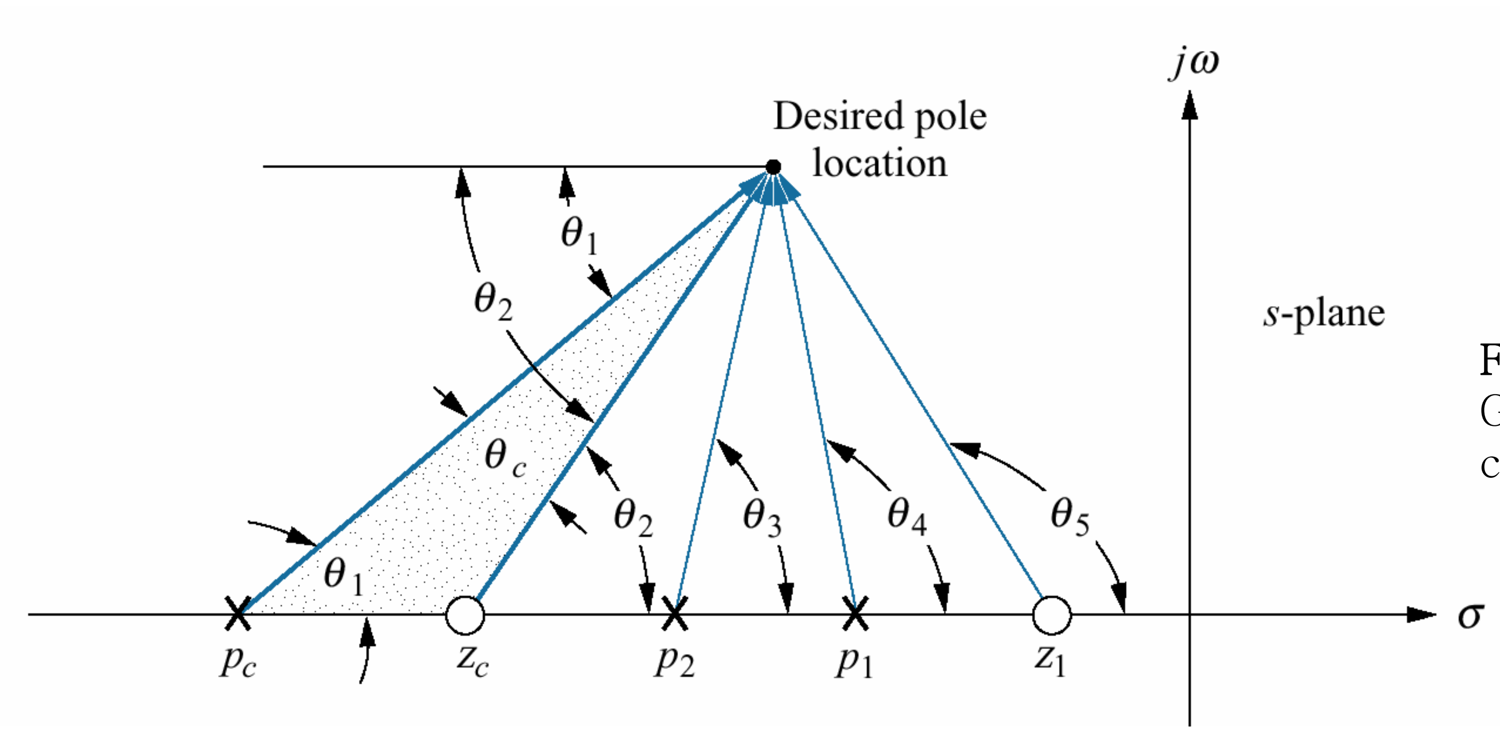
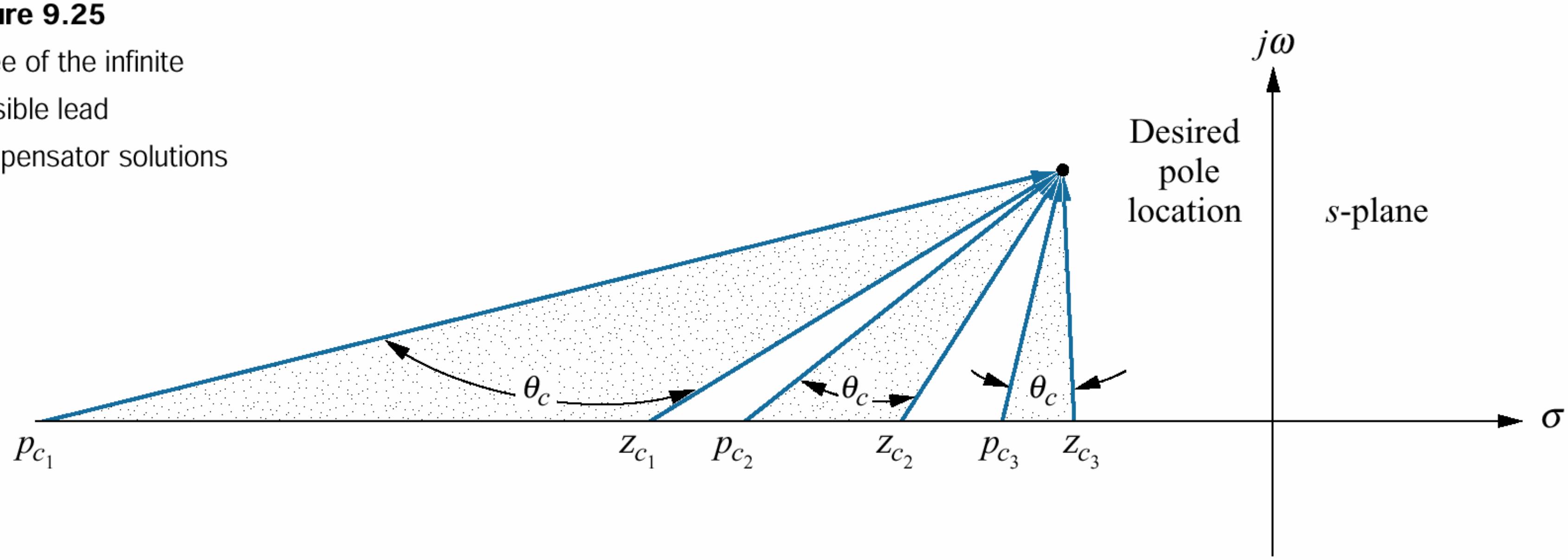


Figure 9.24
Geometry of lead compensation

Now visualize this ray rotating about the desired closed-loop pole location and intersecting the real axis at the compensator pole and zero, as illustrated in figure below.

Figure 9.25

Three of the infinite possible lead compensator solutions



We realize that and infinite number of lead compensators could be used to meet the transient response requirement. For design, we arbitrarily select either a lead compensator pole or zero and find the angular contribution at the design point of this pole or zero along with system open loop poles and zeros. The difference between this angle and 180° is the required contribution of the remaining compensator pole or zero.

Different examples exist in text book.

Lead Compensator Design

PROBLEM: Design three lead compensators for the system of Figure 9.17 that will reduce the settling time by a factor of 2 while maintaining 30% overshoot. Compare the system characteristics between the three designs.

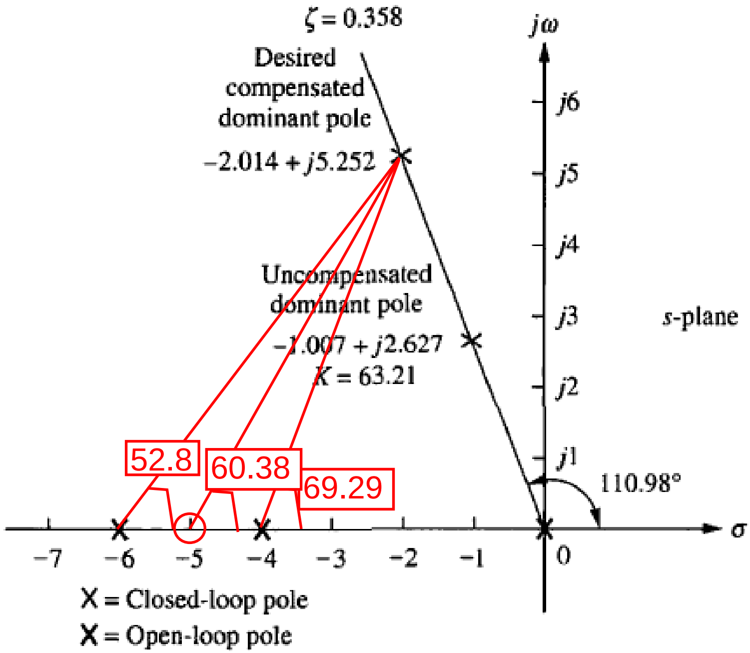


FIGURE 9.26 Lead compensator design, showing evaluation of uncompensated and compensated dominant poles for Example 9.4

SOLUTION: First determine the characteristics of the uncompensated system operating at 30% overshoot to see what the uncompensated settling time is. Since 30% overshoot is equivalent to a damping ratio of 0.358, we search along the $\zeta = 0.358$ line for the uncompensated dominant poles on the root locus, as shown in Figure 9.26. From the pole's real part, we calculate the uncompensated settling time as $T_s = 4/1.007 = 3.972$ seconds. The remaining characteristics of the uncompensated system are summarized in Table 9.4.

Next we find the design point. A twofold reduction in settling time yields $T_s = 3.972/2 = 1.986$ seconds, from which the real part of the desired pole location is $-\zeta\omega_n = -4/T_s = -2.014$. The imaginary part is $\omega_d = -2.014\tan(110.98°) = 5.252$.

We continue by designing the lead compensator. Arbitrarily assume a compensator zero at -5 on the real axis as a possible solution. Using the root locus program, sum the angles from both this zero and the uncompensated system's poles and zeros, using the design point as a test point. The resulting angle is $-172.69°$. The difference between this angle and $180°$ is the angular contribution required from the compensator pole in order to place the design point on the root locus. Hence, an angular contribution of $-7.31°$ is required from the compensator pole.

The geometry shown in Figure 9.27 is used to calculate the location of the compensator pole. From the figure,

$$\frac{5.252}{p_c - 2.014} = \tan 7.31° \tag{9.19}$$

from which the compensator pole is found to be

$$p_c = 42.96 \tag{9.20}$$

The compensated system root locus is sketched in Figure 9.28.

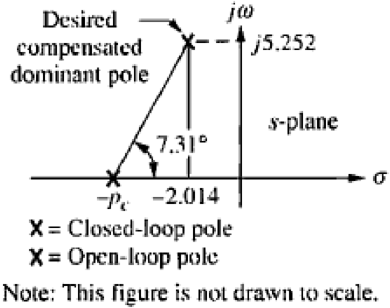


FIGURE 9.27 s-plane picture used to calculate the location of the compensator pole for Example 9.4

TABLE 9.4 Comparison of lead compensation designs for Example 9.4

	Uncompensated	Compensation a	Compensation b	Compensation c
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$	$\frac{K(s+5)}{s(s+4)(s+6)(s+42.96)}$	$\frac{K(s+4)}{s(s+4)(s+6)(s+20.09)}$	$\frac{K(s+2)}{s(s+4)(s+6)(s+8.971)}$
Dominant poles	$-1.007 \pm j2.627$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$
K	63.21	1423	698.1	345.6
ζ	0.358	0.358	0.358	0.358
ω_n	2.813	5.625	5.625	5.625
$\%OS^*$	30 (28)	30 (30.7)	30 (28.2)	30 (14.5)
T_s^*	3.972 (4)	1.986 (2)	1.986 (2)	1.986 (1.7)
T_p^*	1.196 (1.3)	0.598 (0.6)	0.598 (0.6)	0.598 (0.7)
K_v	2.634	6.9	5.791	3.21
$e(\infty)$	0.380	0.145	0.173	0.312
Other poles	−7.986	−43.8, −5.134	−22.06	−13.3, −1.642
Zero	None	−5	None	−2
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK	No pole-zero cancellation

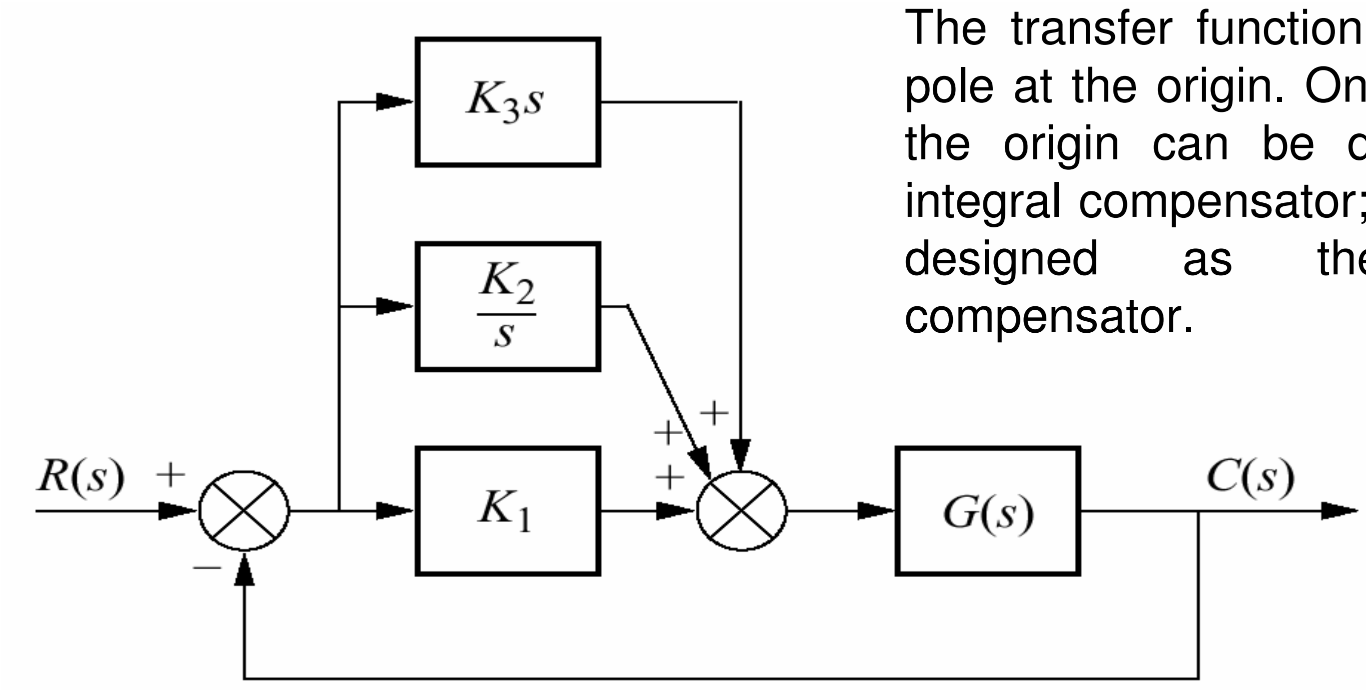
*Simulation results are shown in parentheses.

Improving Steady-State Error and Transient Response

The design can use either active or passive compensators, as previously described. If we design an active PD controller followed by an active PI controller, the resulting compensator is called a proportional-plus-integral-plus-derivative (PID) controller. If we first design a passive lead compensator and then design a passive lag compensator, the resulting compensator is called a lag-lead compensator.

PID Controller Design : A PID controller is shown in figure below. Its transfer function is

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 s + K_2 + K_2 s^2}{s} = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s}$$



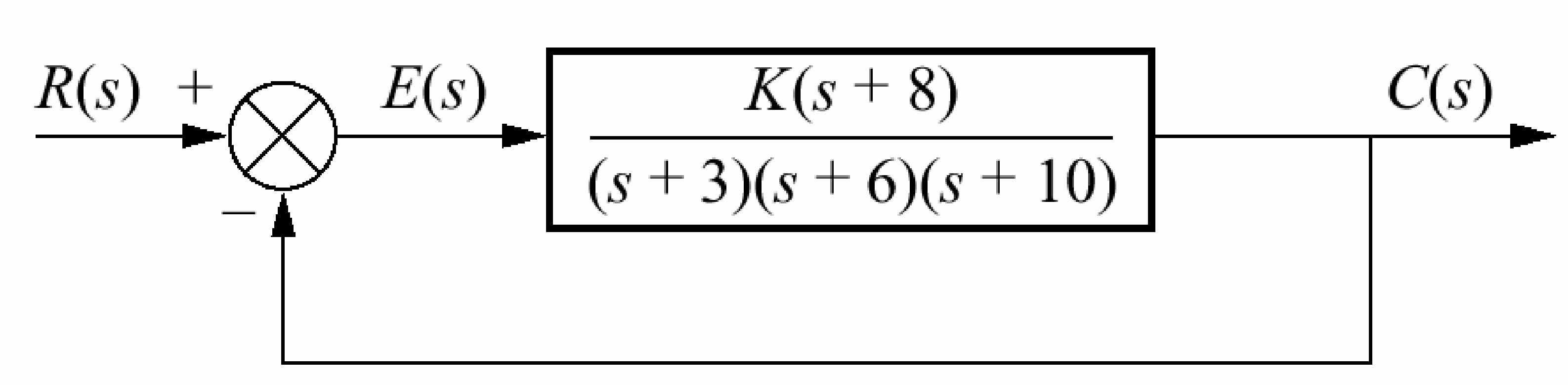
The transfer function has two zeros and a pole at the origin. One zero and the pole at the origin can be designed as the ideal integral compensator; the other zero can be designed as the ideal derivative compensator.

Design technique consists of the following steps :

1. Evaluate the performance of the uncompensated system to determine how much improvements in transient response is required.
2. Design the PD controller to meet the transient response specifications. The design includes the zero location and the loop gain.
3. Simulate the system to be sure all reqirements have been met.
4. Redesign if the simulation shows that the requirements have not been met.
5. Design PI controller to yield the required steady-state error.
6. Determine the gain K_1, K_2, K_3, the PID parameters.
7. Simulate the system to be sure all requirements have been met.
8. Redesign if the simulation shows that the requirements have not been met

Let us look an example.

Example : Given the system of figure below, design a PID controller so that the system can operate with a peak time that is two-thirds that of the uncompensated system at %20 overshoot and with zero steady-state error for a step input.



Solution : **Step 1 :** Let us first evaluate the uncompensated system operating at 20% overshoot. Searching along the 20% overshoot line (ζ=0.456) in root-locus, we find the dominant poles to be -5.415±j10.57 with a gain 121.5. A third pole exists at -8.619. The complete performance of the uncompensated system is shown in the first column of the following table. We estimate that the uncompansated system has a peak time 0.297 second at 20% overshoot(look at the table).

Table 9.5
Predicted characteristics of uncompensated, PD- , and PID- compensated systems

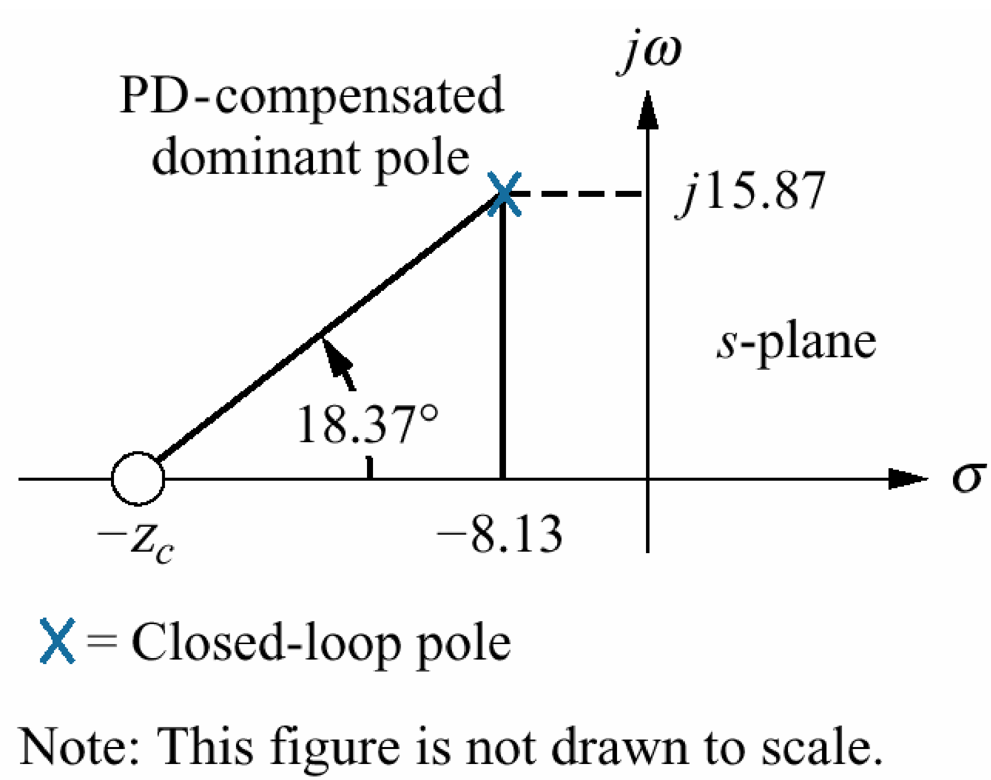
	Uncompensated	**PD-compensated**	**PID-compensated**
Plant and compensator	$\frac{K(s+8)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)(s+0.5)}{(s+3)(s+6)(s+10)s}$
Dominant poles	$-5.415 \pm j10.57$	$-8.13 \pm j15.87$	$-7.516 \pm j14.67$
K	121.5	5.34	4.6
ζ	0.456	0.456	0.456
ω_n	11.88	17.83	16.49
$\%OS$	20	20	20
T_s	0.739	0.492	0.532
T_p	0.297	0.198	0.214
K_p	5.4	13.27	∞
$e(\infty)$	0.156	0.070	0
Other poles	-8.169	-8.079	$-8.099, \ -0.468$
Zeros	-8	$-8, \ -55.92$	$-8, \ -55.92, \ -0.5$
Comments	Second-order approx. OK	Second-order approx. OK	Zero at -55.92 and -0.5 not canceled

Step 2 : To compensate the system reduce the peak time to two-thirds of that of the uncompansated system, we must first find compansated system's dominant pole location. The imaginary part of the compensated dominant pole is

$$\omega_d = \frac{\pi}{T_p} = \frac{\pi}{(2/3)(0.297)} = 15.87$$

Thus the real part of the compansated dominant pole is $\sigma = \dfrac{\omega_d}{\tan 117.13^\circ} = -8.13$

Next we design the compensator. Using the geometry shown in figure,



$$\frac{15.87}{z_c - 8.13} = \tan 18.37^\circ$$

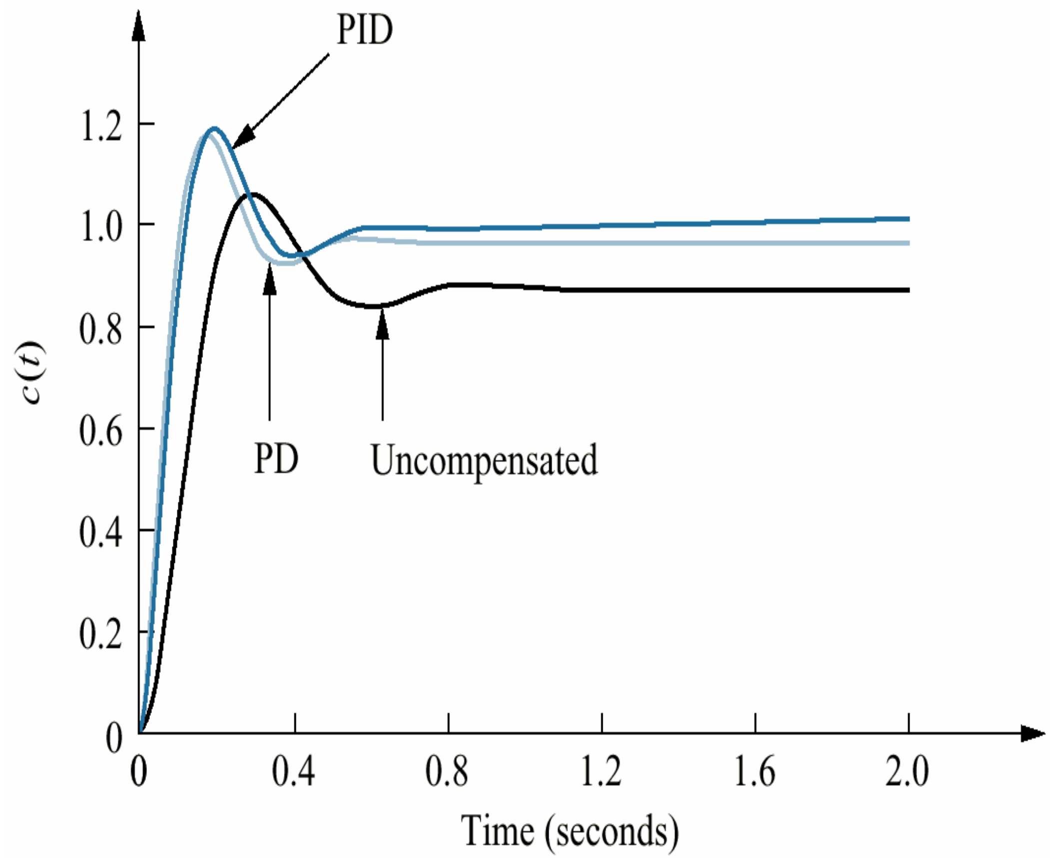
then $z_c = 55.92$

Thus, the PD controller is

$G_{PD} = (s + 55.92)$

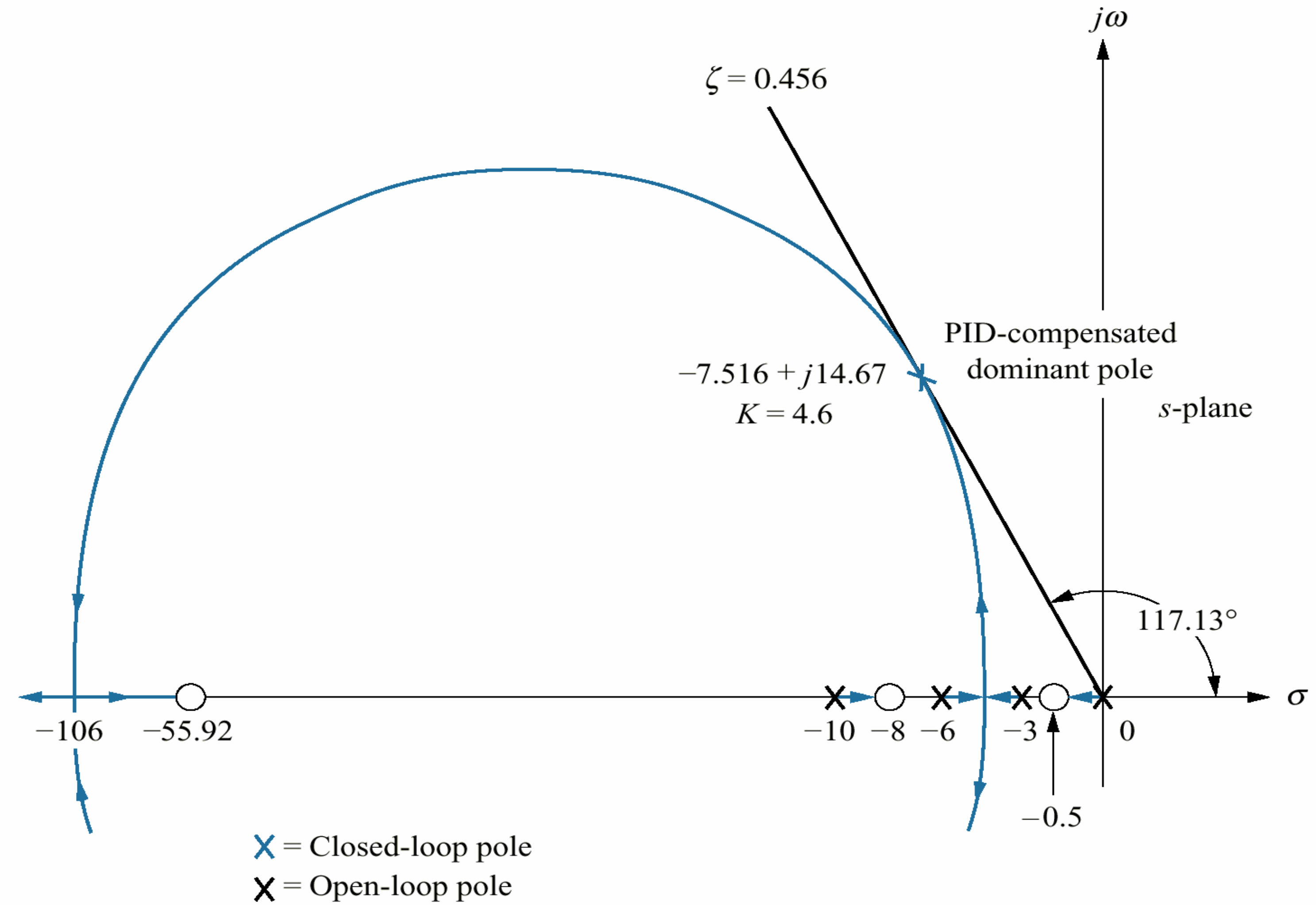
Step 3 and 4 : We simulate the companasated system, as shown in figure below. We see the reduction in peak time and the improvement in steady-state error over the uncompansated system.

Figure 9.35
Step responses for uncompensated, PD-compensated, and PID-compensated systems



Step 5 : Choosing the ideal integral companzator to be $G_{PI}(s)=\dfrac{s+0.5}{s}$, we sketch

The root locus for PID-compensated system, as shown in figure. Searching 0.456 damping ratio line, we find the dominant, second order poles to be -7.516±j14.67, with an associated gain of 4.6 .



Note: This figure is not drawn to scale.

Step 6 : Now we determine the gains K_1, K_2, K_3. We have found G_{PD} and G_{PI}. Thus, the product of the gain and the PID controller is

$$G_{PID}(s) = \frac{K(s+55.92)(s+0.5)}{s} = \frac{4.6(s+55.92)(s+0.5)}{s}$$

$$= \frac{4.6(s^2+56.42s+27.96)}{s}$$

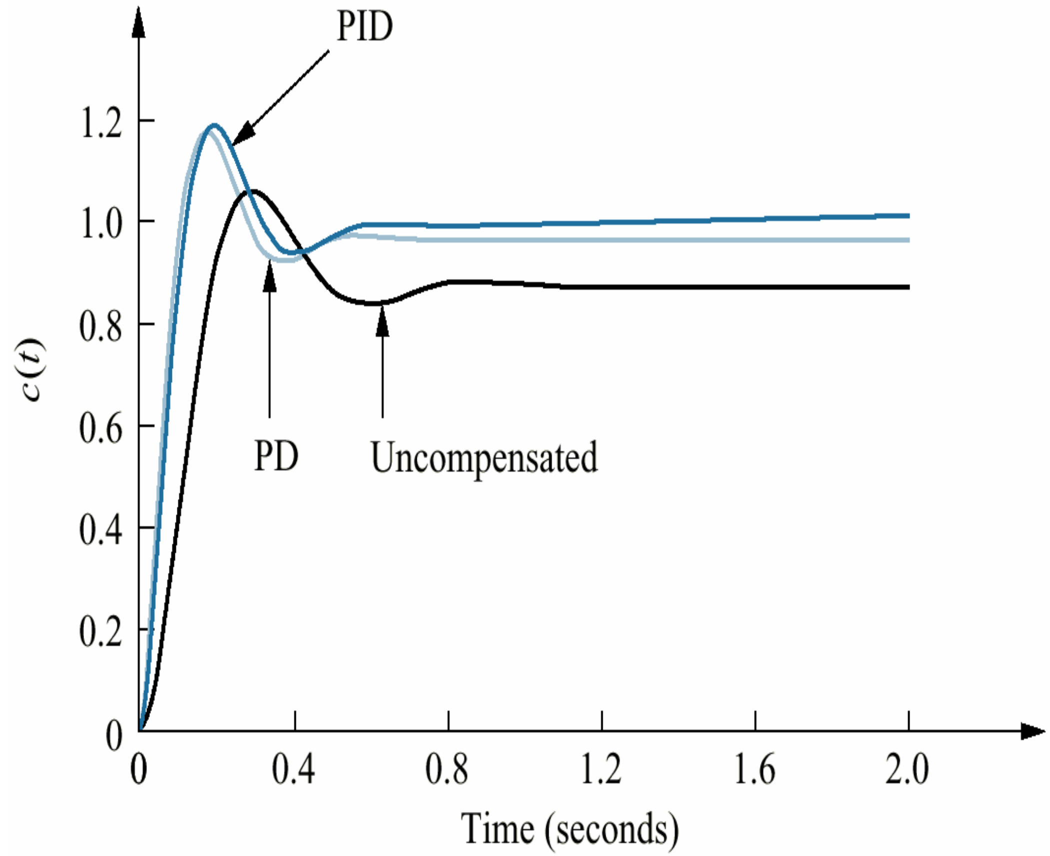
Matching this equation and the transfer function of PID controller we have learned before which is

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 s + K_2 + K_2 s^2}{s} = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s}$$

We obtain K_1=259.5, K_2=128.6 and K_3=4.6

Step 7 and 8 : The following figure summarize our design.

Figure 9.35
Step responses for uncompensated, PD-compensated, and PID-compensated systems

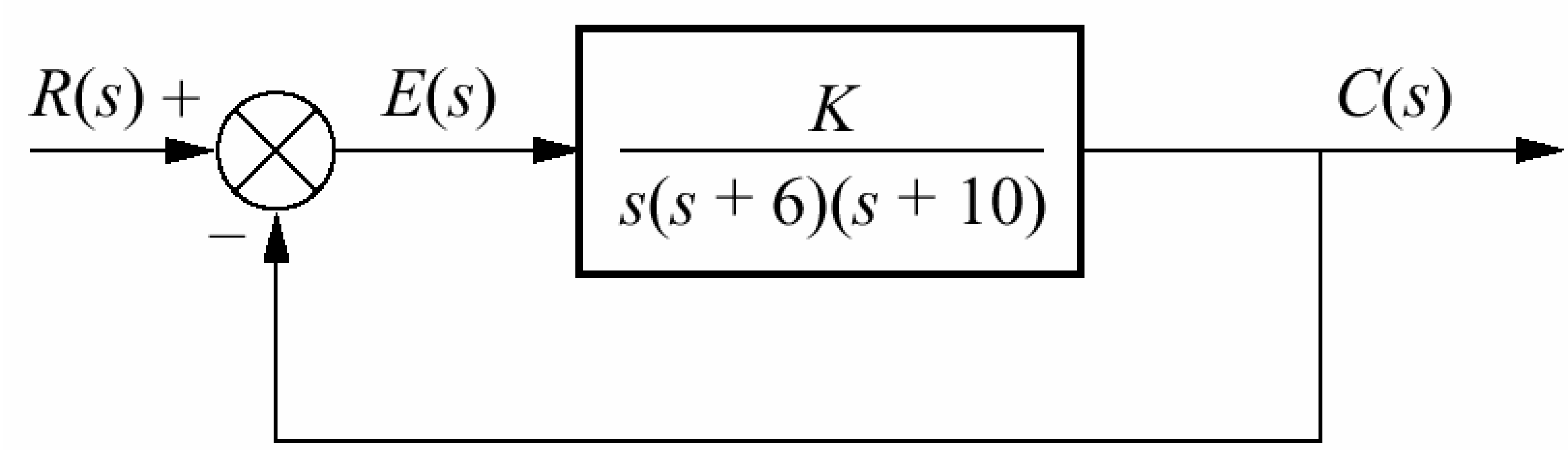


All requirements have been met

Lag-Lead Compensator Design : Another approach to improve steady state error and transient response is lead-lag compensator design. We first design the lead compensator to improve the transient response. Finally we design the lag compensator to meet the steady-state error requirement. The following steps summarize the design procedure :

1. Evaluate the performance of the uncompensated system to determine how much improvements in transient response is required.
2. Design the lead controller to meet the transient response specifications. The design includes the zero location, pole location and the loop gain.
3. Simulate the system to be sure all reqirements have been met.
4. Redesign if the simulation shows that the requirements have not been met.
5. Evaluate the steady-state error performance for the lead compensated system to determine how much more improvement in steady-state error is required.
6. Design the lag companсator to yield the required steady-state error.
7. Simulate the system to be sure all requirements have been met.
8. Redesign if the simulation shows that the requirements have not been met

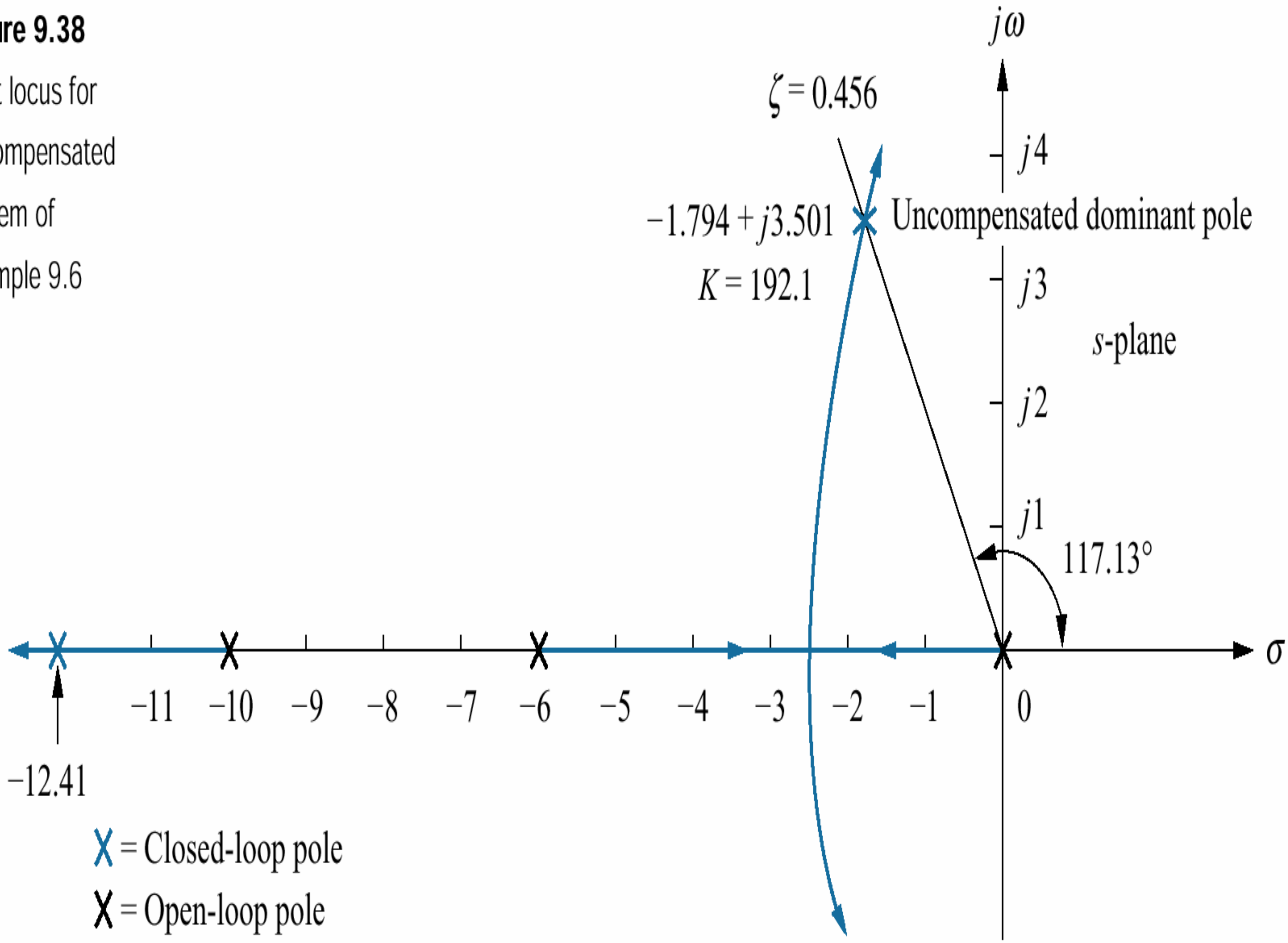
Example : Design a lag-lead compensator for the system of following figurew. So that the system will operate with 20%overshoot and a twofold reduction in settling time. Further, the compensated system will exhibit a tenfold improvement in steady-state error for a ramp input.



Solution : Step 1: First, we evaluate the performance of uncompensated system Searching along the 20% overshoot line (ζ=0.456) in the root locus of system, we find the dominant poles at -1.794±j3.501, with a gain of 192.1. the performance uncompansated system is summarized in following table.

Figure 9.38

Root locus for uncompensated system of Example 9.6

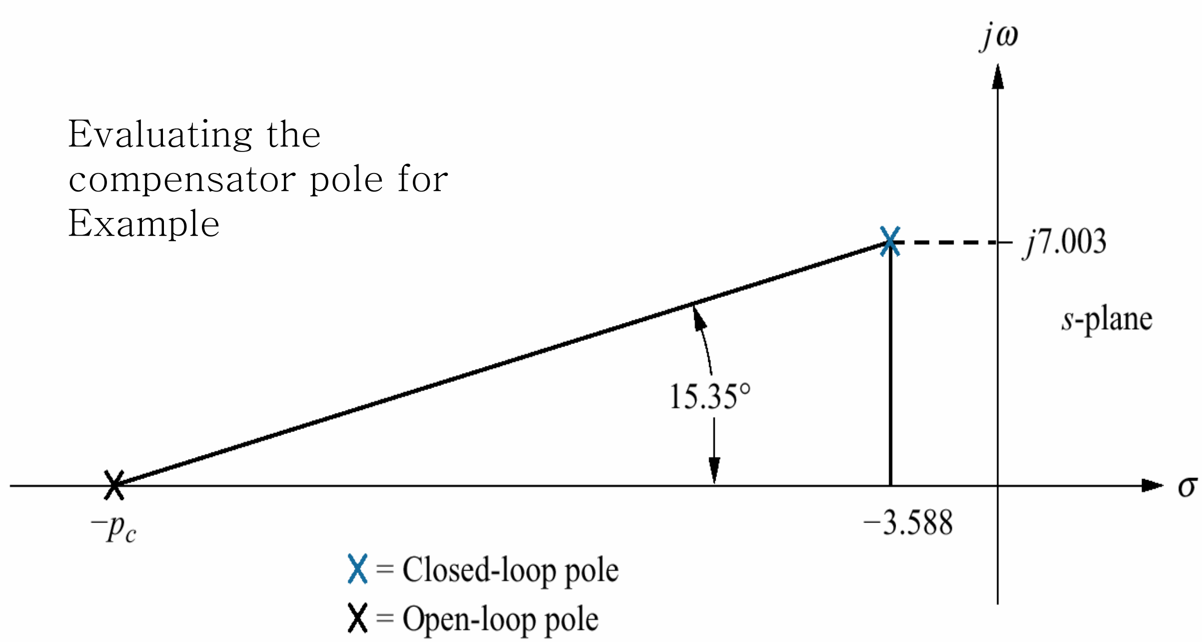


Table

Predicted characteristics of uncompensated, lead-compensated, and lag-lead-compensated systems of Example

	Uncompensated	**Lead-compensated**	**Lag-lead-compensated**
Plant and compensator	$\dfrac{K}{s(s+6)(s+10)}$	$\dfrac{K}{s(s+10)(s+29.1)}$	$\dfrac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}$
Dominant poles	$-1.794 \pm j3.501$	$-3.588 \pm j7.003$	$-3.574 \pm j6.976$
K	192.1	1977	1971
ζ	0.456	0.456	0.456
ω_n	3.934	7.869	7.838
$\%OS$	20	20	20
T_s	2.230	1.115	1.119
T_p	0.897	0.449	0.450
K_v	3.202	6.794	31.92
$e(\infty)$	0.312	0.147	0.0313
Third pole	-12.41	-31.92	-31.91, -0.0474
Zero	None	None	-0.04713
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK

Step2 : Next we design a lead compensator by selecting the location of uncompensated system's dominant poles. In order to realize a twofold reduction of in settling time, the real part of the dominant pole must be increased by a factor of 2, since the settling time is inversely proportional to the real part. Thus, $-\zeta\omega_n = -2(1.794) = -3.588$. the imaginary part of the design point is $\omega_d=\zeta\omega_n \tan 117.13° = 7.003$. Now we design the lead compensator. For this example we select the location of the compensator zero coincident with the the open loop pole at -6. This choice will eliminate a zero and leave the lead compensated system with three poles, the same number that the uncompensated system has. We complete the design by by finding the location of the compensator pole. Using the root locus program, sum the angles to the design point from the uncompansated system's poles and zeros and the compensator zero and get -164.65°. The difference between 180° and this quantity is the angular contribution required from the compensator pole, or -15.35°. Using the geometry shown in figure,

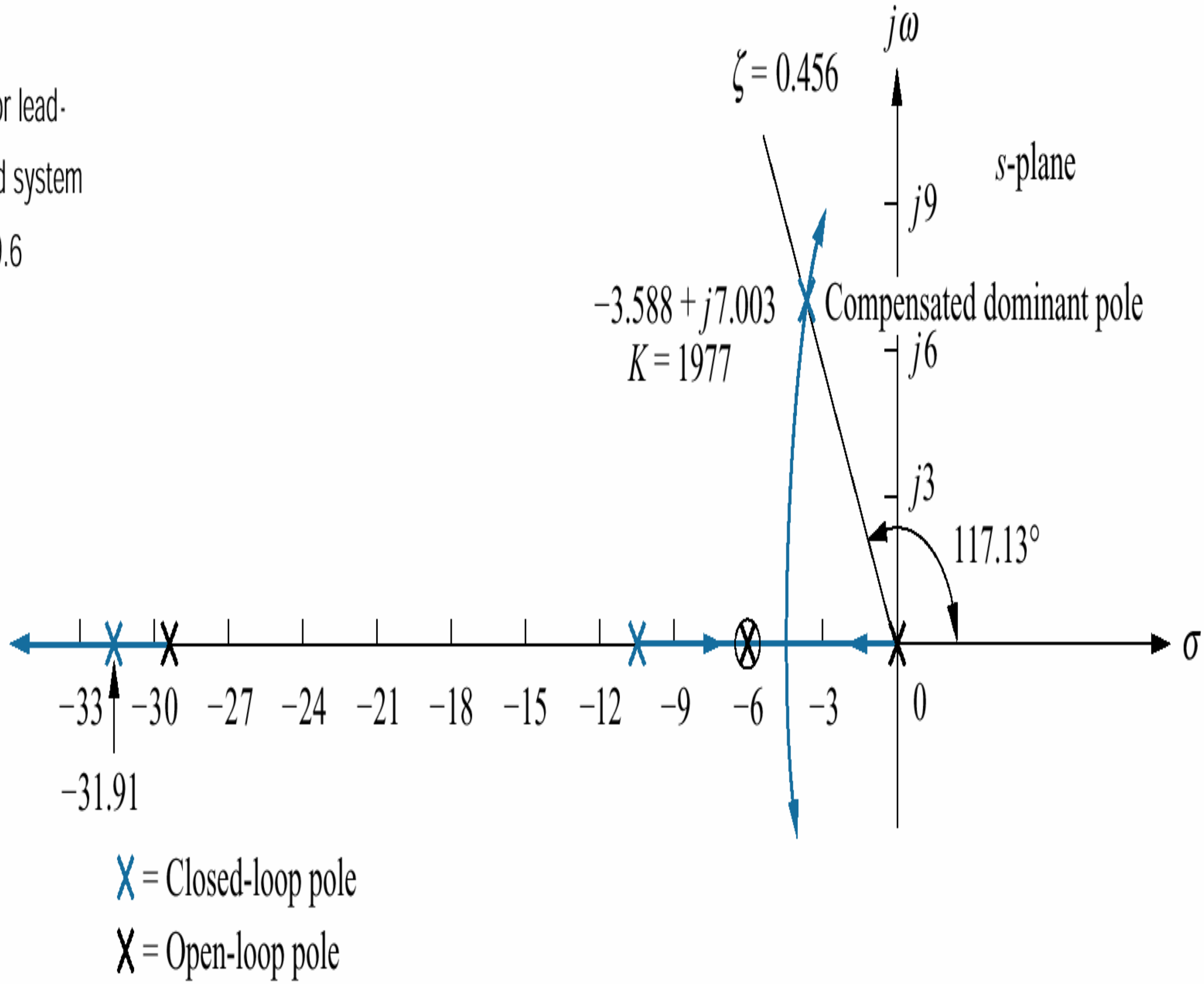


$$\frac{7.003}{p_c - 3.588} = \tan 15.35°$$

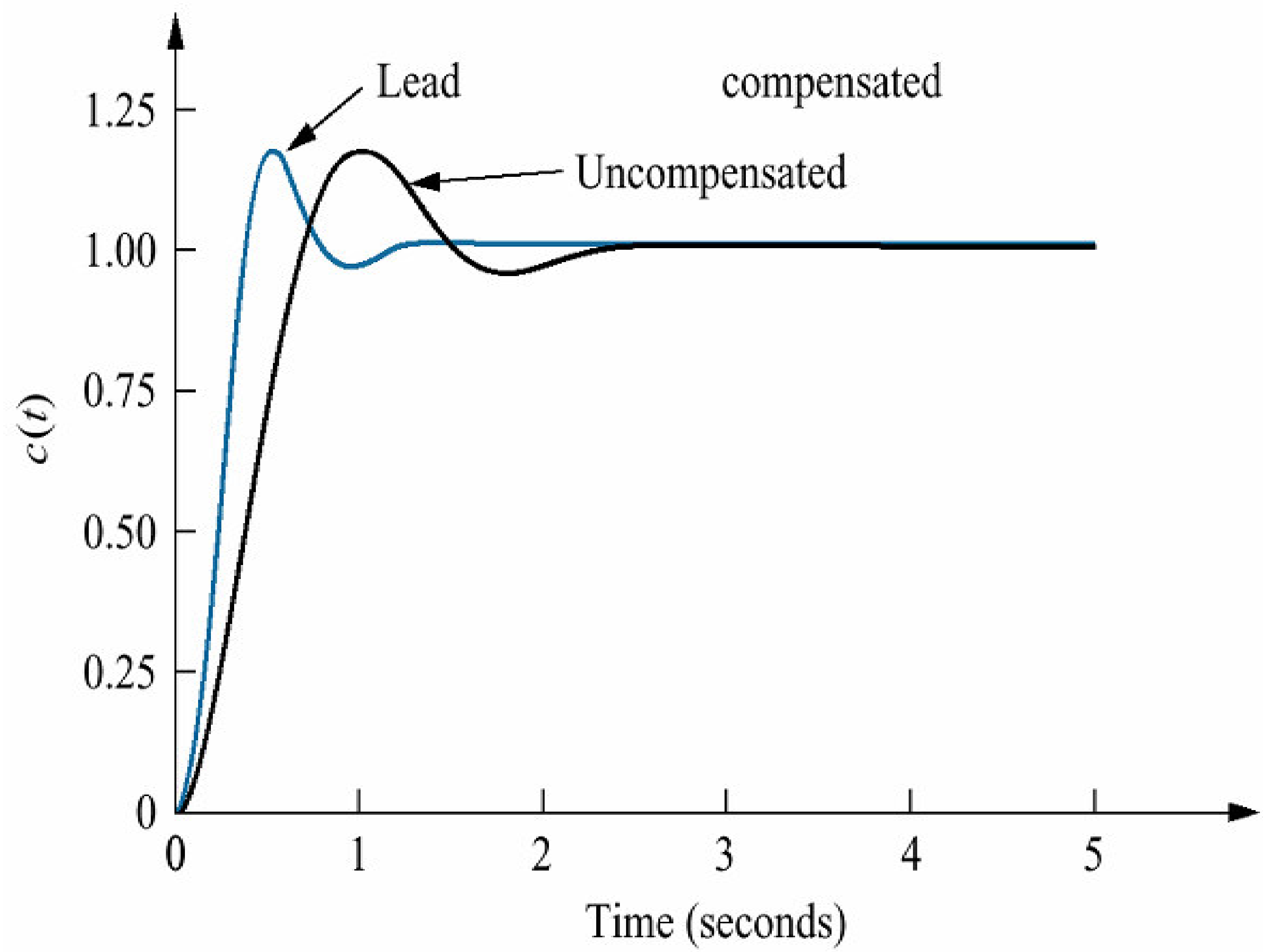
from which the location of compensator pole p_c is found to be -29.1. The complete root locus forthe lead compensation system is sketched in the following figure. The gain setting at the design point is found to be 1977.

Figure 9.40

Root locus for lead-compensated system of Example 9.6



Step3 and 4 : Check the design with a simulation. The result for the lead compensated system is shown in figure and is satisfactory.



Step5 : Continue by designing the lag compensator to improve the steady-state error. Since the uncompensated system open loop transfer function is

$$G_s = \frac{1921}{s(s+6)(s+10)}$$

The static error constant K_v which is inversely proportional to the steady state error is 3.201. Since the open loop transfer function of the lead compensated system is

$$G_s = \frac{1977}{s(s+10)(s+29.1)}$$

The static error constant is 6.794. Thus, the addition of lead compensation has improved the steady state error by factor of 2.122. since the requirements of the problem specified a tenfold improvement, the lag compensator must be designed to improve the steady-state error by a factor of 4.713 (10/2.122=4.713) over the lead compensated system .

Step 6 : We arbitrarily choose the lag compensator pole at 0.01, which then places the lag compensator zero at 0.04713, yielding

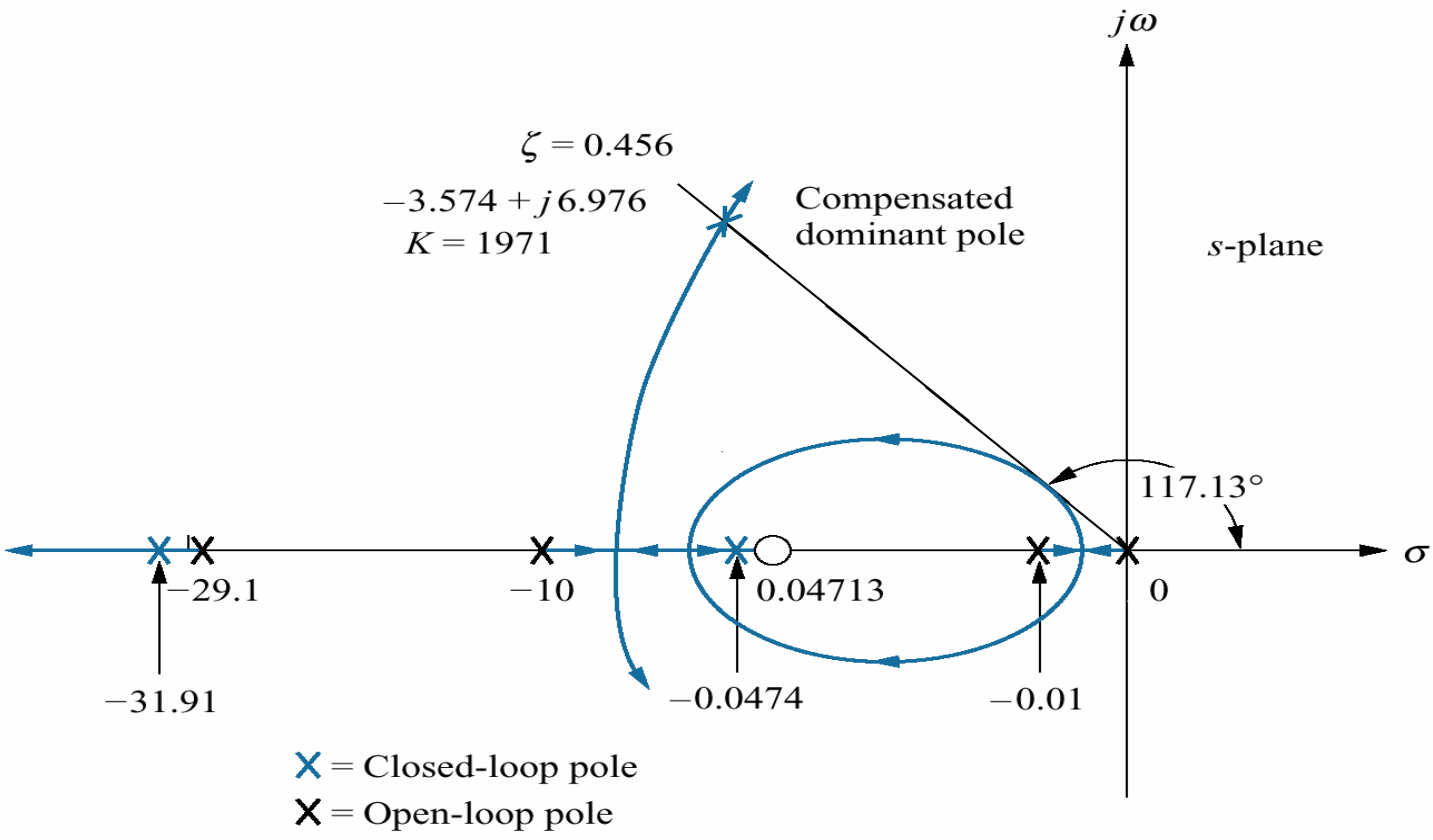
$$G_{lag}(s) = \frac{(s+0.04713)}{(s+0.01)}$$

As the lag compensator. The lag-lead compensated system's open loop transfer function

$$G_{LLC} = \frac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}$$

where the uncompansated system pole at -6 canceled the lead compensator zero at -6.

By drawing the complete root locus for the lag-lead compensated system and by searching along the 0.456 damping ratio line, we find the dominant, the closed loop poles to be at -3.574±j6.976, with a gain of 1971. the lag-lead compansated root locus is shown in figure.

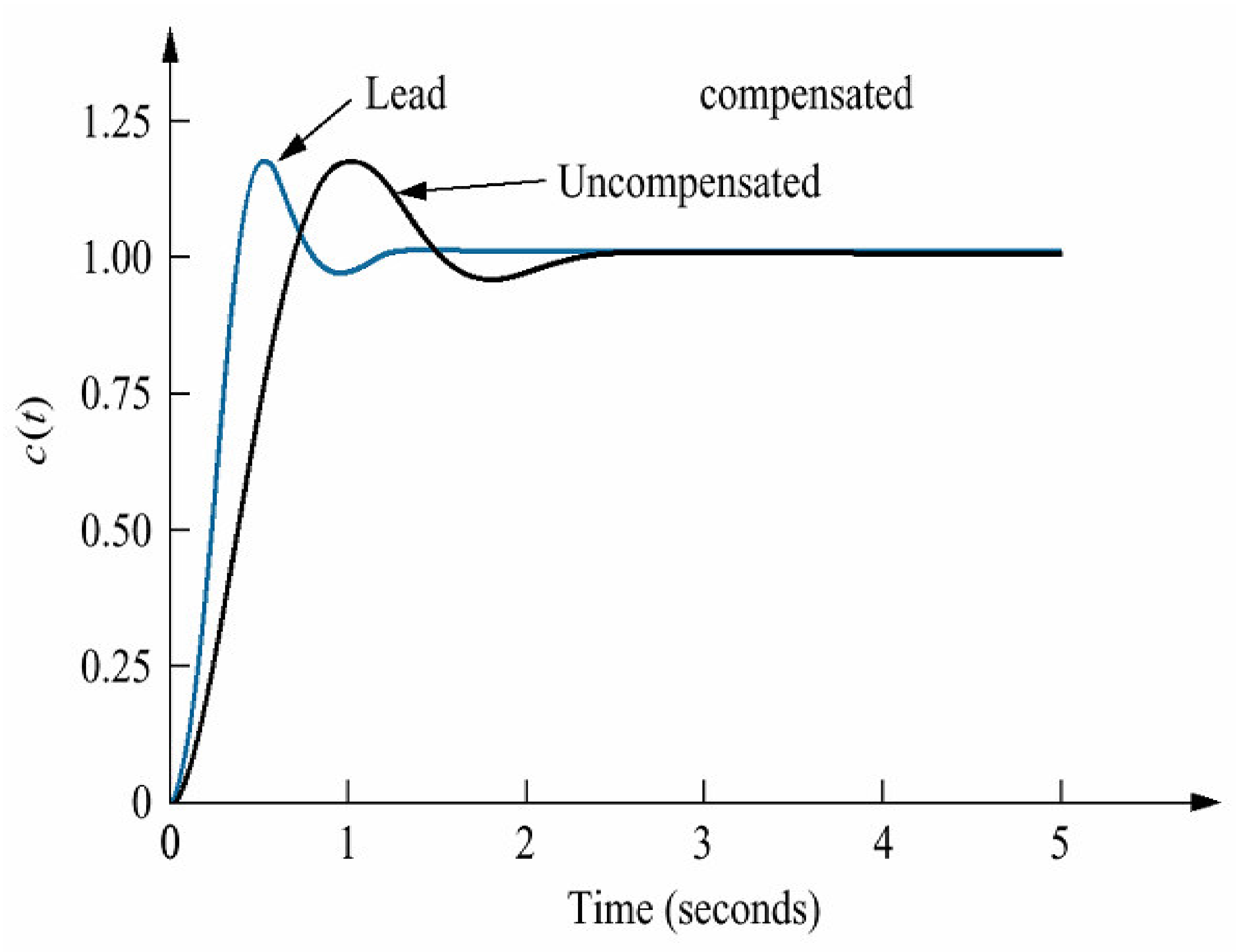


Note: This figure is not drawn to scale.

A summary of our design is shown in following table. Notice that the lag-lead compensation has indeed increased the speed of the system, as witnessed by the settling time or the peak time. The steady-state error for a ramp input has also decreased by about 10 times as ssen from e(∞).

	Uncompensated	**Lead-compensated**	**Lag-lead-compensated**
Plant and compensator	$\frac{K}{s(s+6)(s+10)}$	$\frac{K}{s(s+10)(s+29.1)}$	$\frac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}$
Dominant poles	$-1.794 \pm j3.501$	$-3.588 \pm j7.003$	$-3.574 \pm j6.976$
K	192.1	1977	1971
ζ	0.456	0.456	0.456
ω_n	3.934	7.869	7.838
$\%OS$	20	20	20
T_s	2.230	1.115	1.119
T_p	0.897	0.449	0.450
K_v	3.202	6.794	31.92
$e(\infty)$	0.312	0.147	0.0313
Third pole	-12.41	-31.92	-31.91, -0.0474
Zero	None	None	-0.04713
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK

Step 7 : The final proof of our designs is shown by the simulation of following figures.



Following tables show the summary of cascade compensators.

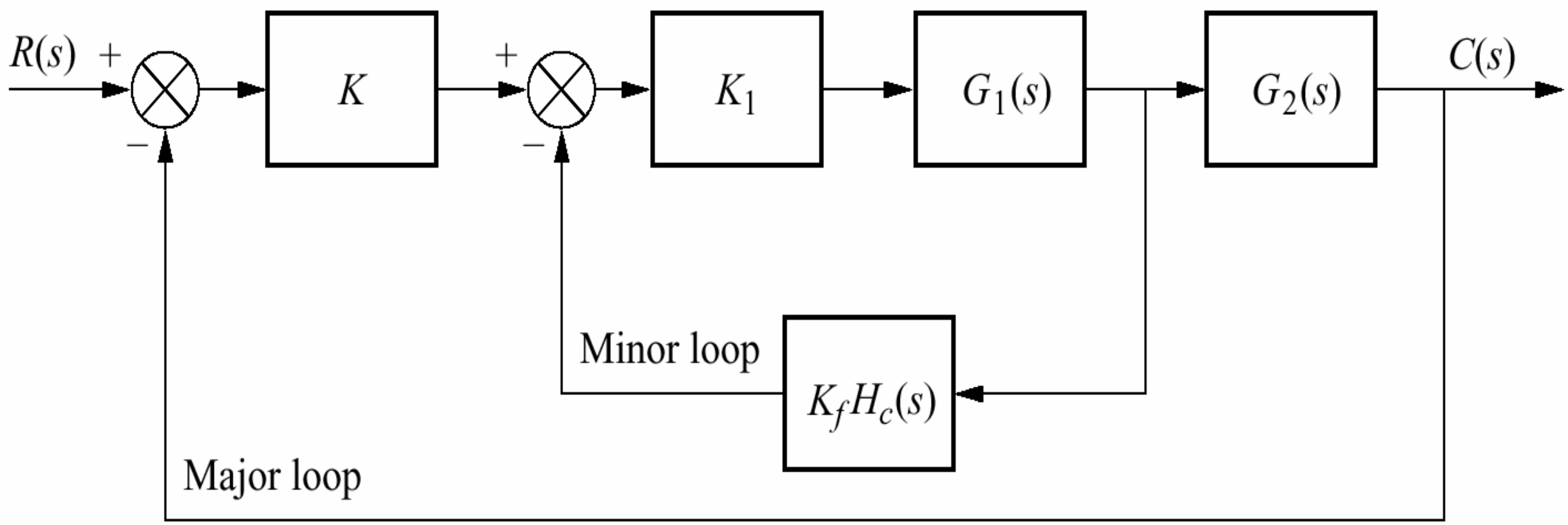
Table 9.7 Types of cascade compensators

Function	Compensator	Transfer function	Characteristics
Improve steady-state error	PI	$K\frac{s+z_c}{s}$	1. Increases system type. 2. Error becomes zero. 3. Zero at $-z_c$ is small and negative. 4. Active circuits are required to implement.
Improve steady-state error	Lag	$K\frac{s+z_c}{s+p_c}$	1. Error is improved but not driven to zero. 2. Pole at $-p_c$ is small and negative. 3. Zero at $-z_c$ is close to, and to the left of, the pole at $-p_c$. 4. Active circuits are not required to implement.
Improve transient response	PD	$K(s+z_c)$	1. Zero at $-z_c$ is selected to put design point on root locus. 2. Active circuits are required to implement. 3. Can cause noise and saturation; implement with rate feedback or with a pole (lead).
Improve transient response	Lead	$K\frac{s+z_c}{s+p_c}$	1. Zero at $-z_c$ and pole at $-p_c$ are selected to put design point on root locus. 2. Pole at $-p_c$ is more negative than zero at $-z_c$. 3. Active circuits are not required to implement.

Improve steady-state error and transient response	PID	$K\dfrac{(s+z_{\text{lag}})(s+z_{\text{lead}})}{s}$	1. Lag zero at $-z_{\text{lag}}$ and pole at origin improve steady-state error. 2. Lead zero at $-z_{\text{lead}}$ improves transient response. 3. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, the origin. 4. Lead zero at $-z_{\text{lead}}$ is selected to put design point on root locus. 5. Active circuits required to implement. 6. Can cause noise and saturation; implement with rate feedback or with an additional pole.
Improve steady-state error and transient response	Lag-lead	$K\dfrac{(s+z_{\text{lag}})(s+z_{\text{lead}})}{(s+p_{\text{lag}})(s+p_{\text{lead}})}$	1. Lag pole at $-p_{\text{lag}}$ and lag zero at $-z_{\text{lag}}$ are used to improve steady-state error. 2. Lead pole at $-p_{\text{lead}}$ and lead zero at $-z_{\text{lead}}$ are used to improve transient response. 3. Lag pole at $-p_{\text{lag}}$ is small and negative. 4. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, lag pole at $-p_{\text{lag}}$. 5. Lead zero at $-z_{\text{lead}}$ and lead pole at $-p_{\text{lead}}$ are selected to put design point on root locus. 6. Lead pole at $-p_{\text{lead}}$ is more negative than lead zero at $-z_{\text{lead}}$. 7. Active circuits are not required to implement.

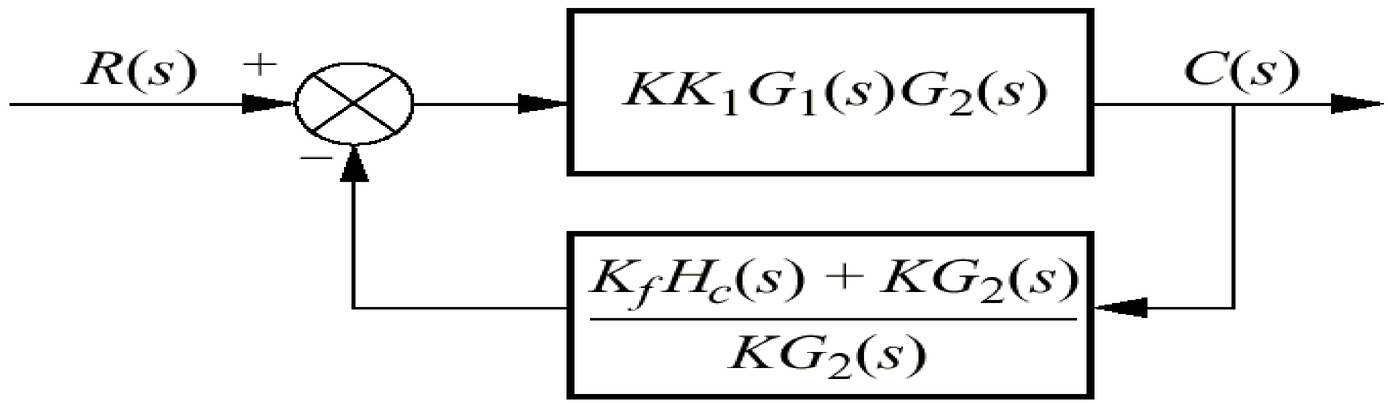
FEEDBACK COMPENSATION

The design procedures for feedback compensation can be more complicated than for cascade compensation. On the other hand, feedback compensation can yield faster response. A configration included feedback compensator is shown in following figure. Note that feedback compensator $H_c(s)$ is placed in the minor loop of a feedback control system.



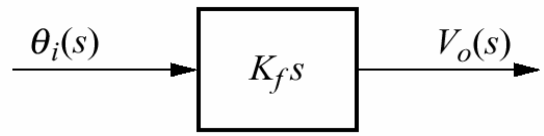
Other configrations arise if we consider K unity, G2(s) unity, or both unity.

The equivalent block diagram is shown in figure below.

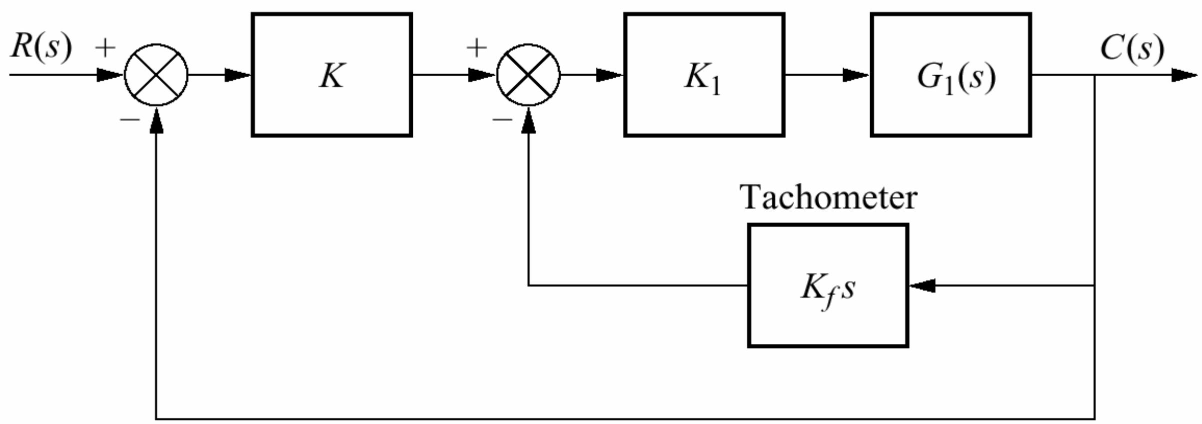


A popular feedback compensator is tachometer as shown in figure below. A tachometer is a voltage generator that yields a voltage output proportional to the input rotational speed.

Figure
a. Transfer function of a tachometer;
b. tachometer feedback compensation



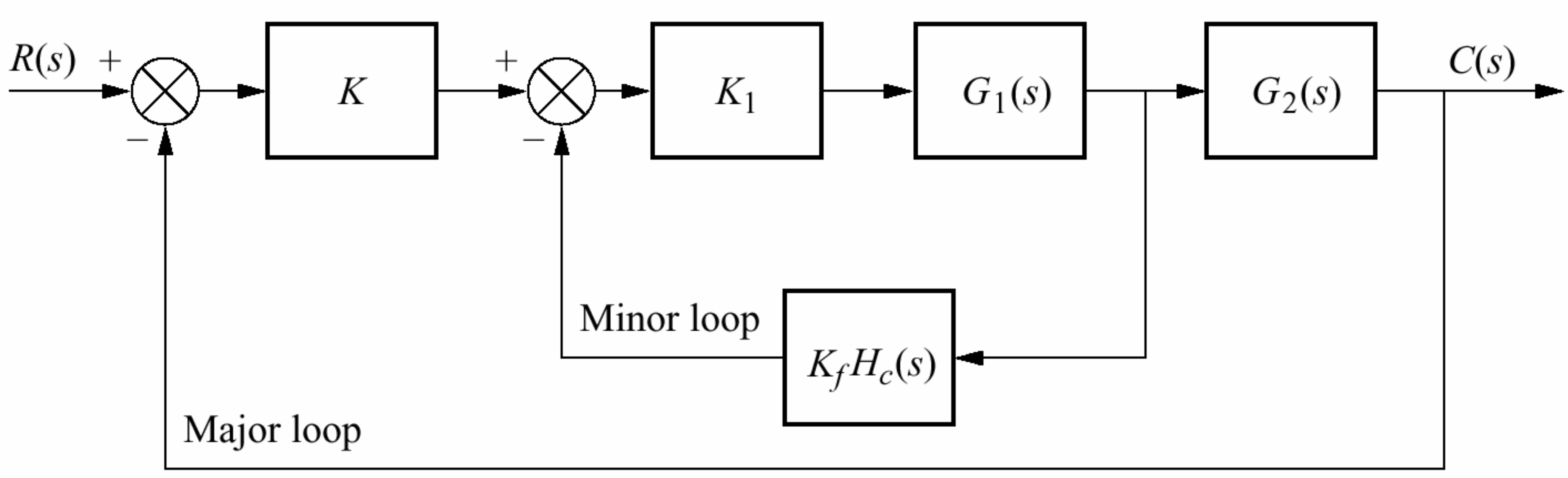
(a)

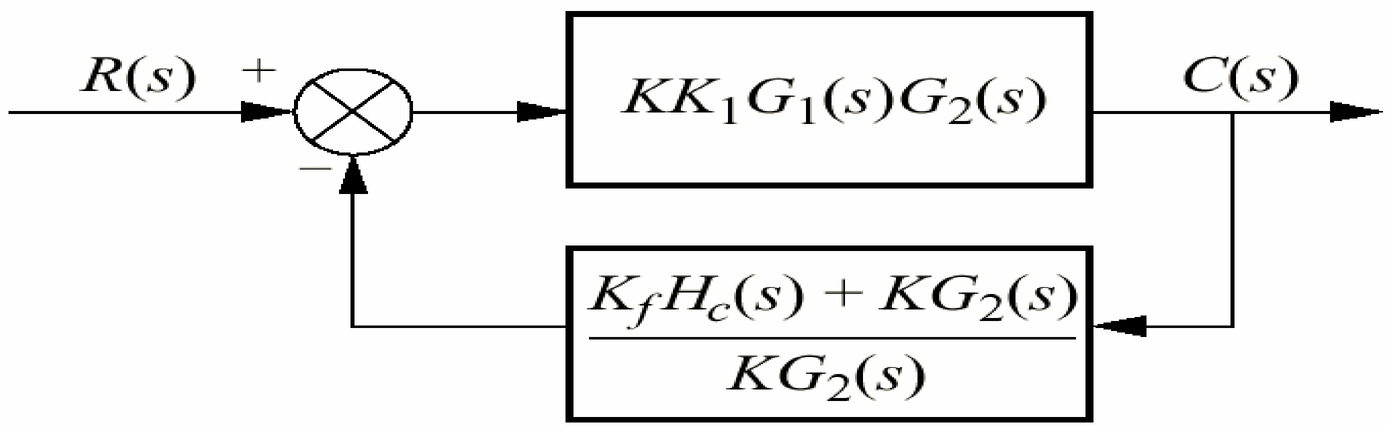


(b)

We now discuss the design procedures. The design of feedback compensation consists of finding the gains, such as K, K_1 and K_f which are seen in the previous block diagram after establishing a dynamic form for H_c. There are two approaches.

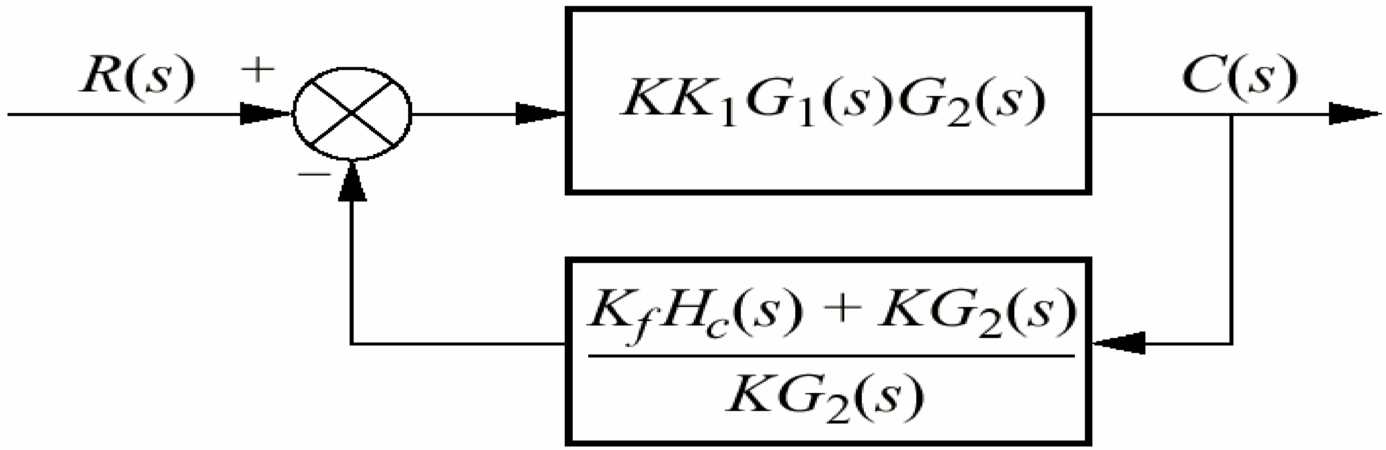
Approach 1 : The first approach consists of reducing the first figure to the second one by pushing K to the right past summing junction, pushing $G_2(s)$ to the left past the pickoff point, and then adding the two feedback paths.





Following figure shows that the loop gain, G(s)H(s), is

$$G(s)H(s) = K_1G_1(s)[K_fH_c(s)+KG_2(s)]$$

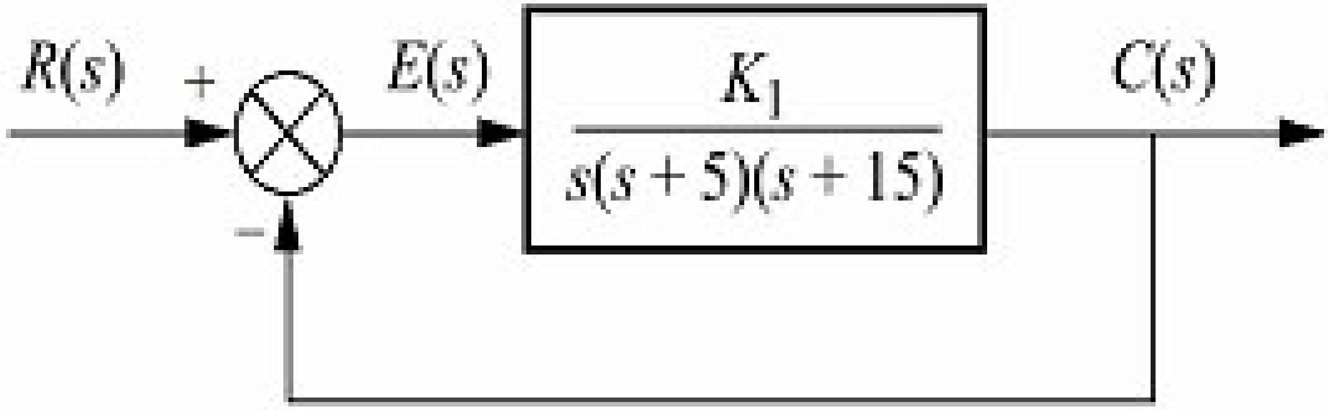


For example, ig $G_2(s)=1$ and minor loop feedback, $K_fH_c(s)$, is a rate sensor, $K_fH_c(s)=K_fs$, then using the formulation above the loop gain is

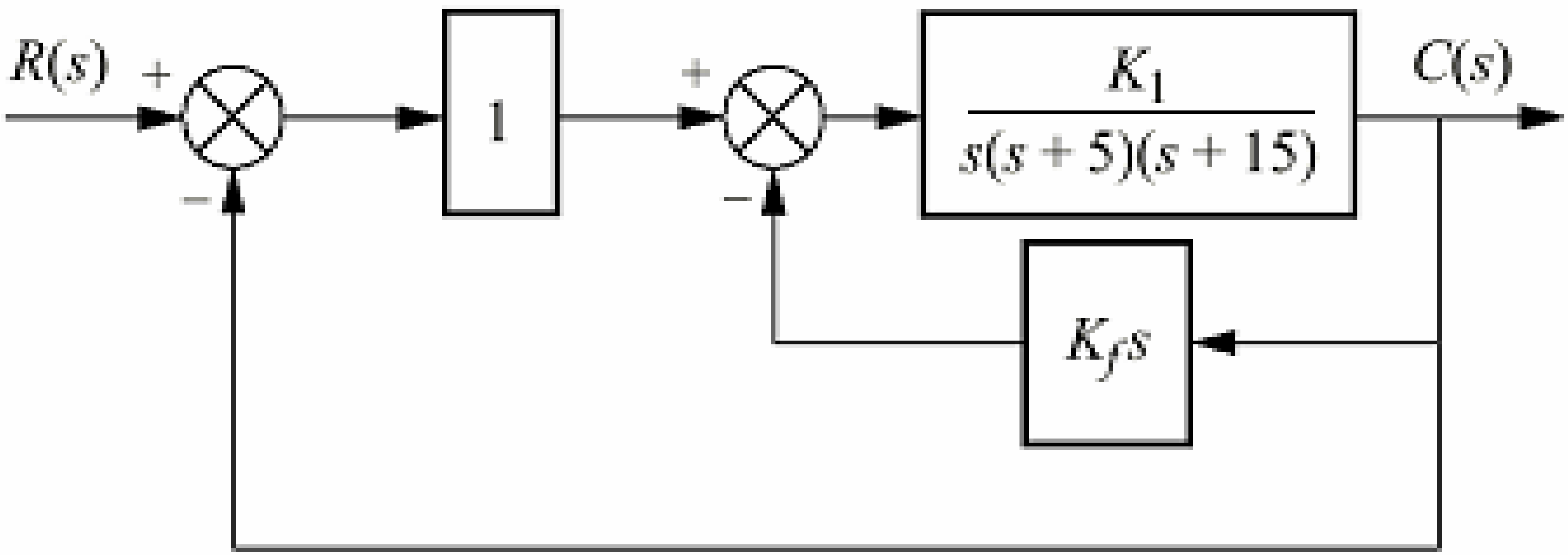
$$G(s)H(s) = K_f K_1 G_1(s)\left(s+\frac{K}{K_f}\right)$$

Thus a zero at $-K/K_f$ is added to the existing open loop poles and zeros. This zero reshapesthe root locus go through the desired design point. A final adjustment of the gain, K_1, yields the desired response. Again, you should verify that this zero is not a closed loop zero. Let us look a numerical example.

Example : Given the system of figure(a), design rate feedback compensation as shown in figure(b) to reduce the settling time by a factor of 4 while continuing to operate the system with 20% overshoot.



(a)

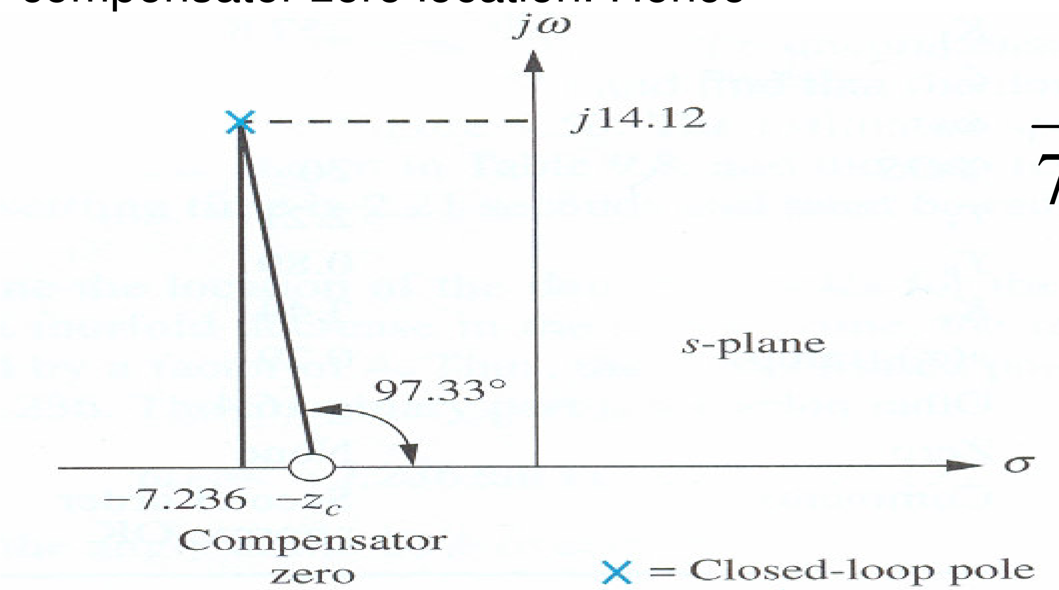


(b)

Solution : First design a PD compensator. For the uncompensated system search along 20%overshoot line (ζ=0.456) and find the dominant poles are that -1.809±j3.531, (you can find this values by sketcing the root locus). The settling time is 2.21 second and must be reduced by a factor of 4 to 0.55 second. Next determine the location of the dominant poles for the compensated system. To achieve a fourfold decrease in the settling time, the real part of the pole must be increased by a factor of 4. Thus the compensated pole has a real part of 4*(-1.809)=-7.236. The imaginary part is then

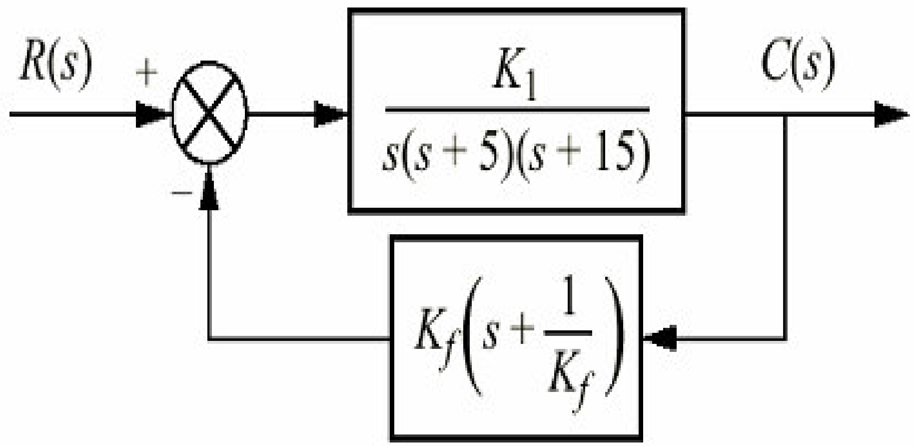
$$\omega_d = -7.236\tan 117.13° = 14.12$$

where 117.13 is the angle of the 20% overshoot line. Using the compensated position of -7.236±j14.12, we sum the angles from the compensated system poles and obtain -277.33°. This angle requires a compensator zero contribution of +97.33° to yield 180° at the design point. The geometry is shown in following figure leads to the calculation of the compensator zero location. Hence

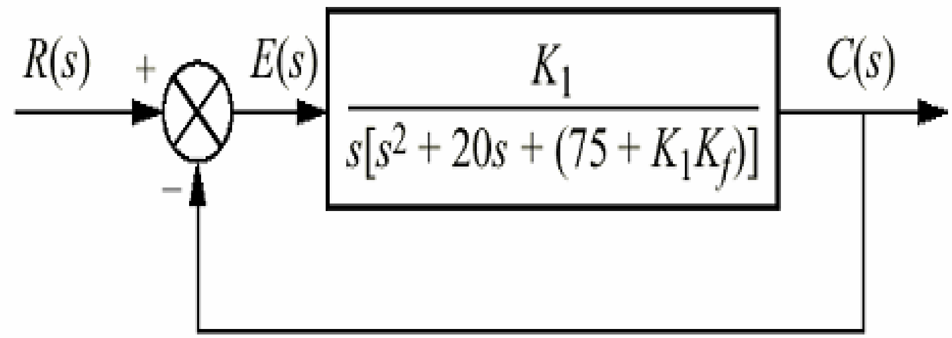


$$\frac{14.12}{7.236 - z_c} = \tan(180° - 97.33°)$$

from which z_c=5.42



(c)



(d)

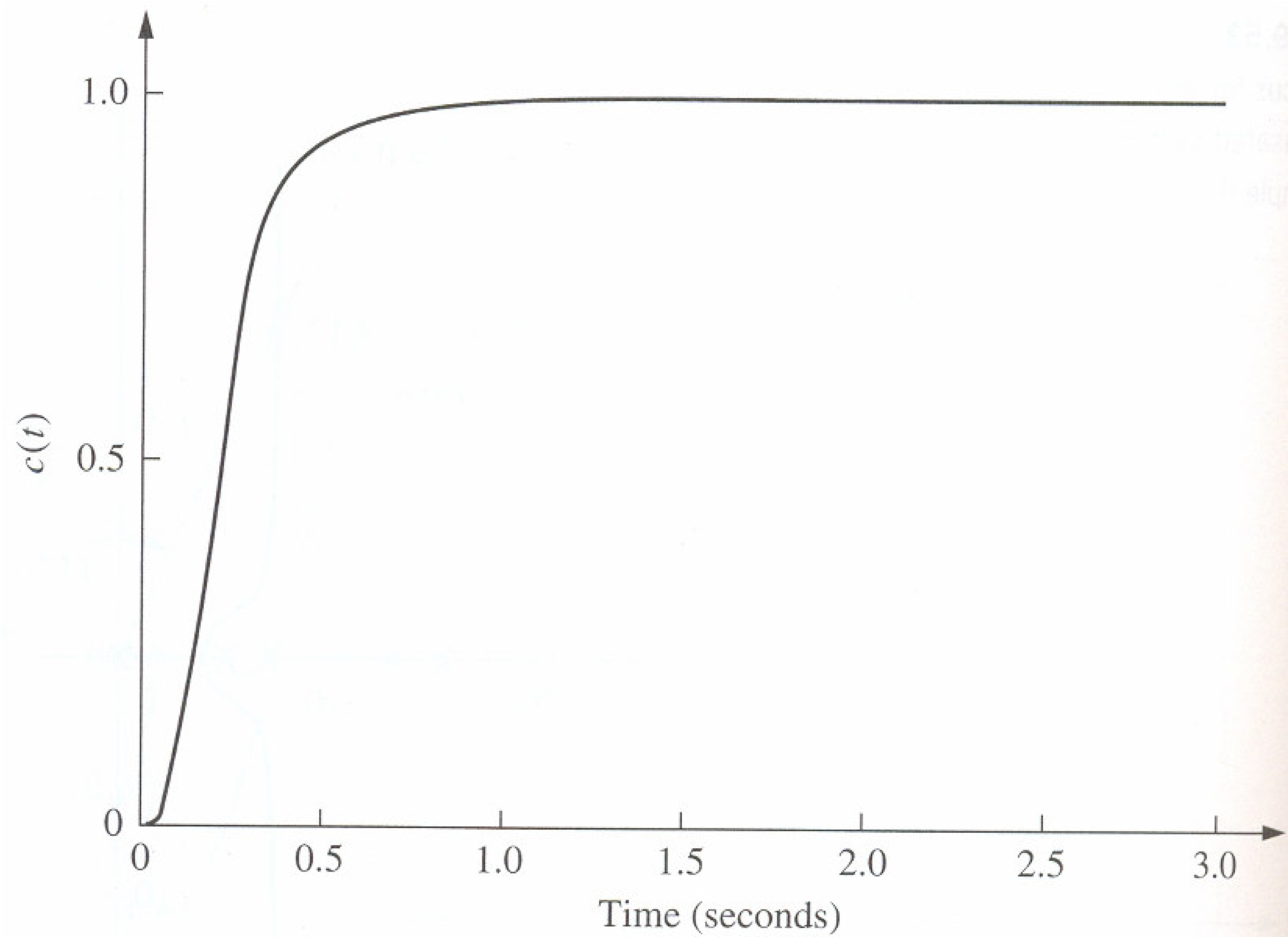
The gain at the design point, which is K_1K_f from figure(c), is found to be 256.7 . Since K_f is the reciprocal of the compensator zero, K_f=0.185. Thus K_1=1388 . In order to evaluate the steady-state eroor characteristic, K_v is found from figure(d) to be

$$K_v = \frac{K_1}{75 + K_1 K_f} = 4.18$$

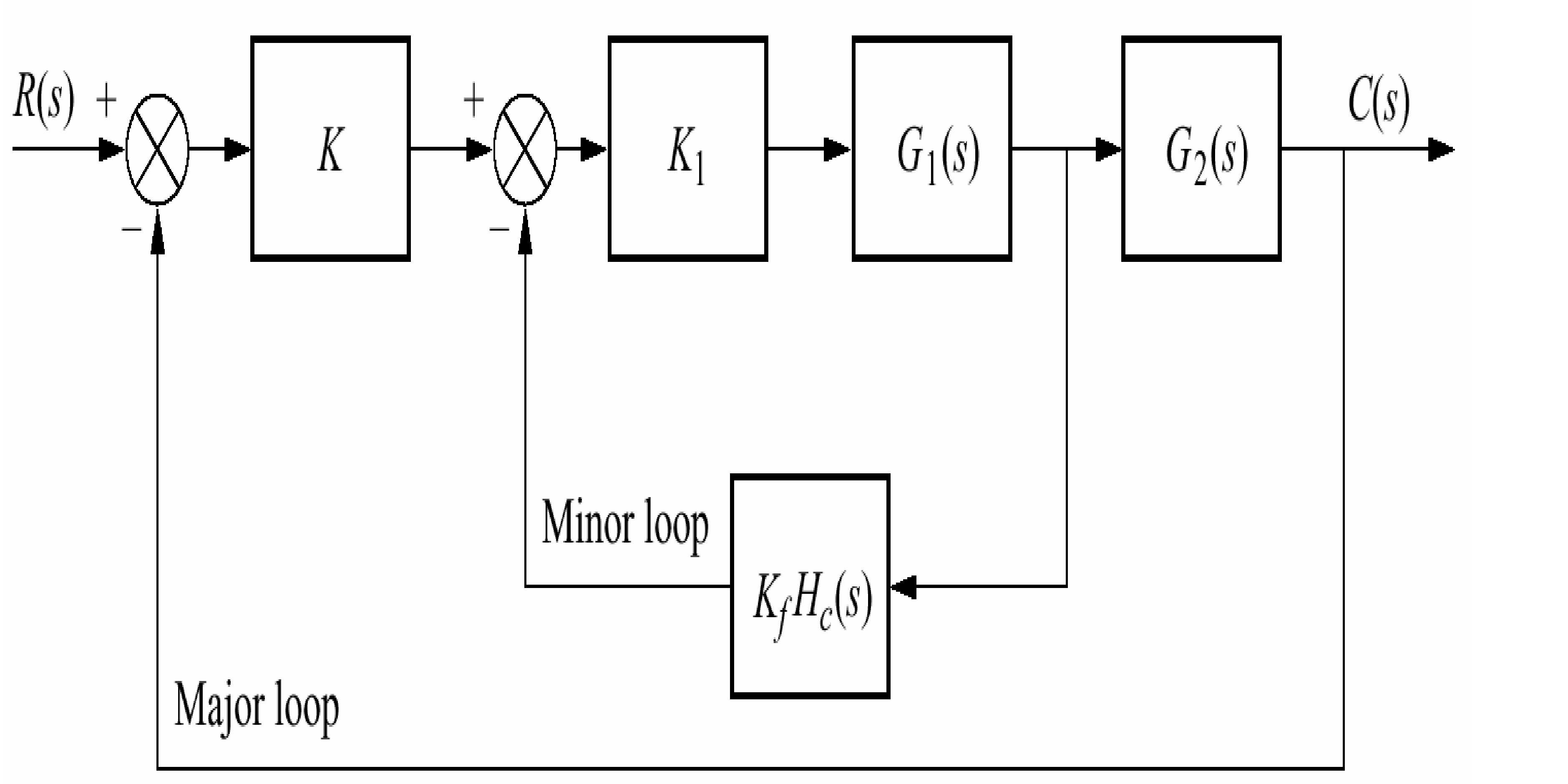
The closed loop transfer function is

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{K_1}{s^3 + 20s^2 + (75 + K_1 K_f)s + K_1}$$

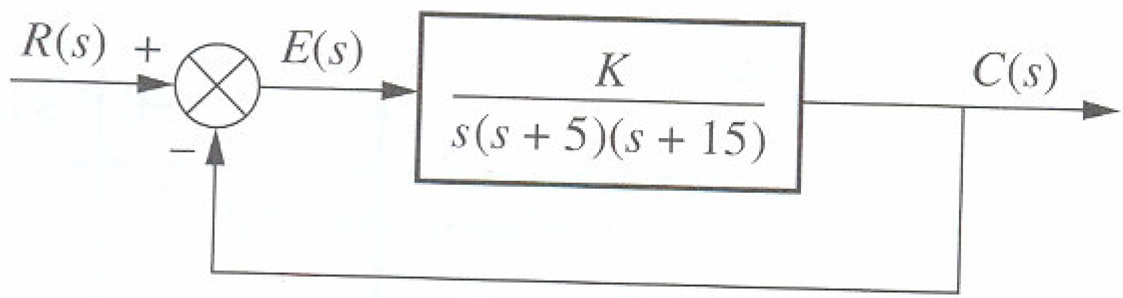
The result of simulation is shown in following figure. Figure shows the design specifications are met.



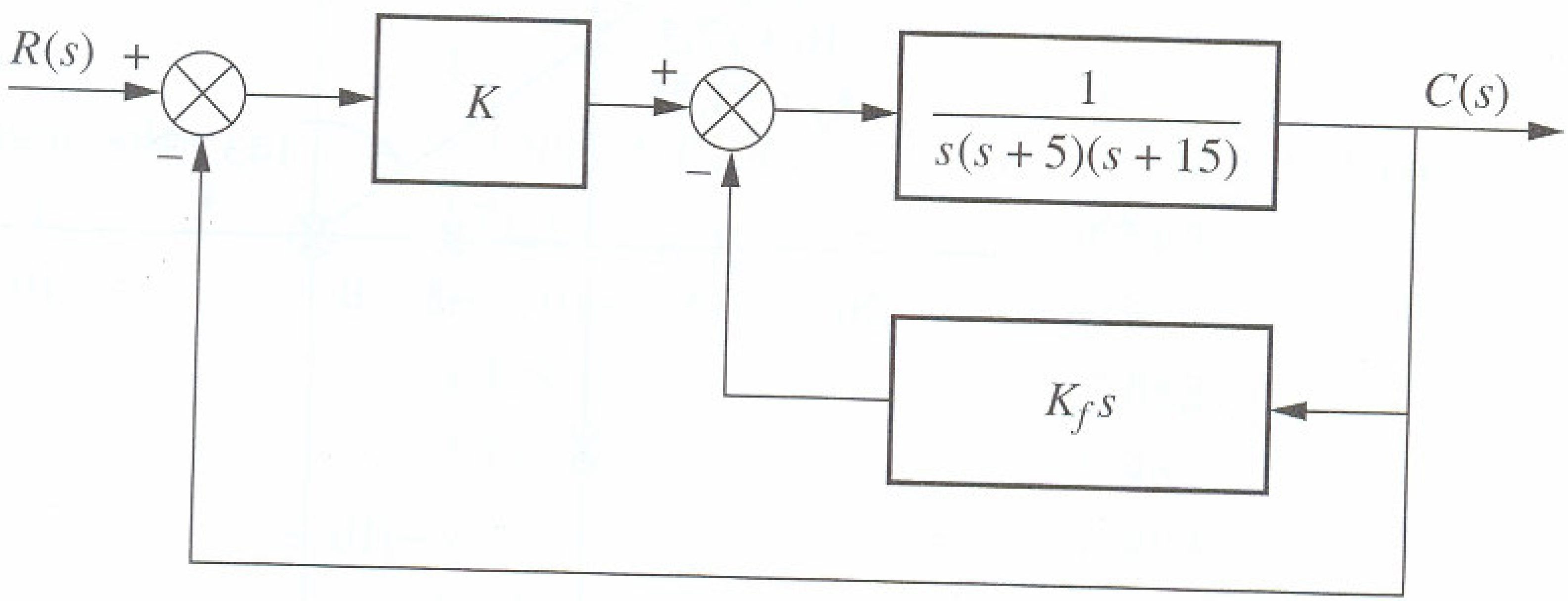
Approach 2 : The second approach allows us to use feedback compensation to design a minor loo's transient response seretely from the closed loop system response. We will see that the minor loop of the figure basically represents a forward path transfer function whose poles can be adjusted with the minor loop gain. These poles then become the open loop poles for the entire control system. In other words, rather than reshaping the root locus with additional poles or zeros, as in cascade compensation, we can actually change the plant's poles through a gain adjustment. Finally, the closed loop poles are set by the loop gain, as in cascade compensation



Example : For the system of following figure(a), design a minor loop feedback compensation, as shown in figure (b), to yield a damping ratio of 0.8 for the minor loop and a damping ratio of 0.6 for the closed loop system.



(a)



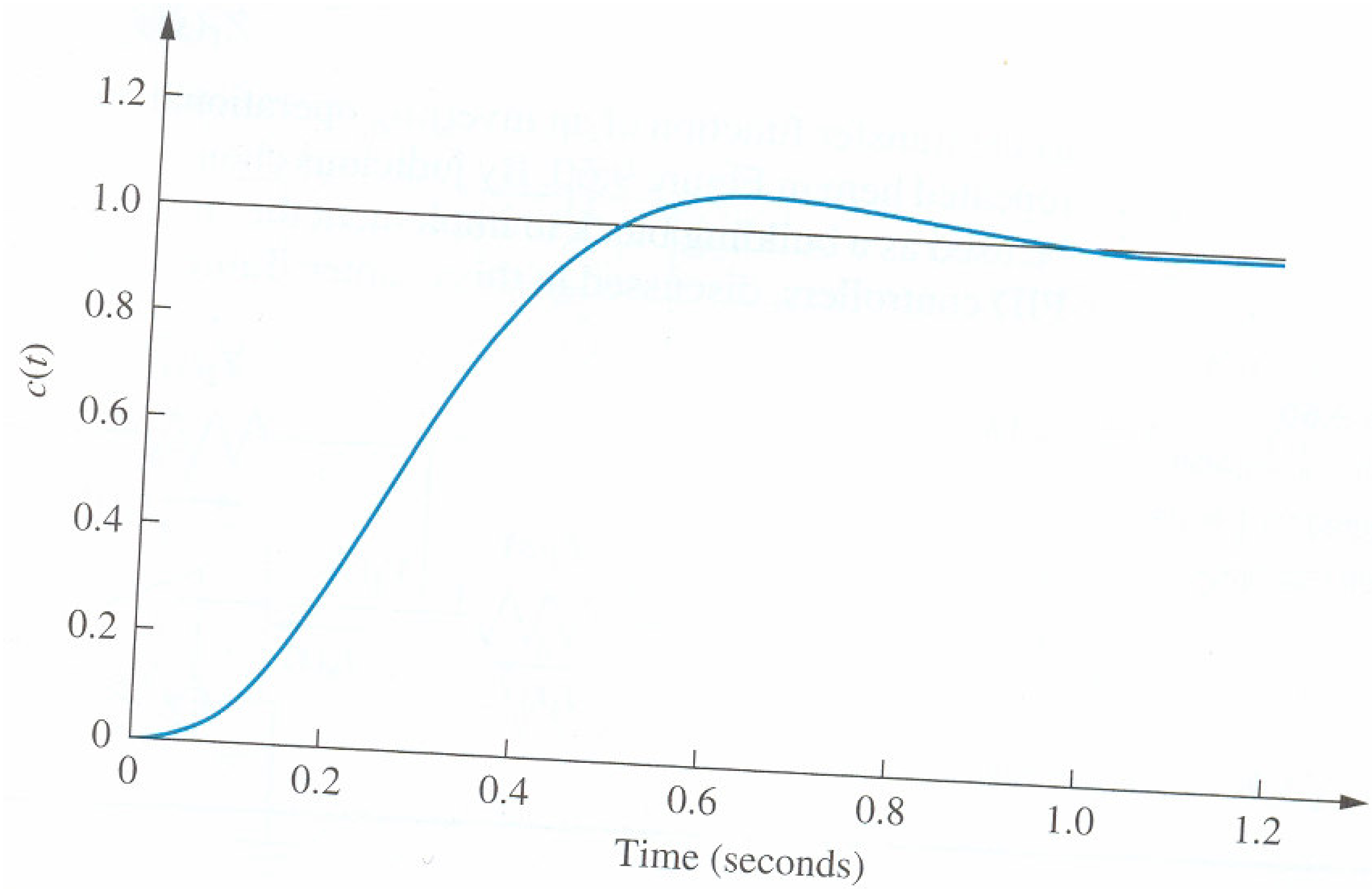
(b)

Solution : The minor loop is defined as the loop containing the plant, 1/[s(s+5)(s+15)], and the feedback compensator, $K_f s$. The value of K_f will be adjusted to set the location of the minor loop poles, and then K will be adjusted to yield the desired closed loop response. The transfer function of the minor loop, $G_{ML}(s)$, is

$$G_{ML}(s) = \frac{1}{s[s^2 + 20s + (75 + K_f)]}$$

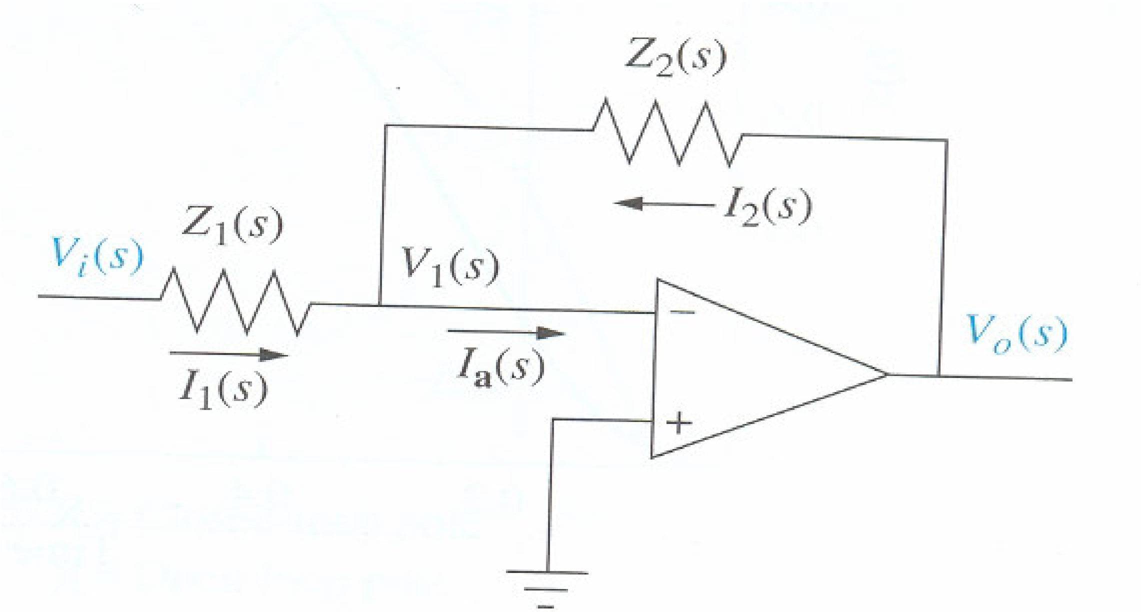
The poles of G_{ML} can be found analytically or via the root locus. If the root locus for the minor loop, where $K_f s/[s(s+5)(s+15)]$ is the open loop transfer function, is sketched, drawing the ζ=0.8 line yields the complex poles at -10±j7.5. The gain K_f, which equals 81.25, places the minor poles in a position to meet the specifications. The poles just found as well as the pole at the orogon , act as openloop poles that gnerate a root locus for varaitons of the gain K. Similarly, the closed loop complex poles are found to be -4.535±j6.046, with a required gain of 624.3. a third pole is at -10.93.

The result of simulation is shown in following figure. Figure shows the design specifications are met.



PHYSICAL REALIZATION OF COMPENSATION

We know the transfer function of an inverting operational amplifier whose configration shown in figure is

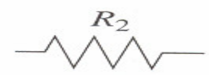
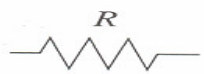
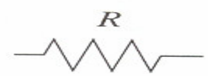
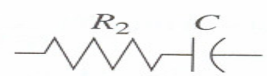
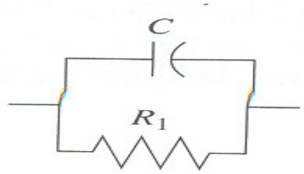
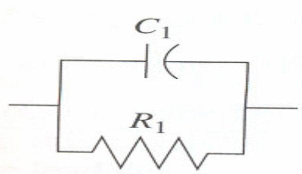
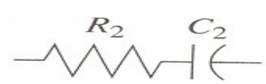
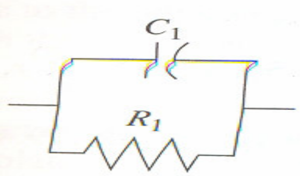
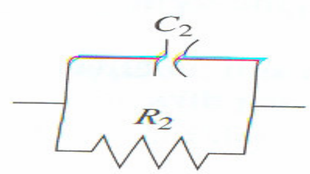
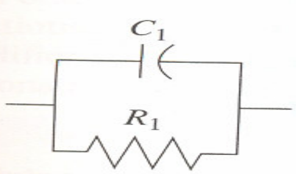
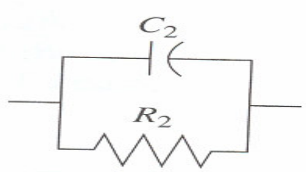


$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

By judicious choice of $Z_1(s)$ and $Z_2(s)$, this circuit can be used as a building block to implement the compensators, such as PID controller. Following table summarizes the realization of all controllers.

Other compensators can be realized by cascading compensators shown in table. For example, a lead-lag compensator can be formed by cascading the lead compensator with lag compensator.

Table 9.10 Active realization of controllers and compensators, using an operational amplifier

Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\dfrac{Z_2(s)}{Z_1(s)}$
Gain			$-\dfrac{R_2}{R_1}$
Integration			$-\dfrac{\frac{1}{RC}}{s}$
Differentiation			$-RCs$
PI controller			$-\dfrac{R_2}{R_1}\dfrac{\left(s+\frac{1}{R_2C}\right)}{s}$
PD controller			$-R_2C\left(s+\dfrac{1}{R_1C}\right)$
PID controller			$-\left[\left(\dfrac{R_2}{R_1}+\dfrac{C_1}{C_2}\right)+R_2C_1s+\dfrac{\frac{1}{R_1C_2}}{s}\right]$
Lag compensation			$-\dfrac{C_1}{C_2}\dfrac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_2C_2 > R_1C_1$
Lead compensation			$-\dfrac{C_1}{C_2}\dfrac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_1C_1 > R_2C_2$

Example : Implement the PID controller has the transfer function $G_c(s) = \dfrac{(s+55.92)(s+0.5)}{s}$

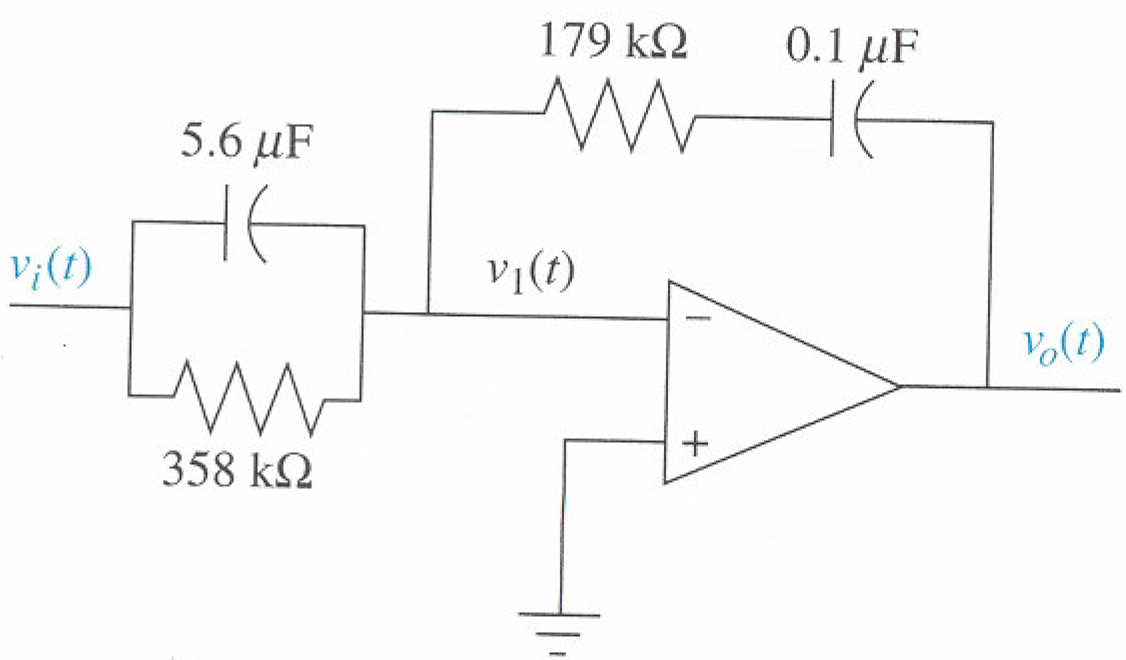
Solution : The transfer function of the controller can be put in the form

$$G_c(s) = s + 56.42 + \frac{27.96}{s}$$

Comparing the PID controller in the table, we obtain following three relationships :

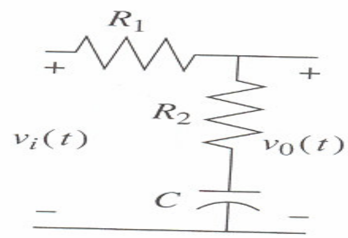
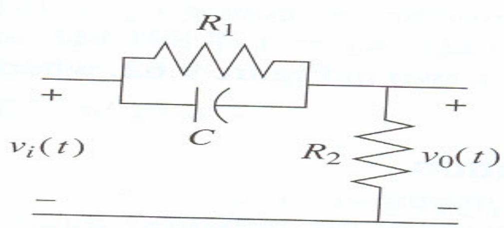
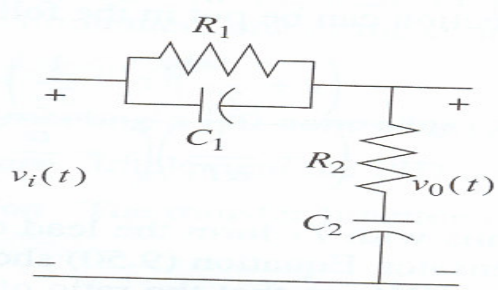
$$\frac{R_2}{R_1} + \frac{C_1}{C_2} = 56.42\ , \qquad R_2 C_1 = 1 \qquad \text{and} \qquad \frac{1}{R_1 C_2} = 27.96$$

Since there are four unknowns and three equations, we arbitrariliy select a practical value for one of the elements. Selecting C_2=0.1 μF, the remaining values are found to be R_1=357.65 kΩ, R_2=178.891 kΩ and C_1=5.59 μF. The complete circuit is shown in following figure, where the circuit element values have been rounded off.



Passive circuit realizition : Lag, lead, and lead lag compensators can also be implemented with pssive network. Following table summarizes the networks and their transfer function.

Table 9.11 Passive realization of compensators

Function	Network	Transfer function, $\frac{V_o(s)}{V_i(s)}$
Lag compensation		$\frac{R_2}{R_1+R_2}\,\frac{s+\frac{1}{R_2C}}{s+\frac{1}{(R_1+R_2)C}}$
Lead compensation		$\frac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C}+\frac{1}{R_2C}}$
Lag-lead compensation		$\frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$

Example : Realize the lead compensator has the transfer function $G_c = \dfrac{s+4}{s+20.09}$

Solution : Comparing the transfer function of a lead network shown in previous table with the compensator transfer function, we obtain the following relationships :

$$\frac{1}{R_1 C} = 4 \qquad \text{and} \qquad \frac{1}{R_1 C} + \frac{1}{R_2 C} = 20.09$$

Hence, $R_1C=0.25$ and $R_2C=0.0622$. Since there are three network elements and two equations, we may select one of the element values arbitrarily. Letting C=1 μF, then R_1=250 kΩ and R_2=62.2 kΩ.