Her bir eklem için tork değerleri:

$$\begin{aligned}\tau_1 &= \left(l_2 l_3 m_3 \mathrm{c}_3 + \tfrac{1}{4} l_2^2 m_2 + l_2^2 m_3 + I_{zz2}\right)\ddot{\theta}_2 - l_3 m_3 \mathrm{s}_3 \left(l_2 + l_1 \mathrm{c}_2\right)\left(\dot{\theta}_3\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3\right) \\ &+ \left(\mathrm{c}_2 l_1 \left(\tfrac{1}{2} l_2 m_2 + l_2 m_3 + \tfrac{1}{2} l_3 m_3 \mathrm{c}_3\right) + \mathrm{c}_3^2 \left(\tfrac{1}{4} l_3^2 m_3 + I_{yy3}\right) + \mathrm{s}_3^2 I_{xx3}\right)\ddot{\theta}_2 \\ &+ \left(l_2 l_3 m_3 \mathrm{c}_3 + I_{zz1} + \tfrac{1}{4} l_1^2 m_1 + \tfrac{1}{4} l_2^2 m_2 + l_2^2 m_3 + I_{zz2} + l_1^2 m_2 + l_1^2 m_3\right)\ddot{\theta}_1 \\ &+ \left(l_1 \mathrm{c}_2 \left(2 l_2 m_3 + l_3 m_3 c_3 + l_2 m_2\right) + \mathrm{c}_3^2 \left(I_{yy3} + \tfrac{1}{4} l_3^2 m_3\right) + \mathrm{s}_3^2 I_{xx3}\right)\ddot{\theta}_1 \\ &- l_1 \mathrm{s}_2 \left(\tfrac{1}{2} l_2 m_2 + m_3 \left(l_2 + \tfrac{1}{2} l_3 \mathrm{c}_3\right)\right)\dot{\theta}_2^2 - \tfrac{1}{2} l_1 l_3 m_3 \mathrm{s}_2 \left(\mathrm{s}_3 \ddot{\theta}_3 + \mathrm{c}_3 \dot{\theta}_3^2\right) \\ &+ 2 \mathrm{c}_3 \mathrm{s}_3 \left(I_{xx3} - \tfrac{1}{4} l_3^2 m_3 - I_{yy3}\right)\left(\dot{\theta}_1\dot{\theta}_3 + \dot{\theta}_2\dot{\theta}_3\right) \\ &- l_1 \mathrm{s}_2 \left(l_2 m_2 + m_3 \left(2 l_2 + l_3 \mathrm{c}_3\right)\right)\dot{\theta}_1\dot{\theta}_2\end{aligned} \tag{3.964}$$

$$\begin{aligned}\tau_2 &= \left(l_2 l_3 m_3 \mathrm{c}_3 + \tfrac{1}{4} l_2^2 m_2 + l_2^2 m_3 + I_{zz2}\right)\left(\ddot{\theta}_2 + \ddot{\theta}_1\right) + \mathrm{s}_2 l_1 l_2 \left(\tfrac{1}{2} m_2 + m_3\right)\dot{\theta}_1^2 \\ &+ \left(\mathrm{c}_3^2 \left(\tfrac{1}{4} l_3^2 m_3 + I_{yy3}\right) + \mathrm{s}_3^2 I_{xx3}\right)\left(\ddot{\theta}_2 + \ddot{\theta}_1\right) - l_2 l_3 m_3 \mathrm{s}_3 \left(\dot{\theta}_1\dot{\theta}_3 + \dot{\theta}_2\dot{\theta}_3\right) \\ &+ \left(l_1 l_2 \mathrm{c}_2 \left(\tfrac{1}{2} m_2 + m_3\right) + \tfrac{1}{2} l_1 l_3 m_3 \mathrm{c}_2 \mathrm{c}_3\right)\ddot{\theta}_1 + \tfrac{1}{2} l_1 l_3 m_3 \mathrm{s}_2 \mathrm{c}_3 \dot{\theta}_1^2 \\ &+ 2 \mathrm{c}_3 \mathrm{s}_3 \left(I_{xx3} - \tfrac{1}{4} l_3^2 m_3 - I_{yy3}\right)\left(\dot{\theta}_1\dot{\theta}_3 + \dot{\theta}_2\dot{\theta}_3\right)\end{aligned} \tag{3.965}$$

$$\begin{aligned}\tau_3 &= I_{zz3}\ddot{\theta}_3 + \mathrm{c}_3 \mathrm{s}_3 \left(I_{yy3} - I_{xx3}\right)\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2 + \dot{\theta}_1^2\right) \\ &+ \tfrac{1}{2} l_3 m_3 \left(g \mathrm{c}_3 + \tfrac{1}{2} l_3 \ddot{\theta}_3 + l_1 \mathrm{s}_3 \left(\mathrm{c}_2 \dot{\theta}_1^2 - \mathrm{s}_2 \ddot{\theta}_1\right)\right) \\ &+ \tfrac{1}{2} l_3 m_3 \mathrm{s}_3 \left(l_2 + \tfrac{1}{2} l_3 \mathrm{c}_3\right)\left(\dot{\theta}_1^2 + 2\dot{\theta}_2\dot{\theta}_1 + \dot{\theta}_2^2\right)\end{aligned} \tag{3.966}$$

Sonuç olarak RN robotunun Lagrange-Euler yöntemi ile bulunan ve denklem 3.499, 3.500 ve 3.501'de görülen tork ifadeleri ile Newton-Euler yöntemi ile bulunan ve denklem 3.964, 3.965 ve 3.966'da görülen tork ifadeleri aynı çıkmıştır.

3.3.14. NR robotunun dinamiğinin Newton-Euler yöntemi ile çıkarılması

NR robotunun her bir bağının kütle merkezi kendi koordinat sistemlerine göre aşağıdaki gibi olur.

$${}^1P_{c_1} = \begin{bmatrix} 0 & \tfrac{1}{2} d_2 & 0 \end{bmatrix}^T \quad {}^2P_{c_2} = \begin{bmatrix} \tfrac{1}{2} l_2 & 0 & 0 \end{bmatrix} \quad {}^3P_{c_3} = \begin{bmatrix} \tfrac{1}{2} l_3 & 0 & 0 \end{bmatrix} \tag{3.967}$$

Robotun ana koordinat sistemi hareket etmediğinden açısal hız ve açısal ivme sıfır olur.

$$ {}^0\omega_0 = 0 \; ; \; {}^0\dot{\omega}_0 = 0 \tag{3.968} $$

Yerçekimi vektörü ana koordinat sisteminin z eksenindedir. Buradan;

$$ {}^0\dot{v}_0 = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T \tag{3.969} $$

Birinci eklem için dışadönük ardışık denklemler:

$$ {}^1\omega_1 = \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \end{bmatrix}^T \; ; \; {}^1\dot{\omega}_1 = \begin{bmatrix} 0 & 0 & \ddot{\theta}_1 \end{bmatrix}^T \tag{3.970} $$

$$ {}^1\dot{v}_1 = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T \tag{3.971} $$

$$ {}^1\dot{v}_{c_1} = \begin{bmatrix} -\frac{1}{2} d_2 \ddot{\theta}_1 & -\frac{1}{2} d_2 \dot{\theta}_1^2 & g \end{bmatrix}^T \tag{3.972} $$

$$ {}^1F_1 = \begin{bmatrix} -\frac{1}{2} d_2 m_1 \ddot{\theta}_1 & -\frac{1}{2} d_2 m_1 \dot{\theta}_1^2 & g m_1 \end{bmatrix}^T \tag{3.973} $$

$$ {}^1N_1 = \begin{bmatrix} 0 & 0 & I_{zz1} \ddot{\theta}_1 \end{bmatrix}^T \tag{3.974} $$

İkinci eklem için dışadönük ardışık denklemler:

$$ \begin{aligned} {}^2\omega_2 &= \begin{bmatrix} -\mathrm{s}_2 \dot{\theta}_1 & -\mathrm{c}_2 \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T \\ {}^2\dot{\omega}_2 &= \begin{bmatrix} -\mathrm{c}_2 \dot{\theta}_1 \dot{\theta}_2 - \mathrm{s}_2 \ddot{\theta}_1 & \mathrm{s}_2 \dot{\theta}_1 \dot{\theta}_2 - \mathrm{c}_2 \ddot{\theta}_1 & \ddot{\theta}_2 \end{bmatrix}^T \end{aligned} \tag{3.975} $$

$$ {}^2\dot{v}_2 = \begin{bmatrix} -g\,\mathrm{s}_2 - d_2\,\mathrm{c}_2 \ddot{\theta}_1 & d_2\,\mathrm{s}_2 \ddot{\theta}_1 - g\,\mathrm{c}_2 & -d_2 \dot{\theta}_1^2 \end{bmatrix}^T \tag{3.976} $$

$$ {}^2\dot{v}_{c_2} = \begin{bmatrix} -g\,\mathrm{s}_2 - \frac{1}{2} l_2 \dot{\theta}_2^2 - \mathrm{c}_2 \left(d_2 \ddot{\theta}_1 + \frac{1}{2} l_2\,\mathrm{c}_2 \dot{\theta}_1^2 \right) \\ \frac{1}{2} l_2 \ddot{\theta}_2 - g\,\mathrm{c}_2 + \mathrm{s}_2 \left(d_2 \ddot{\theta}_1 + \frac{1}{2} l_2\,\mathrm{c}_2 \dot{\theta}_1^2 \right) \\ \frac{1}{2} l_2\,\mathrm{c}_2 \ddot{\theta}_1 - l_2\,\mathrm{s}_2 \dot{\theta}_1 \dot{\theta}_2 - d_2 \dot{\theta}_1^2 \end{bmatrix} \tag{3.977} $$

$$ {}^2F_2 = \begin{bmatrix} -m_2 \left(g\,\mathrm{s}_2 + \frac{1}{2} l_2 \dot{\theta}_2^2 + \mathrm{c}_2 \left(d_2 \ddot{\theta}_1 + \frac{1}{2} l_2\,\mathrm{c}_2 \dot{\theta}_1^2 \right) \right) \\ m_2 \left(\frac{1}{2} l_2 \ddot{\theta}_2 - g\,\mathrm{c}_2 + \mathrm{s}_2 \left(d_2 \ddot{\theta}_1 + \frac{1}{2} l_2\,\mathrm{c}_2 \dot{\theta}_1^2 \right) \right) \\ m_2 \left(\frac{1}{2} l_2\,\mathrm{c}_2 \ddot{\theta}_1 - l_2 \dot{\theta}_1 \dot{\theta}_2\,\mathrm{s}_2 - d_2 \dot{\theta}_1^2 \right) \end{bmatrix} \tag{3.978} $$

$$ {}^2N_2 = \begin{bmatrix} \mathrm{c}_2 \left(I_{yy2} - I_{zz2} - I_{xx2} \right) \dot{\theta}_1 \dot{\theta}_2 - \mathrm{s}_2 I_{xx2} \ddot{\theta}_1 \\ \mathrm{s}_2 \left(I_{yy2} + I_{zz2} - I_{xx2} \right) \dot{\theta}_1 \dot{\theta}_2 - \mathrm{c}_2 I_{yy2} \ddot{\theta}_1 \\ I_{zz2} \ddot{\theta}_2 + \mathrm{c}_2\,\mathrm{s}_2 \left(I_{yy2} - I_{xx2} \right) \dot{\theta}_1^2 \end{bmatrix} \tag{3.979} $$

Üçüncü eklem için dışadönük ardışık denklemler:

$$
\begin{aligned}
{}^3\omega_3 &= \begin{bmatrix} -\mathrm{s}_{(2+3)}\,\dot{\theta}_1 & -\mathrm{c}_{(2+3)}\,\dot{\theta}_1 & \dot{\theta}_2+\dot{\theta}_3 \end{bmatrix}^T \\
{}^3\dot{\omega}_3 &= \begin{bmatrix} -\mathrm{c}_{(2+3)}\left(\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_1\dot{\theta}_3\right)-\mathrm{s}_{(2+3)}\,\ddot{\theta}_1 \\ \mathrm{s}_{(2+3)}\left(\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_1\dot{\theta}_3\right)-\mathrm{c}_{(2+3)}\,\ddot{\theta}_1 \\ \ddot{\theta}_2+\ddot{\theta}_3 \end{bmatrix}
\end{aligned}
\tag{3.980}
$$

$$
{}^3\dot{v}_3 = \begin{bmatrix} -g\,\mathrm{s}_{(2+3)}-l_2\left(\mathrm{c}_3\,\dot{\theta}_2^2-\mathrm{s}_3\,\ddot{\theta}_2\right)-\mathrm{c}_{(2+3)}\left(d_2\ddot{\theta}_1+l_2\,\mathrm{c}_2\,\dot{\theta}_1^2\right) \\ -g\,\mathrm{c}_{(2+3)}+l_2\left(\mathrm{s}_3\,\dot{\theta}_2^2+\mathrm{c}_3\,\ddot{\theta}_2\right)+\mathrm{s}_{(2+3)}\left(d_2\ddot{\theta}_1+l_2\,\mathrm{c}_2\,\dot{\theta}_1^2\right) \\ l_2\,\mathrm{c}_2\,\ddot{\theta}_1-2l_2\,\mathrm{s}_2\,\dot{\theta}_1\dot{\theta}_2-d_2\dot{\theta}_1^2 \end{bmatrix}
\tag{3.981}
$$

$$
{}^3\dot{v}_{c_3} = \begin{bmatrix} {}^3\dot{v}_{c_3(1,1)} & {}^3\dot{v}_{c_3(1,2)} & {}^3\dot{v}_{c_3(1,3)} \end{bmatrix}^T
\tag{3.982}
$$

Matriste,

$$
\begin{aligned}
{}^3\dot{v}_{c_3(1,1)} &= -g\,\mathrm{s}_{(2+3)}-\tfrac{1}{2}l_3\left(\dot{\theta}_2^2+2\dot{\theta}_2\dot{\theta}_3+\dot{\theta}_3^2\right)-l_2\left(\mathrm{c}_3\,\dot{\theta}_2^2-\mathrm{s}_3\,\ddot{\theta}_2\right) \\
&\quad -\mathrm{c}_{(2+3)}\left(d_2\ddot{\theta}_1+l_2\,\mathrm{c}_2\,\dot{\theta}_1^2+\tfrac{1}{2}l_3\,\mathrm{c}_{(2+3)}\,\dot{\theta}_1^2\right) \\
{}^3\dot{v}_{c_3(1,2)} &= -g\,\mathrm{c}_{(2+3)}+\tfrac{1}{2}l_3\left(\ddot{\theta}_2+\ddot{\theta}_3\right)+l_2\left(\mathrm{s}_3\,\dot{\theta}_2^2+\mathrm{c}_3\,\ddot{\theta}_2\right) \\
&\quad +\mathrm{s}_{(2+3)}\left(d_2\ddot{\theta}_1+l_2\,\mathrm{c}_2\,\dot{\theta}_1^2+\tfrac{1}{2}l_3\,\mathrm{c}_{(2+3)}\,\dot{\theta}_1^2\right) \\
{}^3\dot{v}_{c_3(1,3)} &= -\tfrac{1}{2}l_3\left(2\,\mathrm{s}_{(2+3)}\left(\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_1\dot{\theta}_3\right)-\mathrm{c}_{(2+3)}\,\ddot{\theta}_1\right) \\
&\quad -2l_2\,\mathrm{s}_2\,\dot{\theta}_1\dot{\theta}_2-d_2\dot{\theta}_1^2+l_2\,\mathrm{c}_2\,\ddot{\theta}_1
\end{aligned}
$$

$$
{}^3F_3 = \begin{bmatrix} {}^3F_{3(1,1)} & {}^3F_{3(1,2)} & {}^3F_{3(1,3)} \end{bmatrix}^T
\tag{3.983}
$$

Matriste,

$$
\begin{aligned}
{}^3F_{3(1,1)} &= -m_3\left(\mathrm{c}_{(2+3)}\left(d_2\ddot{\theta}_1+l_2\,\mathrm{c}_2\,\dot{\theta}_1^2+\tfrac{1}{2}l_3\,\mathrm{c}_{(2+3)}\,\dot{\theta}_1^2\right)+l_2\left(\mathrm{c}_3\,\dot{\theta}_2^2-\mathrm{s}_3\,\ddot{\theta}_2\right)\right) \\
&\quad +m_3\left(-g\,\mathrm{s}_{(2+3)}-\tfrac{1}{2}l_3\left(\dot{\theta}_2^2+2\dot{\theta}_2\dot{\theta}_3+\dot{\theta}_3^2\right)\right) \\
{}^3F_{3(1,2)} &= m_3\left(\tfrac{1}{2}l_3\left(\ddot{\theta}_2+\ddot{\theta}_3\right)-g\,\mathrm{c}_{(2+3)}+l_2\left(\mathrm{s}_3\,\dot{\theta}_2^2+\mathrm{c}_3\,\ddot{\theta}_2\right)\right) \\
&\quad +m_3\left(\mathrm{s}_{(2+3)}\left(d_2\ddot{\theta}_1+l_2\,\mathrm{c}_2\,\dot{\theta}_1^2+\tfrac{1}{2}l_3\,\mathrm{c}_{(2+3)}\,\dot{\theta}_1^2\right)\right) \\
{}^3F_{3(1,3)} &= m_3\left(l_2\,\mathrm{c}_2\,\ddot{\theta}_1-2l_2\,\mathrm{s}_2\,\dot{\theta}_1\dot{\theta}_2-d_2\dot{\theta}_1^2\right) \\
&\quad -m_3\left(\tfrac{1}{2}l_3\left(2\,\mathrm{s}_{(2+3)}\left(\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_1\dot{\theta}_3\right)-\mathrm{c}_{(2+3)}\,\ddot{\theta}_1\right)\right)
\end{aligned}
$$

$$
{}^{3}N_3 = \begin{bmatrix} \mathrm{c}_{(2+3)}\left(I_{yy3} - I_{zz3} - I_{xx3}\right)\left(\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3\right) - I_{xx3}\,\mathrm{s}_{(2+3)}\,\ddot{\theta}_1 \\ \mathrm{s}_{(2+3)}\left(I_{zz3} - I_{xx3} + I_{yy3}\right)\left(\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3\right) - I_{yy3}\,\mathrm{c}_{(2+3)}\,\ddot{\theta}_1 \\ \mathrm{c}_{(2+3)}\,\mathrm{s}_{(2+3)}\left(I_{yy3} - I_{xx3}\right)\dot{\theta}_1^2 + I_{zz3}\left(\ddot{\theta}_2 + \ddot{\theta}_3\right) \end{bmatrix} \tag{3.984}
$$

Üçüncü eklem için içedönük ardışık denklemler:

$$
{}^{3}f_3 = {}^{3}F_3 \tag{3.985}
$$

$$
{}^{3}n_3 = \begin{bmatrix} {}^{3}n_{3(1,1)} & {}^{3}n_{3(1,2)} & {}^{3}n_{3(1,3)} \end{bmatrix}^T \tag{3.986}
$$

Matriste,

$$
\begin{aligned}
{}^{3}n_{3(1,1)} &= -\mathrm{s}_{(2+3)}\,I_{xx3}\ddot{\theta}_1 + \mathrm{c}_{(2+3)}\left(I_{yy3} - I_{xx3} - I_{zz3}\right)\left(\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3\right) \\
{}^{3}n_{3(1,2)} &= -\mathrm{c}_{(2+3)}\left(I_{yy3} + \tfrac{1}{4}l_3^2 m_3\right)\ddot{\theta}_1 + l_2 l_3 m_3\,\mathrm{s}_2\,\dot{\theta}_1\dot{\theta}_2 + \tfrac{1}{2}l_3 m_3\left(d_2\dot{\theta}_1^2 - l_2\,\mathrm{c}_2\,\ddot{\theta}_1\right) \\
&\quad + \mathrm{s}_{(2+3)}\left(\tfrac{1}{2}l_3^2 m_3 - I_{xx3} + I_{yy3} + I_{zz3}\right)\left(\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3\right) \\
{}^{3}n_{3(1,3)} &= \left(I_{zz3} + \tfrac{1}{4}l_3^2 m_3\right)\left(\ddot{\theta}_2 + \ddot{\theta}_3\right) - \tfrac{1}{2}l_3 m_3 g\,\mathrm{c}_{(2+3)} + \tfrac{1}{2}l_3 m_3 l_2\left(\mathrm{s}_3\,\dot{\theta}_2^2 + \mathrm{c}_3\,\ddot{\theta}_2\right) \\
&\quad + \tfrac{1}{2}l_3 m_3\,\mathrm{s}_{(2+3)}\left(d_2\ddot{\theta}_1 + l_2\,\mathrm{c}_2\,\dot{\theta}_1^2\right) + \mathrm{s}_{(2+3)}\,\mathrm{c}_{(2+3)}\left(\tfrac{1}{4}l_3^2 m_3 - I_{xx3} + I_{yy3}\right)\dot{\theta}_1^2
\end{aligned}
$$

İkinci eklem için içedönük ardışık denklemler:

$$
{}^{2}f_2 = \begin{bmatrix} {}^{2}f_{2(1,1)} & {}^{2}f_{2(1,2)} & {}^{2}f_{2(1,3)} \end{bmatrix}^T \tag{3.987}
$$

Matriste,

$$
\begin{aligned}
{}^{2}f_{2(1,1)} &= -\left(\tfrac{1}{2}l_3 m_3\,\mathrm{c}_2\,\mathrm{c}_{(2+3)} + \mathrm{c}_2^2\,l_2\left(\tfrac{1}{2}m_2 + m_3\right)\right)\dot{\theta}_1^2 - g\,\mathrm{s}_2\left(m_2 + m_3\right) \\
&\quad - d_2\,\mathrm{c}_2\left(m_2 + m_3\right)\ddot{\theta}_1 - \tfrac{1}{2}l_3 m_3\left(\mathrm{s}_3\left(\ddot{\theta}_2 + \ddot{\theta}_3\right) + \mathrm{c}_3\,\dot{\theta}_3^2\right) \\
&\quad - \left(\tfrac{1}{2}l_2 m_2 + \tfrac{1}{2}l_3 m_3\,\mathrm{c}_3 + l_2 m_3\right)\dot{\theta}_2^2 - l_3 m_3\,\mathrm{c}_3\,\dot{\theta}_2\dot{\theta}_3 \\
{}^{2}f_{2(1,2)} &= d_2\,\mathrm{s}_2\left(m_2 + m_3\right)\ddot{\theta}_1 + \left(\tfrac{1}{2}l_2 m_2 + l_2 m_3 + \tfrac{1}{2}l_3 m_3\,\mathrm{c}_3\right)\ddot{\theta}_2 + \tfrac{1}{2}l_3 m_3\,\mathrm{c}_3\,\ddot{\theta}_3 \\
&\quad + \left(2\,\mathrm{c}_2\,\mathrm{s}_2\left(\tfrac{1}{4}l_2 m_2 + \tfrac{1}{2}l_2 m_3\right) + \tfrac{1}{2}l_3 m_3\,\mathrm{s}_2\,\mathrm{c}_{(2+3)}\right)\dot{\theta}_1^2 - g\,\mathrm{c}_2\left(m_2 + m_3\right) \\
&\quad - \tfrac{1}{2}l_3 m_3\,\mathrm{s}_3\left(\dot{\theta}_2^2 + \dot{\theta}_3^2\right) - l_3 m_3\,\mathrm{s}_3\,\dot{\theta}_2\dot{\theta}_3 \\
{}^{2}f_{2(1,3)} &= \left(\mathrm{c}_2\left(\tfrac{1}{2}l_2 m_2 + l_2 m_3\right) + \tfrac{1}{2}l_3 m_3\,\mathrm{c}_{(2+3)}\right)\ddot{\theta}_1 - \mathrm{s}_2\left(2l_2 m_3 + l_2 m_2\right)\dot{\theta}_1\dot{\theta}_2 \\
&\quad - l_3 m_3\,\mathrm{s}_{(2+3)}\left(\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3\right) - d_2\left(m_2 + m_3\right)\dot{\theta}_1^2
\end{aligned}
$$

$$ {}^{2}n_{2} = \begin{bmatrix} {}^{2}n_{2(1,1)} & {}^{2}n_{2(1,2)} & {}^{2}n_{2(1,3)} \end{bmatrix}^{T} \tag{3.988} $$

Matriste,

$$ \begin{aligned} {}^{2}n_{2(1,1)} &= \left(\tfrac{1}{2} l_2 l_3 m_3 \, \mathrm{c}_2 \, \mathrm{s}_3 - \mathrm{s}_2 \, I_{xx2} - \mathrm{c}_3 \, \mathrm{s}_{(2+3)} \, I_{xx3} + \mathrm{s}_3 \, \mathrm{c}_{(2+3)} \left(\tfrac{1}{4} l_3^2 m_3 + I_{yy3}\right)\right) \ddot{\theta}_1 - l_2 l_3 m_3 \, \mathrm{s}_2 \, \mathrm{s}_3 \, \dot{\theta}_2 \dot{\theta}_1 \\ &\quad + \left(\mathrm{c}_3 \, \mathrm{c}_{2+3} \left(I_{yy3} - I_{xx3} - I_{zz3}\right) - \mathrm{s}_3 \, \mathrm{s}_{(2+3)} \left(\tfrac{1}{2} l_3^2 m_3 - I_{xx3} + I_{yy3} + I_{zz3}\right)\right)\left(\dot{\theta}_2 \dot{\theta}_1 + \dot{\theta}_3 \dot{\theta}_1\right) \\ &\quad - \tfrac{1}{2} d_2 l_3 m_3 \, \mathrm{s}_3 \, \dot{\theta}_1^2 + \mathrm{c}_2 \left(I_{yy2} - I_{xx2} - I_{zz2}\right) \dot{\theta}_2 \dot{\theta}_1 \\ {}^{2}n_{2(1,2)} &= -\mathrm{c}_2 \, I_{yy2} \ddot{\theta}_1 + \mathrm{s}_2 \left(I_{yy2} - I_{xx2} + I_{zz2}\right) \dot{\theta}_1 \dot{\theta}_2 - \tfrac{1}{2} l_2 m_2 \left(\tfrac{1}{2} l_2 \, \mathrm{c}_2 \, \ddot{\theta}_1 - l_2 \, \mathrm{s}_2 \, \dot{\theta}_1 \dot{\theta}_2 - d_2 \dot{\theta}_1^2\right) \\ &\quad - l_2 m_3 \left(l_2 \, \mathrm{c}_2 \, \ddot{\theta}_1 - 2 l_2 \, \mathrm{s}_2 \, \dot{\theta}_1 \dot{\theta}_2 - d_2 \dot{\theta}_1^2 - \tfrac{1}{2} l_3 \left(2 \, \mathrm{s}_{(2+3)} \left(\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3\right) - \mathrm{c}_{(2+3)} \, \ddot{\theta}_1\right)\right) \\ &\quad + \mathrm{s}_3 \left(\mathrm{c}_{(2+3)} \left(I_{yy3} - I_{xx3} - I_{zz3}\right)\left(\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1 \dot{\theta}_3\right) - \mathrm{s}_{(2+3)} \, I_{xx3} \ddot{\theta}_1\right) \\ {}^{2}n_{2(1,3)} &= -l_3 m_3 \, \mathrm{s}_3 \left(\tfrac{1}{2} l_2 \dot{\theta}_3^2 + l_2 \dot{\theta}_2 \dot{\theta}_3\right) + \left(\tfrac{1}{4} l_3^2 m_3 - I_{xx3} + I_{yy3}\right)\left(\mathrm{c}_2 \, \mathrm{s}_2 \left(\mathrm{c}_3^2 - \mathrm{s}_3^2\right) + \mathrm{c}_3 \, \mathrm{s}_3 \left(\mathrm{c}_2^2 - \mathrm{s}_2^2\right)\right) \dot{\theta}_1^2 \\ &\quad + \left(l_2 l_3 m_3 \, \mathrm{c}_3 + \tfrac{1}{4} l_2^2 m_2 + l_2^2 m_3 + \tfrac{1}{4} l_3^2 m_3 + I_{zz2} + I_{zz3}\right) \ddot{\theta}_2 + \left(\tfrac{1}{2} l_2 l_3 m_3 \, \mathrm{c}_3 + \tfrac{1}{4} l_3^2 m_3 + I_{zz3}\right) \ddot{\theta}_3 \\ &\quad + \left(\tfrac{1}{2} d_2 l_2 m_2 \, \mathrm{s}_2 + d_2 l_2 m_3 \, \mathrm{s}_2 + \tfrac{1}{2} d_2 l_3 m_3 \, \mathrm{s}_{(2+3)}\right) \ddot{\theta}_1 - g \left(l_2 \, \mathrm{c}_2 \left(\tfrac{1}{2} m_2 + m_3\right) + \tfrac{1}{2} l_3 m_3 \, \mathrm{c}_{(2+3)}\right) \\ &\quad + \left(\tfrac{1}{2} l_2 l_3 m_3 \left(\mathrm{c}_2 \, \mathrm{s}_{(2+3)} + \mathrm{s}_2 \, \mathrm{c}_{(2+3)}\right) + \mathrm{c}_2 \, \mathrm{s}_2 \left(\tfrac{1}{4} l_2^2 m_2 + l_2^2 m_3 - I_{xx2} + I_{yy2}\right)\right) \dot{\theta}_1^2 \end{aligned} $$

Birinci eklem için içedönük ardışık denklemler:

$$ {}^{1}f_{1} = \begin{bmatrix} {}^{1}f_{1(1,1)} & {}^{1}f_{1(1,2)} & {}^{1}f_{1(1,3)} \end{bmatrix}^{T} \tag{3.989} $$

Matriste,

$$ \begin{aligned} {}^{1}f_{1(1,1)} &= -\left(l_2 \, \mathrm{s}_2 \left(\tfrac{1}{2} m_2 + m_3\right) + \tfrac{1}{2} l_3 m_3 \, \mathrm{s}_{(2+3)}\right) \ddot{\theta}_2 - \left(l_2 \, \mathrm{c}_2 \left(\tfrac{1}{2} m_2 + m_3\right) + \tfrac{1}{2} l_3 m_3 \, \mathrm{c}_{(2+3)}\right) \dot{\theta}_1^2 \\ &\quad - \left(l_2 \, \mathrm{c}_2 \left(\tfrac{1}{2} m_2 + m_3\right) + \tfrac{1}{2} l_3 m_3 \, \mathrm{c}_{(2+3)}\right) \dot{\theta}_2^2 - d_2 \left(\tfrac{1}{2} m_1 + m_2 + m_3\right) \ddot{\theta}_1 \\ &\quad - \tfrac{1}{2} l_3 m_3 \left(\mathrm{c}_{(2+3)} \, \dot{\theta}_3^2 + \mathrm{s}_{(2+3)} \, \ddot{\theta}_3\right) - l_3 m_3 \, \mathrm{c}_{(2+3)} \, \dot{\theta}_2 \dot{\theta}_3 \\ {}^{1}f_{1(1,2)} &= \left(l_2 \, \mathrm{c}_2 \left(\tfrac{1}{2} m_2 + m_3\right) + \tfrac{1}{2} l_3 m_3 \, \mathrm{c}_{(2+3)}\right) \ddot{\theta}_1 - l_2 \, \mathrm{s}_2 \left(m_2 + 2 m_3\right) \dot{\theta}_2 \dot{\theta}_1 \\ &\quad - l_3 m_3 \, \mathrm{s}_{(2+3)} \left(\dot{\theta}_2 \dot{\theta}_1 + \dot{\theta}_3 \dot{\theta}_1\right) - d_2 \left(\tfrac{1}{2} m_1 + m_2 + m_3\right) \dot{\theta}_1^2 \\ {}^{1}f_{1(1,3)} &= \mathrm{s}_{(2+3)} \left(\tfrac{1}{2} l_3 m_3 \dot{\theta}_3^2 + l_3 m_3 \dot{\theta}_2 \dot{\theta}_3\right) + \left(l_2 \, \mathrm{s}_2 \left(\tfrac{1}{2} m_2 + m_3\right) + \tfrac{1}{2} l_3 m_3 \, \mathrm{s}_{(2+3)}\right) \dot{\theta}_2^2 \\ &\quad + g \left(m_1 + m_2 + m_3\right) - \tfrac{1}{2} l_3 m_3 \, \mathrm{c}_{(2+3)} \left(\ddot{\theta}_2 + \ddot{\theta}_3\right) - l_2 \, \mathrm{c}_2 \left(\tfrac{1}{2} m_2 + m_3\right) \ddot{\theta}_2 \end{aligned} $$

$$ {}^{1}n_{1} = \begin{bmatrix} {}^{1}n_{1(1,1)} & {}^{1}n_{1(1,2)} & {}^{1}n_{1(1,3)} \end{bmatrix}^{T} \tag{3.990} $$

Matriste,

$$
\begin{aligned}
{}^{1}n_{1(1,1)} = & -\left(\tfrac{1}{2}m_2+m_3\right)d_2 l_2\, \mathrm{c}_2\, \ddot{\theta}_2 + g d_2\left(\tfrac{1}{2}m_1+m_2+m_3\right) + d_2 l_3 m_3\, \mathrm{s}_{(2+3)}\left(\dot{\theta}_2\dot{\theta}_3+\tfrac{1}{2}\dot{\theta}_3^2\right) \\
& -\left(\tfrac{1}{2}l_2 l_3 m_3\, \mathrm{c}_3+\tfrac{1}{4}l_3^2 m_3 + I_{zz3}\right)\dot{\theta}_3\dot{\theta}_1 - \left(d_2 l_2\, \mathrm{s}_2\left(\tfrac{1}{2}m_2+m_3\right)+\tfrac{1}{2}d_2 l_3 m_3\, \mathrm{s}_{(2+3)}\right)\dot{\theta}_1^2 \\
& +\left(\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\left(\mathrm{c}_3^2-\mathrm{s}_3^2\right)-4\,\mathrm{c}_2\,\mathrm{c}_3\,\mathrm{s}_2\,\mathrm{s}_3\right)\left(\tfrac{1}{4}l_3^2 m_3 - I_{xx3}+I_{yy3}\right)\left(\dot{\theta}_2\dot{\theta}_1+\dot{\theta}_3\dot{\theta}_1\right) \\
& -\tfrac{1}{2}d_2 l_3 m_3\, \mathrm{c}_{(2+3)}\left(\ddot{\theta}_3+\ddot{\theta}_2\right)+\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\left(\tfrac{1}{4}l_2^2 m_2 + l_2^2 m_3 - I_{xx2}+I_{yy2}\right)\dot{\theta}_2\dot{\theta}_1 \\
& +\left(\tfrac{1}{2}l_2 l_3 m_3\left(\mathrm{c}_2\,\mathrm{s}_{(2+3)}+\mathrm{s}_2\,\mathrm{c}_{(2+3)}\right)+\mathrm{c}_2\,\mathrm{s}_2\left(\tfrac{1}{4}l_2^2 m_2 + l_2^2 m_3 - I_{xx2}+I_{yy2}\right)\right)\ddot{\theta}_1 \\
& +\left(\left(\mathrm{c}_2\,\mathrm{s}_2\left(\mathrm{c}_3^2-\mathrm{s}_3^2\right)+\mathrm{c}_3\,\mathrm{s}_3\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\right)\left(\tfrac{1}{4}l_3^2 m_3 - I_{xx3}+I_{yy3}\right)\right)\ddot{\theta}_1 \\
& -\left(\tfrac{1}{4}l_2^2 m_2 + l_2^2 m_3 + \tfrac{1}{4}l_3^2 m_3 + I_{zz2} + I_{zz3} + l_2 l_3 m_3\, \mathrm{c}_3\right)\dot{\theta}_2\dot{\theta}_1 \\
& +\left(\mathrm{c}_3\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)-2\,\mathrm{c}_2\,\mathrm{s}_2\,\mathrm{s}_3\right)\left(l_2 l_3 m_3\dot{\theta}_2\dot{\theta}_1+\tfrac{1}{2}l_2 l_3 m_3\dot{\theta}_3\dot{\theta}_1\right) \\
& +\left(d_2 l_2\, \mathrm{s}_2\left(\tfrac{1}{2}m_2+m_3\right)+\tfrac{1}{2}d_2 l_3 m_3\, \mathrm{s}_{(2+3)}\right)\dot{\theta}_2^2 \\
{}^{1}n_{1(1,2)} = & \left(\mathrm{c}_2\,\mathrm{s}_2\left(l_2 l_3 m_3\, \mathrm{c}_3+\tfrac{1}{4}l_2^2 m_2 + l_2^2 m_3 - I_{xx2}+I_{yy2}+\left(\tfrac{1}{4}l_3^2 m_3 + I_{yy3} - I_{xx3}\right)\left(\mathrm{c}_3^2-\mathrm{s}_3^2\right)\right)\right)\dot{\theta}_1^2 \\
& +\left(\tfrac{1}{2}l_2 l_3 m_3\, \mathrm{c}_3+\tfrac{1}{4}l_3^2 m_3 + I_{zz3}\right)\ddot{\theta}_3+\left(l_2 l_3 m_3\, \mathrm{c}_3+\tfrac{1}{4}l_2^2 m_2 + l_2^2 m_3 + \tfrac{1}{4}l_3^2 m_3 + I_{zz2} + I_{zz3}\right)\ddot{\theta}_2 \\
& +\left(d_2 l_2\, \mathrm{s}_2\left(\tfrac{1}{2}m_2+m_3\right)+\tfrac{1}{2}d_2 l_3 m_3\, \mathrm{s}_{(2+3)}\right)\ddot{\theta}_1 - g\left(\tfrac{1}{2}l_3 m_3\, \mathrm{c}_{(2+3)}+\mathrm{c}_2\, l_2\left(m_3+\tfrac{1}{2}m_2\right)\right) \\
& -l_2 l_3 m_3\, \mathrm{s}_3\left(\tfrac{1}{2}\dot{\theta}_3^2+\dot{\theta}_2\dot{\theta}_3\right)+\left(\mathrm{c}_3\,\mathrm{s}_3\left(\tfrac{1}{4}l_3^2 m_3 + I_{yy3} - I_{xx3}\right)+\tfrac{1}{2}l_2 l_3 m_3\, \mathrm{s}_3\right)\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\dot{\theta}_1^2 \\
{}^{1}n_{1(1,3)} = & \left(2\,\mathrm{c}_2\,\mathrm{s}_2\left(I_{xx2}-\tfrac{1}{4}l_2^2 m_2 - l_2^2 m_3 - I_{yy2}\right)-l_2 l_3 m_3\left(\mathrm{c}_2\,\mathrm{s}_{(2+3)}+\mathrm{s}_2\,\mathrm{c}_{(2+3)}\right)\right)\dot{\theta}_1\dot{\theta}_2 \\
& +\left(\mathrm{c}_2^2\left(I_{yy2}+\tfrac{1}{4}l_2^2 m_2 + l_2^2 m_3\right)+\mathrm{s}_2^2\, I_{xx2}+I_{xx3}\,\mathrm{s}_{(2+3)}^2\right)\ddot{\theta}_1+\tfrac{1}{2}d_2 l_3 m_3\, \mathrm{s}_{(2+3)}\,\ddot{\theta}_3 \\
& +\left(\left(I_{yy3}+\tfrac{1}{4}l_3^2 m_3\right)\mathrm{c}_{(2+3)}^2+l_2 l_3 m_3\, \mathrm{c}_2\, \mathrm{c}_{(2+3)}+I_{zz1}+d_2^2\left(\tfrac{1}{4}m_1+m_2+m_3\right)\right)\ddot{\theta}_1 \\
& +\left(2\left(I_{xx3}-\tfrac{1}{4}l_3^2 m_3 - I_{yy3}\right)\left(\mathrm{c}_2\,\mathrm{s}_2\left(\mathrm{c}_3^2-\mathrm{s}_3^2\right)+\mathrm{c}_3\,\mathrm{s}_3\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\right)\right)\dot{\theta}_1\dot{\theta}_3 \\
& -2\left(\tfrac{1}{4}l_3^2 m_3 - I_{xx3}+I_{yy3}\right)\left(\mathrm{c}_2\,\mathrm{s}_2\left(\mathrm{c}_3^2-\mathrm{s}_3^2\right)+\mathrm{c}_3\,\mathrm{s}_3\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\right)\dot{\theta}_1\dot{\theta}_2 \\
& +d_2 l_3 m_3\, \mathrm{c}_{(2+3)}\,\dot{\theta}_2\dot{\theta}_3+\left(d_2 l_2\, \mathrm{s}_2\left(\tfrac{1}{2}m_2+m_3\right)+\tfrac{1}{2}d_2 l_3 m_3\, \mathrm{s}_{(2+3)}\right)\ddot{\theta}_2 \\
& +\tfrac{1}{2}d_2 l_3 m_3\, \mathrm{c}_{(2+3)}\,\dot{\theta}_3^2+\left(d_2 l_2\, \mathrm{c}_2\left(\tfrac{1}{2}m_2+m_3\right)+\tfrac{1}{2}d_2 l_3 m_3\, \mathrm{c}_{(2+3)}\right)\dot{\theta}_2^2 \\
& -\frac{1}{2}l_2 l_3 m_3\left(\mathrm{c}_2\,\mathrm{s}_{(2+3)}+\mathrm{s}_2\,\mathrm{c}_{(2+3)}+\mathrm{s}_3\right)\dot{\theta}_1\dot{\theta}_3
\end{aligned}
$$

Her bir eklem için tork değerleri:

$$\begin{aligned}\tau_1 &= \left(2\,c_2\,s_2\left(I_{xx2}-\tfrac{1}{4}l_2^2m_2-l_2^2m_3-I_{yy2}\right)-l_2l_3m_3\left(c_2\,s_{(2+3)}+s_2\,c_{(2+3)}\right)\right)\dot\theta_1\dot\theta_2\\ &+\left(\left(I_{yy3}+\tfrac{1}{4}l_3^2m_3\right)c_{(2+3)}^2+l_2l_3m_3\,c_2\,c_{(2+3)}+I_{zz1}+d_2^2\left(\tfrac{1}{4}m_1+m_2+m_3\right)\right)\ddot\theta_1\\ &+\left(c_2^2\left(I_{yy2}+\tfrac{1}{4}l_2^2m_2+l_2^2m_3\right)+s_2^2\,I_{xx2}+I_{xx3}\,s_{(2+3)}^2\right)\ddot\theta_1+\tfrac{1}{2}d_2l_3m_3\,s_{(2+3)}\ddot\theta_3\\ &+\left(2\left(I_{xx3}-\tfrac{1}{4}l_3^2m_3-I_{yy3}\right)\left(c_2\,s_2\left(c_3^2-s_3^2\right)+c_3\,s_3\left(c_2^2-s_2^2\right)\right)\right)\dot\theta_1\dot\theta_3\\ &-2\left(\tfrac{1}{4}l_3^2m_3-I_{xx3}+I_{yy3}\right)\left(c_2\,s_2\left(c_3^2-s_3^2\right)+c_3\,s_3\left(c_2^2-s_2^2\right)\right)\dot\theta_1\dot\theta_2\\ &+d_2l_3m_3\,c_{(2+3)}\dot\theta_2\dot\theta_3+\left(d_2l_2\,s_2\left(\tfrac{1}{2}m_2+m_3\right)+\tfrac{1}{2}d_2l_3m_3\,s_{(2+3)}\right)\ddot\theta_2\\ &+\tfrac{1}{2}d_2l_3m_3\,c_{(2+3)}\dot\theta_3^2+\left(d_2l_2\,c_2\left(\tfrac{1}{2}m_2+m_3\right)+\tfrac{1}{2}d_2l_3m_3\,c_{(2+3)}\right)\dot\theta_2^2\\ &-\frac{1}{2}l_2l_3m_3\left(c_2\,s_{(2+3)}+s_2\,c_{(2+3)}+s_3\right)\dot\theta_1\dot\theta_3\end{aligned} \tag{3.991}$$

$$\begin{aligned}\tau_2 &= \left(\tfrac{1}{4}l_3^2m_3-I_{xx3}+I_{yy3}\right)\left(c_2\,s_2\left(c_3^2-s_3^2\right)+c_3\,s_3\left(c_2^2-s_2^2\right)\right)\dot\theta_1^2\\ &-l_3m_3\,s_3\left(\tfrac{1}{2}l_2\dot\theta_3^2+l_2\dot\theta_2\dot\theta_3\right)+\tfrac{1}{2}l_2l_3m_3\left(c_2\,s_{(2+3)}+s_2\,c_{(2+3)}\right)\dot\theta_1^2\\ &+\left(l_2l_3m_3\,c_3+\tfrac{1}{4}l_2^2m_2+l_2^2m_3+\tfrac{1}{4}l_3^2m_3+I_{zz2}+I_{zz3}\right)\ddot\theta_2\\ &+\left(\tfrac{1}{2}d_2l_2m_2\,s_2+d_2l_2m_3\,s_2+\tfrac{1}{2}d_2l_3m_3\,s_{(2+3)}\right)\ddot\theta_1\\ &+c_2\,s_2\left(\tfrac{1}{4}l_2^2m_2+l_2^2m_3-I_{xx2}+I_{yy2}\right)\dot\theta_1^2\\ &-g\left(l_2\,c_2\left(\tfrac{1}{2}m_2+m_3\right)+\tfrac{1}{2}l_3m_3\,c_{(2+3)}\right)\\ &+\left(\tfrac{1}{2}l_2l_3m_3\,c_3+\tfrac{1}{4}l_3^2m_3+I_{zz3}\right)\ddot\theta_3\end{aligned} \tag{3.992}$$

$$\begin{aligned}\tau_3 &= \tfrac{1}{2}l_3m_3l_2\left(s_3\,\dot\theta_2^2+c_3\,\ddot\theta_2\right)+\tfrac{1}{2}l_3m_3\,s_{(2+3)}\left(d_2\ddot\theta_1+l_2\,c_2\,\dot\theta_1^2\right)\\ &+\left(I_{zz3}+\tfrac{1}{4}l_3^2m_3\right)\left(\ddot\theta_2+\ddot\theta_3\right)-\tfrac{1}{2}l_3m_3g\,c_{(2+3)}\\ &+s_{(2+3)}\,c_{(2+3)}\left(\tfrac{1}{4}l_3^2m_3-I_{xx3}+I_{yy3}\right)\dot\theta_1^2\end{aligned} \tag{3.993}$$

Sonuç olarak NR robotunun Lagrange-Euler yöntemi ile bulunan ve denklem 3.537, 3.538 ve 3.539'da görülen tork ifadeleri ile Newton-Euler yöntemi ile bulunan ve denklem 3.991, 3.992 ve 3.993'de görülen tork ifadeleri aynı çıkmıştır.

3.3.15. NN robotunun dinamiğinin Newton-Euler yöntemi ile çıkarılması

NN robotunun her bir bağının kütle merkezi kendi koordinat sistemlerine göre aşağıdaki gibi olur.