BERNOULLİ DENKLEMİ

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Bernoulli Denklemi

1. Sıkıştırılamayan (sıvı)
2. Sürtünmesiz (ideal)
3. Stasyoner (kararlı)

Potansiyel enerji – $E_p = mgz$ [J]

Kinetik enerji – $E_k = \frac{mv^2}{2}$ [J]

Basınç enerjisi – $E_b = \frac{m \cdot p}{\rho}$ [J]

Toplam enerji $\sum E = E_p + E_k + E_b =$

$$= mgz + \frac{mv^2}{2} + \frac{mp}{\rho}$$

Bernoulli yasasına göre, $\sum E = \text{sabit}$

$$mgz + \frac{mv^2}{2} + \frac{mp}{\rho} = \text{sabit}$$

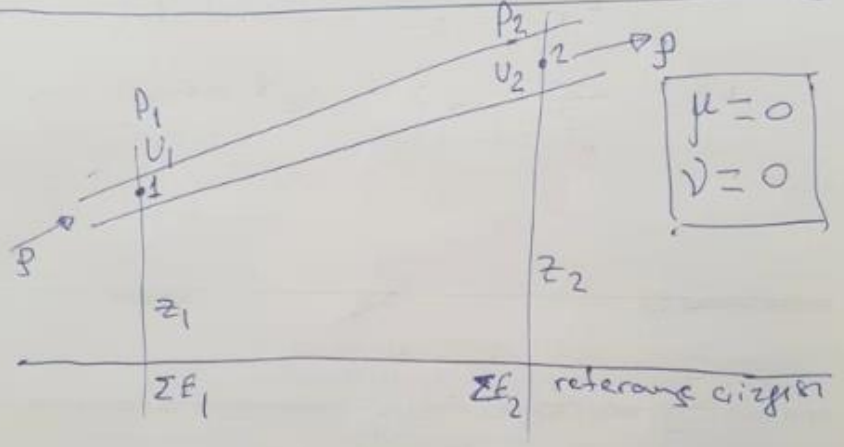
Akışkanın birim ağırlığına gelen enerji yük adlandırılır. Yük cinsinden sıvılarda Bernoulli denklemi

$$z + \frac{v^2}{2g} + \frac{p}{\rho g} = \text{sabit} \quad [M]$$

z = sabit için,

$$\frac{U^2}{2g} + \frac{P}{\rho g} = \text{sabit}$$

> Akış hızı (U) ile basınç (P) ters orantılıdır.



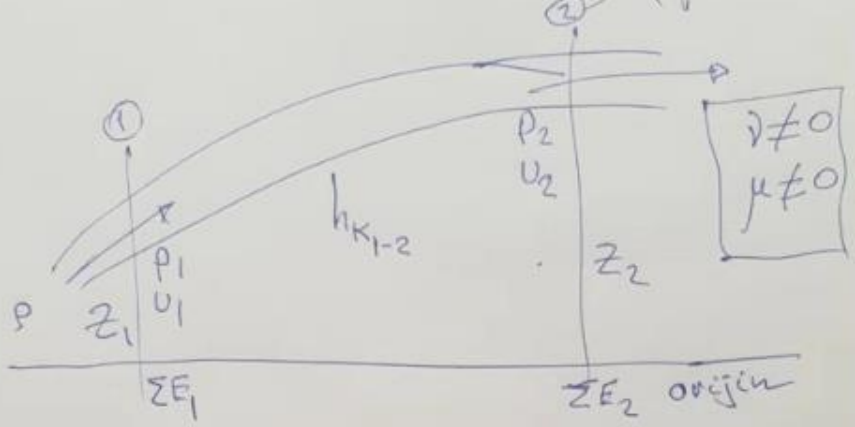
$$\Sigma E_1 = \Sigma E_2$$

$$z_1 + \frac{P_1}{\rho g} + \frac{U_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{U_2^2}{2g}$$

Kütlenin korunumu kanunu ve Bernoulli denklemlerinden çıkan önemli sonuçlar birleştirilerek 3. bir önemli sonuç elde edilebilir:

> Kesit alanı (A) ile basınç (P) doğru orantılıdır.

Sürtünmeli akışkan için; ($\mu \neq 0$) ③



$$\Sigma E_1 = \Sigma E_2 + h_{K_{1-2}}$$

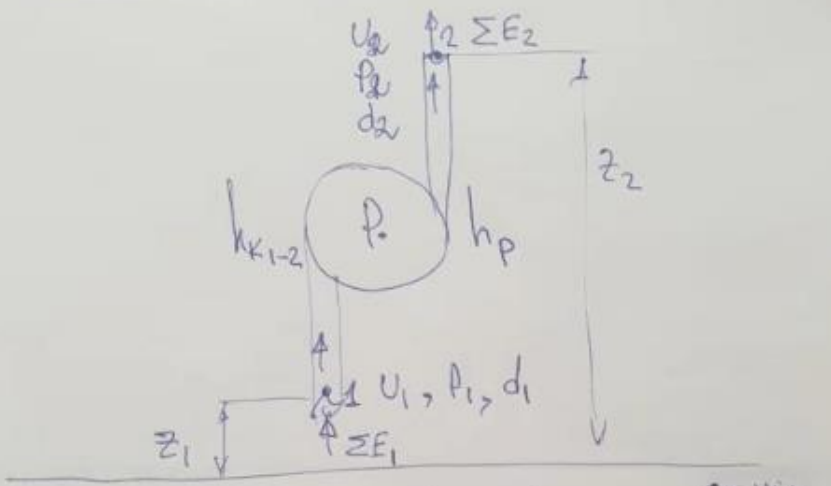
$$Z_1 + \frac{U_1^2}{2g} + \frac{P_1}{\rho g} = Z_2 + \frac{U_2^2}{2g} + \frac{P_2}{\rho g} + h_{K_{1-2}}$$

$h_{K_{1-2}} = K \cdot \frac{U_2^2}{2g}$ 1 ve 2 noktaları arasındaki yük kaybı.

K – kayıp yük katsayısıdır.

$$Z_1 + \frac{U_1^2}{2g} + \frac{P_1}{\rho g} = Z_2 + \frac{U_2^2}{2g} + \frac{P_2}{\rho g} + K \cdot \frac{U_2^2}{2g}$$

Aralarında Pompa bulunan iki nokta için Bernoulli Denklemi



$$\Sigma E_1 = \Sigma E_2 + h_{K1-2} - h_p$$

$$z_1 + \frac{v_1^2}{2g} + \frac{p_1}{\rho g} = z_2 + \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + K \cdot \frac{v_2^2}{2g} - h_p$$

h_p - Pompanın akışkanın birim ağırlığına kazandırdığı enerjidir.

Pompanın akışkana kazandırdığı güç:

$$W_a = \rho \cdot g \cdot \dot{V} \cdot h_p$$

Pompa verimi:

$$\eta = \frac{W_a}{W_{mil}}$$

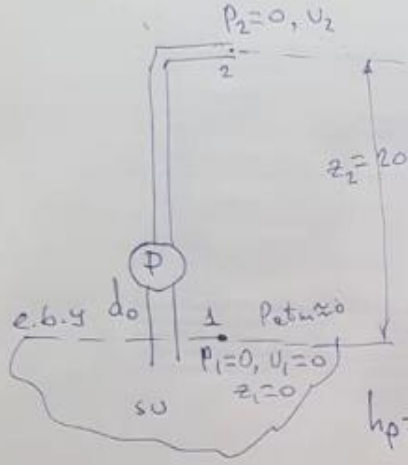
$\dot{V}$ - hacimsel debidir

4. soru:

$\dot{V} = 0{,}0012\ m^3/s$, $\rho = 1000\ kg/m^3$, $\pi = 3{,}14$, $g \simeq 10\ m/s^2$, $z_2 = 20\ m$

$\eta = \%65$, $h_{k1-2} = 3 \cdot \frac{U_2^2}{2g}$, $d_0 = 5\ cm$, $d_2 = 2{,}4\ cm$

bilindiğine göre, pompanın mil gücünü bulunuz.



1 ve 2 noktaları için Bernoulli denkleminin göre,

$$z_1 + \frac{P_1}{\rho g} + \frac{U_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{U_2^2}{2g} + 3\frac{U_2^2}{2g} - h_p$$

yazılabilir.

$$h_p = z_2 + \frac{4U_2^2}{2g} = z_2 + \frac{2U_2^2}{g}$$

$\dot{V} = A_2 \cdot U_2$

$$U_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi \cdot \left(\frac{d_2}{2}\right)^2} = \frac{0{,}0012}{3{,}14 \cdot \left(\frac{0{,}024}{2}\right)^2} = 3{,}654\ m/s$$

$$h_p = z_2 + \frac{2U_2^2}{g} = 20 + \frac{2 \cdot (3{,}654)^2}{10} = 20 + 1{,}409 = 21{,}409\ m$$

$$W_a = \rho \cdot g \cdot \dot{V} \cdot h_p = 1000 \cdot 10 \cdot 0{,}0012 \cdot 21{,}409 = 256{,}908\ W$$

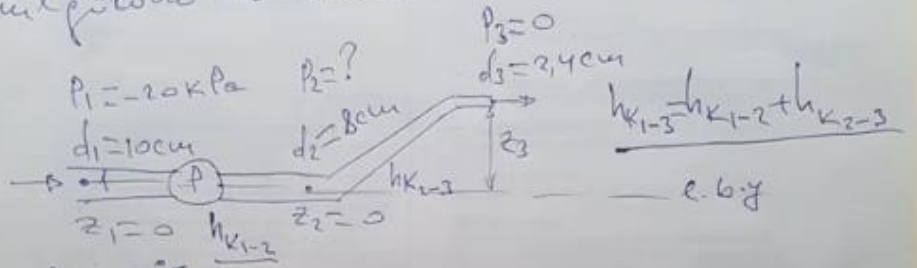
$$W_{mil} = \frac{W_a}{\eta} = \frac{256{,}908}{0{,}65} = 395{,}243\ W$$

5. soru:

$P_1 = -20\,kPa$, $d_1 = 10cm$, $d_2 = 8cm$, $d_3 = 2,4cm$, $z_3 = 15\,m$

$\dot{V} = 0,0012\,m^3/s$, $\pi = 3,14$, $g = 10\,m/s^2$, $\gamma = 0,7$, $h_{K1-2} = 3 \cdot \frac{U_2^2}{2g}$,

$h_{K2-3} = 1,2 \cdot \frac{U_3^2}{2g}$ olduğuna göre pompanın milindeki basıncı.



$$U_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi(\frac{d_1}{2})^2} = \frac{0,0012}{3,14(\frac{0,1}{2})^2} = 0,153\,m/s$$

$$U_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi \cdot (\frac{d_2}{2})^2} = \frac{0,0012}{3,14(\frac{0,08}{2})^2} = 0,239\,m/s$$

$$U_3 = \frac{\dot{V}}{A_3} = \frac{\dot{V}}{\pi(\frac{d_3}{2})^2} = \frac{0,0012}{3,14 \cdot (\frac{0,024}{2})^2} = 2,654\,m/s$$

1 ve 3 noktaları için Bernoulli denklemine göre,

$$z_1 + \frac{P_1}{\rho g} + \frac{U^2}{2g} = z_3 + \frac{P_3}{\rho g} + \frac{U_3^2}{2g} + \frac{3U_2^2}{2g} + \frac{1,2U_3^2}{2g} - h_p$$

$$h_p = z_3 + \frac{2,2 \cdot U_3^2 + 3U_2^2 - U_1^2}{2g} - \frac{P_1}{\rho g} =$$

$$= 15 + \frac{2{,}2\,(2{,}654)^2 + 3\cdot(0{,}239)^2 - (0{,}153)^2}{2\cdot 10} - \frac{-20\cdot 10^3}{1000\cdot 10} =$$

$$= 15 + \frac{15{,}496 + 0{,}171 - 0{,}023}{20} + \frac{20\cdot 10^3}{10000} =$$

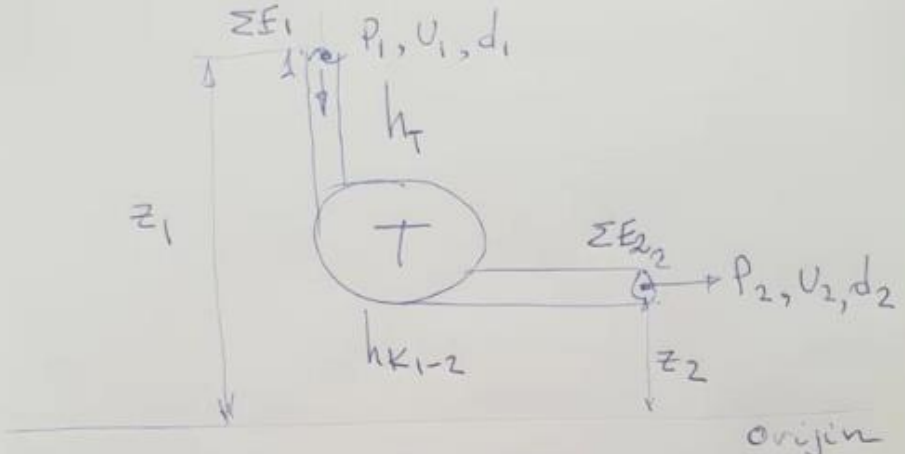
$$= 15 + 0{,}782 + 2 = 17{,}782 \text{ m}$$

$$W_a = \rho \cdot g \cdot \dot{V} \cdot h_p = 1000\cdot 10\cdot 0{,}002\cdot 17{,}782 =$$

$$= 213{,}384 \text{ W}$$

$$W_{mil} = \frac{W_a}{\eta} = \frac{213{,}384}{0{,}7} = 304{,}834 \text{ W}$$

Aralarında Türbin Bulunan İki Nokta için Bernoulli Denklemi



$$\Sigma E_1 = \Sigma E_2 + h_{K1-2} + h_T$$

$$z_1 + \frac{U_1^2}{2g} + \frac{P_1}{\rho g} = z_2 + \frac{U_2^2}{2g} + \frac{P_2}{\rho g} + K\cdot\frac{U_2^2}{2g} + h_T$$

h_T - Türbinin akışkanın birim ağırlığından çektiği enerjidir (Türbin düşüsü).

Türbinin hareketli akışkandan aldığı güç: $W_a = \rho \cdot g \cdot \dot{V} \cdot h_T$

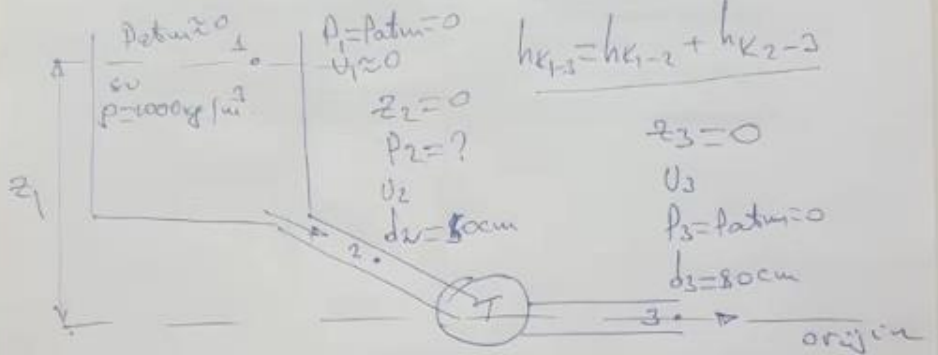
$\dot{V}$ - hacimsel debi

Türbin verimi:

$$\eta = \frac{W_{mil}}{W_a}$$

6. soru:

$P_{atm} \approx 0$, $z_1 = 25m$, $d_2 = 50cm$, $d_3 = 80cm$, $g = 10 m/s^2$, $\pi = 3,14$, $h_{K_{1-2}} = 3\frac{v_2^2}{2g}$, $h_{K_{2-3}} = 1,5\frac{v_3^2}{2g}$, $\dot{V} = 1,0048 m^3/s$ $\eta = 0,55$ olduğuna göre türbinin mil gücünü hesaplayınız.



$h_{K_{1-3}} = h_{K_{1-2}} + h_{K_{2-3}}$

$$v_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi \cdot (\frac{d_2}{2})^2} = \frac{1,0048}{3,14(\frac{0,5}{2})^2} = 5,12 m/s$$

$$v_3 = \frac{\dot{V}}{A_3} = \frac{\dot{V}}{\pi(\frac{d_3}{2})^2} = \frac{1,0048}{3,14(\frac{0,8}{2})^2} = 2 m/s$$

1 ve 3 noktaları için Bernoulli denklemine göre,

$$z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_3 + \frac{P_3}{\rho g} + \frac{v_3^2}{2g} + \frac{3v_2^2}{2g} + \frac{1,5v_3^2}{2g} + h_T$$

$$h_T = z_1 - \frac{2,5v_3^2 + 3v_2^2}{2g} = 25 - \frac{2,5\cdot(2,1)^2 + 3\cdot(5,12)^2}{2\cdot 10} =$$

$$= 25 - \frac{10 + 131,072}{20} = 25 - 7,054 = 17,946 \text{ m}$$

$$W_A = \rho g \dot{V} h_T = 1000 \cdot 10 \cdot 1,0048 \cdot 17,946 =$$

$$= 180325,427 \text{ W}$$

$$W_{mil} = \eta \cdot W_A = 0,55 \cdot 180325,426 =$$

$$= 99178,985 \text{ W}$$

8. soru:

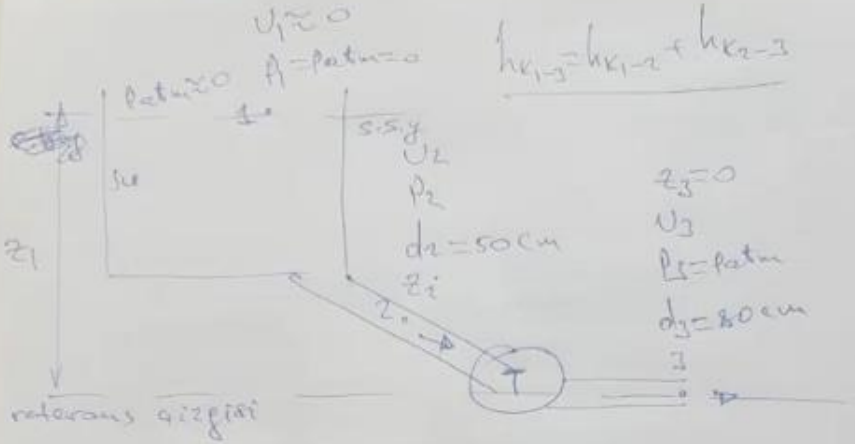
$p_{atm}=0$, $d_2=50cm$, $d_3=80cm$, $g=10\ m/s^2$,

$\pi=3,14$, $h_{K_{1-2}}=1,2\cdot\frac{U_2^2}{2g}$, $h_{K_{2-3}}=3\frac{U_3}{2g}$,

$\dot{V}=1,0048\ m^3/s$, $\eta=\%60$, $\rho=1000\ kg/m^3$,

$W_{mil}=120.000\ W$ da bilmeri için

$z_1=?$ bulunur.



$$h_{K_{1-3}} = h_{K_{1-2}} + h_{K_{2-3}}$$

$$v_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi\left(\frac{d_2}{2}\right)^2} = \frac{1,0048}{3,14 \cdot \left(\frac{0,5}{2}\right)^2} = 5,12\ m/s$$

$$v_3 = \frac{\dot{V}}{A_3} = \frac{\dot{V}}{\pi\left(\frac{d_3}{2}\right)^2} = \frac{1,0048}{3,14 \cdot \left(\frac{0,8}{2}\right)^2} = 2\ m/s$$

$$W_A = \frac{W_{mil}}{\eta} = \frac{120000}{0,60} = 200000\ W$$

$$h_T = \frac{W_A}{\rho \cdot g \cdot \dot{V}} = \frac{200000}{1000 \cdot 10 \cdot 1,0048} = 19,905\ m$$

$$z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = z_3 + \frac{p_3}{\rho g} + \frac{v_3^2}{2g} + \frac{1,2 v_2^2}{2g} + \frac{3 v_3^2}{2g} + h_T$$

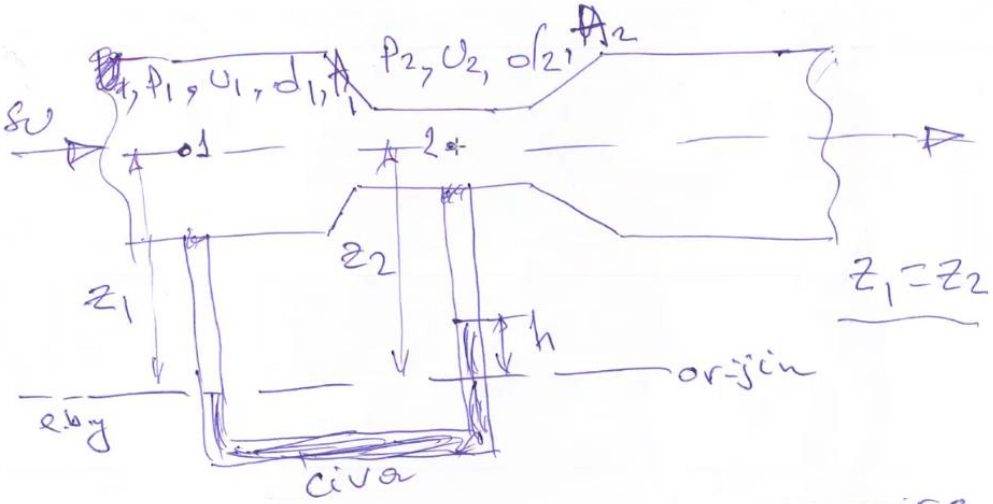
(14)

$$z_1 = \frac{4v_3^2 + 1{,}2v_2^2}{2g} + h_T = \frac{4 \cdot 2^2 + 1{,}2 \cdot (5{,}12)^2}{2 \cdot 10} + 19{,}905 =$$

$$= \frac{16 + 31{,}457}{20} + 19{,}905 = 2{,}373 + 19{,}905 =$$

$$= 22{,}278 \text{ m}$$

1.soru: $\pi = 3{,}14$, $g \simeq 10\ m/s^2$, $\rho = 1000\ kg/m^3$, $d_1 = 10cm$, $d_2 = 5cm$, $\rho_c = 13600\ kg/m^3$, $h = 20cm$ olduklarına göre venturimetreden geçen suyun hacimsel debisini hesaplayınız.



Kütlenin korunumu yasasına göre,

$$\dot{V}_1 = \dot{V}_2$$

$$A_1 \cdot U_1 = A_2 \cdot U_2 \Rightarrow U_1 = \frac{A_2}{A_1} \cdot U_2 = \frac{\pi\left(\frac{d_2}{2}\right)^2}{\pi\left(\frac{d_1}{2}\right)^2} \cdot U_2 =$$

$$= \left(\frac{d_2}{d_1}\right)^2 \cdot U_2 = \left(\frac{5}{10}\right)^2 \cdot U_2 = 0{,}25\,U_2 \qquad \boxed{U_1 = 0{,}25 \cdot U_2}$$

Diferansiyel manometreden,

$$P_1 + \rho g z_1 - \rho_c \cdot g h - \rho g (z_2 - h) = P_2$$

$$P_1 - P_2 = -\rho g z_1 + \rho_c g h + \rho g z_2 - \rho g h =$$

$$= gh(\rho_c - \rho) + \rho g \cancel{(z_2 - z_1)}^{0} = gh \cdot (\rho_c - \rho) =$$

$$= 10 \cdot 0{,}2 \cdot (13600 - 1000) = 25200 \text{ Pa}$$

1. ve 2. kesitler için Bernoulli denklemine göre,

$$\cancel{z_1} + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \cancel{z_2} + \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho} = \frac{v_2^2 - v_1^2}{2} = \frac{v_2^2 - (0{,}25 v_2)^2}{2} = \frac{0{,}9375 v_2^2}{2}$$

$$\frac{25200}{1000} = \frac{0{,}9375 v_2^2}{2}$$

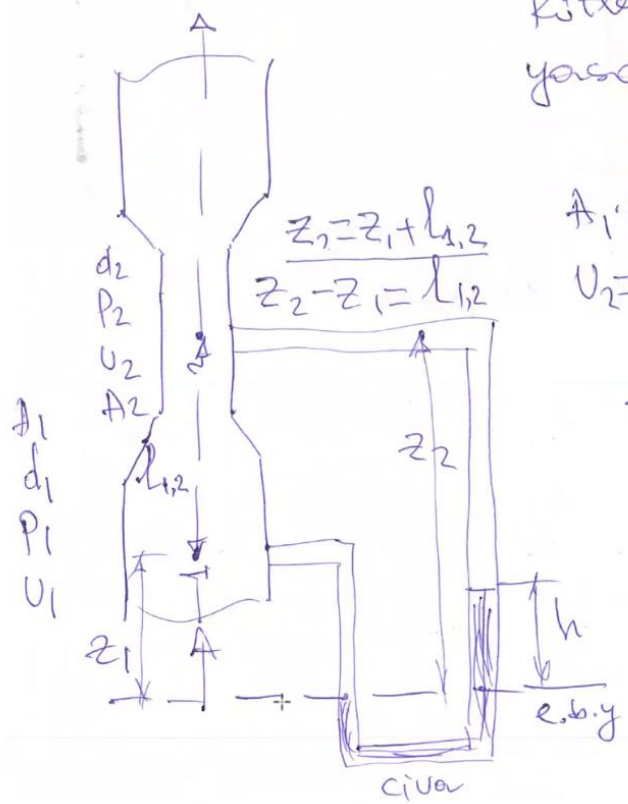
$$50{,}4 = 0{,}9375 v_2^2$$

$$v_2^2 = 53{,}76 \qquad v_2 = 7{,}821 \text{ m/s}$$

$$\dot{V} = \dot{V}_2 = \pi \left(\frac{d_2}{2}\right)^2 \cdot v_2 = 3{,}14 \cdot \left(\frac{0{,}05}{2}\right)^2 \cdot 7{,}821 =$$

$$= 0{,}015 \text{ m}^3/\text{s}$$

2. soru: $\pi = 3,14$, $g \simeq 10\ m/s^2$, $\rho_c = 13600\ kg/m^3$, $\rho = 1000\ kg/m^3$, $d_1 = 10cm$, $d_2 = 5cm$, $h = 25cm$, $l_{1,2} = 20cm$, $h_{K1-2} = 3 \cdot \frac{U_2^2}{2g}$ verildiğine göre venturi metreden geçen suyun hacimsel debisini hesaplayınız.



Kütlenin korunumu yasasına göre,

$$\dot{V}_1 = \dot{V}_2$$

$$A_1 \cdot U_1 = A_2 \cdot U_2$$

$$U_2 = \frac{A_1}{A_2} \cdot U_1 = \left(\frac{d_1}{d_2}\right)^2 \cdot U_1 =$$

$$= \left(\frac{10}{5}\right)^2 \cdot U_1 = 4 \cdot U_1$$

$$\boxed{U_2 = 4 \cdot U_1}$$

Diferansiyel manometreden,

$$P_1 + \rho g z_1 - \rho_c g h + \rho g (z_2 - h) = P_2$$

$$P_1 - P_2 = -\rho g z_1 + \rho_c g h + \rho g z_2 - \rho g h =$$

$$= \rho g (z_2 - z_1) + g h (\rho_c - \rho) =$$

$$= 1000 \cdot 10 \cdot 0{,}2 + 10 \cdot 0{,}25 \cdot (13600 - 1000) =$$

$$= 2000 + 31500 = \underline{33500} \text{ Pa}$$

1. ve 2. kesitler için Bernoulli denklemine göre,

$$\Sigma E_1 = \Sigma E_2 + h_{K_{1-2}}$$

$$z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + 3 \cdot \frac{v_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = (z_2 - z_1) + \frac{v_2^2 + 3v_2^2 - v_1^2}{2g} =$$

$$= z_2 - z_1 + \frac{4v_2^2 - v_1^2}{2g} = z_2 - z_1 + \frac{4 \cdot 16 v_1^2 - v_1^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = z_2 - z_1 + \frac{63 v_1^2}{2 \cdot g}$$

$$\frac{33500}{1000 \cdot 10} = 0{,}2 + \frac{63 \cdot v_1^2}{2 \cdot 10}$$

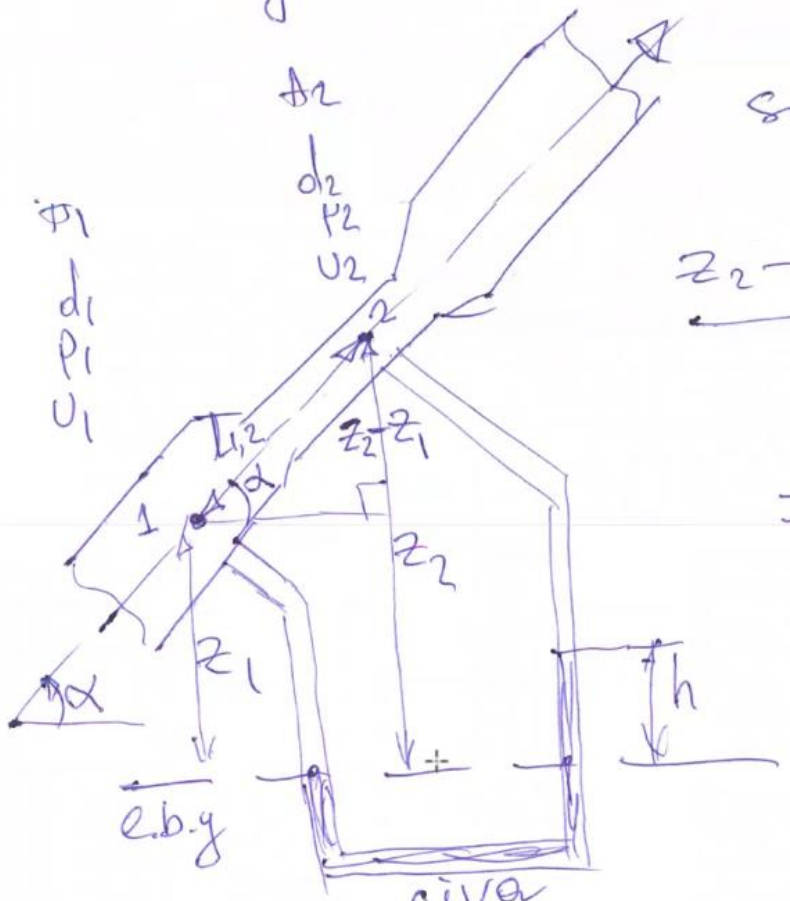
$$3,35 = 0,2 + 3,15\,v_1^2$$

$$3,15 = 3,15\,v_1^2 \qquad v_1 = 1\ m/s$$

$$\dot{V} = \dot{V}_1 = A_1 \cdot v_1 = \pi\left(\frac{d_1}{2}\right)^2 \cdot v_1 = 3,14 \cdot \left(\frac{0,1}{2}\right)^2 \cdot 1 =$$

$$= 0,008\ m^3/s$$

3.soru: $\pi = 3,14$, $g \cong 10 m/s^2$, $\rho_c = 13500 kg/m^3$
$\rho = 1000 kg/m^3$, $d_1 = 20cm$, $d_2 = 10cm$,
$h = 30cm$, $l_{1,2} = 50cm$, $\alpha = 30°$, $h_{K_{1-2}} = 5 \cdot \frac{U_2^2}{2g}$
olduklarına göre venturimetreden geçen suyun hacimsel debisini bulalım.



$$\sin\alpha = \frac{z_2 - z_1}{l_{1,2}}$$

$$z_2 - z_1 = l_{1,2} \cdot \sin\alpha =$$

$$= 50 \cdot \sin 30° =$$

$$= 50 \cdot \frac{1}{2} = 25cm =$$

$$= 0,25m$$

Kütlenin korunumu yasına göre, (6)

$\dot{V}_1 = \dot{V}_2 \Rightarrow A_1 \cdot U_1 = A_2 \cdot U_2 \Rightarrow U_2 = \left(\frac{d_1}{d_2}\right)^2 \cdot U_1 =$

$= \left(\frac{20}{10}\right)^2 \cdot U_1 = 4 \cdot U_1$

$$\boxed{U_2 = 4U_1}$$

Diferansiyel manometreden,

$P_1 + \rho g z_1 - \rho_c g h - \rho g (z_2 - h) = P_2$

$P_1 - P_2 = gh(\rho_c - \rho) + \rho g (z_2 - z_1) =$

$= 10 \cdot 0{,}3 \cdot (13500 - 1000) + 1000 \cdot 10 \cdot 0{,}25 =$

$= 37500 + 2500 = 40000 \text{ Pa}$

1. ve 2. kesitler için Bernoulli denklemiinden,

$$z_1 + \frac{P_1}{\rho g} + \frac{U_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{U_2^2}{2g} + h_{K1-2}$$

$$\frac{P_1 - P_2}{\rho g} = z_2 - z_1 + \frac{U_2^2 - U_1^2}{2g} + \frac{5U_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = z_2 - z_1 + \frac{6 \cdot (4U_1^2) - U_1^2}{2g}$$

(7)

$$\frac{p_1 - p_2}{\rho g} = z_2 - z_1 + \frac{9 \cdot U_1^2}{2g}$$

$$\frac{40000}{1000 \cdot 10} = 0{,}25 + \frac{9 \cdot U_1^2}{2 \cdot 10}$$

$$4 = 0{,}25 + 4{,}75 \cdot U_1^2$$

$$U_1^2 = 0{,}79 \qquad U_1 = 0{,}19 \ m/s$$

$$\dot{V} = \dot{V}_1 = A_1 \cdot U_1 = \pi \cdot \left(\frac{d_1}{2}\right)^2 \cdot U_1 =$$

$$= 3{,}14 \cdot \left(\frac{0{,}2}{2}\right)^2 \cdot 0{,}19 = 0{,}006 \ m^3/s$$