OTOMATİK KONTROL I SINAV FORMÜL KAĞIDI

2.Freakans Uzayında Modelleme

2.2 Laplace Dönüşümü

Laplace transform table

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^n+1}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

Laplace transform theorems

Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)] = F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s)-f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s)-sf(0-)-f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s)-\sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

2.3 Transfer Fonksiyonu

$$a_n\frac{d^nc(t)}{dt^n}+a_{n-1}\frac{d^{n-1}c(t)}{dt^{n-1}}+\cdots+a_0c(t)=b_m\frac{d^mr(t)}{dt^m}+b_{m-1}\frac{d^{m-1}r(t)}{dt^{m-1}}+\cdots+b_0r(t)$$

$$a_ns^nC(s)+a_{n-1}s^{n-1}C(s)+\cdots+a_0C(s)+\text{initial condition terms involving } c(t) = b_ms^mR(s)+b_{m-1}s^{m-1}R(s)+\cdots+b_0R(s)+\text{initial condition terms involving } r(t)$$

$$(a_ns^n+a_{n-1}s^{n-1}+\cdots+a_0)C(s)=(b_ms^m+b_{m-1}s^{m-1}+\cdots+b_0)R(s)$$

$$\boxed{\frac{C(s)}{R(s)}=G(s)=\frac{(b_ms^m+b_{m-1}s^{m-1}+\cdots+b_0)}{(a_ns^n+a_{n-1}s^{n-1}+\cdots+a_0)}}$$

$$\boxed{C(s)=R(s)G(s)}$$

2.4 Elektriksel Devrelerin Transfer Fonksiyonu

TABLE 2.3 Voltage-current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s)=V(s)/I(s)$	Admittance $Y(s)=I(s)/V(s)$
Capacitor	$v(t)=\frac{1}{C}\int_0^1 i(\tau)d\tau$	$i(t)=C\frac{dv(t)}{dt}$	$v(t)=\frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t)=Ri(t)$	$i(t)=\frac{1}{R}v(t)$	$v(t)=R\frac{dq(t)}{dt}$	R	$\frac{1}{R}=G$
Inductor	$v(t)=L\frac{di(t)}{dt}$	$i(t)=\frac{1}{L}\int_0^1 v(\tau)d\tau$	$v(t)=L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), C – F (farads), R – Ω (ohms), G – Ω (mhos), L – H (henries).

First, take the Laplace transform of the equations in the voltage-current column of Table 2.3 assuming zero initial conditions.

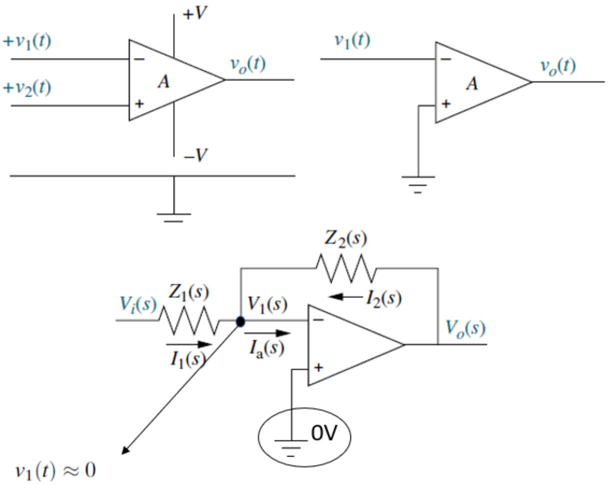
For the capacitor, $\boxed{V(s)=\frac{1}{Cs}I(s)}$

For the resistor, $\boxed{V(s)=RI(s)}$

For the inductor, $\boxed{V(s)=LsI(s)}$

Now define the following transfer function: $\boxed{\frac{V(s)}{I(s)}=Z(s)}$

Inverting Operational Amplifier



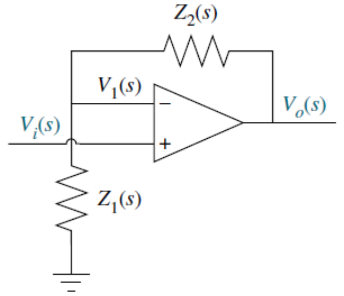
$v_1(t) \approx 0$

$I_1(s) = V_i(s)/Z_1(s)$

$-I_2(s) = -V_o(s)/Z_2(s)$

$$\boxed{\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}}$$

Noninverting Operational Amplifier



$$V_o(s) = A(V_i(s) - V_1(s))$$

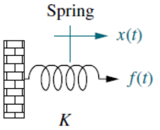
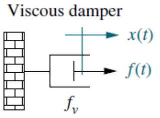
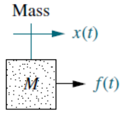
$$V_1(s) = \frac{Z_1(s)}{Z_1(s)+Z_2(s)} V_o(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{A}{1 + AZ_1(s)/(Z_1(s)+Z_2(s))}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{Z_1(s)+Z_2(s)}{Z_1(s)}}$$

2.5 Doğrusal Mekanik Sistemlerin Transfer Fonksiyonları

TABLE 2.4 Force-velocity, force-displacement, and impedance translational relationships for springs, viscous dampers, and mass

Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring  $x(t)$ $f(t)$ K	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper  $x(t)$ $f(t)$ f_v	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass  $x(t)$ M $f(t)$	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m(newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).

Taking the Laplace transform of the force-displacement column in Table 2.4, we obtain

for the spring, $\boxed{F(s) = KX(s)}$

for the viscous damper, $\boxed{F(s) = f_v sX(s)}$

for the mass, $\boxed{F(s) = Ms^2X(s)}$

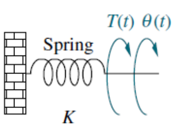
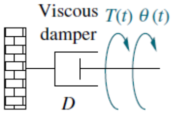
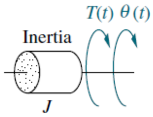
If we define impedance for mechanical components as

$$\boxed{Z_M(s) = \frac{F(s)}{X(s)}}$$

$$\boxed{F(s) = Z_M(s)X(s)}$$

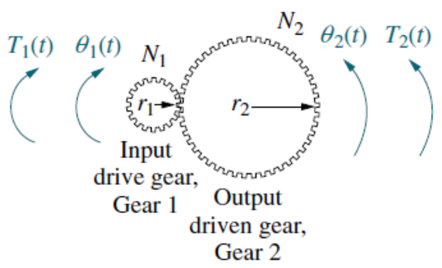
2.6 Dönen Mekanik Sistemlerin Transfer Fonksiyonları

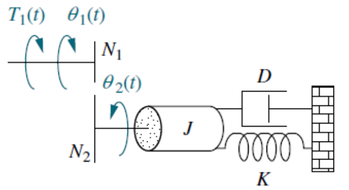
Torque-angular velocity, torque-angular displacement, and impedance rotational relationships for springs, viscous dampers, and inertia

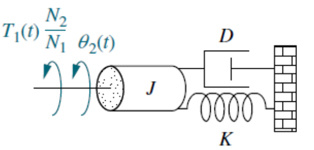
Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
Spring  $T(t)$ $\theta(t)$ K	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
Viscous damper  $T(t)$ $\theta(t)$ D	$T(t) = D\omega(t)$	$T(t) = D\frac{d\theta(t)}{dt}$	Ds
Inertia  $T(t)$ $\theta(t)$ J	$T(t) = J\frac{d\omega(t)}{dt}$	$T(t) = J\frac{d^2\theta(t)}{dt^2}$	Js^2

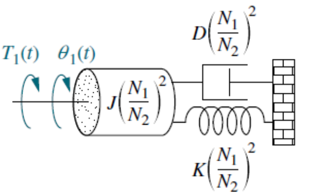
Note: The following set of symbols and units is used throughout this book: $T(t)$ – N-m (newton-meters), $\theta(t)$ – rad(radians), $\omega(t)$ – rad/s(radians/second), K – N-m/rad(newton- meters/radian), D – N-m-s/rad (newton- meters-seconds/radian). J – kg-m^2(kilograms-meters^2 – newton-meters-seconds^2/radian).

2.7 Dişili Sistemler için Transfer Fonksiyonları









$$r_1\theta_1 = r_2\theta_2$$

$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

$$T_1\theta_1 = T_2\theta_2$$

$$\frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}$$

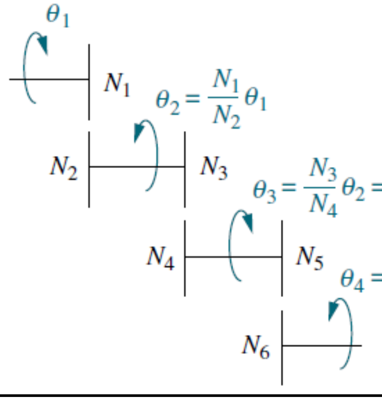
$$(Js^2 + Ds + K)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

$$(Js^2 + Ds + K)\frac{N_1}{N_2}\theta_1(s) = T_1(s)\frac{N_2}{N_1}$$

$$\left[J\left(\frac{N_1}{N_2}\right)^2 s^2 + D\left(\frac{N_1}{N_2}\right)^2 s + K\left(\frac{N_1}{N_2}\right)^2\right]\theta_1(s) = T_1(s)$$

$$\left(\frac{\text{Number of teeth of gear on } \textit{destination} \text{ shaft}}{\text{Number of teeth of gear on } \textit{source} \text{ shaft}}\right)^2$$

$$\theta_1 \rightarrow \boxed{\frac{N_1}{N_2}} \rightarrow \theta_2 \qquad T_1 \rightarrow \boxed{\frac{N_2}{N_1}} \rightarrow T_2$$

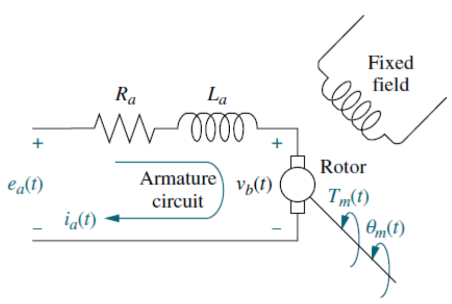


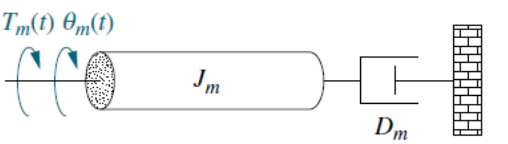
$$\theta_2 = \frac{N_1}{N_2}\theta_1$$

$$\theta_3 = \frac{N_3}{N_4}\theta_2 = \frac{N_1 N_3}{N_2 N_4}\theta_1$$

$$\theta_4 = \frac{N_5}{N_6}\theta_3 = \frac{N_1 N_3 N_5}{N_2 N_4 N_6}\theta_1$$

2.8 Elektromekanik Sistemlerin Transfer Fonksiyonları





$$E_a(s) \rightarrow \boxed{G(s)} \rightarrow \theta_m(s)$$

$$v_b(t) = K_b\frac{d\theta_m(t)}{dt}$$

$$V_b(s) = K_b s\theta_m(s)$$

$$R_a I_a(s) + L_a s I_a(s) + V_b(s) = E_a(s)$$

$$T_m(s) = K_t I_a(s)$$

$$I_a(s) = \frac{1}{K_t}T_m(s)$$

$$\frac{(R_a + L_a s)T_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s)$$

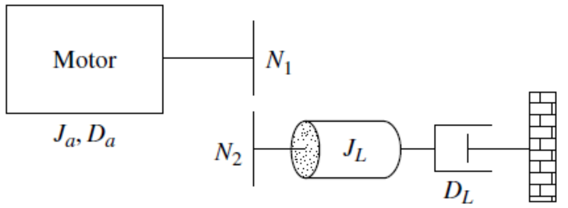
$$T_m(s) = (J_m s^2 + D_m s)\theta_m(s)$$

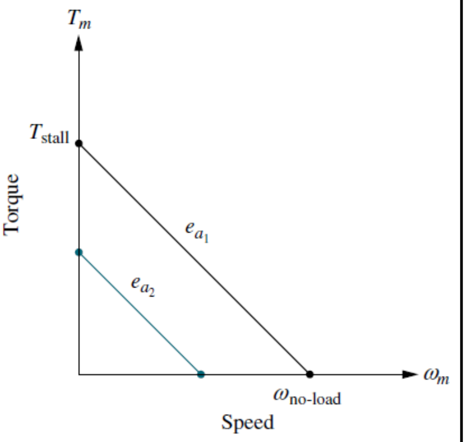
$$\frac{(R_a + L_a s)(J_m s^2 + D_m s)\theta_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s)$$

$$\left[\frac{R_a}{K_t}(J_m s + D_m) + K_b\right]s\theta_m(s) = E_a(s)$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K}{s(s+\alpha)}$$





$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2; \; D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2$$

$$\frac{R_a}{K_t}T_m(s) + K_b s\theta_m(s) = E_a(s)$$

$$\frac{R_a}{K_t}T_m(t) + K_b\omega_m(t) = e_a(t)$$

$$\frac{R_a}{K_t}T_m + K_b\omega_m = e_a$$

$$T_m = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a}e_a$$

$$T_{\text{stall}} = \frac{K_t}{R_a}e_a$$

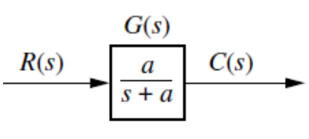
$$\omega_{\text{no-load}} = \frac{e_a}{K_b}$$

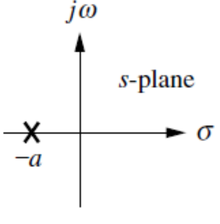
$$\frac{K_t}{R_a} = \frac{T_{\text{stall}}}{e_a}$$

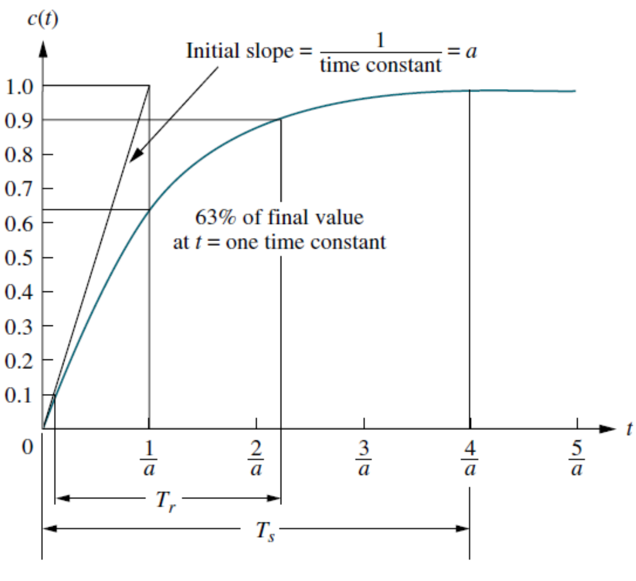
$$K_b = \frac{e_a}{\omega_{\text{no-load}}}$$

4.Zaman Cevabı

4.3 Birinci Mertebe Sistemler







$$C(s) = R(s)G(s) = \frac{a}{s(s+a)}$$

$$c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$$

Zaman Sabiti, $T_C = 1/a$

Yükselme Zamanı, $T_r = \frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a}$

Yerleşme Zamanı, $T_s = \frac{4}{a}$

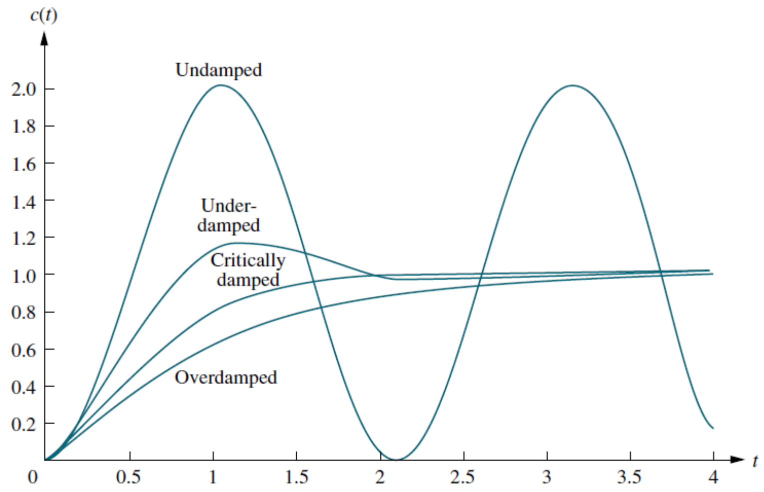
4.4 İkinci Mertebe Sistemlere Giriş

1. *Overdamped* (Aşırı Sönümlü) *responses*
   Poles: (Δ > 0) Two real at $-\sigma_1$, $-\sigma_2$
   $c(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$

2. *Underdamped* (Sönümlü) *responses*
   Poles: (Δ < 0) Two complex at $-\sigma_d \pm j\omega_d$
   $c(t) = Ae^{-\sigma_d t}\cos(\omega_d t - \phi)$

3. *Undamped* (Sönümsüz Sistem) *responses*
   Poles: (Δ < 0) Two imaginary at $\pm j\omega_1$
   $c(t) = A\cos(\omega_1 t - \phi)$

4. *Critically damped* (Kritik Sönümlü) *responses*
   Poles: (Δ = 0) Two real at $-\sigma_1$
   $c(t) = K_1 e^{-\sigma_1 t} + K_2 t e^{-\sigma_1 t}$



4.5 Genel İkinci Mertebe Sistemler

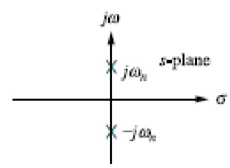
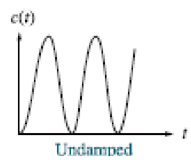
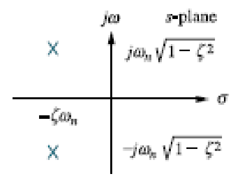
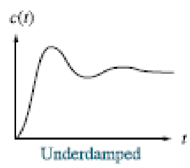
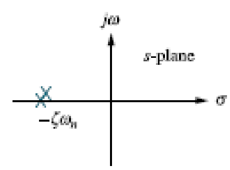
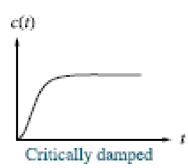
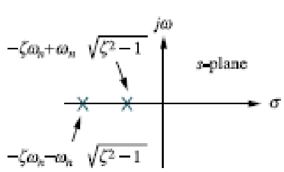
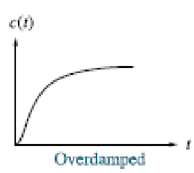
$$G(s) = \frac{b}{s^2 + as + b} \quad , \quad G(s) = \frac{b}{s^2 + b}$$

$$\boxed{\begin{aligned} \omega_n &= \sqrt{b} \\ b &= \omega_n^2 \end{aligned}}$$

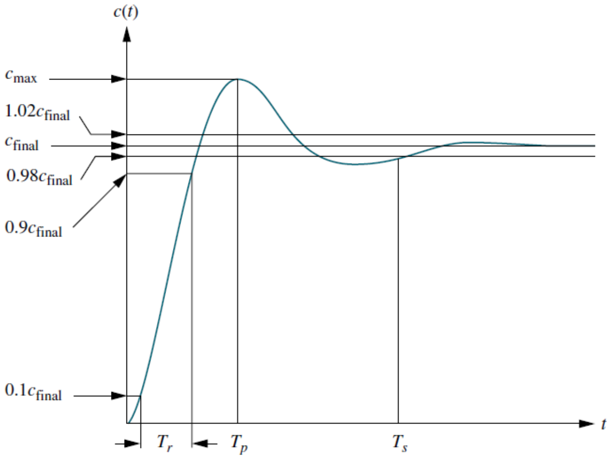
$$\boxed{\begin{aligned} a &= 2\zeta\omega_n \\ \zeta &= \frac{a}{2\sqrt{b}} \end{aligned}}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

ζ	Poles	Step response
0		 (Sönümsüz Sistem)
$0 < \zeta < 1$		 (Sönümlü)
$\zeta = 1$		 (Kritik Sönümlü)
$\zeta > 1$		 (Aşırı Sönümlü)

4.6 Sönümlü İkinci Mertebe Sistemler



T_p Tepe Zamanı $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

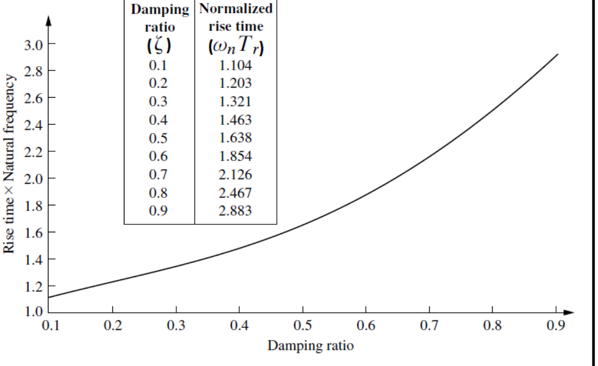
%OS Yüzde Aşım Oranı $\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$

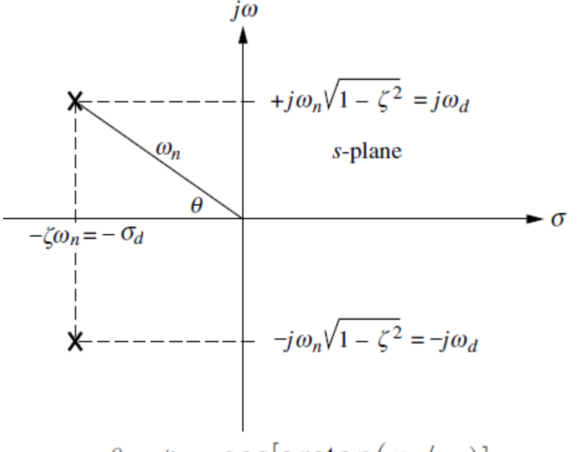
$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

T_s Yerleşme Zamanı $T_s = \dfrac{4}{\zeta\omega_n}$

T_r Yükselme Zamanı

Damping ratio (ζ)	Normalized rise time ($\omega_n T_r$)
0.1	1.104
0.2	1.203
0.3	1.321
0.4	1.463
0.5	1.638
0.6	1.854
0.7	2.126
0.8	2.467
0.9	2.883





$$\cos\theta = \zeta = \cos[\arctan(\omega_d/\sigma_d)]$$

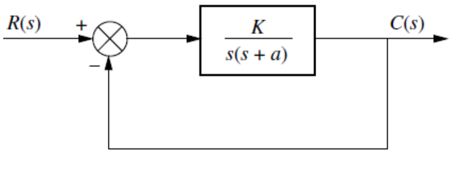
$$\omega_n = \sqrt{\omega_d^2 + \sigma_d^2}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d}$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{\pi}{\sigma_d}$$

5.Karmaşık Sistemlerin Sadeleştirilmesi

5.3 Geri Besleme Sistemlerin Tasarımı Ve Analizleri



$$T(s) = \frac{K}{s^2 + as + K}$$

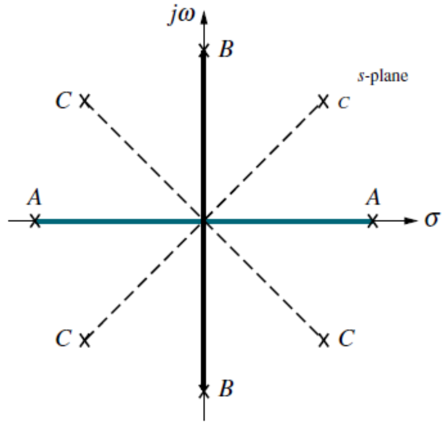
6.Kararlılık

6.2 Routh-Hurwitz Kriteri

$$R(s) \longrightarrow \boxed{\frac{N(s)}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}} \longrightarrow C(s)$$

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

6.3 Routh-Hurwitz Kriteri: Özel Durum



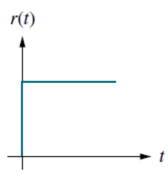
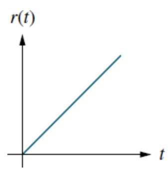
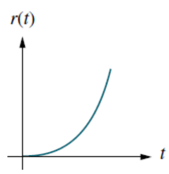
A: Real and symmetrical about the origin

B: Imaginary and symmetrical about the origin

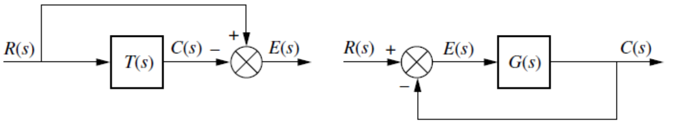
C: Quadrantal and symmetrical about the origin

7.Kararlı Hal Hataları

7.1 Giriş

Waveform	Name	Physical interpretation	Time function	Laplace transform
	Step	Constant position	1	$\frac{1}{s}$
	Ramp	Constant velocity	t	$\frac{1}{s^2}$
	Parabola	Constant acceleration	$\frac{1}{2}t^2$	$\frac{1}{s^3}$

7.2 Bütün Geri Besleme Sistemleri İçin Kararlı Durum Hatası



$$E(s) = R(s) - C(s) \qquad C(s) = R(s)T(s) \;\Rightarrow\; E(s) = R(s)[1 - T(s)]$$

$T(s) = \dfrac{G(s)}{1+G(s)}$ yazarsak $E(s) = \dfrac{R(s)}{1+G(s)}$ olur

$$e(\infty) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$e(\infty) = \lim_{s\to\infty} sR(s)[1 - T(s)]$$

$$e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

Step Input

$R(s) = 1/s$

$$e(\infty) = e_{\text{step}}(\infty) = \lim_{s\to 0} \frac{s(1/s)}{1+G(s)} = \frac{1}{1+\lim_{s\to 0} G(s)}$$

Ramp Input

$R(s) = 1/s^2$

$$e(\infty) = e_{\text{ramp}}(\infty) = \lim_{s\to 0} \frac{s(1/s^2)}{1+G(s)} = \lim_{s\to 0} \frac{1}{s+sG(s)} = \frac{1}{\lim_{s\to 0} sG(s)}$$

Parabolic Input

$R(s) = 1/s^3$

$$e(\infty) = e_{\text{parabola}}(\infty) = \lim_{s\to 0} \frac{s(1/s^3)}{1+G(s)} = \lim_{s\to 0} \frac{1}{s^2+s^2G(s)} = \frac{1}{\lim_{s\to 0} s^2G(s)}$$

7.3 Statik Hata Sabitleri ve Sistem Tipleri

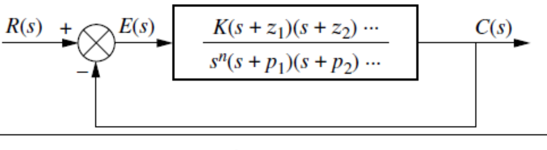
Static Error Constants

position constant, (step) $\quad K_p = \lim_{s\to 0} G(s)$

velocity constant, (ramp) $\quad K_v = \lim_{s\to 0} sG(s)$

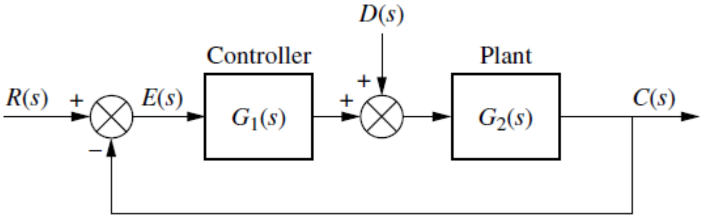
acceleration constant, (parabolic) $\quad K_a = \lim_{s\to 0} s^2G(s)$

System Type



Input	Steady-state error formula	Type 0 Static error constant	Type 0 Error	Type 1 Static error constant	Type 1 Error	Type 2 Static error constant	Type 2 Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

7.5 Bozulmalar için Kararlı Durum Hatası



$$E(s) = R(s) - C(s) \quad \Rightarrow \quad C(s) = R(s) - E(s)$$

$$C(s) = [\,E(s)G_1(s) + D(s)\,]\,G_2(s)$$

$$R(s) - E(s) = E(s)G_1(s)G_2(s) + D(s)G_2(s)$$

$$R(s) - D(s)G_2(s) = E(s)\,[\,G_1(s)G_2(s) + 1\,]$$

$$E(s) = \frac{1}{1 + G_1(s)G_2(s)}R(s) - \frac{G_2(s)}{1 + G_1(s)G_2(s)}D(s)$$

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{s}{1 + G_1(s)G_2(s)}R(s) - \lim_{s\to 0}\frac{sG_2(s)}{1 + G_1(s)G_2(s)}D(s)$$
$$= e_R(\infty) + e_D(\infty)$$

$$e_R(\infty) = \lim_{s\to 0}\frac{s}{1 + G_1(s)G_2(s)}R(s)$$

$$e_D(\infty) = -\lim_{s\to 0}\frac{sG_2(s)}{1 + G_1(s)G_2(s)}D(s)$$

$D(s) = 1/s$ (step disturbance) yazılırsa

$$e_D(\infty) = -\frac{1}{\lim\limits_{s\to 0}\dfrac{1}{G_2(s)} + \lim\limits_{s\to 0} G_1(s)}$$