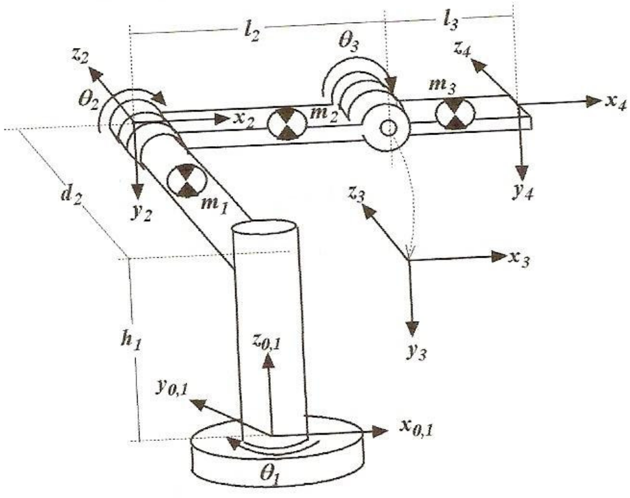
3.2.14. NR robotunun dinamiğinin Lagrange-Euler yöntemi ile çıkarılması

NR robotunun eklem düzenleşimi ve sembolik kütle gösterimi Şekil 3.14'de ve bu düzenleşime göre elde edilmiş DH parametreleri Tablo 3.14'de verilmiştir.



Şekil 3.14: NR robotunun eklem düzenleşimi ve sembolik kütle gösterimi

Tablo 3.14: NR robotunun DH parametreleri

i	θ_i	α_{i-1}	a_{i-1}	d_i
1	θ_1	0	0	h_1
2	θ_2	–90	0	d_2
3	θ_3	0	l_2	0
4	0	0	l_3	0

Tablo 3.14'deki verilerden yararlanarak NR robotunun dönüşüm matrisleri aşağıdaki gibi elde edilir.

$$
{}^{0}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.502}
$$

$$
{}^{1}_{2}T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.503}
$$

$$
{}^{2}_{3}T = \begin{bmatrix} c_3 & -s_3 & 0 & l_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.504}
$$

$$
{}^{3}_{4}T = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.505}
$$

$$
{}^{0}_{4}T = \begin{bmatrix} c_1\, c_{(2+3)} & -c_1\, s_{(2+3)} & -s_1 & l_2\, c_1\, c_2 - d_2\, s_1 + l_3\, c_1\, c_{(2+3)} \\ s_1\, c_{(2+3)} & -s_1\, s_{(2+3)} & c_1 & d_2\, c_1 + l_2\, c_2\, s_1 + l_3\, s_1\, c_{(2+3)} \\ -s_{(2+3)} & -c_{(2+3)} & 0 & h_1 - l_2\, s_2 - l_3\, s_{(2+3)} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.506}
$$

İlk eklem için değişkenler aşağıdaki gibi bulunur:

$$
\Delta h_1 = [0 \quad d_2 / 2 \quad 0 \quad 1]^T \tag{3.507}
$$

$$
I_1 = \begin{bmatrix} c_1^2\, I_{xx1} + s_1^2\, I_{yy1} & c_1\, s_1 \left(I_{xx1} - I_{yy1} \right) & 0 \\ c_1\, s_1 \left(I_{xx1} - I_{yy1} \right) & c_1^2\, I_{yy1} + s_1^2\, I_{xx1} & 0 \\ 0 & 0 & I_{zz1} \end{bmatrix} \tag{3.508}
$$

$$
h_1 = \left[-\tfrac{1}{2} d_2\, s_1 \quad \tfrac{1}{2} d_2\, c_1 \quad h_1 \quad 1 \right]^T \tag{3.509}
$$

$$
b_1 = [0 \quad 0 \quad 1]^T \tag{3.510}
$$

$$
J_1 = \begin{bmatrix} -\tfrac{1}{2} d_2\, c_1 & -\tfrac{1}{2} d_2\, s_1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{3.511}
$$

$$
A_1 = \begin{bmatrix} -\tfrac{1}{2} d_2\, c_1 & 0 & 0 \\ -\tfrac{1}{2} d_2\, s_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.512}
$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{3.513}$$

$$D(\theta_1) = \begin{bmatrix} I_{zz1} + \frac{1}{4} d_2^2 m_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.514}$$

İkinci eklem için değişkenler aşağıdaki gibi bulunur:

$$\Delta h_2 = [\, l_2 / 2 \quad 0 \quad 0 \quad 1]^T \tag{3.515}$$

$$I_2 = \begin{bmatrix} \mathrm{s}_1^2 I_{zz2} + \mathrm{c}_1^2 \left(\mathrm{c}_2^2 I_{xx2} + \mathrm{s}_2^2 I_{yy2} \right) & I_{2(1,2)} & \mathrm{c}_1 \mathrm{c}_2 \mathrm{s}_2 \left(I_{yy2} - I_{xx2} \right) \\ \mathrm{c}_1 \mathrm{s}_1 \left(\mathrm{c}_2^2 I_{xx2} + \mathrm{s}_2^2 I_{yy2} - I_{zz2} \right) & I_{2(2,2)} & \mathrm{c}_2 \mathrm{s}_1 \mathrm{s}_2 \left(I_{yy2} - I_{xx2} \right) \\ \mathrm{c}_1 \mathrm{c}_2 \mathrm{s}_2 \left(I_{yy2} - I_{xx2} \right) & I_{2(3,2)} & \mathrm{c}_2^2 I_{yy2} + \mathrm{s}_2^2 I_{xx2} \end{bmatrix} \tag{3.516}$$

Matriste,

$$\begin{aligned} I_{2(1,2)} &= \mathrm{c}_1 \mathrm{s}_1 \left(\mathrm{c}_2^2 I_{xx2} + \mathrm{s}_2^2 I_{yy2} - I_{zz2} \right) \\ I_{2(2,2)} &= \mathrm{c}_1^2 I_{zz2} + \mathrm{s}_1^2 \left(\mathrm{c}_2^2 I_{xx2} + \mathrm{s}_2^2 I_{yy2} \right) \\ I_{2(3,2)} &= \mathrm{c}_2 \mathrm{s}_1 \mathrm{s}_2 \left(I_{yy2} - I_{xx2} \right) \end{aligned}$$

$$h_2 = \left[\tfrac{1}{2} l_2 \mathrm{c}_1 \mathrm{c}_2 - d_2 \mathrm{s}_1 \quad d_2 \mathrm{c}_1 + \tfrac{1}{2} l_2 \mathrm{c}_2 \mathrm{s}_1 \quad h_1 - \tfrac{1}{2} l_2 \mathrm{s}_2 \quad 1 \right]^T \tag{3.517}$$

$$b_2 = [-\mathrm{s}_1 \quad \mathrm{c}_1 \quad 0]^T \tag{3.518}$$

$$J_2 = \begin{bmatrix} -d_2 \mathrm{c}_1 - \frac{1}{2} l_2 \mathrm{c}_2 \mathrm{s}_1 & \frac{1}{2} l_2 \mathrm{c}_1 \mathrm{c}_2 - d_2 \mathrm{s}_1 & 0 & 0 & 0 & 1 \\ -\frac{1}{2} l_2 \mathrm{c}_1 \mathrm{s}_2 & -\frac{1}{2} l_2 \mathrm{s}_1 \mathrm{s}_2 & -\frac{1}{2} l_2 \mathrm{c}_2 & -\mathrm{s}_1 & \mathrm{c}_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{3.519}$$

$$A_2 = \begin{bmatrix} -d_2 \mathrm{c}_1 - \frac{1}{2} l_2 \mathrm{c}_2 \mathrm{s}_1 & -\frac{1}{2} l_2 \mathrm{c}_1 \mathrm{s}_2 & 0 \\ \frac{1}{2} l_2 \mathrm{c}_1 \mathrm{c}_2 - d_2 \mathrm{s}_1 & -\frac{1}{2} l_2 \mathrm{s}_1 \mathrm{s}_2 & 0 \\ 0 & -\frac{1}{2} l_2 \mathrm{c}_2 & 0 \end{bmatrix} \tag{3.520}$$

$$B_2 = \begin{bmatrix} 0 & -\mathrm{s}_1 & 0 \\ 0 & \mathrm{c}_1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{3.521}$$

$$D(\theta_2)=\begin{bmatrix} m_2\left(d_2^2+\frac{1}{4}l_2^2\,\mathrm{c}_2^2\right)+\mathrm{c}_2^2\,I_{yy2}+\mathrm{s}_2^2\,I_{xx2} & \frac{1}{2}d_2l_2m_2\,\mathrm{s}_2 & 0 \\ \frac{1}{2}d_2l_2m_2\,\mathrm{s}_2 & \frac{1}{4}l_2^2m_2+I_{zz2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.522}$$

Üçüncü eklem için değişkenler aşağıdaki gibi bulunur:

$$\Delta h_3=[\,l_3/2 \quad 0 \quad 0 \quad 1\,]^T \tag{3.523}$$

$$I_3=\begin{bmatrix} I_{3(1,1)} & I_{3(1,2)} & \mathrm{c}_1\,\mathrm{c}_{(2+3)}\,\mathrm{s}_{(2+3)}\left(I_{yy3}-I_{xx3}\right) \\ I_{3(2,1)} & I_{3(2,2)} & \mathrm{s}_1\,\mathrm{c}_{(2+3)}\,\mathrm{s}_{(2+3)}\left(I_{yy3}-I_{xx3}\right) \\ I_{3(3,1)} & I_{3(3,2)} & \mathrm{c}_{(2+3)}^2\,I_{yy3}+\mathrm{s}_{(2+3)}^2\,I_{xx3} \end{bmatrix} \tag{3.524}$$

Matriste,

$$I_{3(1,1)}=\mathrm{s}_1^2\,I_{zz3}+\mathrm{c}_1^2\left(\mathrm{c}_{(2+3)}^2\,I_{xx3}+\mathrm{s}_{(2+3)}^2\,I_{yy3}\right)$$

$$I_{3(1,2)}=I_{3(2,1)}=\mathrm{c}_1\,\mathrm{s}_1\left(\mathrm{c}_{(2+3)}^2\,I_{xx3}+\mathrm{s}_{(2+3)}^2\,I_{yy3}-I_{zz3}\right)$$

$$I_{3(2,2)}=\mathrm{c}_1^2\,I_{zz3}+\mathrm{s}_1^2\left(\mathrm{c}_{(2+3)}^2\,I_{xx3}+\mathrm{s}_{(2+3)}^2\,I_{yy3}\right)$$

$$I_{3(3,1)}=\mathrm{c}_1\,\mathrm{c}_{(2+3)}\,\mathrm{s}_{(2+3)}\left(I_{yy3}-I_{xx3}\right)$$

$$I_{3(3,2)}=\mathrm{s}_1\,\mathrm{c}_{(2+3)}\,\mathrm{s}_{(2+3)}\left(I_{yy3}-I_{xx3}\right)$$

$$h_3=\left[\,h_{3(1,1)} \quad d_2\,\mathrm{c}_1+l_2\,\mathrm{c}_2\,\mathrm{s}_1+\tfrac{1}{2}l_3\,\mathrm{s}_1\,\mathrm{c}_{(2+3)} \quad h_1-l_2\,\mathrm{s}_2-\tfrac{1}{2}l_3\,\mathrm{s}_{(2+3)} \quad 1\,\right]^T \tag{3.525}$$

Matriste,

$$h_{3(1,1)}=l_2\,\mathrm{c}_1\,\mathrm{c}_2-d_2\,\mathrm{s}_1+\tfrac{1}{2}l_3\,\mathrm{c}_1\,\mathrm{c}_{(2+3)}$$

$$b_3=[-\mathrm{s}_1 \quad \mathrm{c}_1 \quad 0]^T \tag{3.526}$$

$$J_3=\begin{bmatrix} J_{3(1,1)} & J_{3(1,2)} & 0 & 0 & 0 & 1 \\ J_{3(2,1)} & J_{3(2,2)} & -l_2\,\mathrm{c}_2-\frac{1}{2}l_3\,\mathrm{c}_{(2+3)} & -\mathrm{s}_1 & \mathrm{c}_1 & 0 \\ -\frac{1}{2}l_3\,\mathrm{c}_1\,\mathrm{s}_{(2+3)} & -\frac{1}{2}l_3\,\mathrm{s}_1\,\mathrm{s}_{(2+3)} & -\frac{1}{2}l_3\,\mathrm{c}_{(2+3)} & -\mathrm{s}_1 & \mathrm{c}_1 & 0 \end{bmatrix}^T \tag{3.527}$$

Matriste,

$$J_{3(1,1)}=-d_2\,\mathrm{c}_1-l_2\,\mathrm{c}_2\,\mathrm{s}_1-\tfrac{1}{2}l_3\,\mathrm{s}_1\,\mathrm{c}_{(2+3)}$$

$$J_{3(1,2)}=-d_2\,\mathrm{s}_1+l_2\,\mathrm{c}_1\,\mathrm{c}_2+\tfrac{1}{2}l_3\,\mathrm{c}_1\,\mathrm{c}_{(2+3)}$$

$$J_{3(2,1)} = -l_2\,c_1\,s_2 - \tfrac{1}{2} l_3\,c_1\,s_{(2+3)}$$
$$J_{3(2,2)} = -l_2\,s_1\,s_2 - \tfrac{1}{2} l_3\,s_1\,s_{(2+3)}$$

$$A_3 = \begin{bmatrix} A_{3(1,1)} & -l_2\,c_1\,s_2 - \frac{1}{2} l_3\,c_1\,s_{(2+3)} & -\frac{1}{2} l_3\,c_1\,s_{(2+3)} \\ A_{3(2,1)} & -l_2\,s_1\,s_2 - \frac{1}{2} l_3\,s_1\,s_{(2+3)} & -\frac{1}{2} l_3\,s_1\,s_{(2+3)} \\ 0 & -l_2\,c_2 - \frac{1}{2} l_3\,c_{(2+3)} & -\frac{1}{2} l_3\,c_{(2+3)} \end{bmatrix} \tag{3.528}$$

Matriste,

$$A_{3(1,1)} = -d_2\,c_1 - l_2\,c_2\,s_1 - \tfrac{1}{2} l_3\,s_1\,c_{(2+3)}$$
$$A_{3(2,1)} = l_2\,c_1\,c_2 - d_2\,s_1 + \tfrac{1}{2} l_3\,c_1\,c_{(2+3)}$$

$$B_3 = \begin{bmatrix} 0 & -s_1 & -s_1 \\ 0 & c_1 & c_1 \\ 1 & 0 & 0 \end{bmatrix} \tag{3.529}$$

$$D(\theta_3) = \begin{bmatrix} D(\theta_3)_{(1,1)} & D(\theta_3)_{(1,2)} & \frac{1}{2} d_2 l_3 m_3\,s_{(2+3)} \\ D(\theta_3)_{(2,1)} & D(\theta_3)_{(2,2)} & m_3\left(\frac{1}{2} l_2 l_3\,c_3 + \frac{1}{4} l_3^2\right) + I_{zz3} \\ D(\theta_3)_{(3,1)} & D(\theta_3)_{(3,2)} & \frac{1}{4} l_3^2 m_3 + I_{zz3} \end{bmatrix} \tag{3.530}$$

Matriste,

$$D(\theta_3)_{(1,1)} = m_3\left(d_2^2 + l_2 l_3\,c_2\,c_{(2+3)} + l_2^2\,c_2^2 + \tfrac{1}{4} l_3^2\,c_{(2+3)}^2\right) + c_{(2+3)}^2\,I_{yy3} + s_{(2+3)}^2\,I_{xx3}$$
$$D(\theta_3)_{(1,2)} = D(\theta_3)_{(2,1)} = m_3\left(d_2 l_2\,s_2 + \tfrac{1}{2} d_2 l_3\,s_{(2+3)}\right)$$
$$D(\theta_3)_{(2,2)} = m_3\left(l_2 l_3\,c_3 + l_2^2 + \tfrac{1}{4} l_3^2\right) + I_{zz3}$$
$$D(\theta_3)_{(3,1)} = \tfrac{1}{2} d_2 l_3 m_3\,s_{(2+3)}$$
$$D(\theta_3)_{(3,2)} = m_3\left(\tfrac{1}{2} l_2 l_3\,c_3 + \tfrac{1}{4} l_3^2\right) + I_{zz3}$$

Robotun toplam kütle matrisi:

$$D(q) = \begin{bmatrix} D(q)_{(1,1)} & D(q)_{(1,2)} & \frac{1}{2} d_2 l_3 m_3\,s_{(2+3)} \\ D(q)_{(2,1)} & D(q)_{(2,2)} & m_3\left(\frac{1}{2} l_2 l_3\,c_3 + \frac{1}{4} l_3^2\right) + I_{zz3} \\ D(q)_{(3,1)} & D(q)_{(3,2)} & \frac{1}{4} l_3^2 m_3 + I_{zz3} \end{bmatrix} \tag{3.531}$$

Matriste,

$$D(q)_{(1,1)} = I_{zz1} + \tfrac{1}{4} d_2^2 m_1 + c_2^2\, I_{yy2} + s_2^2\, I_{xx2} + c_{(2+3)}^2\, I_{yy3} + m_2 \left(d_2^2 + \tfrac{1}{4} l_2^2\, c_2^2 \right)$$
$$+\, m_3 \left(l_2 l_3\, c_2\, c_{(2+3)} + d_2^2 + l_2^2\, c_2^2 + \tfrac{1}{4} l_3^2\, c_{(2+3)}^2 \right) + s_{(2+3)}^2\, I_{xx3}$$
$$D(q)_{(1,2)} = D(q)_{(2,1)} = \tfrac{1}{2} d_2 l_2 m_2\, s_2 + m_3 \left(d_2 l_2\, s_2 + \tfrac{1}{2} d_2 l_3\, s_{(2+3)} \right)$$
$$D(q)_{(2,2)} = \tfrac{1}{4} l_2^2 m_2 + I_{zz2} + I_{zz3} + m_3 \left(l_2 l_3\, c_3 + l_2^2 + \tfrac{1}{4} l_3^2 \right)$$
$$D(q)_{(3,1)} = \tfrac{1}{2} d_2 l_3 m_3\, s_{(2+3)}$$
$$D(q)_{(3,2)} = I_{zz3} + m_3 \left(\tfrac{1}{2} l_2 l_3\, c_3 + \tfrac{1}{4} l_3^2 \right)$$

Robotun her bir eklemine etki eden hız bağlaşım matrisleri, C_1, C_2, C_3 ve robotun toplam coriolis ve merkezkaç kuvvet vektörü, C şu şekilde bulunur:

$$C_1 = \begin{bmatrix} 0 & 0 & 0 \\ C_{1(2,1)} & C_{1(2,2)} & \tfrac{1}{2} d_2 l_3 m_3\, c_{(2+3)} \\ C_{1(3,1)} & \tfrac{1}{2} d_2 l_3 m_3\, c_{(2+3)} & \tfrac{1}{2} d_2 l_3 m_3\, c_{(2+3)} \end{bmatrix} \qquad (3.532)$$

Matriste,

$$C_{1(2,1)} = 2 c_2\, s_2 \left(I_{xx2} - I_{yy2} - \tfrac{1}{4} l_2^2 m_2 - l_2^2 m_3 \right) - l_2 l_3 m_3 \left(2 c_2\, c_3\, s_2 + s_3 \left(c_2^2 - s_2^2 \right) \right)$$
$$+ \left(2 c_2\, s_2 \left(c_3^2 - s_3^2 \right) + 2 c_3\, s_3 \left(c_2^2 - s_2^2 \right) \right) \left(I_{xx3} - I_{yy3} - \tfrac{1}{4} l_3^2 m_3 \right)$$
$$C_{1(2,2)} = \tfrac{1}{2} d_2 l_2 m_2\, c_2 + d_2 l_2 m_3\, c_2 + \tfrac{1}{2} d_2 l_3 m_3\, c_{(2+3)}$$
$$C_{1(3,1)} = \left(2 c_2\, s_2 \left(c_3^2 - s_3^2 \right) + 2 c_3\, s_3 \left(c_2^2 - s_2^2 \right) \right) \left(I_{xx3} - I_{yy3} - \tfrac{1}{4} l_3^2 m_3 \right)$$
$$- \tfrac{1}{2} l_2 l_3 m_3 \left(2 c_2\, c_3\, s_2 + s_3 \left(c_2^2 - s_2^2 \right) \right) - \tfrac{1}{2} l_2 l_3 m_3\, s_3$$

$$C_2 = \begin{bmatrix} C_{2(1,1)} & C_{2(1,2)} & -\tfrac{1}{4} d_2 l_3 m_3\, c_{(2+3)} \\ C_{2(2,1)} & 0 & 0 \\ C_{2(3,1)} & -l_2 l_3 m_3\, s_3 & -\tfrac{1}{2} l_2 l_3 m_3\, s_3 \end{bmatrix} \qquad (3.533)$$

Matriste,

$$C_{2(1,1)} = \tfrac{1}{2} l_2 l_3 m_3 \left(2 c_2\, c_3\, s_2 + s_3 \left(c_2^2 - s_2^2 \right) \right) + c_2\, s_2 \left(I_{yy2} - I_{xx2} + \tfrac{1}{4} l_2^2 m_2 + l_2^2 m_3 \right)$$
$$+ \left(I_{yy3} - I_{xx3} + \tfrac{1}{4} l_3^2 m_3 \right) \left(c_2\, s_2 \left(c_3^2 - s_3^2 \right) + c_3\, s_3 \left(c_2^2 - s_2^2 \right) \right)$$
$$C_{2(1,2)} = -\tfrac{1}{4} d_2 l_2 m_2\, c_2 - \tfrac{1}{2} d_2 l_2 m_3\, c_2 - \tfrac{1}{4} d_2 l_3 m_3\, c_{(2+3)}$$
$$C_{2(2,1)} = \tfrac{1}{4} d_2 l_2 m_2\, c_2 + \tfrac{1}{2} d_2 l_2 m_3\, c_2 + \tfrac{1}{4} d_2 l_3 m_3\, c_{(2+3)}$$
$$C_{2(3,1)} = \tfrac{1}{4} d_2 l_3 m_3\, c_{(2+3)}$$

$$C_3 = \begin{bmatrix} C_{3(1,1)} & -\frac{1}{4} d_2 l_3 m_3 \, c_{(2+3)} & -\frac{1}{4} d_2 l_3 m_3 \, c_{(2+3)} \\ \frac{1}{4} d_2 l_3 m_3 \, c_{(2+3)} & \frac{1}{2} l_2 l_3 m_3 \, s_3 & \frac{1}{4} l_2 l_3 m_3 \, s_3 \\ \frac{1}{4} d_2 l_3 m_3 \, c_{(2+3)} & -\frac{1}{4} l_2 l_3 m_3 \, s_3 & 0 \end{bmatrix} \tag{3.534}$$

Matriste,

$$C_{3(1,1)} = \left(c_2 \, s_2 \left(c_3^2 - s_3^2 \right) + c_3 \, s_3 \left(c_2^2 - s_2^2 \right) \right) \left(I_{yy3} - I_{xx3} + \frac{1}{4} l_3^2 m_3 \right) + \frac{1}{2} l_2 l_3 m_3 \, c_2 \, s_{(2+3)}$$

$$C = \begin{bmatrix} C_{(1,1)} \\ C_{(2,1)} \\ C_{(3,1)} \end{bmatrix} \tag{3.535}$$

Matriste,

$$\begin{aligned} C_{(1,1)} &= -\frac{1}{2} l_2 l_3 m_3 \left(2 c_2 \, c_3 \, s_2 + 2 c_2^2 \, s_3 \right) \dot{\theta}_3 \dot{\theta}_1 - l_2 l_3 m_3 \left(2 c_2 \, c_3 \, s_2 + s_3 \left(c_2^2 - s_2^2 \right) \right) \dot{\theta}_2 \dot{\theta}_1 \\ &\quad + \left(\frac{1}{2} d_2 l_2 m_2 \, c_2 + d_2 l_2 m_3 \, c_2 + \frac{1}{2} d_2 l_3 m_3 \, c_{(2+3)} \right) \dot{\theta}_2^2 + d_2 l_3 m_3 \, c_{(2+3)} \, \dot{\theta}_2 \dot{\theta}_3 \\ &\quad + \left(2 c_2 \, s_2 \left(c_3^2 - s_3^2 \right) + 2 c_3 \, s_3 \left(c_2^2 - s_2^2 \right) \right) \left(I_{xx3} - I_{yy3} - \frac{1}{4} l_3^2 m_3 \right) \dot{\theta}_2 \dot{\theta}_1 \\ &\quad + \left(2 c_2 \, s_2 \left(c_3^2 - s_3^2 \right) + 2 c_3 \, s_3 \left(c_2^2 - s_2^2 \right) \right) \left(I_{xx3} - I_{yy3} - \frac{1}{4} l_3^2 m_3 \right) \dot{\theta}_3 \dot{\theta}_1 \\ &\quad + 2 c_2 \, s_2 \left(I_{xx2} - I_{yy2} - \frac{1}{4} l_2^2 m_2 - l_2^2 m_3 \right) \dot{\theta}_2 \dot{\theta}_1 + \frac{1}{2} d_2 l_3 m_3 \, c_{(2+3)} \, \dot{\theta}_3^2 \\ C_{(2,1)} &= \left(\frac{1}{2} l_2 l_3 m_3 \left(2 c_2 \, c_3 \, s_2 + s_3 \left(c_2^2 - s_2^2 \right) \right) + c_2 \, s_2 \left(I_{yy2} - I_{xx2} + \frac{1}{4} l_2^2 m_2 + l_2^2 m_3 \right) \right) \dot{\theta}_1^2 \\ &\quad + \left(I_{yy3} - I_{xx3} + \frac{1}{4} l_3^2 m_3 \right) \left(c_2 \, s_2 \left(c_3^2 - s_3^2 \right) + c_3 \, s_3 \left(c_2^2 - s_2^2 \right) \right) \dot{\theta}_1^2 \\ &\quad - m_3 l_2 l_3 \, s_3 \, \dot{\theta}_2 \dot{\theta}_3 - \frac{1}{2} l_2 l_3 m_3 \, s_3 \, \dot{\theta}_3^2 \\ C_{(3,1)} &= \left(\left(c_2 \, s_2 \left(c_3^2 - s_3^2 \right) + c_3 \, s_3 \left(c_2^2 - s_2^2 \right) \right) \left(I_{yy3} - I_{xx3} + \frac{1}{4} l_3^2 m_3 \right) \right) \dot{\theta}_1^2 \\ &\quad + \frac{1}{2} l_2 l_3 m_3 \, c_2 \, s_{(2+3)} \, \dot{\theta}_1^2 + \frac{1}{2} l_2 l_3 m_3 \, s_3 \, \dot{\theta}_2^2 \end{aligned}$$

Yerçekimi vektörü:

$$G(q) = \begin{bmatrix} 0 & G(q)_{(1,2)} & -\frac{1}{2} g l_3 m_3 \, c_{(2+3)} \end{bmatrix}^T \tag{3.536}$$

Matriste,

$$G(q)_{(1,2)} = g \left(m_3 \left(\frac{1}{2} l_3 \left(s_2 \, s_3 - c_2 \, c_3 \right) - l_2 \, c_2 \right) - \frac{1}{2} l_2 m_2 \, c_2 \right)$$

Sonuç olarak robotun her bir eklemine etki eden tork aşağıdaki gibi bulunur:

$$\begin{aligned}
\tau_1 =& -\tfrac{1}{2}l_2l_3m_3\left(2\,\mathrm{c}_2\,\mathrm{c}_3\,\mathrm{s}_2+2\,\mathrm{c}_2^2\,\mathrm{s}_3\right)\dot{\theta}_3\dot{\theta}_1-l_2l_3m_3\left(2\,\mathrm{c}_2\,\mathrm{c}_3\,\mathrm{s}_2+\mathrm{s}_3\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\right)\dot{\theta}_2\dot{\theta}_1\\
&+\left(m_2\left(d_2^2+\tfrac{1}{4}l_2^2\,\mathrm{c}_2^2\right)+m_3\left(l_2l_3\,\mathrm{c}_2\,\mathrm{c}_{(2+3)}+d_2^2+l_2^2\,\mathrm{c}_2^2+\tfrac{1}{4}l_3^2\,\mathrm{c}_{(2+3)}^2\right)\right)\ddot{\theta}_1\\
&+\left(\tfrac{1}{2}d_2l_2m_2\,\mathrm{c}_2+d_2l_2m_3\,\mathrm{c}_2+\tfrac{1}{2}d_2l_3m_3\,\mathrm{c}_{(2+3)}\right)\dot{\theta}_2^2+d_2l_3m_3\,\mathrm{c}_{(2+3)}\,\dot{\theta}_2\dot{\theta}_3\\
&+\left(I_{zz1}+\tfrac{1}{4}d_2^2m_1+\mathrm{c}_2^2\,I_{yy2}+\mathrm{s}_2^2\,I_{xx2}+\mathrm{c}_{(2+3)}^2\,I_{yy3}+\mathrm{s}_{(2+3)}^2\,I_{xx3}\right)\ddot{\theta}_1\\
&+\left(2\,\mathrm{c}_2\,\mathrm{s}_2\left(I_{xx2}-I_{yy2}-\tfrac{1}{4}l_2^2m_2-l_2^2m_3\right)\right)\dot{\theta}_2\dot{\theta}_1+\tfrac{1}{2}d_2l_3m_3\,\mathrm{c}_{(2+3)}\,\dot{\theta}_3^2\\
&+\left(\tfrac{1}{2}d_2l_2m_2\,\mathrm{s}_2+m_3\left(d_2l_2\,\mathrm{s}_2+\tfrac{1}{2}d_2l_3\,\mathrm{s}_{(2+3)}\right)\right)\ddot{\theta}_2+\tfrac{1}{2}d_2l_3m_3\,\mathrm{s}_{(2+3)}\,\ddot{\theta}_3\\
&+\left(2\,\mathrm{c}_2\,\mathrm{s}_2\left(\mathrm{c}_3^2-\mathrm{s}_3^2\right)+2\,\mathrm{c}_3\,\mathrm{s}_3\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\right)\left(I_{xx3}-I_{yy3}-\tfrac{1}{4}l_3^2m_3\right)\dot{\theta}_2\dot{\theta}_1\\
&+\left(2\,\mathrm{c}_2\,\mathrm{s}_2\left(\mathrm{c}_3^2-\mathrm{s}_3^2\right)+2\,\mathrm{c}_3\,\mathrm{s}_3\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\right)\left(I_{xx3}-I_{yy3}-\tfrac{1}{4}l_3^2m_3\right)\dot{\theta}_3\dot{\theta}_1
\end{aligned}\tag{3.537}$$

$$\begin{aligned}
\tau_2 =& \left(I_{yy3}-I_{xx3}+\tfrac{1}{4}l_3^2m_3\right)\left(\mathrm{c}_2\,\mathrm{s}_2\left(\mathrm{c}_3^2-\mathrm{s}_3^2\right)+\mathrm{c}_3\,\mathrm{s}_3\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\right)\dot{\theta}_1^2\\
&+\left(\tfrac{1}{2}d_2l_2m_2\,\mathrm{s}_2+m_3\left(d_2l_2\,\mathrm{s}_2+\tfrac{1}{2}d_2l_3\,\mathrm{s}_{(2+3)}\right)\right)\ddot{\theta}_1-\tfrac{1}{2}l_2l_3m_3\,\mathrm{s}_3\,\dot{\theta}_3^2\\
&+\left(\tfrac{1}{4}l_2^2m_2+I_{zz2}+I_{zz3}+m_3\left(l_2l_3\,\mathrm{c}_3+l_2^2+\tfrac{1}{4}l_3^2\right)\right)\ddot{\theta}_2\\
&+g\left(m_3\left(\tfrac{1}{2}l_3\,\mathrm{s}_2\,\mathrm{s}_3-\tfrac{1}{2}l_3\,\mathrm{c}_2\,\mathrm{c}_3-l_2\,\mathrm{c}_2\right)-\tfrac{1}{2}l_2m_2\,\mathrm{c}_2\right)\\
&+\left(I_{zz3}+m_3\left(\tfrac{1}{2}l_2l_3\,\mathrm{c}_3+\tfrac{1}{4}l_3^2\right)\right)\ddot{\theta}_3-m_3l_2l_3\,\mathrm{s}_3\,\dot{\theta}_2\dot{\theta}_3\\
&+\mathrm{c}_2\,\mathrm{s}_2\left(I_{yy2}-I_{xx2}+\tfrac{1}{4}l_2^2m_2+l_2^2m_3\right)\dot{\theta}_1^2\\
&+\tfrac{1}{2}l_2l_3m_3\left(2\,\mathrm{c}_2\,\mathrm{c}_3\,\mathrm{s}_2+\mathrm{s}_3\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\right)\dot{\theta}_1^2
\end{aligned}\tag{3.538}$$

$$\begin{aligned}
\tau_3 =& \tfrac{1}{2}d_2l_3m_3\,\mathrm{s}_{(2+3)}\,\ddot{\theta}_1+\left(I_{zz3}+m_3\left(\tfrac{1}{2}l_2l_3\,\mathrm{c}_3+\tfrac{1}{4}l_3^2\right)\right)\ddot{\theta}_2+\left(\tfrac{1}{4}l_3^2m_3+I_{zz3}\right)\ddot{\theta}_3\\
&+\left(\left(\mathrm{c}_2\,\mathrm{s}_2\left(\mathrm{c}_3^2-\mathrm{s}_3^2\right)+\mathrm{c}_3\,\mathrm{s}_3\left(\mathrm{c}_2^2-\mathrm{s}_2^2\right)\right)\left(I_{yy3}-I_{xx3}+\tfrac{1}{4}l_3^2m_3\right)\right)\dot{\theta}_1^2\\
&+\tfrac{1}{2}l_2l_3m_3\,\mathrm{c}_2\,\mathrm{s}_{(2+3)}\,\dot{\theta}_1^2+\tfrac{1}{2}l_2l_3m_3\,\mathrm{s}_3\,\dot{\theta}_2^2-\tfrac{1}{2}gl_3m_3\,\mathrm{c}_{(2+3)}
\end{aligned}\tag{3.539}$$

3.2.15. NN robotunun dinamiğinin Lagrange-Euler yöntemi ile çıkarılması

NN robotunun eklem düzenleşimi ve sembolik kütle gösterimi Şekil 3.15'de ve bu düzenleşime göre elde edilmiş DH parametreleri Tablo 3.15'de verilmiştir.