14 – Örnek 3.2.1 e çalış.

Example 3.21

Determine the z-transform and the ROC of the signal

$$x(n) = [3(2^n) - 4(3^n)]u(n)$$

Solution If we define the signals

$$x_1(n) = 2^n u(n)$$

and

$$x_2(n) = 3^n u(n)$$

then $x(n)$ can be written as

$$x(n) = 3x_1(n) - 4x_2(n)$$

According to (3.2.1). its z-transform is

$$X(z) = 3X_1(z) - 4X_2(z)$$

From (3.1.7) we recall that

$$\alpha^n u(n) \overset{z}{\longleftrightarrow} \frac{1}{1-\alpha z^{-1}} \qquad \text{ROC: } |z| > |\alpha| \qquad (3.2.2)$$

By setting $\alpha = 2$ and a = 3 in *(3.2.2)*. we obtain

$$x_1(n) = 2^n u(n) \longleftrightarrow X_1(z) = \frac{1}{1-2z^{-1}} \qquad \text{ROC: } |z| > 2$$

$$x_2(n) = 3^n u(n) \longleftrightarrow X_2(z) = \frac{1}{1-3z^{-1}} \qquad \text{ROC: } |z| > 3$$

The intersection of the ROC of $X_1(z)$ and $X_2(z)$ is $|z| > 3$. Thus the overall transform $X(z)$ is

$$X(z) = \frac{3}{1-2z^{-1}} - \frac{4}{1-3z^{-1}} \qquad \text{ROC: } |z| > 3$$

16 – founer transform birleşimini(Convergence of the Fourier Transform) figure 4.14 üzerinden açıklayınız.

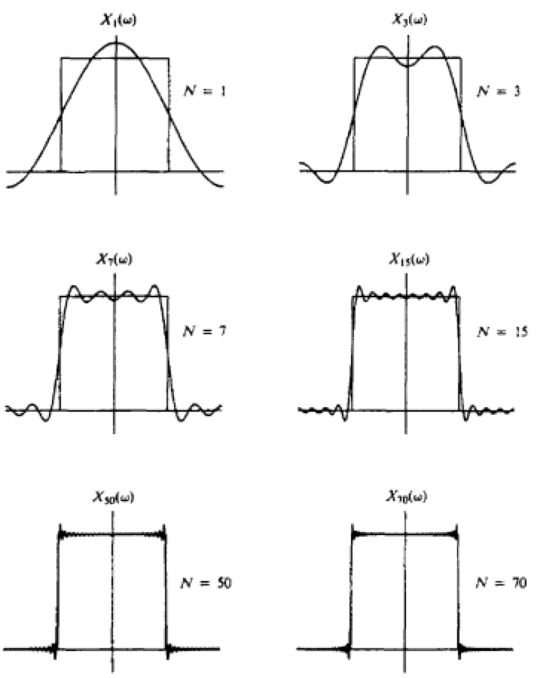

Figure 4.14 Illustration of convergence of the Fourier transform and the Gibbs phenomenon at the point of discontinuity.

15 – Örnek 4.2.1 e çalış.

Example 4.2.1

Determine the spectra of the signals

(a) $x(n) = \cos\sqrt{2}\pi n$

(b) $x(n) = \cos \pi n/3$

(c) $x(n)$ is periodic with period N = 4 and

$$x(n) = \{\underset{\uparrow}{1}, 1, 0, 0\}$$

Solution

(a) For $\omega_0 = \sqrt{2}\pi$, we have $f_0 = 1/\sqrt{2}$. Since f_0 is not a rational number. the signal is not periodic. Consequently. this signal cannot be expanded in a Fourier series. Nevertheless. the signal does possess a spectrum. Its spectral content consists of the single frequency component at $\omega = \omega_0 = \sqrt{2}\pi$.

(b) In this case $f_0 = \frac{1}{6}$ and hence $x(n)$ is periodic with fundamental period N = 6. From (4.2.8) we have

$$c_k = \frac{1}{6}\sum_{n=0}^{5} x(n)e^{-j2\pi kn/6} \qquad k = 0, 1, \ldots, 5$$

However. $x(n)$ can be expressed as

$$x(n) = \cos\frac{2\pi n}{6} = \frac{1}{2}e^{j2\pi n/6} + \frac{1}{2}e^{-j2\pi n/6}$$

which is already in the form of the exponential Fourier series in **(4.2.7).** In comparing the two exponential terms in $x(n)$ with (4.2.7), it is apparent that $c_1 = \frac{1}{2}$. The second exponential in $x(n)$ corresponds to the term k = −1 in (4.2.7). However. this term can also be written as

$$e^{-j2\pi n/6} = e^{j2\pi(5-6)n/6} = e^{j2\pi(5n)/6}$$

which means that $c_{-1} = c_5$. But this is consistent with (4.2.9), and our previous observation that the Fourier series coefficients form a periodic sequence of period N. Consequently. we conclude that

$$c_0 = c_2 = c_3 = c_4 = 0$$

$$c_1 = \frac{1}{2} \qquad c_5 = \frac{1}{2}$$

(c) From (4.2.8). we have

$$c_k = \frac{1}{4}\sum_{n=0}^{3} x(n)e^{-j2\pi kn/4} \qquad k = 0, 1, 2, 3$$

or

$$c_k = \frac{1}{4}(1 + e^{-j\pi k/2}) \qquad k = 0, 1, 2, 3$$

For $k = 0, 1, 2, 3$ we obtain

$$c_0 = \frac{1}{2} \qquad c_1 = \frac{1}{4}(1 - j) \qquad c_2 = 0 \qquad c_3 = \frac{1}{4}(1 + j)$$

The magnitude and phase spectra are

$$|c_0| = \frac{1}{2} \qquad |c_1| = \frac{\sqrt{2}}{4} \qquad |c_2| = 0 \qquad |c_3| = \frac{\sqrt{2}}{4}$$

$$\measuredangle c_0 = 0 \qquad \measuredangle c_1 = -\frac{\pi}{4} \qquad \measuredangle c_2 = \text{undefined} \qquad \measuredangle c_3 = \frac{\pi}{4}$$

Figure 4.10 illustrates the spectral content of the signals in (b) and (c).

17 – Fourier Trensformun Z-Transform ile ilişkisini eşitili 4.5.56 üzerinden açıklayınız.

$$X(z)|_{z=re^{j\omega}} = \sum_{n=-\infty}^{\infty} [x(n)r^{-n}]e^{-j\omega n}$$

18 – sinyellerin spektrol karakter. alçalı, orta ve yüksek frekans sinyellerini x(F) ve x(w) dönüşümlerini çizerek açıklayınız.

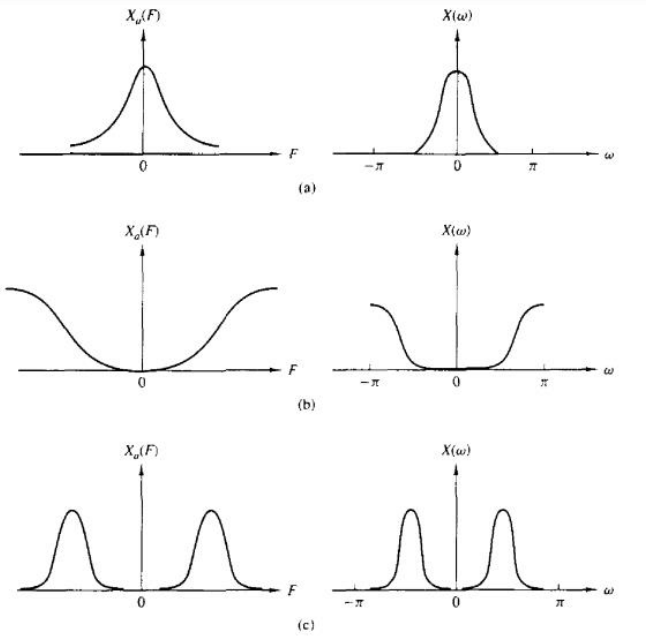

Figure 4.25 (a) Low-frequency. (b) high-frequency. and (c) medium-frequency signals.

19 – Elektroretinogram, Pneumogram, ve ses sinyellerinin frekans aralığını yazınız (tablo 4-1)

TABLE 4.1 FREQUENCY RANGES OF SOME BIOLOGICAL SIGNALS

Type of Signal	Frequency Range (Hz)
Electroretinogram[a]	0–20
Electronystapmogram[b]	0–20
Pneumogram[c]	0–40
Electrocardiogram (ECG)	0–100
Electroencephalogram (EEG)	0–100
Electromyogram[d]	10–200
Sphygmomanogram[e]	0–200
Speech	100–4000

[a] A graphic recording of retina characteristics.
[b] A graphic recording of involuntary movement of the eyes.
[c] A graphic recording of respiratory activity.
[d] A graphic recording of muscular action. such as muscular contraction.
[e] A recording of blood pressure.

20 – Örnek 4,4,1 e çalış.

Example 4.4.1

Determine the output sequence of the system with impulse response

$$h(n) = (\tfrac{1}{2})^n u(n) \tag{4.4.6}$$

when the input is the complex exponential sequence

$$x(n) = Ae^{j\pi n/2} \qquad -\infty < n < \infty$$

Solution First we evaluate the Fourier transform of the impulse response $h(n)$, and then we use (4.4.5) to determine $y(n)$. From Example 4.2.3 we recall that

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n)e^{-j\omega n} = \frac{1}{1-\frac{1}{2}e^{-j\omega}} \tag{4.4.7}$$

At $\omega = \pi/2$, (4.4.7) yields

$$H\left(\frac{\pi}{2}\right) = \frac{1}{1+j\frac{1}{2}} = \frac{2}{\sqrt{5}}e^{-j26.6^\circ}$$

and therefore the output is

$$y(n) = A\left(\frac{2}{\sqrt{5}}e^{-j26.6^\circ}\right)e^{j\pi n/2}$$
$$y(n) = \frac{2}{\sqrt{5}}Ae^{j(\pi n/2-26.6^\circ)} \qquad -\infty < n < \infty \tag{4.4.8}$$

This example clearly illustrates that the only effect of the system on the input signal is to scale the amplitude by $2/\sqrt{5}$ and shift the phase by -26.6°. Thus the output is also a complex exponential of frequency $\pi/2$, amplitude $2A/\sqrt{5}$, and phase -26.6°.

If we alter the frequency of the input signal. the effect of the system on the input also changes and hence the output changes. In particular. if the input sequence is a complex exponential of frequency π. that is,

$$x(n) = Ae^{j\pi n} \qquad -\infty < n < \infty \tag{4.4.9}$$

then, at $\omega = \pi$,

$$H(\pi) = \frac{1}{1-\frac{1}{2}e^{-j\pi}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$