CASE STUDY: DEUTSCHE BANK FINDING RELATIVE VALUE TRADES

ADVANCED FIXED INCOME AND CREDIT

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1 Question 1: Bootstraping

1.1 Bootstraping to find the zero-coupon yields

Solve for the prices of discount bonds given the market prices of coupon bonds and identify the zero-coupon yields from the prices of zero-coupon bonds.

The objective of Question 1.1 is to derive the prices of discount (zero-coupon) bonds from the observed market prices of coupon-bearing Treasury securities and, subsequently, to obtain the corresponding zero-coupon yields. The data used in this analysis are taken from Exhibit 1: Prices and Coupon Rates of Various U.S. Treasury Bonds on August 15, 2003, which lists coupon rates, maturities, and current prices for a broad set of U.S. Treasury bonds.

The first step involves organizing the bond data by maturity. The reference date in this case is set to 15 August 2003, and each bond's maturity is expressed in years as the difference between its maturity date and the reference date divided by 365.25. Each bond has a notional value of 100, pays semiannual coupons, and is priced using clean prices observed in the market. The coupon payment for bond i is given by

$$c_i = \frac{\text{Coupon Rate}}{100} \times \frac{100}{m},$$

where $m = 2$ denotes the semiannual payment frequency.

To determine discount factors, we construct a system of linear equations linking each bond's price to the present value of its cash flows. Let R_j denote the discount factor applicable to the j-th cash-flow date, and let P_i be the market price of bond i. Since bonds are ordered by increasing maturity, each bond's cash flows depend only on discount factors corresponding to earlier or equal maturities. The resulting system can therefore be written in matrix form as:

$$\begin{bmatrix} N + c_1 & 0 & 0 & \dots & 0 \\ c_2 & N + c_2 & \dots & 0 & \\ c_3 & c_3 & N + c_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_n & c_n & c_n & c_n & N + c_n \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ \vdots \\ R_N \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_N \end{bmatrix}$$

Because this cash-flow matrix is lower triangular with non-zero diagonal entries, it is invertible, and the vector of discount factors can be obtained as $R = C^{-1} \times P$.

Once discount factors are known, the zero-coupon yield for each maturity can be extracted using the standard compounding relation:

$$R_i = \frac{1}{(1 + \frac{y_i}{m})^{mT_i}},$$

which can be rearranged to yield:

$$y_i = m \left(R_i^{-\frac{1}{mT_i}} - 1 \right).$$

In this expression, y_i represents the annualized yield corresponding to the maturity T_i. This formula is applied to every discount factor derived through the bootstrapping procedure, producing a complete vector of zero-coupon yields.

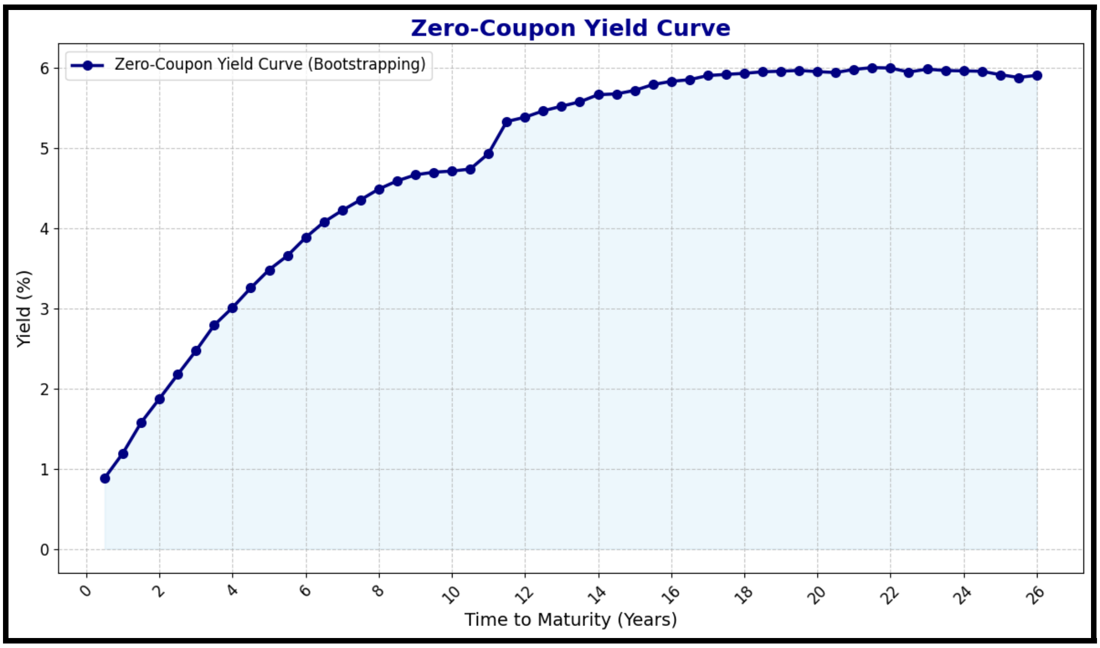

Figure 1: Zero-Coupon Yields : Bootstrapping against DB Model

The resulting yield curve in **Figure 1** is upward-sloping, starting near 0.9%, rising to 4.7% at 10 years, and then jumping to 5.3% at 11 years. It then flattens, peaking around 6.0% at 20–22 years. This shape is consistent with a normal yield curve. The procedure assumes semiannual coupons, zero accrued interest (clean prices are used), and aligned coupon dates. Despite these simplifications, the bootstrapped curve provides a reliable term structure estimate, though the 11-year jump is a notable anomaly.

1.2 Comparaison with deutsche bank model

Compare the zero-coupon yields from Deutsche Bank's model (in Exhibit 4 of the case) to the market zero-coupon yields, and identify which zero-coupon bonds Deutsche Bank *should buy, and which ones they should sell.*

The goal of Question 1.2 is to compare the market zero-coupon yields obtained through bootstrapping with Deutsche Bank's model-based yields presented in Exhibit 4. The analysis identifies the yield differences between the two curves and determines which zero-coupon bonds appear cheap or rich relative to Deutsche Bank's internal model.

Let $y^{mkt}(T)$ denote the market-implied zero-coupon yield from the bootstrapping procedure, and $y^{DB}(T)$ the yield predicted by Deutsche Bank's model for the same maturity T. The yield differential, expressed in basis points, is defined as:

$$\Delta y(T) = [y^{mkt}(T) - y^{DB}(T)] \times 100$$

Since a higher yield corresponds to a lower bond price, a positive $\Delta y(T)$ indicates that the market zero-coupon bond is undervalued relative to the model (i.e., it is cheap and should be bought), while a negative value implies that the market price is rich compared to the model (i.e., it should be sold).

The two curves are plotted in **Figure 2**, which shows that both the bootstrapped and Deutsche Bank curves closely overlap, indicating that the model captures the general shape of the market term structure.

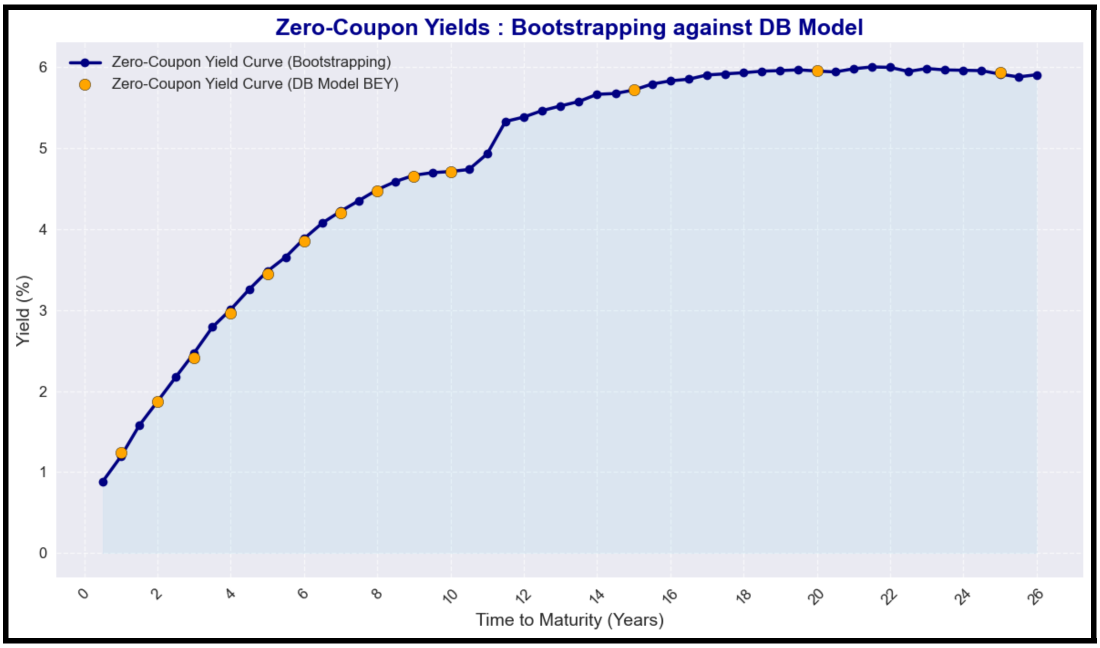

Figure 2: Zero-Coupon Yield Curve - Bootstrapping vs DB Model

However, small deviations exist across maturities. These differences are summarized numerically in **Table 1**, which lists the discount factors, market zero-coupon yields, Deutsche Bank's model yields, and the resulting differences in basis points.

Table 1: Zero-Coupon Yield Data - Bootstrapping vs DB Model

Maturity (years)	Discount Factor	Zero-Coupon Yields (%)	Model Prediction (BEY)	Difference (bps)
1.0	0.988176	1.192962	1.2443	-5.133781
2.0	0.963406	1.872718	1.8727	0.001766
3.0	0.928970	2.471112	2.4110	6.011193
4.0	0.887418	3.008378	2.9665	4.187825
5.0	0.841559	3.479914	3.4454	3.451365
6.0	0.794000	3.881711	3.8557	2.601106
7.0	0.746507	4.220336	4.1996	2.073624
8.0	0.701204	4.486530	4.4677	1.883032
9.0	0.660381	4.663971	4.6528	1.117126
10.0	0.627754	4.710684	4.7107	-0.001643
15.0	0.429353	5.716684	5.7160	0.068410
20.0	0.309470	5.951302	5.9517	-0.039772
25.0	0.232986	5.912825	5.9315	-1.867540

As seen in the data, the model slightly underestimates market yields for intermediate maturities. Specifically, for maturities between three and nine years, the market yields exceed the model yields by between 1 and 6 basis points, with the largest deviation observed at the three-year maturity (+6.01 bps). For shorter maturities, such as one year, the model yield is higher than the market yield (–5.13 bps), suggesting that short-term bonds appear rich relative to the model. Beyond ten years, differences become negligible, fluctuating around zero, except for the 25-year maturity where the model yield slightly exceeds the market yield (–1.87 bps).

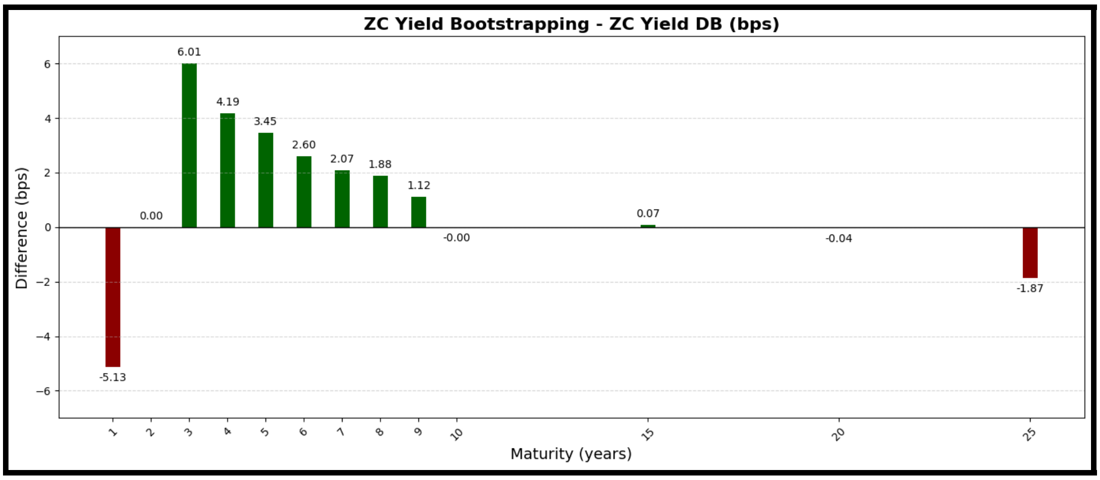

Figure 3: Zero-Coupon Yield Difference - Bootstrapping vs DB Model

These variations are illustrated in **Figure 3**, which plots the yield spread $\Delta y(T)$ across maturities. The positive green bars indicate maturities where the market yield is higher (and bonds are undervalued), while the negative red bars indicate the opposite. The chart confirms that the model undervalues bonds in the medium segment of the curve (three to nine years) and overvalues short- and very long-term bonds.

From an arbitrage perspective, Deutsche Bank should therefore buy the zero-coupon bonds maturing between three and nine years, where market yields exceed model yields, and sell the short-term (one-year) and long-term (twenty-five-year) zero-coupon bonds, where model yields are higher. The two- and ten-year maturities show almost perfect alignment between market and model values, suggesting no clear arbitrage opportunity.

1.3 Arbitrage analysis

Is this strategy (of buying some bonds and selling others) a risk-free arbitrage strategy?

The strategy outlined in Question 1.2 involves buying zero-coupon bonds that appear undervalued relative to Deutsche Bank's model and selling those that appear overvalued. The idea is to profit from small yield differences between the market and the model by offsetting long and short positions that should, in theory, converge over time.

In reality, this is not risk-free arbitrage. The observed yield gaps, generally between one and six basis points, are too small to represent genuine mispricing. They mostly reflect liquidity conditions, estimation noise, and minor modeling differences rather than exploitable price discrepancies. The bootstrapped curve reflects actual market quotes on that date, while Deutsche Bank's curve is a theoretical construct based on specific modeling assumptions. Even small mismatches in coupon timing, convexity, or bond liquidity can explain these basis-point differences.

In practice, transaction costs, bid-ask spreads, and balance-sheet charges would easily outweigh the potential gains. Moreover, positions would remain exposed to shifts in the overall yield curve and to model risk if the market does not move toward the theoretical values.

Overall, the deviations between the two curves indicate relative-value differences, not pure arbitrage. Buying and selling based on these discrepancies could generate profits if the spreads close, but it remains a directional and model-dependent trade, not a risk-free one.

2 Question 2: Cubic Splines

2.1 Estimation of the Cubic Splines Function

Estimate the cubic splines' function that uses knots 2-, 5-, 10-, 15- and 20-year zero-coupon yields. Describe succinctly the methodology and document any choice made in the estimation. Plot the discount function (i.e., the cubic splines based ZC yields) against the zero-coupon yields obtained in Q1.1.

To obtain a smooth and realistic representation of the term structure, we estimated the cubic spline interpolation based on the zero-coupon yields we derived in Q1.1.

As required in the question, we used five interior knots at 2, 5, 10, 15, and 20 years. These points divide the maturity spectrum into several segments short-term, medium-term, and long-term where each segment is modeled by its own third-degree polynomial.

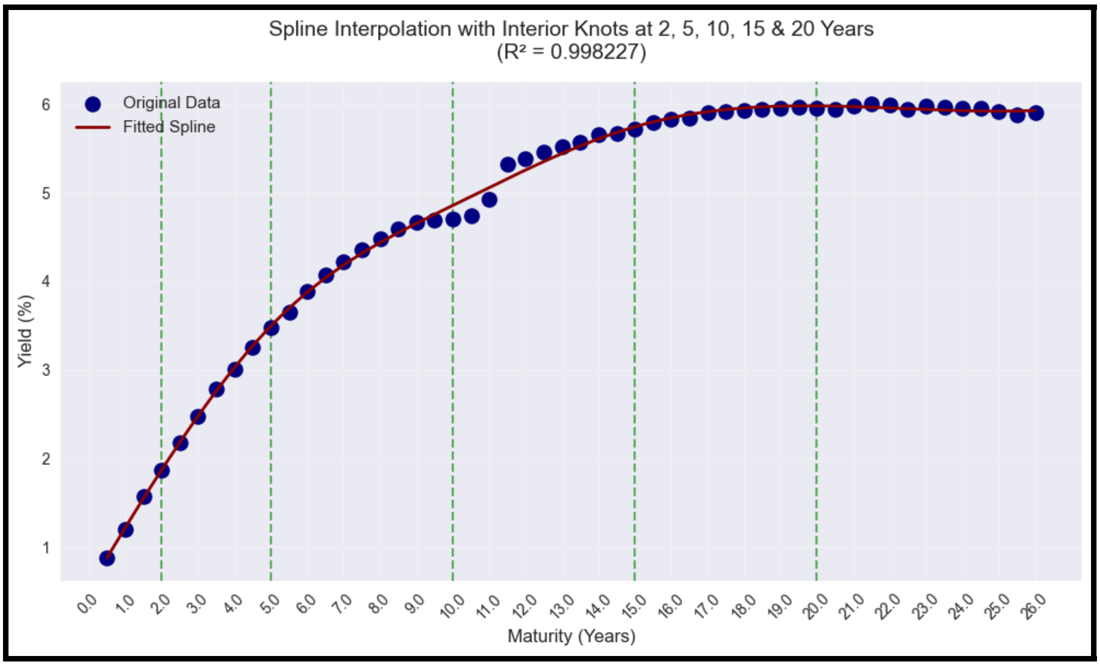

Figure 4: Cubic Splines Interpolation vs Bootstrapped data

For each maturity interval $[x_i, x_{i+1}]$ (green vertical lines in Figure 2.1), the spline takes the form:

$$y(t) = a_i + b_i(t - x_i) + c_i(t - x_i)^2 + d_i(t - x_i)^3$$

The coefficients (a_i, b_i, c_i, d_i) are determined so that the curve is:

1. Continuous in yield level,
2. Continuous in its first derivative (slope),
3. Continuous in its second derivative (curvature) at every knot.

In practice, we used the function `cubic_spline()` to estimate these coefficients numerically, ensuring that the interpolated curve minimizes local fitting errors.

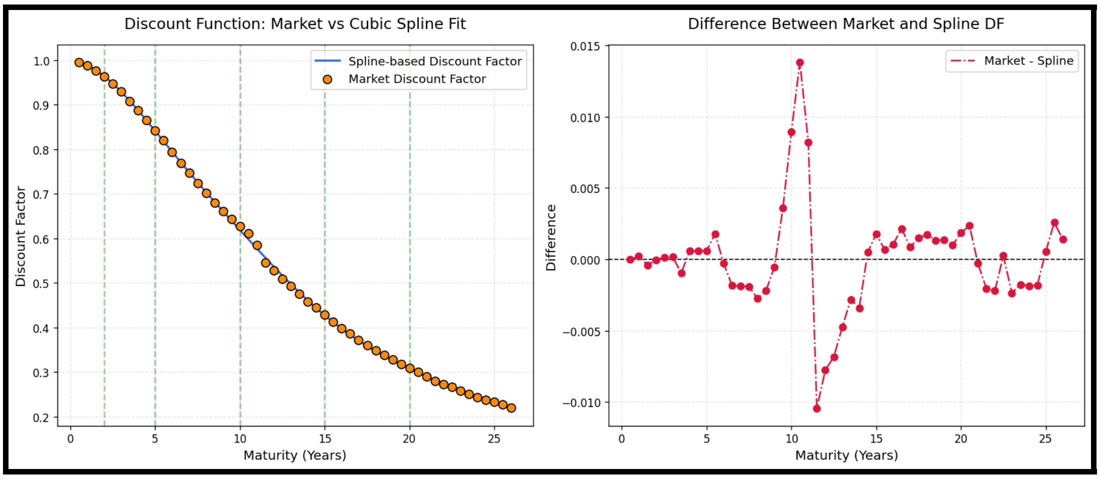

Figure 5: Cubic Splines Interpolation vs Bootstrapped data - Difference

In **Figure 5**, we compare the market discount factors obtained from bootstrapping with those estimated from the cubic spline fit. The left plot shows that both curves almost perfectly overlap, confirming that the spline provides an accurate and smooth approximation of the market term structure. The right plot displays the small residuals between the two, which remain close to zero except for minor deviations around the knot points (near 10 years). These differences occur because the spline coefficients were estimated through linear regression, minimizing the overall squared error rather than forcing the curve to pass exactly through each point. This approach ensures a smooth and realistic yield curve, especially at the short and long ends, without overfitting local variations.

2.2 Discussion on Spline-based vs Bootstrapped Yields

Discuss when using those new 'spline-based' zero-coupon yields estimates instead of the one implied by the bootstrapped zero-coupon bonds in Q1.1 is appropriate. Please justify and discuss whether this affects the conclusions reached in Q1.2.

Using the spline-based zero-coupon yields instead of the bootstrapped ones is appropriate when we need a smooth and continuous representation of the yield curve, especially for pricing instruments that mature between observed data points. Unlike the bootstrapped curve, which fits each bond exactly, the spline approach connects the data smoothly and eliminates small market irregularities. This makes it more appropriate for applications like pricing bonds with non-standard maturities, estimating forward rates, or performing risk and sensitivity analysis.

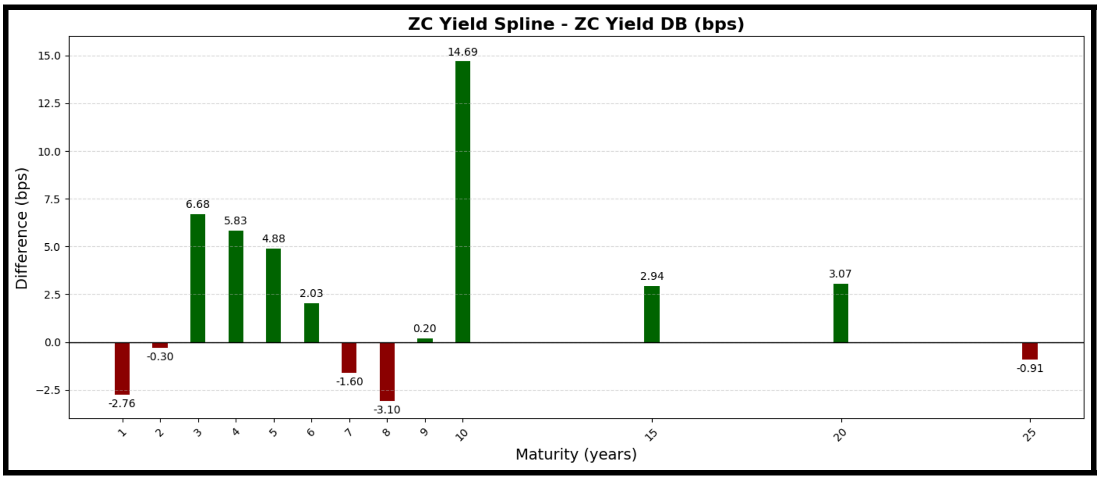

Figure 6: Cubic Splines Interpolation vs Bootstrapped data - Difference in bps

As we can see from **Figure 6**, the spline-based yields remain very close to the bootstrapped ones, with only small differences up to around 15 basis points near the 10-year maturity. These differences come from the smoothing effect of the spline, which balances the overall shape of the curve instead of matching each market point perfectly.

In terms of trading signals, the main conclusions from Q1.2 still hold. The maturities we would buy or sell remain mostly the same (we have a change for maturities 2, 7, 8, 10 and 20), even if the size of the deviations changes slightly. In other words, the spline curve gives us a cleaner and more stable view of the yield curve, but it doesn't fundamentally change our interpretation of where the market is over- or undervalued.

So overall, we can say that bootstrapping is more precise for detecting arbitrage opportunities, while spline fitting is more reliable for analysis, pricing, and long-term decision-making.

2.3 Re-estimation with Two Knots

Re-estimate the cubic splines' function only using as knots 5- and 20-year zero-coupon yields. Plot the discount function against the one obtained in Q2.1. Discuss how changing the knots affects the estimated discount function.

In this question, we re-estimated the cubic spline using only two knots, placed at 5 and 20 years, instead of the five knots used earlier (2, 5, 10, 15, and 20 years). Our goal was to see how reducing the number of knots would affect the smoothness and accuracy of the yield curve.

As shown in **Figure 7**, both splines fit the market data well, but the two-knot spline (red) is clearly smoother. This is confirmed quantitatively by its lower total curvature (0.552 vs. 0.616), indicating fewer directional changes along the curve.

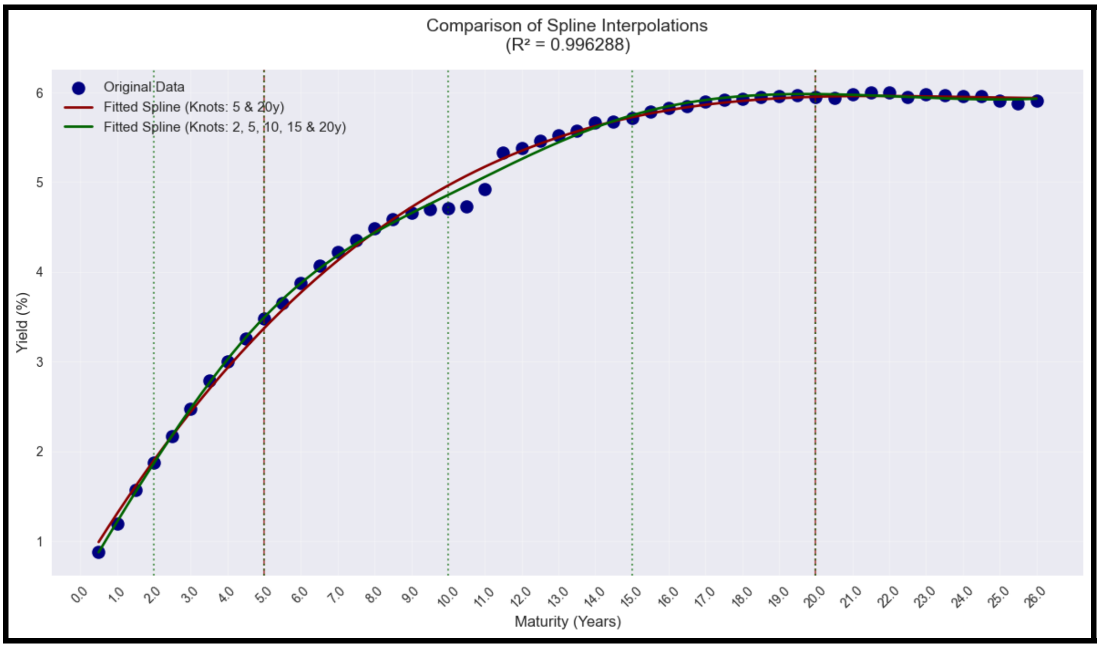

Figure 7: Cubic Splines Interpolations vs Bootstrapped data

The R^2 remains very high (around 0.996), showing that the 2-knot spline still explains almost all of the variation in yields.

3 Question 3: Nelson-Siegel Model

3.1 Estimation of the Nelson-Siegel-Svensson Model

Estimate the discount function according to the Nelson-Siegel-Svensson model. Describe succinctly the methodology and document any choice made in the estimation. Plot it against the zero-coupon yields obtained in Q1.1.

The objective of Question 3.1 is to estimate the discount function using the Nelson-Siegel-Svensson (NSS) model and to compare the resulting fitted curve with the market zero-coupon yields obtained through bootstrapping in Question 1.1. The Nelson-Siegel-Svensson framework extends the traditional Nelson-Siegel model by introducing two exponential decay terms, allowing for greater flexibility in capturing complex yield curve shapes such as humps or dips in medium maturities.

The functional form of the NSS spot rate $r(t)$ is:

$$r(t) = \beta_0 + \beta_1 \frac{1 - e^{-t/\lambda_1}}{t/\lambda_1} + \beta_2 \left(\frac{1 - e^{-t/\lambda_1}}{t/\lambda_1} - e^{-t/\lambda_1} \right) + \beta_3 \left(\frac{1 - e^{-t/\lambda_2}}{t/\lambda_2} - e^{-t/\lambda_2} \right)$$

In this specification, β_0 represents the long-term level of the yield curve, β_1 captures the short-term slope, β_2 drives medium-term curvature, and β_3 together with λ_2 controls any additional curvature in the long end of the curve. The decay parameters λ_1 and λ_2 determine how quickly the exponential terms diminish as maturity increases, shaping the location and steepness of the curve's humps.

To estimate the model, the six parameters $(\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2)$ were obtained by minimizing the sum of squared errors between the NSS-implied yields and the observed market zero-coupon yields from Q1.1. This nonlinear least squares optimization was implemented using the maturities expressed in years and the yields in decimal form. Reasonable initial guesses and upper/lower parameter bounds were applied to ensure numerical stability and to prevent economically implausible solutions, such as negative long-term levels or decay constants close to zero. The resulting fitted NSS curve is plotted against the bootstrapped zero-coupon yields in **Figure 8**.

The resulting fitted NSS curve (red line) is plotted against the bootstrapped zero-coupon yields (blue dots) in **Figure 8**. The fit is visually close for most maturities, but the smooth, parametric NSS curve is structurally unable to capture the sharp, non-linear jump in the market data observed between 10 and 12 years. At this point, the market yields (blue dots) leap from approximately 4.7% to over 5.3%. The model (red line) cannot replicate this jump and is forced to smooth through the anomaly, creating a

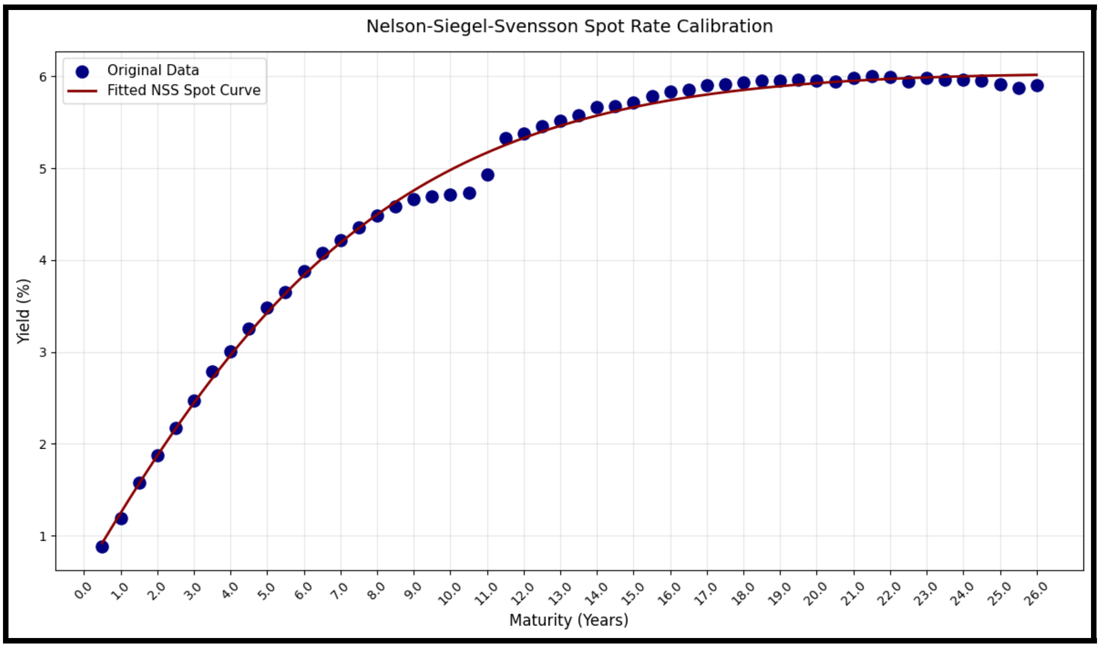

Figure 8: NSS Spot Rate Calibration

visible gap where the market yields are clearly higher than the fitted NSS yields.

To examine the discount function implied by this fit, the NSS spot rates were transformed into discount factors according to:

$$D(t) = \left(1 + \frac{r(t)}{m}\right)^{-mt}$$

where $m = 2$ denotes the semiannual compounding frequency. The comparison between the market and model-based discount functions is displayed in **Figure 9**.

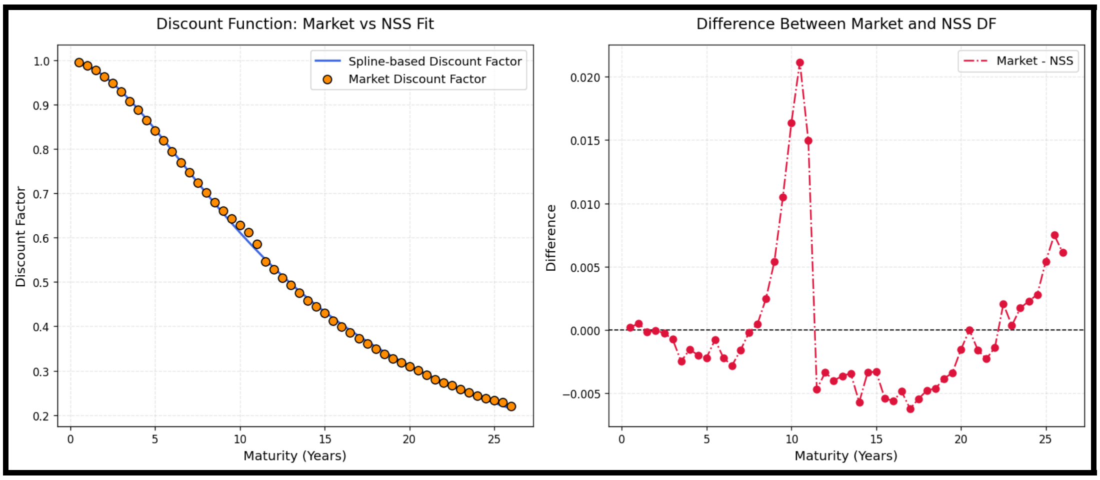

Figure 9: Discount Function and Differences: Market vs Nelson-Siegel-Svensson Fit

The comparison of the discount functions in **Figure 9** highlights the financial consequence of this model-fitting error. While the left panel shows the two discount factor curves are almost indistinguishable at a glance, the right panel, which plots the residual difference (Market – NSS), reveals a dramatic estimation error.

Instead of being centered near zero, the residuals spike to a peak of +0.022 (a 2.2 percentage point difference) precisely around the 11–12-year maturity. This spike, which is orders of magnitude larger than the errors at other maturities, is not a "minor deviation"; it is the most significant feature of the plot. It confirms that the NSS model has failed to accurately price the bonds corresponding to the 11–12-year jump, resulting in a large pricing residual.

3.2 Comparison with the Spline Model

Compare the fit of the two models from Q2.1 and Q3.1. Discuss the differences.

The cubic spline model from Q2.1 and the Nelson-Siegel-Svensson (NSS) model from Q3.1 both aim to reproduce the market yield curve, but they differ fundamentally in flexibility, smoothness, and economic interpretation, leading to a clear trade-off.

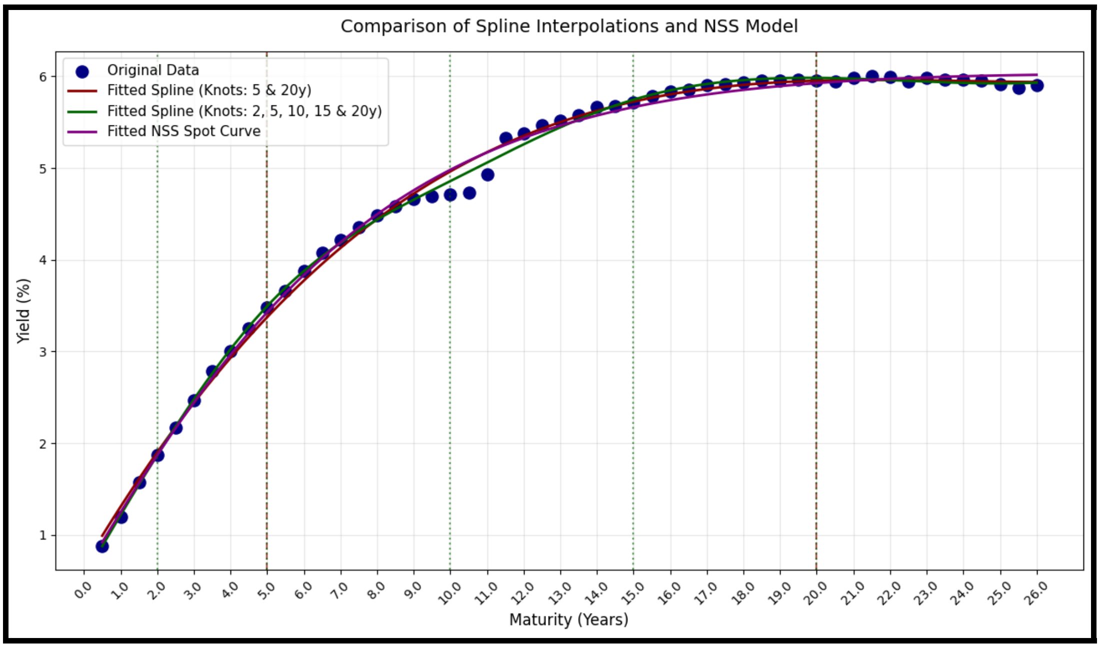

Figure 10: Comparison of Spline Interpolations and NSS Model

As seen in **Figure 10**, the market data (blue dots) exhibits a sharp, non-linear jump at the 11-year maturity. The smooth, parametric NSS model (purple) is unable to fit this anomaly and smooths right through it, creating a large, visible error. In contrast, the more flexible 5-knot spline (green) bends to capture this jump, resulting in a much closer in-sample fit at that specific point. The 2-knot spline (red) offers a compromise, being smoother than the 5-knot spline but underfitting the mid-maturity segment.

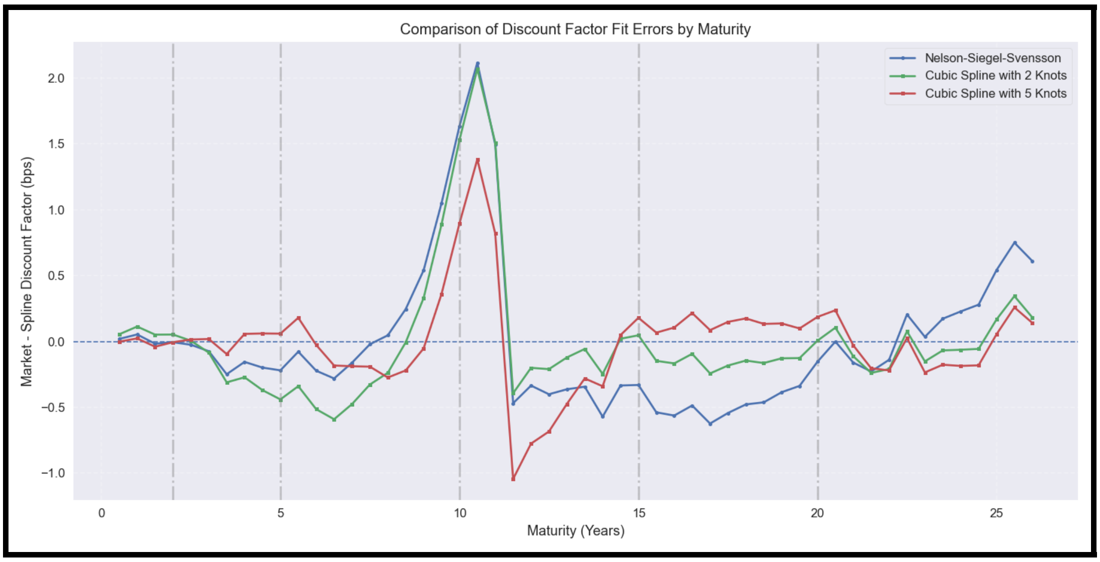

Figure 11: Discount Factor Fit Errors by Maturity

The error plot, **Figure 11**, quantifies this trade-off. The 11-year jump causes a massive pricing error (a +0.022 residual) for the NSS model (blue), its largest error by far. The 5-knot spline (green) fits this jump much better, with a smaller peak error (around +0.014). However, this flexibility comes at a cost. In other parts of the curve (e.g., 14–18 years), the 5-knot spline "overfits" or "wiggles," producing larger local errors than the more stable and smooth NSS model.

In summary, the spline model provides a better in-sample fit by having the flexibility to capture market anomalies like the 11-year jump. The NSS model, while providing a smoother and more economically interpretable curve, fails to price this anomaly, leading to significant localized pricing errors.

3.3 Factor Loadings and Contributions

Estimate and plot the factor loadings and contributions from the different factors affecting the shape of the spot zero-coupon curve. Explain their key drivers.

The Nelson-Siegel-Svensson (NSS) model decomposes the spot curve into four distinct components, each defined by a factor loading (its shape) and a fitted parameter (its magnitude and direction). The loadings, shown in **Figure 12**, determine how each factor's influence is distributed across maturities, while the contributions, seen in **Figure 13**, show the final impact on the yield curve.

The level component is the simplest, with a constant loading of 1 across all maturities. Its contribution is a flat line at approximately +5.9%, representing the fitted β_0 parameter. This factor acts as the long-term anchor for the entire curve. The slope component

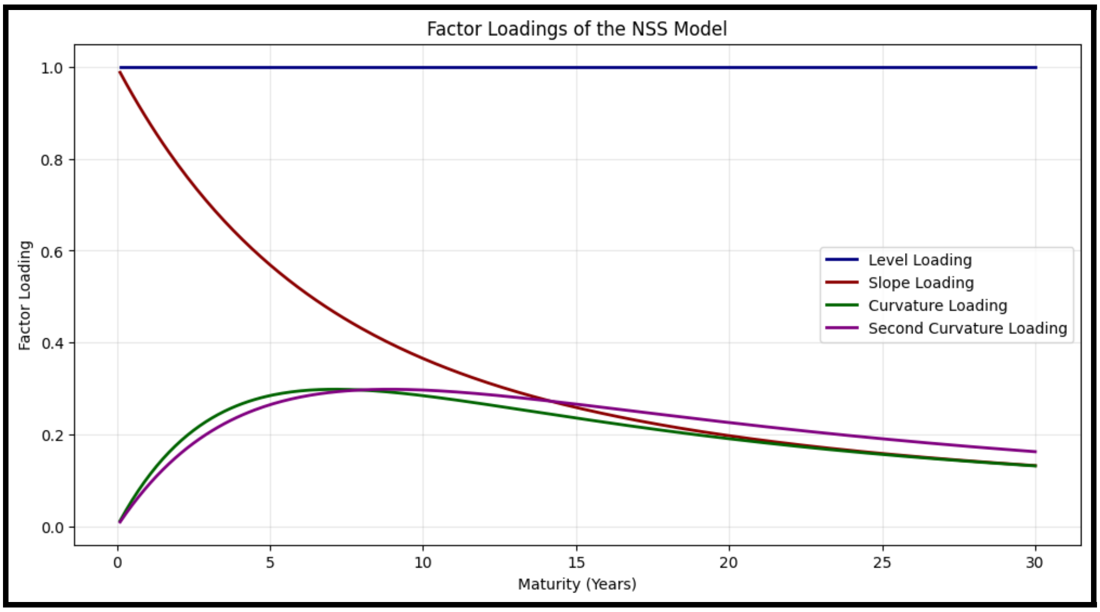

Figure 12: Factor Loadings of the NSS Model

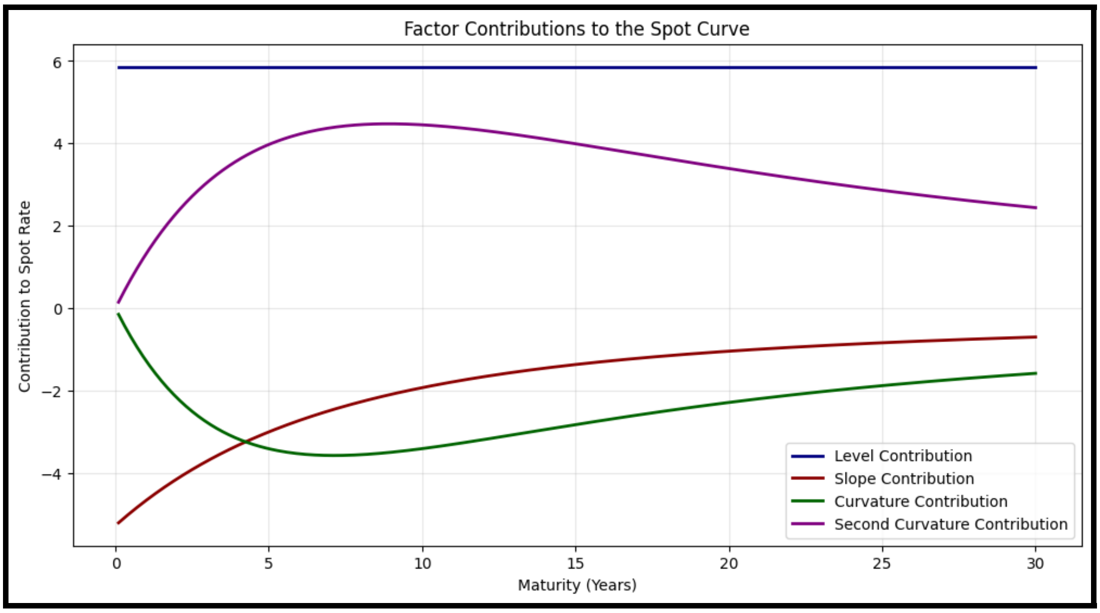

Figure 13: Factor Contributions to the Spot Curve

has a loading that starts at 1 and decays exponentially to 0, giving it maximum influence on short-term rates. Its contribution plot shows a large negative value (approx. -5) at the short end, implying its fitted parameter, β_1, is negative. This factor is responsible for pulling the short-term rates down from the long-term level, creating the curve's steep upward slope.

The shape of the curve's "belly" is controlled by the two curvature factors. The first curvature loading is hump-shaped, peaking in the medium term. Its contribution, however, is an inverted hump with a negative peak (approx. -3.5), meaning its β_2 parameter is negative and acts to pull down mid-term yields. This is countered by the second curvature component, which has a broader, positive hump-shaped contribution peaking around 8–12 years. This positive β_3 parameter adds a "hump" to the medium-to-long end, providing the flexibility needed to fit the market data.

In summary, the observed spot curve is the sum of these four contributions: a high long-term level (β_0) is pulled down at the short end by a negative slope factor (β_1), and the mid-section is then fine-tuned by the two opposing curvature factors (β_2 and β_3) to match the market's specific shape.

4 Question 4: Hedging and Key Rate Duration Analysis

4.1 10-Year Rate Shock and Yield Curve Impact

To estimate the sensitivity of any instrument or portfolio to the 10-year rate changes, i.e., its 10-year key rate duration (KRD), you will shock the 10-year rate by 50bps. Discuss what the impact is on the zero-coupon yield curve estimated in Q2.1 and plot the difference.

In this question, we shocked the 10-year zero-coupon yield by +50 basis points to evaluate how sensitive the estimated yield curve is to a local rate change. This method corresponds to the 10-year Key Rate Duration (KRD).

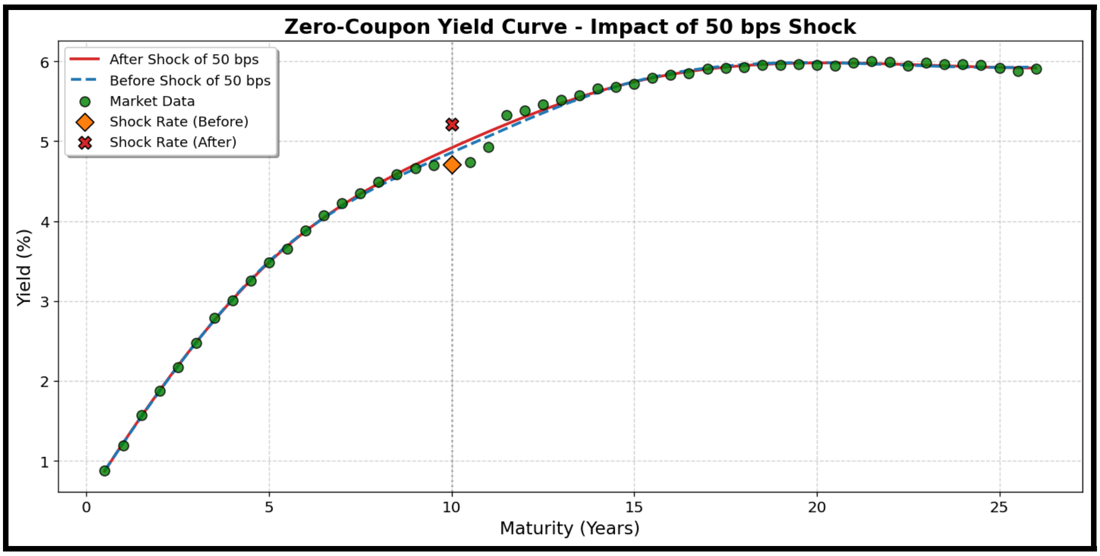

Figure 14: Cubic Splines Interpolations after 50 bp shock of the 10-year rate

As shown in **Figure 14**, when we raise the 10-year yield, the curve moves slightly upward around that point, creating a small bump in the medium-term area. The effect is mainly local, but not completely isolated since the spline produces a smooth and continuous curve, the neighboring maturities (around 8 to 12 years) also adjust slightly to maintain a natural shape. The short- and long-term parts of the curve, however, stay almost unchanged.

Figure 15 shows the difference between the shocked and baseline cubic-spline yield curves. The plot highlights how the +50 bp perturbation at the 10-year knot propagates across maturities. The largest deviation appears around the 10-year point, where the rate was directly shocked, while smaller opposite-signed adjustments occur at neighboring maturities. These oscillations reflect the smoothness constraint of the cubic spline,

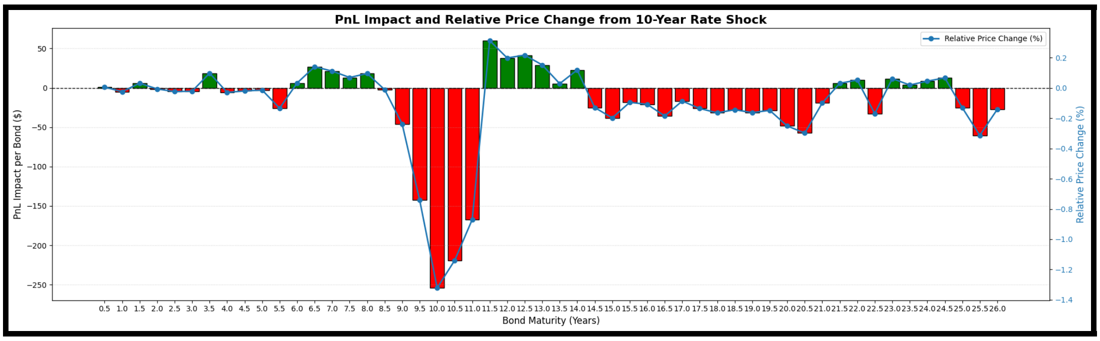

Figure 15: Difference between Cubic Splines before and after 50 bp shock of the 10-year rate

which redistributes the shock slightly along the curve to maintain continuity and differentiability. Overall, the impact remains localized and well-behaved, confirming that the spline captures the intended 10-year key-rate sensitivity without distorting the global term-structure shape.

4.2 Portfolio Exposure to the 10-Year Rate Shock

Assume you own a $1 million portfolio that is equally invested in all bonds. What is the KRD of your portfolio to this scenario? Describe your approach to estimating this exposure.

In this question, we consider a $1,000,000 portfolio equally invested across all bonds included in Exhibit 1. Each bond, therefore, represents the same weight in the portfolio. To compute the Key Rate Duration (KRD) with respect to the 10-year point, we begin by pricing all bonds using the zero-coupon yield curve obtained from the cubic spline interpolation developed in Question 2.1.

Next, we introduce a +50 basis point shock to the 10-year zero rate while keeping the spline structure consistent. We then reprice all bonds using the new (shocked) discount factors, resulting in a new total portfolio value of $998,905.48. The difference between the two valuations,

$$\Delta V = V_{\text{before shock}} - V_{\text{after shock}} = -1,094.52,$$

represents the portfolio's loss following the rate shock.

From this change in value, we derive the portfolio's 10-year KRD. The Dollar Value of 1 Basis Point (DV01) is:

$$DV01 = \frac{\Delta V}{50} = -21.89 \text{ USD/bp}.$$

Metric	Value ($)
Initial Portfolio Value	1,000,000.00
PnL Impact from +50bps Shock	-1,094.52
Final Portfolio Value After Shock	998,905.48

Table 2: Portfolio Value and PnL after 10-Year Rate Shock

Assuming a linear relationship and disregarding convexity effects, this provides a reasonable local sensitivity estimate. Meaning that for every 1 basis point increase in the 10-year zero rate, the portfolio loses about \$21.89.

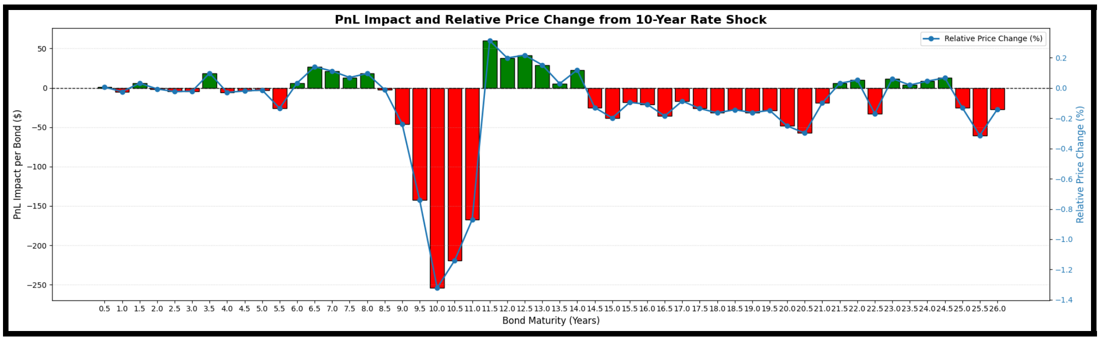

Figure 16: Portfolio PnL Loss after 50 bp shock of the 10-year rate / Bond

As shown in **Figure 16**, the majority of the portfolio's P&L loss clearly comes from bonds with maturities around 10 years, which is exactly where the +50 bps shock was applied. These mid-term bonds experience the strongest price declines, as reflected by the large negative red bars. In contrast, shorter and longer-term bonds are only marginally affected, showing that the impact of the shock remains localized and relatively stable around the targeted maturity.

4.3 Key Rate Durations for All Maturities

Estimate and plot the KRDs of your portfolio for all maturities, when applying the same shock individually to all zero-coupon yields. Plot and interpret the results.

In this section, we estimate the Key Rate Duration (KRD) of the portfolio across all maturities by applying a +50 basis point shock individually to each zero-coupon yield node. For each maturity, we bump the corresponding zero rate by 50 bps, refit the cubic spline curve (using the same five knots as before: 2, 5, 10, 15, and 20 years), and reprice the entire portfolio using the new discount factors. The change in the portfolio's value after each shock gives us the dollar KRD for that specific maturity.

The results in **Figure 17** reveal that when the shock is applied to mid-term maturities, the perturbation slightly extends to neighboring points due to the spline's smoothness

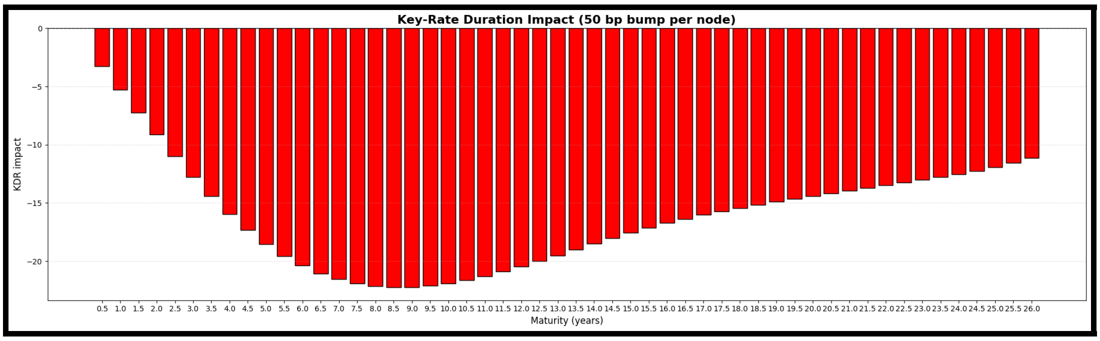

Figure 17: Key Rate Durations across Maturities (+50 bps per node)

constraint. Each local bump in this central region affects not only its own maturity but also nearby segments, creating a cumulative impact across multiple nodes.

As a result, the aggregated impact of these overlapping local shocks in the mid-curve is larger than that observed when the shocks are applied to the extremes of the curve, where the spline remains more anchored and isolated. This explains why the KRD values reach their maximum magnitude exactly at the duration of the portfolio.

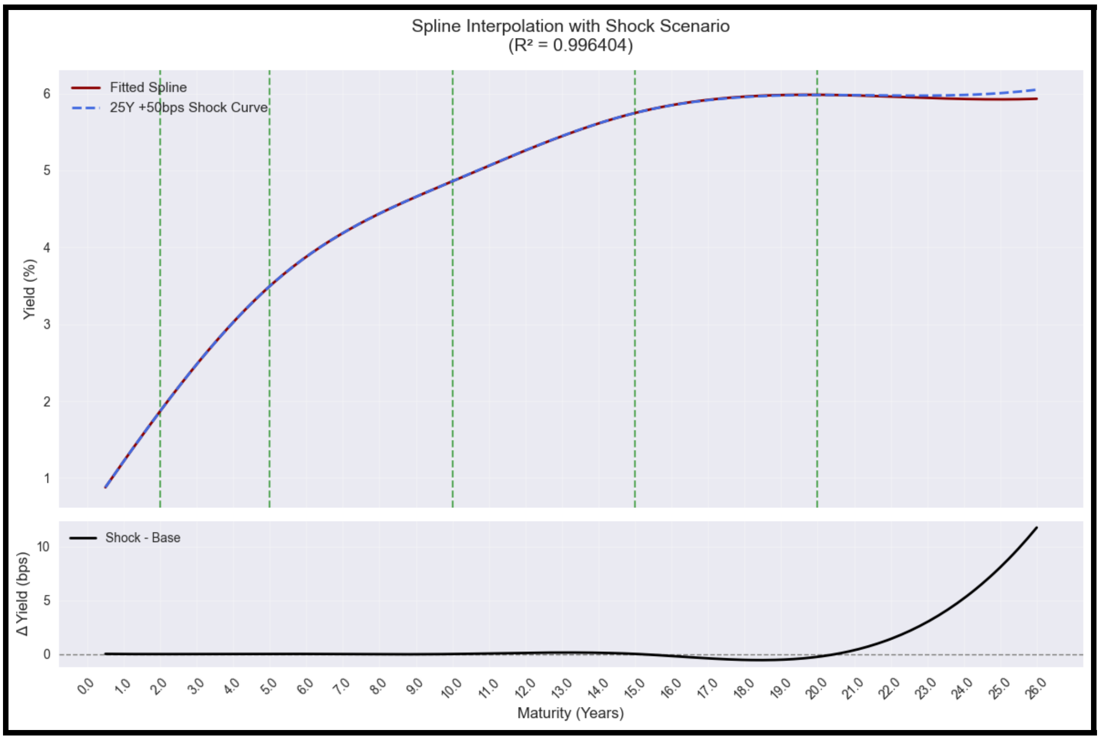

Figure 18: Key Rate Duration Impact for +50 bps Shock of 25-y rate

Finally, specific shock scenarios are analyzed in **Figures 18 to 20**:

- A +50 bps shock at the **long end (25 years)** causes a very localized adjustment with minimal propagation to other maturities.
- A +50 bps shock at the **short end (1 year)** also results in a localized movement,

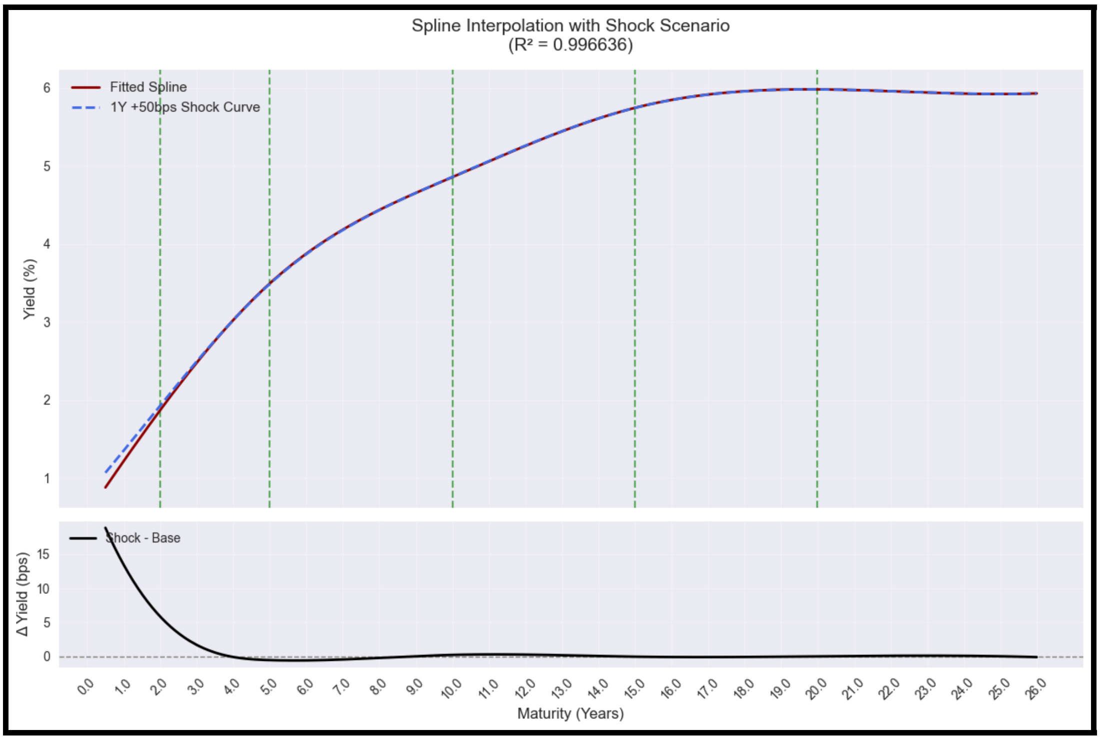

Figure 19: Key Rate Duration Impact for +50 bps Shock of 1-y rate

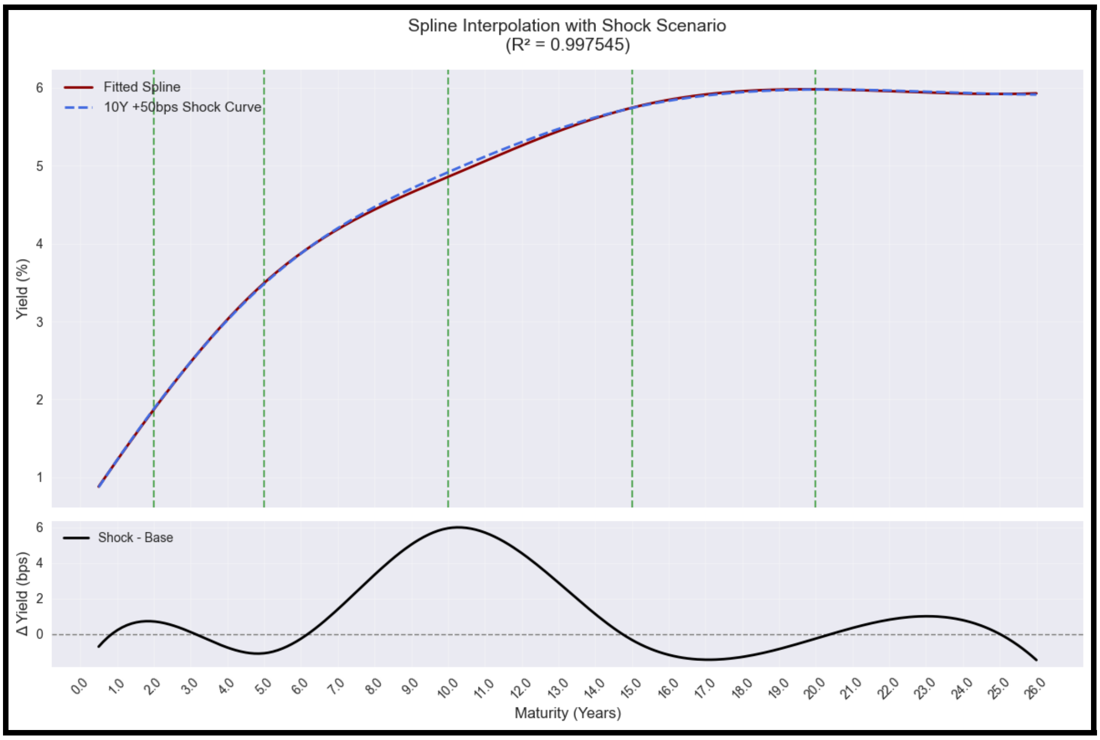

Figure 20: Key Rate Duration Impact for +50 bps Shock of 10-y rate

isolated from the rest of the curve.

- A +50 bps shock at the **10-year maturity** generates a broader deformation of the curve, as the perturbation spreads symmetrically through the mid-to-long region to maintain spline continuity.

5 QUESTION 5: PORTFOLIO STRATEGY

5.1 Steepening of the Zero-Coupon Yield Curve

In this question, we model the expected **steepening** of the zero-coupon yield curve based on the bootstrapped curve obtained previously. Economically, this steepening represents the market's expectation that long-term interest rates will rise faster than short-term ones. Mathematically, we apply the following adjustment to each maturity point T:

$$y^T_{\text{steepen}} = y^T + \left(\frac{T-\bar{T}}{\bar{T}} \times 0.0050\right) + \varepsilon, \qquad \varepsilon \sim \mathcal{N}(0, 0.0025),$$

where $\bar{T}$ is the average bond maturity in the sample and ε is a random noise term.

The implementation proceeds in three main steps. First, we compute the **deterministic component** of the steepening, which adjusts the original zero-coupon yields proportionally to their distance from $\bar{T}$, producing a smooth and systematic inclination of the curve. Second, we generate a **stochastic component** (normally distributed noise) to simulate local market fluctuations and random yield variations. Finally, we combine both effects to obtain the **simulated steepened curve**, which captures both the structural trend and random deviations.

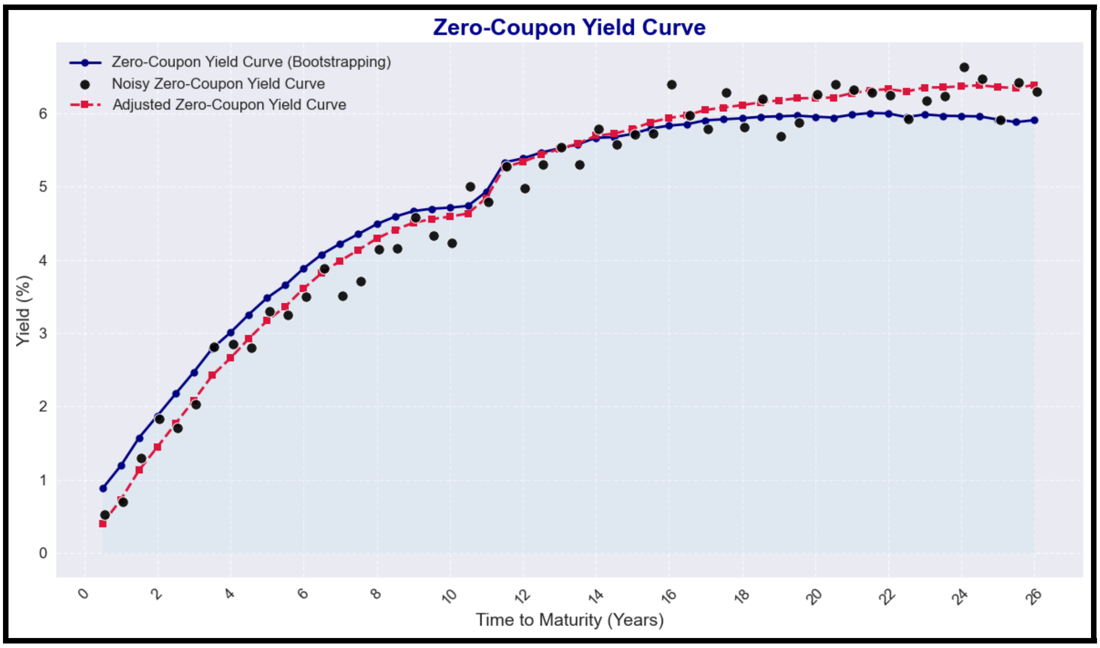

Figure 21: Zero-Coupon Yield Curve: Bootstrapped (blue), Adjusted Deterministic (red dashed), and Noisy Simulated (black points).

Figure 21 illustrates these results. The blue line represents the original bootstrapped zero-coupon yield curve, the red dashed line corresponds to the deterministic steepened version, and the black points show the noisy simulated yields. We observe that long-term maturities increase more significantly than short-term ones, resulting in a steeper yield curve, while the dispersion of black points reflects the random noise component around this theoretical expectation.

5.2 Re-estimation of the Discount Function using the Nelson-Siegel-Svensson Model

In this question, we re-estimate the **discount function** using the Nelson–Siegel–Svensson (NSS) methodology, which extends the original Nelson–Siegel model by introducing two decay parameters to better capture the curvature of the yield curve. The objective is to fit a smooth functional form to the previously obtained (and now steepened) zero-coupon yields, and then to revalue all coupon-paying bonds accordingly.

Mathematically, the NSS model expresses the yield curve as:

$$y(\tau) = \beta_0 + \beta_1 \frac{1 - e^{-\tau/\lambda_1}}{\tau/\lambda_1} + \beta_2 \left(\frac{1 - e^{-\tau/\lambda_1}}{\tau/\lambda_1} - e^{-\tau/\lambda_1} \right) + \beta_3 \left(\frac{1 - e^{-\tau/\lambda_2}}{\tau/\lambda_2} - e^{-\tau/\lambda_2} \right),$$

where β_0 captures the long-term level of interest rates, β_1 the short-term slope, and β_2, β_3 the medium-term curvature components, while λ_1 and λ_2 control the exponential decay rates.

Calibration and Interpretation. The parameters $(\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2)$ were estimated through non-linear least squares fitting on the steepened zero-coupon data obtained in Q5.1. The resulting fitted NSS spot curve is shown in **Figure 22**. We observe that the NSS model closely matches the shocked data while producing a smooth, monotonic yield structure that avoids the local irregularities observed in the raw bootstrapped points.

Bond Valuation and Mispricing Analysis. Using the fitted NSS spot curve, we re-derived the corresponding **discount factors** and revalued all coupon-paying bonds in the sample. By comparing their theoretical NSS-implied prices to their actual market prices, we identified which bonds appeared *cheap* (undervalued) or *expensive* (overvalued) under the steepening scenario.

Figure 23 presents this analysis: green bars correspond to bonds priced below their theoretical NSS values (i.e., offering potential value to investors), while red bars represent bonds priced above their theoretical fair value. Overall, shorter and medium-term bonds tend to appear slightly underpriced, whereas longer maturities (beyond 20 years) are relatively expensive since investors demand higher returns for long durations(steeper curve).

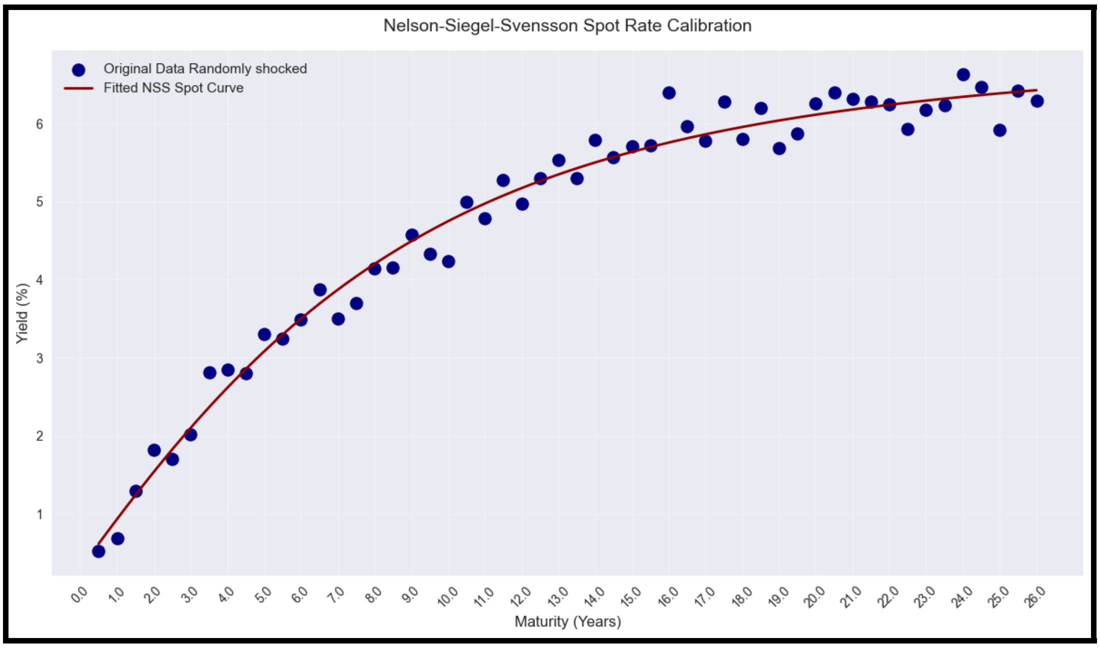

Figure 22: Nelson–Siegel–Svensson Spot Rate Calibration (Figure 22). The dark red curve represents the fitted NSS spot curve, while the blue points correspond to the randomly shocked zero-coupon yields.

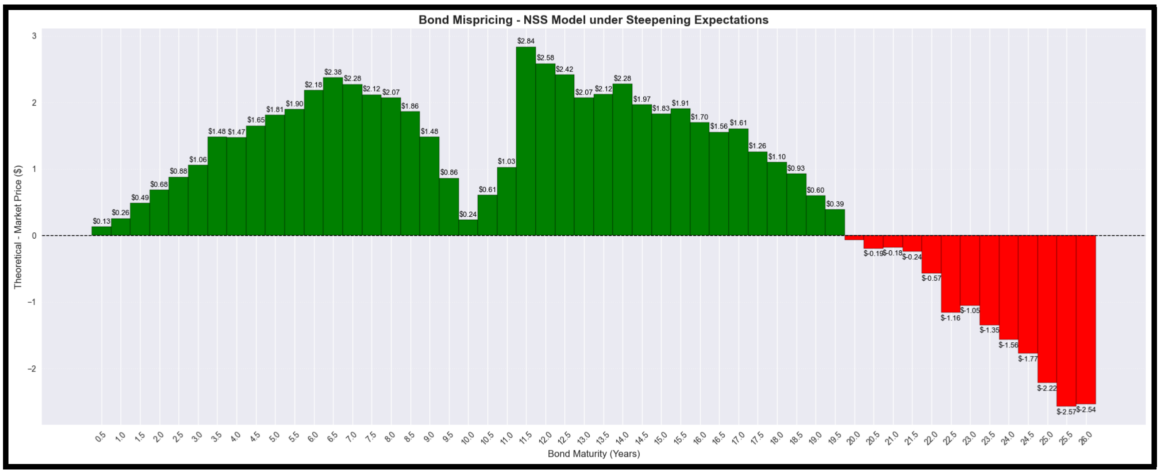

Figure 23: Bond Mispricing under the NSS Model (Figure 23). Green bars indicate underpriced bonds (theoretical price > market price), while red bars denote overpriced ones.

Discussion. The NSS model successfully captures the overall shape of the steepened yield curve while providing a stable framework for discounting future cash flows. The mispricing analysis highlights how market dynamics adjust to the expected steepening: short-term bonds benefit from slight undervaluation (reflecting liquidity and demand), while long-term bonds are penalized as investors anticipate further rate increases. This approach not only refines the discount function estimation but also offers a practical way to identify potential trading opportunities within the bond portfolio.

5.3 Zero-Cost Long/Short Strategy to Harvest Mispricing

We build a diversified **zero-cost long/short** portfolio to exploit the mispricings identified in Q5.2. Let w_i be the portfolio weight (in \$ per \$ of initial wealth) on bond i with market price P_i, modified duration D_i^{mod}, and curve shock Δy_i (from Q5.1). Under a small-yield-shift approximation, the scenario P&L contribution is

$$\alpha_i \;=\; -\,D_i^{\text{mod}}\,P_i\,\Delta y_i, \qquad \text{and we maximize } \sum_i \alpha_i\,w_i$$

subject to: (i) *zero-cost* $\sum_i P_i w_i = 0$, (ii) diversification bounds $-5\% \leq w_i \leq 5\%$, (iii) gross leverage $\sum_i |w_i| \leq 200\%$, (iv) portfolio duration within the market band $\big|\sum_i D_i^{\text{mod}} w_i\big| \leq \overline{D}_{\text{market}}$. This yields a well-spread selection of longs (undervalued bonds) and shorts (overvalued bonds), while keeping interest-rate risk contained.

Allocation and expected return. The optimized portfolio invests **\$100 long** across the cheapest names and **\$100 short** across the most expensive ones, with per-bond positions capped at ±5% of wealth (max \$5 long or short per bond), ensuring diversification. Under the steepening scenario, the **expected return** is **6.14%** on the \$100 capital, i.e. an expected P&L of approximately **\$6.14** with no financing costs.

Key diagnostics. The table below summarizes the main optimization outputs.

Status	`optimal`
Scenario P&L (expected)	**6.1402%**
Zero-cost check	2.487×10^{-14}
Total long (USD)	\$100
Total short (USD)	−\$100
Gross exposure	167.95% (cap: 200%)
# of holdings	35 (18 long / 17 short)
Min / Max position	−5.00% / 5.00%
Portfolio duration	−3.117045 ($\leq$ market $\lvert 7.956154 \rvert$)

Reading the positioning. Long positions concentrate where theoretical values exceed market prices (positive mispricing), while shorts target bonds whose market prices stand above NSS-implied values (negative mispricing). This mapping between mispricing and portfolio weights is shown in **Figure 24**.

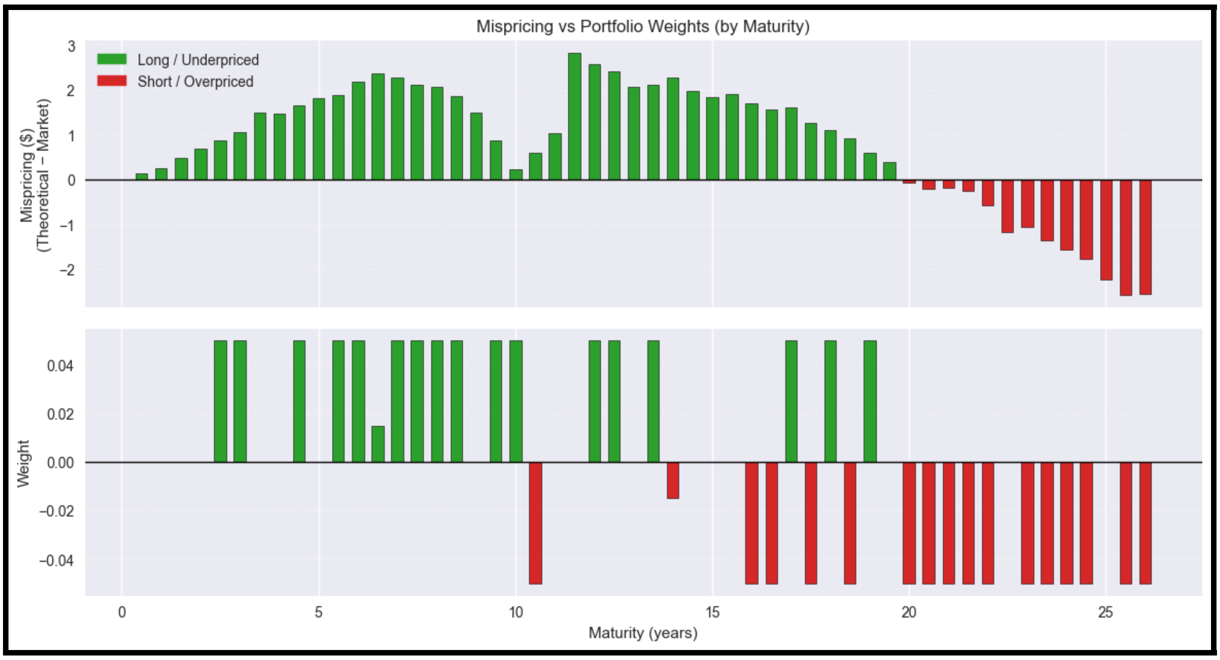

Figure 24: Mispricing vs. Portfolio Weights (Figure 24). Top: mispricing (theoretical − market). Bottom: optimized weights. Green = long/underpriced; red = short/overpriced.

6 PYTHON LABS: TRUMP TARIFFS' THREATS

This analysis examines the impact of Trump's trade policies on U.S. Treasury yields using daily constant-maturity data from FRED, covering maturities from 3 months to 30 years. The data was processed by forward-filling non-trading days to ensure continuity. Trade-related events were categorized for the first term (2017-2020) into "actions," "news," and a "global" category that gathers the two previous with a separate comprehensive list for the 2025 second term. Each event was mapped to the nearest trading day, and yield changes were examined within symmetric windows around those dates.

For all figures and tables comparing event and non-event periods, a ±10 business-day window was applied to compute the average "normal" daily yield change per maturity (**Table 3**). We removed the ±10 business-day window around each event to exclude the impact of the events from the data when computing the average "normal" daily yield change per maturity. We chose 10 business days to ensure sufficient data remained for analysis.

Maturity	Mean Daily Yield Change
3M	0.0011
6M	0.0011
1Y	0.0009
2Y	0.0008
3Y	0.0007
5Y	0.0006
7Y	0.0005
10Y	0.0006
20Y	0.0006
30Y	0.0006

Table 3: Mean Daily Yield Changes Outside Event Windows.

A longer ±30-day window was used as asked for visualizing the dynamic response in the heatmaps (the average effect of a "Trump event" over ±30 days). The abnormal yield change was defined as the difference between the corresponding change during non-event periods, computed separately by maturity and averaged across the window and the average "normal" yield change.

6.1 Investigate how markets have reacted to Trump's announcements/actions. Discuss.

The results reveal a clear maturity-dependent reaction to trade events. As shown in **Figure 25**, abnormal yield changes increase steadily along the curve, from about 0.007 for short maturities to nearly 0.06 for 30-year bonds (2nd mandate). This steepening

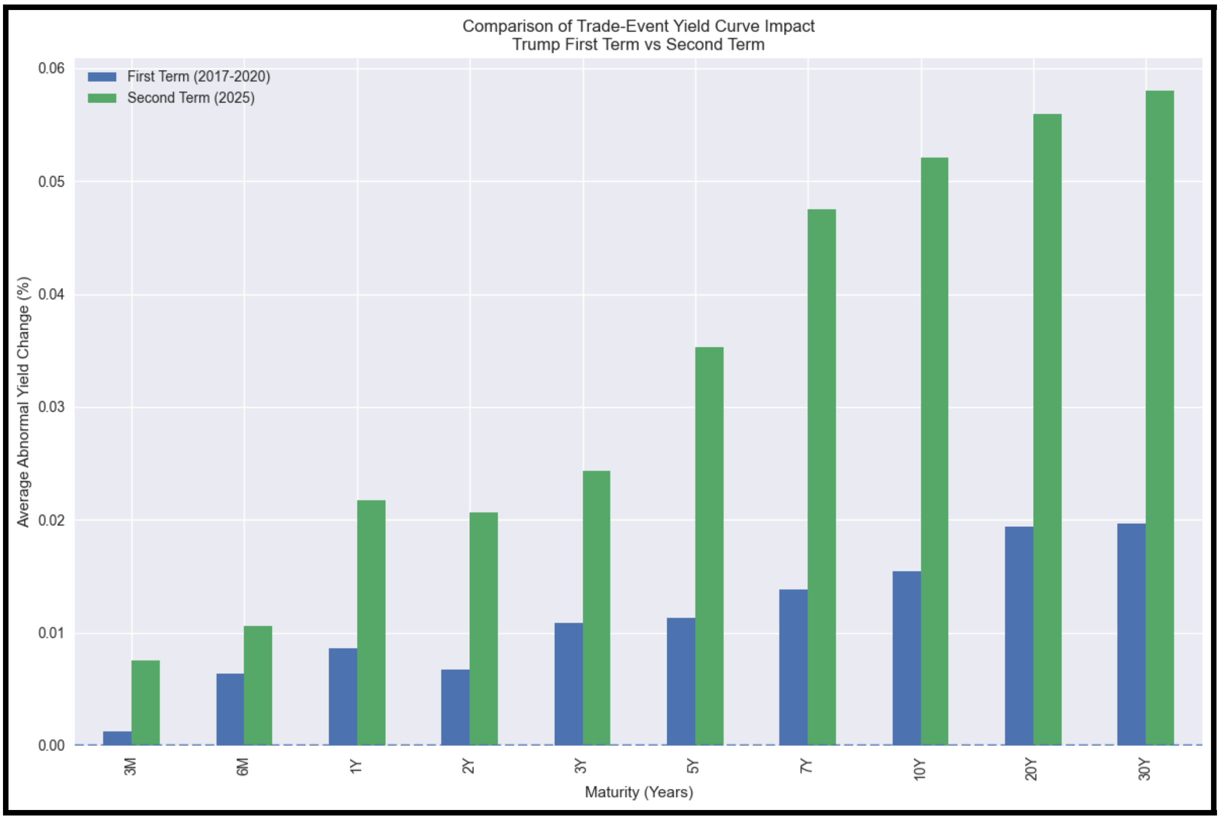

Figure 25: Comparison of Trade-Event Yield Curve Impact: Trump First Term vs Second Term

indicates that longer-term rates were far more sensitive to trade shocks, suggesting upward revisions in long-term rate expectations and a higher term premium demanded by investors. Those abnormal average yield changes are as defined before so we already subtracted the "normal" average yield changes computed in **Table 3**.

A comparison between the two presidential mandates provides further insight. **Figure 25** makes clear that the yield response in 2025 was substantially stronger than during 2017-2020, across all maturities. The more pronounced and uniform steepening likely reflects the broader and more aggressive tariff measures introduced during the second term.

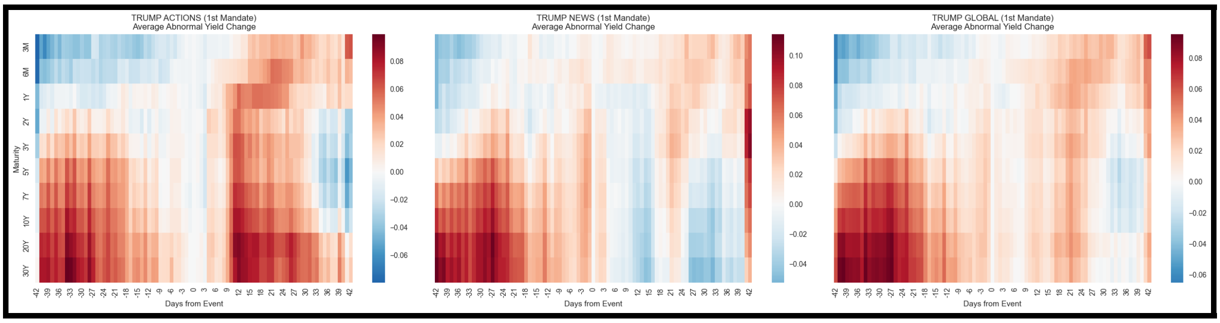

Figure 26: Heatmaps of Average Abnormal Yield Change Around Events, First Mandate

The heatmap in **Figure 26** illustrates how yield changes evolved around trade events within a ±30-day business days window. During the first term, reactions to concrete trade actions stayed positive for several days after each event, indicating a sustained adjustment at longer maturities. By contrast, yield movements following news announcements were weaker and short-lived, suggesting limited market impact from rhetoric alone. The combined "GLOBAL" panel shows a pattern similar to that of "ACTIONS," reinforcing that actual policy implementations, rather than announcements, were the main drivers of yield dynamics.

Taken together, the evidence suggests that trade policy shocks under both mandates systematically lifted U.S. Treasury yields, with the impact intensifying at longer maturities and reaching its peak during the 2025 term. Markets appeared to price in higher long-run interest rates and risk premia in response to renewed and expanded trade tensions.

6.2 Extend your analysis to the different yield curve components (slope, level, curvature). Discuss

Breaking down the yield curve reaction into its level, slope, and curvature components provides insight into whether Trump's trade announcements shifted the entire curve or altered its shape. **Figure 27** summarizes the average abnormal movements of these components within a ±30-day window around the announcements.

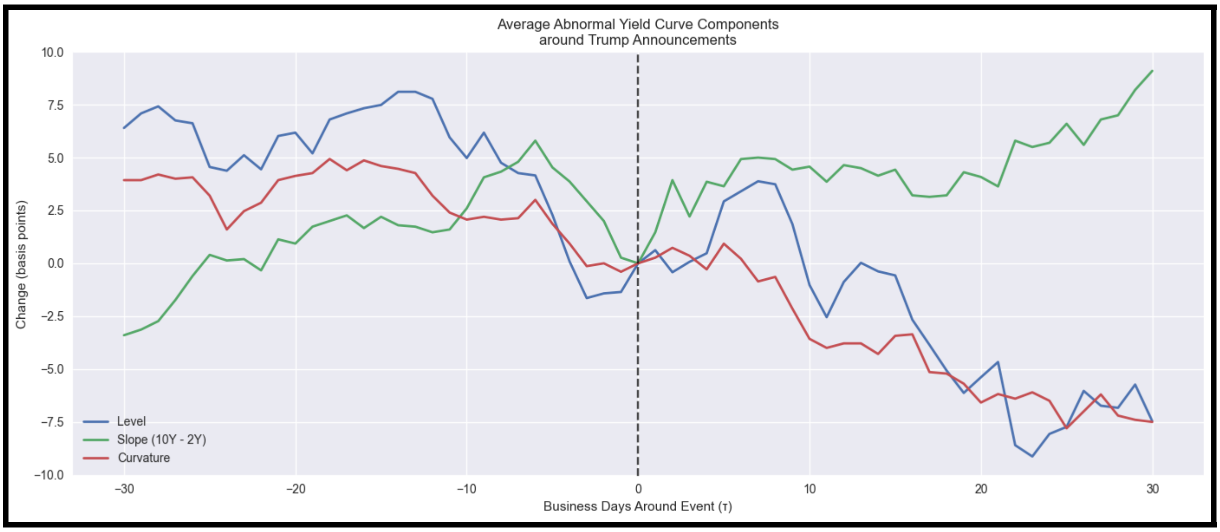

Figure 27: Average Abnormal Yield Curve Components around Trump Announcements

Before the events, all components fluctuate slightly above zero, but their dynamics diverge afterward. The level component declines after the announcements, indicating a broad fall in yields consistent with a flight-to-quality reaction. In contrast, the slope rises steadily, suggesting that short-term yields fell more than long-term ones, reflecting expectations of potential monetary easing in response to trade tensions. The curvature component also declines, implying that mid-term maturities moved more strongly than either end of the curve and that the central segment temporarily flattened.

Overall, the decomposition indicates that Trump's trade announcements led to an overall drop in yields but a steeper term structure between the two- and ten-year maturities, consistent with short-term risk aversion and longer-term policy optimism.

6.3 Same question as Q1.1 when it comes to the reaction of targeted countries. Discuss

The analysis of retaliation or response events by countries targeted by U.S. trade measures, summarized in **Figure 28**, shows the average abnormal yield changes within a ±30-business-day window.

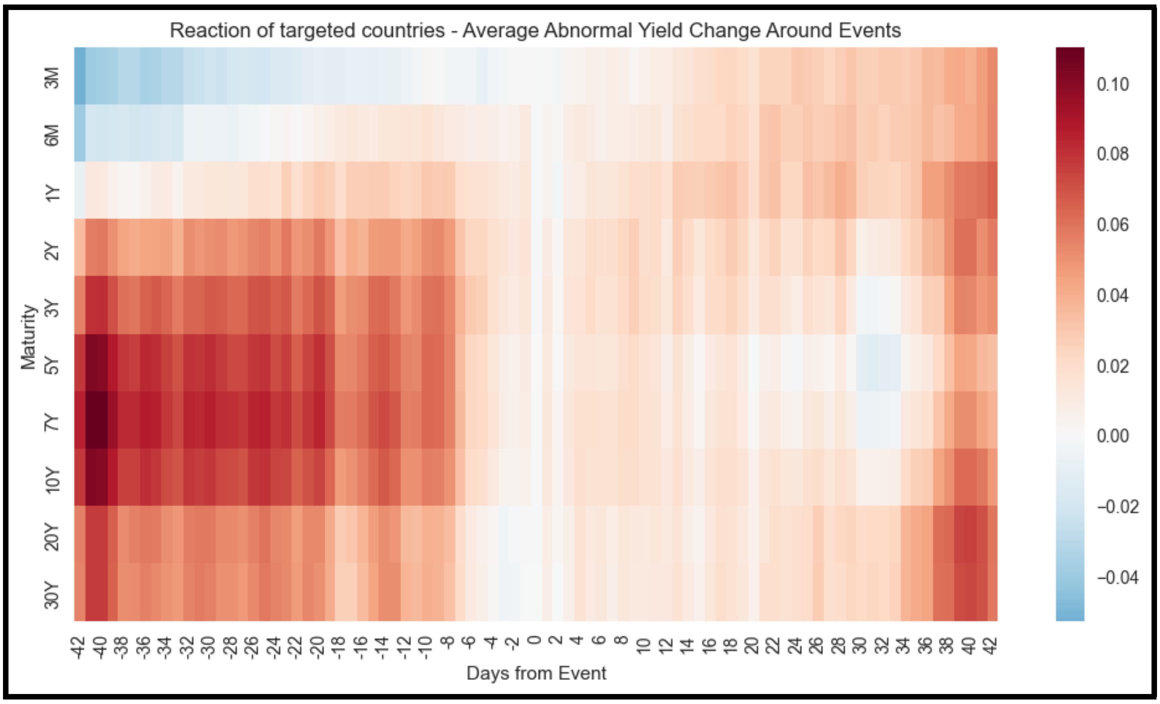

Figure 28: Reaction of Targeted Countries - Average Abnormal Yield Change Around Events

Unlike the pattern observed for Trump's own trade actions, the strongest reactions appear before the event date. Between roughly -25 and -10 business days, yields on maturities between two and ten years rise sharply, shown by the deep red cluster in the heatmap. This suggests that markets anticipated countermeasures in advance, pricing in higher uncertainty and inflation risk before the official announcements.

As the event date approaches, the reaction fades and stabilizes shortly afterward. The post-event period shows mostly neutral tones, indicating limited additional adjustment once retaliations were confirmed. The most pronounced responses occur in the five- to ten-year range, reaching abnormal increases of about 8 to 10 basis points, while short maturities remain largely unaffected and the very long end reacts only mildly.

These findings indicate that markets incorporated expectations of foreign retaliation ahead of time, generating a pre-event steepening concentrated in the medium segment of the curve. Once those measures were formally announced, yields adjusted little further, reflecting that the news had already been priced in.

6.4 Have the reactions been different in the first vs second mandate? Discuss

The comparison between Trump's first and second mandates shows a marked shift in how U.S. Treasury yields responded to trade-related events. As seen in **Figure 29**, reactions during the second term were stronger, earlier, and more volatile across maturities.

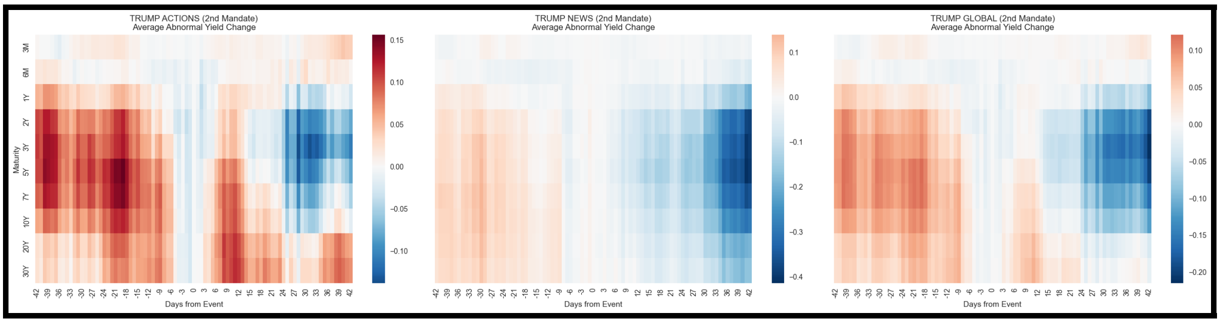

Figure 29: Comparison of Yield Reactions Across Event Types During Trump's Second Mandate: Actions, News, and Global Events

In the second mandate, yield increases tended to occur well before the official announcements, suggesting that markets anticipated policy actions and priced in higher uncertainty ahead of time. Once measures were announced, this pattern reversed: yields declined across the curve, reflecting a flight-to-quality response and rising expectations of monetary support.

News-related events produced a similar but broader effect, with yield drops spreading across all maturities even in the absence of concrete policy moves. When all events are combined, the overall response is dominated by these post-announcement declines, particularly along the medium and long segments of the curve.

Relative to 2017–2020, the 2025 term shows three clear differences: larger market swings, stronger anticipatory reactions, and a reversal in post-event behavior. Whereas yields previously rose after tariff implementations, they now fall sharply, suggesting that investors interpret renewed trade tensions as a sign of economic weakness rather than inflationary pressure.