

Magnetic Materials, Permanent Magnets and Demagnetization

METU

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1 Intro

Book starts with explaining the relation between magnetism and

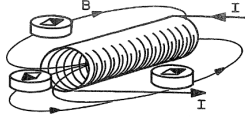
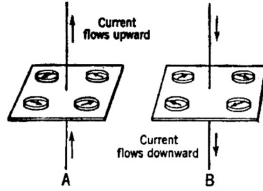
- Maxwell's Equations
- Classical Electron Theory
- Quantum Mechanics

1.1 Magnetic Fields: Observation

1.1.1 Current-Carrying Wire

A current, passing through a wire, causes magnetic field around itself. This field conforms with the right-hand rule.

1.1.2 Solenoid

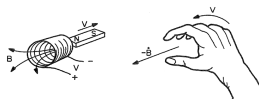


Earth's magnetic field is believed to be formed as a result of currents flowing in it's molten core.

1.1.3 Voltage Induced in a Coil

According to Lenz' law, change in flux in a coil induces voltage to complement the reduction.

$$v = -\frac{dB}{dt}$$



It does not try to complement the diminishing flux such that the flux Φ passing through the coil is same/constant. According to the law, if the change in flux is small but rapid, this will result with a high induced voltage.

This induced voltage enables the operation of generators and transformers, and a material behavior known as *diamagnetism*.

While Faraday's law tells us the magnitude of the EMF produced, Lenz's law tells us the direction that current will flow.

$$\mathcal{E} = -\frac{d\Phi}{dt} \quad (2)$$

1.2 Constitutive Relations

Electric field may cause separation between positive and negative charges within a material. Electric dipole moment is the measure of this separation, in other terms, polarization measure of this form.

$$\mathbf{p}_e = qd \quad [\text{Cm}] \quad (3)$$

where \mathbf{p}_e is the electric dipole moment, q is charge and d is the displacement. This is the equation for one dipole. To achieve a density equation,

$$\frac{N}{V}p_e = np_e = P \quad [\text{C/m}^2] \quad (4)$$

where, N is the number of electric dipoles, V is the volume dipoles take up and P is macroscopic dipole moment density. Vector form of macroscopic dipole moment density \mathbf{P} is linked with electric field \mathbf{E} as follows,

$$P = \chi_e E \quad (5)$$

where, χ_e is electric susceptibility. Finally, electric displacement field is

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_0 \mathbf{E} + \chi_e \mathbf{E} = (\epsilon_0 + \chi_e) \mathbf{E} = \epsilon \mathbf{E} \quad (6)$$

Now, in a similar manner to electric field E , applied magnetic field H provokes responses on materials. A magnetic field causes change in magnetic dipole moment p_m [Am²] of a material. Here, magnetic dipole moment is described as a dipole with infinitesimal size. Magnetization M is defined as macroscopic magnetic dipole moment density, that is

$$\frac{N}{V} p_m = n p_m = M \quad [\text{A/m}] \quad (7)$$

where, N is the number of magnetic dipoles, V is the volume dipoles take up and M is macroscopic magnetic dipole moment density. Vector form of macroscopic magnetic dipole moment density \mathbf{M} is linked with magnetic field \mathbf{H} as follows,

$$M = \chi_m H \quad [\text{A/m}] \quad (8)$$

where, χ_m is magnetic susceptibility. Finally, magnetic flux density is

$$B = \mu_0(H + M) = \mu_0(H + \chi_m H) = \mu_0(1 + \chi_m)H = \mu H \quad [\text{T, Wb/m}^2] \quad (9)$$

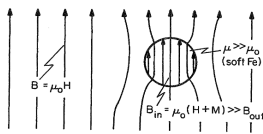
The relation $\mu_0 \mu_r = \mu$ [H/m] suggests that relative permeability from above relationship is

$$\mu_r = (1 + \chi_m) \quad (10)$$

Magnetic material's response to magnetic field H , is magnetization M . Here, H is the cause and M is the material effect.

$$B = \mu_0 H + \mu_0 M \quad [\text{T, Wb/m}^2] \quad (11)$$

As can be seen in Eq. 9, 11, magnetic flux density B is the overall result including both external flux $\mu_0 H$ and the materials response $\mu_0 M$. Faraday's Law states that its the change of magnetic flux density B w.r.t time $-dB/dT$ that gives rise to an electric field E or a voltage V . In terms of magnetic circuits, ferromagnetic materials provide a path with low reluctance, much like a low resistance resistor to current. As a result, these materials draw the magnetic flux and amplify it with their magnetization.



Note: naval mines (magnetic) utilizes this principle. They are set up as the mine's sensor measures Earth's magnetic field. If a ship (high permeability material $\mu \gg \mu_0$) is nearby the mine, magnetic field over the mine is drawn towards the ship. Mine senses this and explodes.

1.3 Maxwell's Equations

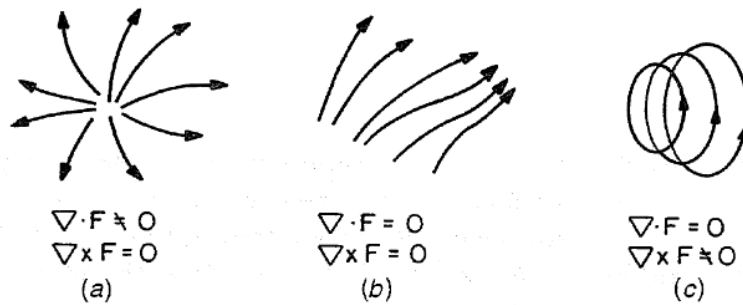
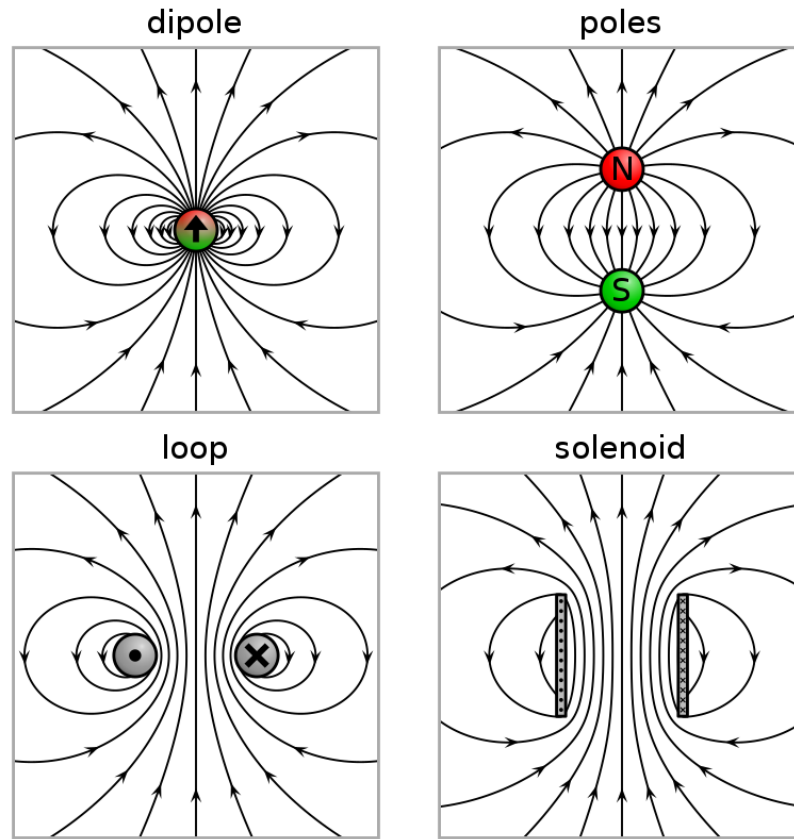
1.3.1 Preliminary Mathematics

Diverging and curling fields are depicted in Fig. ??.

Divergence expresses that net flux through an enclosed surface is determined by the sources/sinks enclosed by the surface. In Fig. ?? a, divergence of the field is not zero, meaning that if a surface enclosing the center of the field is considered, there is a field source at the middle, and the net flux is not zero, it is outwards. However, in Fig. ?? b and c, any enclosed surface have a zero net flux, meaning there is no source/sink enclosed by the surface.

In mathematical terms, divergence theorem states that surface integral of a vector field over a closed surface is equal to the volume integral of the divergence over the region inside the surface.

$$\iiint_V (\nabla \cdot \mathbf{F}) dV = \oint_S (\mathbf{F} \cdot \mathbf{n}) dS. \quad (12)$$



Gauss's Theorem $\int (\nabla \cdot \mathbf{F}) d^3x = \int \mathbf{F} \cdot d\mathbf{A}$

Stokes Theorem $\int (\nabla \times \mathbf{F}) \cdot d\mathbf{A} = \int \mathbf{F} \cdot d\mathbf{l}$

Gauss's Law for Electric Fields Electric field always moves from a positive charge to a negative charge. In other words, electric charges are sources/sinks for electric fields. There can be electric monopoles, while both positive and negative charges can be obtained individually.

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon} \quad (13)$$

$$\iiint_V (\nabla \cdot \mathbf{E}) dV = \frac{1}{\epsilon} \iiint_V \rho dV \quad (14)$$

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon} \iiint_V \rho dV \quad (15)$$

Gauss's Law for Magnetic Fields

$$\nabla \cdot \mathbf{B} = 0 \quad (16)$$

There is no source for a magnetic flux [**Pyrhonen**]. All the flux loops are closed. The flux entering an object must also leave the object. These remain to be true unless a magnetic monopole is discovered.

$$\iiint_V (\nabla \cdot \mathbf{B}) dV = 0 \quad (17)$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0 \quad (18)$$

$\nabla \cdot \mathbf{B} = 0$ implies there can be no net outflow of \mathbf{B} over any closed surface, therefore no sources of \mathbf{B} , no magnetic monopoles. Magnetic poles always come in pairs, usually designated north and south, called dipoles.

Faraday's Induction Law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (19)$$

$$\oint_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \oint_S \mathbf{B} \cdot d\mathbf{S} \quad (20)$$

$$\int \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt} \quad (21)$$

where, $\Phi = \mathbf{B} \cdot \mathbf{A}$

Magnetic field \mathbf{B} changing in time (time dependent) $-\partial \mathbf{B} / \partial t$ causes a spatially rotating electric field \mathbf{E} . Maxwell-Faraday law vs Lenz' law A change in current \mathbf{J} gives rise to a change in magnetic field \mathbf{B} . This change in magnetic field \mathbf{B} induces an electromotive force EMF opposing the change in current \mathbf{J}

Ampere's Law

$$\oint B \cdot dl = \mu_0 \int J \cdot dA = \mu_0 J \quad (22)$$

$$\nabla \times \mathbf{B} = \mu_0 J + \frac{\mu_0 \epsilon \partial \mathbf{E}}{\partial t} \quad (23)$$

Either a current density $\mu_0 J$ or an electric polarization current $\mu_0 \epsilon \partial \mathbf{E} / \partial t$ lead to a circulating magnetic field $\nabla \times \mathbf{B} \neq 0$.

1.4 Magnetism and Currents

1.4.1 Magnetic Field about a Straight Current-Carrying Wire

A qualitative measurement for the magnetic flux density \mathbf{B} around a straight current-carrying wire, we use Ampere's law: $\oint B \cdot dl = \mu_0 \int J \cdot dA = \mu_0 J$ (neglecting the electric displacement term). Applying this to the problem depicted at Fig. ??, we get

$$B \cdot dl = 2\pi R B \quad (24)$$

$$\mu_0 \int J \cdot dA = \mu_0 J = \mu_0 I \quad (25)$$

Magnetic flux density B has a uniform magnitude over the line at points equidistant to the wire, therefore, line integral of B gives $2\pi R B$. Therefore, the resulting magnetic flux density B is (μ_0 has a unit of [H/m])

$$B = \frac{\mu_0 I}{2\pi R} \quad \left[\frac{HA}{m^2}, \frac{Wb}{m^2}, T \right] \quad (26)$$

In terms of magnetic field $H = B / \mu_0$

$$H = \frac{I}{2\pi R} \quad (27)$$

Note: through the book by O’Handley [[**ohandley*modern*2000**]], the term H is used in behalf of magnetic fields sourced by macroscopic currents, e.g. by magnetic charges. The term B is used for magnetic fields when microscopic currents contribute to the flux density, especially in situation where magnetization is important.



$$\mathbf{B} = \frac{\mu_0 I}{2\pi R} \quad (28)$$

$$\mathbf{H} = \frac{I}{2\pi R} \quad (29)$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} \quad (30)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad (31)$$

$$\nabla \times \mathbf{B} = \frac{1}{r} \frac{\partial}{\partial r} [r B_\theta(r)] = \mu_0 \mathbf{J} \quad (32)$$

$$\int_0^R \left\{ \frac{\partial}{\partial r} [r B_\theta(r)] = \mu_0 \mathbf{J} \right\} dr \quad (33)$$

$$R B_\theta(R) = \mu_0 \mathbf{J} \frac{R^2}{2} \quad (34)$$

$$B_\theta(R) = \mu_0 \mathbf{J} \frac{R}{2} \quad (35)$$

$$\mathbf{J} = \frac{I}{\pi R^2} \quad (36)$$

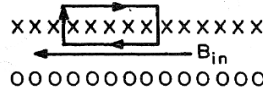
$$B_\theta(R) = \mu_0 \frac{\mathbf{I}}{2\pi R} \quad (37)$$

$$\mathbf{H} = \frac{B_\theta(R)}{\mu_0} = \frac{\mathbf{I}}{2\pi R} \quad (38)$$

1.4.2 Moment of a Current Loop

The source that leads to atomic magnetic moment p_m are microscopic current loops inside the material.

To find the magnetic flux density B , Ampere’s law is utilized to a solenoid, as depicted in Fig. ??.



$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int \mathbf{J} \cdot d\mathbf{A} = \mu_0 N I \quad (39)$$

Here, the book assumes that the B is only non-zero on the integral line inside the solenoid. Therefore,

$$\oint \mathbf{B} \cdot d\mathbf{l} = B_{in} l \quad (40)$$

where, l is the length of the solenoid. As a result,

$$B_{in}l = \mu_0 NI \quad (41)$$

$$B_{in} = \frac{\mu_0 NI}{l} H_{in} = \frac{NI}{l} \quad (42)$$

where, the term N/l represents the number of coils the the line of integration encloses, and I is the current each of these coils carry.

1.4.3 Origin of Atomic Magnetic Moment

The study done above for a magnetic flux density generated by a solenoid can be seen as the origin of the atomic magnetic moment in a ferromagnetic material. If external magnetic field is zero, i.e. $H = 0$, then the magnetic flux density B is

$$B = \mu_0(H + M) = \mu_0 M \quad (43)$$

$$M = \left(\frac{N}{l}\right)I \quad (44)$$

where, magnetization M is replaced by magnetic field H stated as the field resulting from a solenoid. Furthermore, $M = np_m$ and $n = N/V = N/Al$. Therefore,

$$M = \frac{N}{Al}p_m = \frac{N}{l}I \quad (45)$$

$$p_m = AI \quad (46)$$

Calculating Magnetism in Hydrogen Atom w.r.t. Bohr Model sourced by electron intrinsic angular momentum (spin of electrons)

According to the discussion above, magnetic moment of a hydrogen atom with 1 electron is $p_m = AI$, where A is the area the electron circulates and I is the current representation of the electron. The area can be found by πr_0^2 and the current can be stated as $I = e(\omega/2\pi)$. These are according to Bohr's atomic model and the resulting values are mere approximations.

Angular velocity of the electron can be stated as $\omega = v/r_0$, and the current becomes $I = e(v/2\pi r_0)$. Electron's velocity can be derived from its energy, that is $E = \frac{1}{2}mv^2$. Resulting magnetic moment is

$$\mu_m = p_m = IA \approx e \left(\frac{\omega}{2\pi} \pi r_0^2 \right) \quad (47)$$

$$\approx e \left(\frac{v}{2\pi r_0} \pi r_0^2 \right) \quad (48)$$

$$\approx 5.7 \times 10^{-24} \quad [\text{Am}^2] \quad (49)$$

where, E is the electronic energy of the 1s electron in hydrogen. If a hydrogenic material with $n \approx 10^{29}/m^3$ atoms per unit volume is considered, with each atom behaving like a magnetic dipole. Then the magnetization can be calculated as

$$M = n\mu_m = 5.7 \times 10^5 \quad (50)$$

$$B = \mu_0(H + M) = \mu_0 M = 0.72 \quad [\text{T}] \quad (51)$$

This is the case in which all the atoms are aligned towards the same direction. This value is called the saturation magnetization $B_s = \mu_0 M_s$. The books states that the saturation magnetization B_s is about 2.2[T] for Fe, 1.7[T] for Co and 0.6[T] for Ni.

1.5 Types of Magnetism

1.5.1 Weak Magnetism

Let's take the following equality into consideration:

$$B = \mu_0(H + M) = \mu_0 H + \mu_0 M \quad (52)$$

Here, the resulting magnetic field has two components. $\mu_0 H$ component is due to the magnetic field intensity applied externally, and $\mu_0 M$ component is due to the alignment of atomic magnetic moments within the material.

Magnetic susceptibility χ_m is usually the indicator for weak magnetic responses. Weak magnetic responses occur in materials with susceptibility between $\pm 10^{-4}$ to 10^{-6} . In these type of materials, majority of the magnetic field \mathbf{B} is sourced by the applied magnetic field intensity \mathbf{H} and the remaining part is sourced by the magnetization \mathbf{M} . This is because the magnetic moments are not fully, not even significantly aligned. (For hydrogen, magnetization sources the magnetic field of 1 T when fully aligned, 10^{-6} T when not aligned). This is because of the thermal energy in the material, $k_B T$.

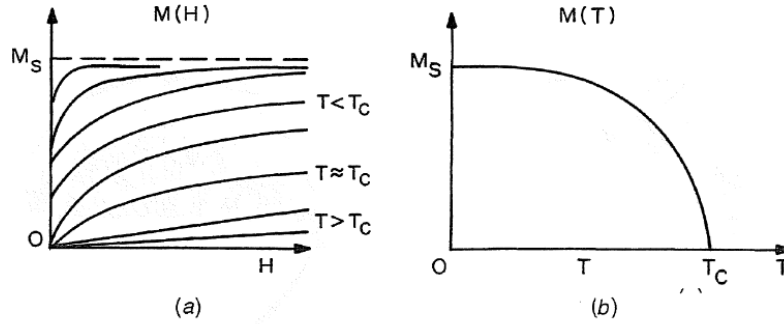
The potential energy U of the magnetic moment μ_m in an applied magnetic field B is

$$U = -\mu_m \cdot B \quad (53)$$

Additionally, thermal energy at room temperature is $k_B T$. If, the magnetic energy is small relative to thermal energy, then the field presents a weak, linear effect on aligning the moments.

1.5.2 Ferromagnetism

Ferromagnetic materials are characterized by a long-range ordering of their atomic moments, even in the absence of an external field [O'Handley]. This ordering, spontaneous alignment of magnetic moments, is observed to vanish above a certain temperature, called Curie temperature T_C .



Relative magnetic permeability $\mu_r = \mu/\mu_0 = 1 + \chi_m$ is used to describe magnetic response of ferromagnetic materials to magnetic field intensity H .

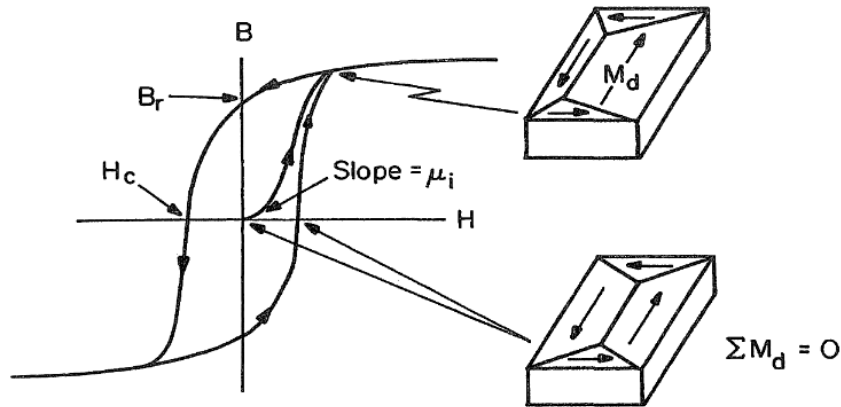
In ferromagnetic materials, relatively small values of magnetic field intensity H (e.g. $H \approx 100$ A/m) lead to high magnetic field B (e.g. $B = 1 - 2$ T) values.

1.5.3 Antimagnetism and Ferrimagnetism

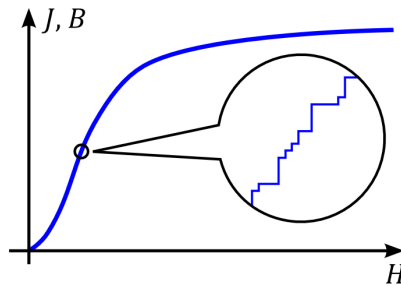
1.6 Technical Magnetic Materials

1.6.1 B-H Loops and Magnetic Domains

Starting with a magnetic material at demagnetized state, that is $B = 0, H = 0$. Increasing the applied magnetic field, applying a weak field to the material leads to certain motions on domain walls such that volume of the domains having the largest component of magnetization M parallel to applied field H . Initial permeability $\mu_i = (B/H)_{H \approx 0}$ is defined from the initial induction B produced in response to the weak magnetic field intensity H . Initial permeability μ_i can be as high as 10^{-1} ($\mu_0 \mu_r = \mu_i, \mu_r \approx 10^5$) in some materials. At magnetic field intensities H a little higher, materials permeability increases to its maximum μ_{max} . At this point, the domain wall motion is at its peak. At higher magnetic field intensities H , when this motion is finished, there often remain domains with nonzero magnetization orthogonal to the applied field. Magnetization in these domains must also be rotated towards the applied field to achieve minimum potential energy. When these two motions: domain wall motion and orthogonal magnetization rotation are finished, the material comes to a static state of magnetic saturation, where $B_s = \mu_0(H + M_s)$.



Now, as the applied magnetic field intensity H decreases, magnetization rotates back to its easy direction first. Rotation is generally a lossless process, without hysteresis. As H decreases further, domain walls recede. Here, the domain walls jump abruptly from one local minimum energy to the next. This is called Barkhausen jumps. During domain wall motion, energy is lost. Therefore, domain wall motions (Barkhausen jumps) are lossy, irreversible processes. As a result, B-H loop opens up (area between the curves for increasing and decreasing magnetic field intensity increases), that is called hysteresis, when lossy magnetization processes are involved. When the applied magnetic field intensity decreases to zero, induction remaining in the material is called residual induction B_r , and magnetization remaining in the material is called remanence magnetization M_r . An magnetic field intensity shall be applied to the material in reverse direction, to get such magnetized material to present a magnetic field $B = 0$ again. This reverse field is called coercive field H_c . It can be seen as a measure of how easy to get the material magnetized. The field needed to restore magnetization M to zero is called intrinsic coercivity $H_{c,intrinsic}$.



1.6.2 Soft Magnetic Materials

Soft magnetic materials are classified as materials in which the domain wall motion and domain magnetization rotation occurs in weak fields (relatively low magnetic field intensities), e.g. $H_c \leq 10^3$ A/m.

Note: Earth produces a magnetic field of 0.4 Oe, or 30 A/m.

1.6.3 Hard Magnetic Materials

Some magnetic materials, once magnetized, resist demagnetization. These materials may have defects which strongly impede domain wall motion, or they may consist of single domain particles with high magnetic anisotropy that the direction of magnetization changes with very large fields. These materials are called hard magnets, or permanent magnets. Because they resist demagnetization in the presence of a negative field, hard magnets exert a repelling force against the source of a negative imposed field. Thus, permanent magnets store energy like a spring; they exert a restoring force without contact. The energy stored in a permanent magnet is related to the area inside the second quadrant of the B-H loop. The (BN)_{max}, quality factor commonly used to rate permanent magnets is the maximum value of the BM product along the B-H curve in second quadrant. Hard

magnets are used in many motors and actuators; they also find applications in frictionless bearings, microwave generators, and lenses for charged particle machines.

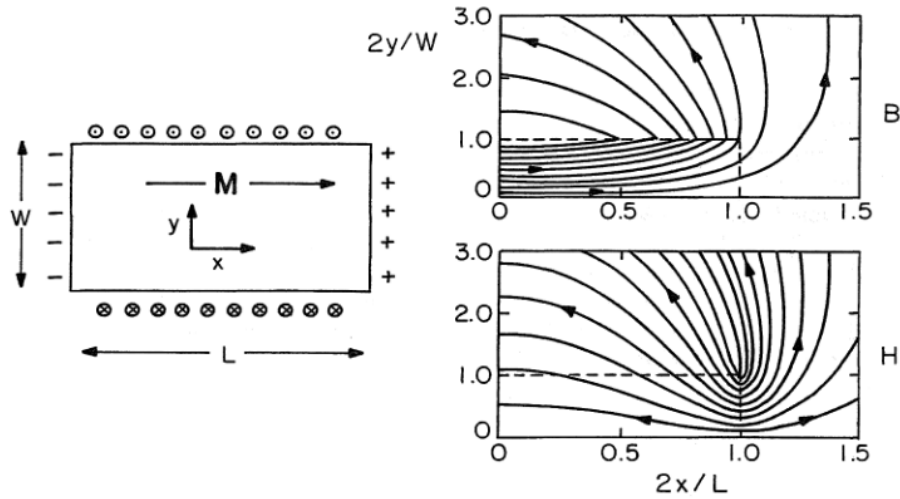
1.6.4 Demagnetizing Field

The answer is in the examination of airgap surfaces. Free poles occur at the surfaces of magnetized materials from which the flux passes through. These free poles (north) emanate magnetic field H which tries to reach to free poles (south). Here, magnetic pole strength per surface area is given as

$$\sigma = \mathbf{M} \cdot \mathbf{n} \quad (54)$$

where, \mathbf{M} is magnetization and given as $= np_m = (N/V)P_m$, and \mathbf{n} is the normal vector.

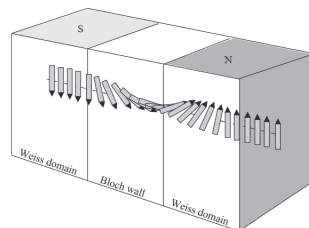
Magnetic pole strength per surface area σ sets up magnetic field on both sides of a surface. Answer to the question that is asked before, why magnetic fields at the airgap and on the PM are in opposite direction, is here. From the surface with north poles emerges magnetic field H to go to the opposing airgap surface, which is with south poles. Some magnetic field goes through the airgap, resulting with airgap flux, and some go through the PM material. Magnetic field emanates through the material is called demagnetizing field.



As can be seen in Fig. ??, the magnetic flux density B obeys the Ampere's law, is formed according to current loops depicted on top and bottom of the material.

2 Magnetic Materials of a Rotating Machine: Pyrhonen 3.6

In ferromagnetic materials, there are magnetized domains called Weiss domains (named after Pierre Weiss) and the transition region between these domains called Bloch walls (named after Felix Bloch). As depicted in Fig ??, in Bloch wall the magnetization rotates around normal to the domain wall, in contrast to Neel wall (though one of two types transition regions between Weiss domains, Neel wall is not mentioned in engineering books). This transition is rather abrupt. To give some values, size of Weiss domains range starting at 100 nm and up, and Bloch walls range between 10 – 100 nm.



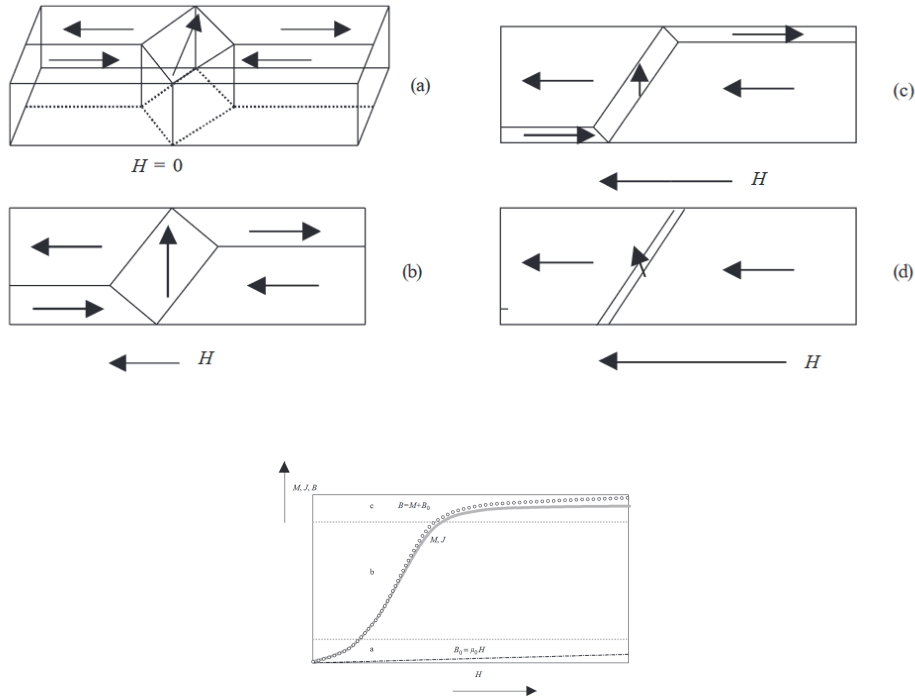
Magnetization of a ferromagnetic material requires an external magnetic field H . Two independent processes happen during the magnetization. These processes can be seen in Fig. ??.

First process is that at weak (relative to the levels of field required for other process) external fields, Bloch walls starts to move. This movement happens in such fashion that volume of Weiss domains which have the same magnetization direction as the external field starts to increase, and the volume of Weiss domains which have the opposite magnetization direction starts to decrease.

Other process is that at strong external fields, Weiss domains with magnetization direction normal to the external field are altered such that their magnetization direction starts to rotate towards the external field. This process requires such high external fields that, in some soft magnetic materials, Bloch wall movements are almost completed before the domain magnetization rotation happens.

Under the influence of an external fields low enough, Bloch walls only move slightly, and return back to their original position once the external field is removed. However, if higher external fields are applied, Bloch walls can not go back to their original position. This kind of Bloch wall displacement is called Barkhausen jumps. These jumps are irreversible, and they are the reason for ferromagnetic hysteresis.

In ferromagnetic materials, magnetization can be described in 3 phases. In first phase, only reversible Bloch wall motions occur. In second phase, Barkhausen jumps occur. In third phase, all the Weiss domains are magnetized in the direction of external field, and magnetic saturation settles.



2.1 Losses in Iron Circuits: Pyrhonen 3.6.2

2.1.1 Hysteresis Losses

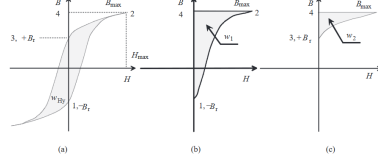
$$w_1 = \int_{-B_r}^{B_{max}} H dB \quad (55)$$

$$w_2 = \int_{B_{max}}^B H dB \quad (56)$$

$$W_{Hy} = V \oint H dB \quad (57)$$

$$P_{Hy} = fVw_{Hy} \quad (58)$$

$$P_{Hy} = \eta fV B_{max}^k \quad (59)$$



2.1.2 Eddy Current Losses

Faraday's law states that alternating flux induces voltages. In a conductor, these induced voltages result with eddy current. The term 'eddy' comes from some analogy with fluid dynamics, and Leon Foucault is credited to be the one who discovered eddy currents.

$$dI = \frac{E}{R} = \frac{2\pi f \hat{B}_m w x dx}{\sqrt{2} \rho} \quad (60)$$

$$P_{Fe, Ft} = \frac{wh\pi^2 f^2 d^3 \hat{B}_m^2}{6\rho} = \frac{V\pi^2 f^2 d^2 \hat{B}_m^2}{6\rho} \quad (61)$$

3 Permanent Magnets in Rotating Machines: Pyrhonen 3.7

3.1 Operating Point of a Permanent Magnet Circuit: Pyrhonen 3.7.3

To examine the operating point of a permanent magnet circuit, let's consider a permanent magnet material in the shape of a closed ring. The material is magnetized to saturation.

Magnetization: a copper wire may be wound around the ring to form 1000 turns. Then, Ampere's law, Eq. 22 states:

$$\oint \mathbf{H} \cdot d\mathbf{l} = NI = H2\pi R \quad (62)$$

if the ring has a radius of $R = 10$ cm, then the current needed to supply 600 kA/m is

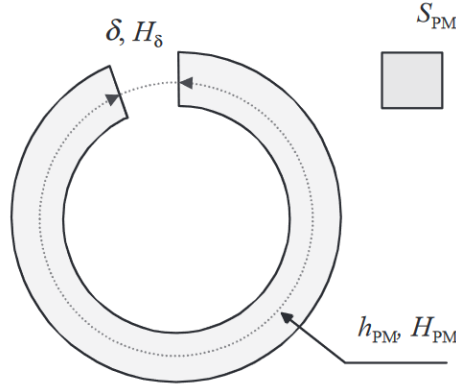
$$I = \frac{H2\pi R}{N} = \frac{600,000 \times 2\pi \times 10 \times 10^{-3}}{1000} = 37.7 A \quad (63)$$

The magnetizing equipment is removed. Now, as the external field is removed, PM supplies the remanent flux density $B = B_r$ at zero field $H = 0$. At this moment, the operating point is on magnetic flux (induction) axis.

Magnetic Circuit: PM + Air gap A section of the ring is cut out, as depicted in Fig. ?? . If Ampere's law, Eq. 22 is again applied,

$$\oint \mathbf{H} \cdot d\mathbf{l} = H_{PM} h_{PM} + H_{\delta} \delta = 0 \quad (64)$$

because there is no current enclosed by the ring, $\int_A \mathbf{J} \cdot d\mathbf{A} = 0$. As the airgap length increases, demagnetizing field H increases; therefore, PM magnetic flux density decreases. The slope of the load line increases and its interception with the recoil curve, i.e. the operating point, moves towards left.



Magnetic Circuit: PM + Air gap + Load a coil is wound around the magnetic circuit. It is loaded with current I . The magnetic field path $d\mathbf{l}$ encloses N conductors, all of which carry a current I . Therefore, $\oint_A \mathbf{J} \cdot d\mathbf{A} = \pm NI$. As a result, Ampere's Law is applied to the system as follows.

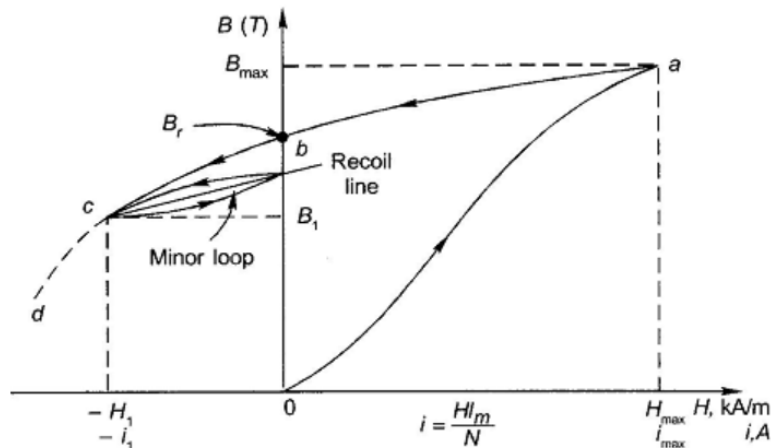
$$\oint \mathbf{H} \cdot d\mathbf{l} = H_{PM}h_{PM} + H_\delta\delta = \pm NI \quad (65)$$

where N is coil number of turns. The \pm sign depends on the coil winding.

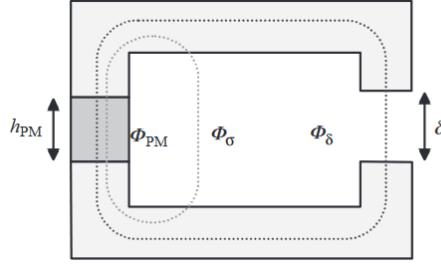
Loading the coil with current does not change the slope of the load line, but moves the line horizontally. If the magnetic field generated by the coil opposes the PM magnetic flux density, i.e. is in the same direction as PM demagnetizing field, then the load line, and therefore the operating point, moves to left. As a result, the operating point moves left as well. In contrast to this, if the coil is wound such that it's magnetic field is in the same direction as the PM magnetic flux density, i.e. opposes the PM demagnetizing field, then the load line, and therefore the operating point, moves to right.

As long as the operating point stays on the right side of the knee point, the resulting demagnetization is reversible, meaning that as the demagnetizing field on the PM fades, the PM again supplies it's original remanent magnetic flux density B_r . This can be called as partial reversible demagnetization. If the demagnetizing field increases too much, the operating point surpasses the knee. After this, the PM irreversibly loses some fraction of its magnetization, and the recoil curve declines to a position lower than the original recoil curve. This is called partial irreversible demagnetization. To achieve the original remanent magnetic field B_r , the PM needs to be magnetized again.

The recoil curve is not exactly straight. Therefore, even though the operation along the recoil curve is reversible, the operation itself incorporates hysteresis losses. A mere representation of this is depicted in Fig. ???. However, PM hysteresis losses are scarce and insignificant during normal operation in rotating electric machines [pyrhonen'design'2014].



Magnetic Circuit: PM + Soft Magnetic Material + Airgap Permanent magnets are expensive materials and they require intricate work to forge. Therefore, PMs are usually utilized with soft magnetic materials in applications. One such application and the resulting magnetic circuit is given in Fig. ??



Leakage flux and air gap flux are originated from the magnetic flux supplied by the PM. The permeance of the path with the air gap is much higher than the path of the leakage flux. Therefore,

PM sources the magnetic flux in this system. The path that consists of the soft magnetic material and the air gap is the path with the highest permeance. Therefore, majority of the magnetic flux travels through the air gap. Remaining small fraction of magnetic flux that does not travel through the air gap is called leakage flux.

The optimum operating point of a PM is at which the energy product $|BH|$ is at maximum.

Magnetic Circuit Representation of PM

3.2 Demagnetization of Permanent Magnets: Pyrhonen 3.7.4

There are 2 major reasons the operating point of a PM may pass the knee and demagnetize: the load is too high, or the temperature is too high.

Load Based Demagnetization Demagnetization happens if the operating point of the PM is on the nonlinear part of the B-H curve at second quadrant. PM temperature and the magnetic field intensity on the PM are 2 major actors in PM demagnetization process.

B-H curve for NdFeB magnet VACODYM 677TP can be seen in Fig. ?. As can be seen, -1000 kA/m on this PM, at 20° does no irreversible

Temperature Based Demagnetization

A Appendix

From here, we can relate PM magnetic field to air gap magnetic field as $H_{PM} = -H_\delta \delta / h_{PM}$. Here, negative sign suggests the fields at the air gap and on PM are opposing to each other. Magnetic field on the PM opposing the PM magnetizing direction is called demagnetizing field, explained in Sec. 1.6.4.

In addition to Eq. 64, H_δ can be written as

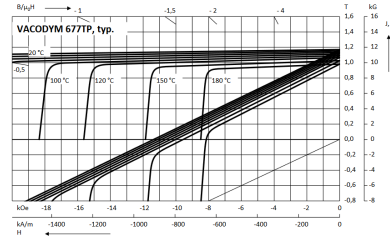
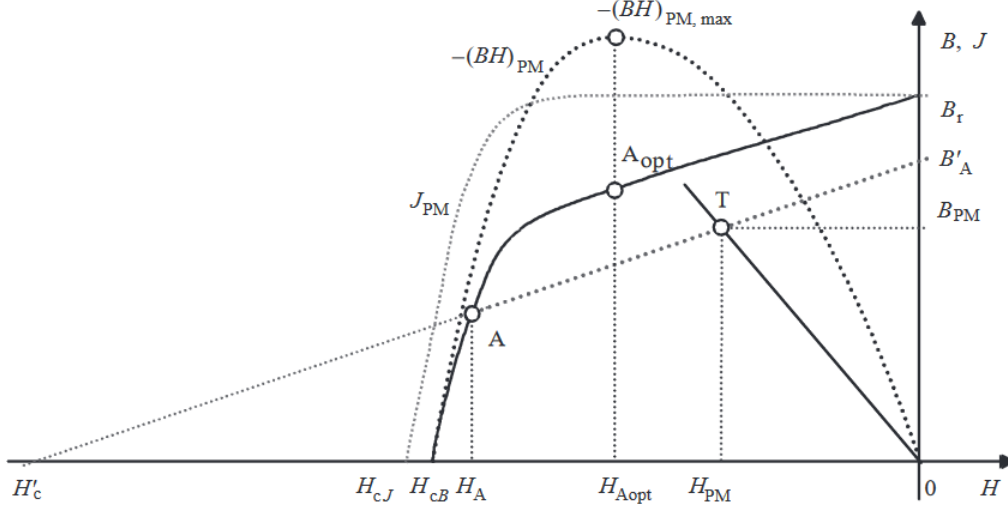
$$H_\delta = B_\delta / \mu_0 \quad (66)$$

where, B_δ is air gap magnetic flux density. Considering a uniform ring structure, where cross-section area of PM and air gap is equal $A_{PM} = A_\delta$ and assuming no leakage flux in the system, air gap magnetic flux density equals to PM magnetic flux density $B_{PM} = B_\delta$. Furthermore, PM magnetic flux density B_{PM} is related to the demagnetizing field H_{PM} by

$$B_{PM} = B_r + \mu_0 \mu_r H_{PM} \quad (67)$$

then, Eq. 64 becomes

$$\frac{B_r}{\mu_r} = \left(\frac{1}{\mu_r} + \frac{\delta}{h_{PM}} \right) B_{PM} \quad (68)$$



describing magnetic flux density supplied by the PM in relation with the PM and air gap lengths. Moreover, arranging Eq. 64

$$H_{\delta} = -\frac{H_{PM}h_{PM}}{\delta} \quad (69)$$

$$\mu_0 H_{\delta} = -\mu_0 \frac{H_{PM}h_{PM}}{\delta} \quad (70)$$

$$B_{PM} = B_{\delta} = \mu_0 H_{\delta} \quad (71)$$

$$\frac{B_{PM}}{H_{PM}} = -\mu_0 \frac{h_{PM}}{\delta} \quad (72)$$

which constitutes the load line $0-T$ at no-load $NI = 0$. Let's increase the air gap length slowly, starting from an initial value of $\delta = 0$. As air gap length increases, operating point moves along the curve at 2nd quadrant, starting from point B_r . As the air gap length δ increases, demagnetizing field increases as well. At a certain point $H = H_{Aopt}$, the energy stored in the PM becomes highest as $|BH|$ becomes maximum. At a higher demagnetizing field of H_A , operating point moves along what it seem to be a 'knee' on Fig. ?? . If the operating point does not pass this 'knee', then the operation is reversible, meaning that as the demagnetizing field lowers, the operating point moves back on the same curve. However, if the operating point does not pass this 'knee' as in the case where operating point is at point A, then the operation is irreversible. A new recoil curve, $A-B'_A$, is drawn. Hereafter, the remanent flux density is B'_A , lower than B_r .

It is clear that as air gap length increases, PM flux density decreases. Also, the demagnetizing curve has a slope of $\mu_0\mu_r$. What this indicates is that PM operation over reversible demagnetizing curve does not solely consists of external field, but there are processes going on within the PM material. These processes are a combination of magnetization rotation (reversible) and Barkhausen jumps (irreversible, causes hysteresis).

If air gap length exceeds some level such that the resulting demagnetizing field on the PM surpasses the level that correspond to the knee point (point where the slope of the curve increases abruptly), then the magnet is irreversibly demagnetized. Without a load acting on the magnetic

circuit, this kind of a demagnetization in PM is highly unlikely. Equation for load line is given as

$$\frac{B_m}{H_m} = -\mu_0 \frac{A_g}{A_m} \frac{l_m}{l_g} \quad (73)$$

B Questions

What is the energy stored in the PM at $B = 0$ and $H \neq 0$

There can be no magnetic monopoles, however we talk about surface free poles. What is the explanation? Isn't this against current loop unit magnetic source explanation?

Can zero magnetic field $H = 0$ condition is satisfied only when a ring magnet is considered?

After saturation, is slope equal to μ_0 ?

Mechanics behind the PM normal operation hysteresis?

Is a permanent magnet a ferromagnetic material with large hysteresis loop, high coercivity?