MIDDLE EAST TECHNICAL UNIVERSITY

ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

EE 361 Electromechanical Energy Conversion I

Midterm-1 Examination

11 November 2019

Duration: 110 minutes

Attempt all questions.
Show all your calculations for full credit.
Please mention it clearly if you continue your solutions on an extra page.

Student Name: Solutions

Student ID: ____________________

Q1 (30 pts)	
Q2 (35 pts)	
Q3 (35 pts)	
Total	

Q1) (30 pts) We have an electromechanical energy converter application where we want to have a constant flux density in the air-gap. We would like compare different magnetic circuits for this application. First, a system with current excitation is analyzed. Second, a magnetic circuit with the air-gap flux density is designed by replacing the coil with a permanent magnet (PM).

Dimensions and characteristics of the system are given below:
g = 1.5 mm (length of **each** air-gap)
$l_{plunger}$= 5 mm (length of the plunger)
A_g= A_{core} =25 mm^2 (cross sectional area of the air-gaps and plunger)
$\mu_0 = 4\pi 10^{-7}$ (H/m)
N= 130 (number of turns)

Assumptions:

- Magnetic core material and plunger have a very high permeability so we can assume $\mu \rightarrow \infty$.
- All the leakage and fringing flux components are ignored.

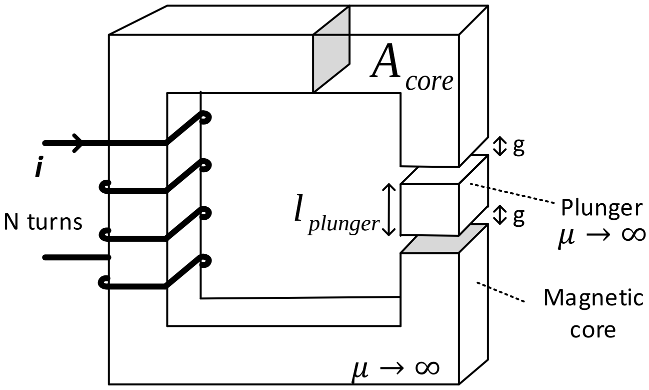


Fig. 1. C-shaped magnetic core with coil.

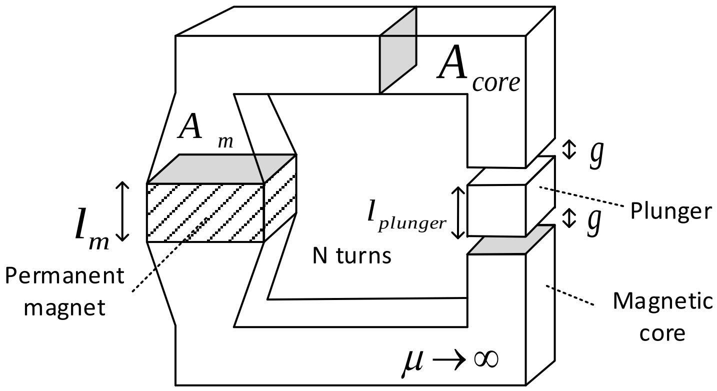


Fig. 2. C-shaped magnetic core with permanent magnet.

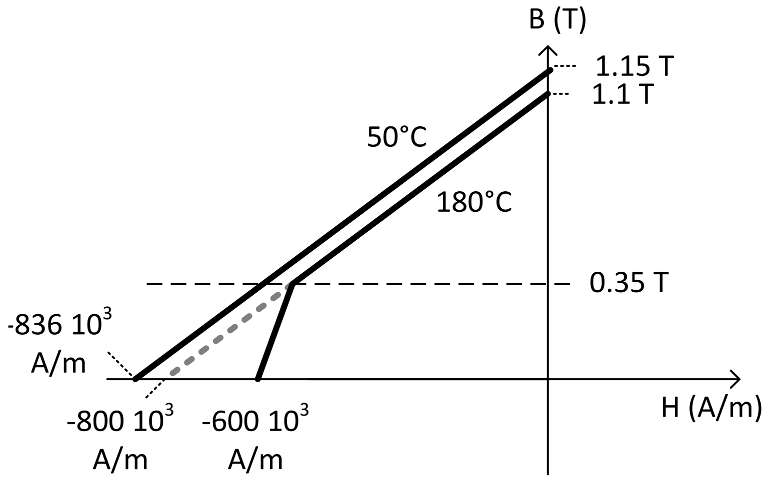


Fig. 3. B-H characteristics of the permanent magnet material at 50 °C and 180 °C.

a. Calculate the required coil current (i) to get 0.7 T in the air-gap. (5 points)

$\oint H \cdot dl = Ni$ $\quad \underbrace{H_c \cancel{l_c}}_{0} + 2H_g g = Ni$ $\quad B_g = 0.7T$

$$i = \frac{1}{N}\left(2 \cdot \frac{B_g}{\mu_0} \cdot g\right) = \frac{1}{130}\left(2 \times \frac{0.7}{4\pi 10^{-7}} \cdot 1.5\,10^{-3}\right) = 12.85A$$

b. Coil is replaced with a NdFeB high energy density permanent magnet from which the characteristics are shown in Fig. 2. System operates at 180 °C. Calculate the permanent magnet length l_m to get an air-gap flux density of 0.7 T by keeping $A_m = A_g$. Does the permanent magnet operate at its optimum operating point? (5 points)

$H_m l_m + 2H_g \cdot g = Ni = 0$

$B_m A_m = B_g A_g$ for $A_m = A_g \Rightarrow B_m = B_g = 0.7T$

$B_m = \frac{-1.1}{800\,10^3} H_m + 1.1$ (PM chrac.) therefore $H_m = -290909$ A/m

$$l_m = -\frac{B_m}{H_m} \cdot \frac{1}{\mu_0}(2 \times g) = \frac{-0.7}{-290909} \cdot \frac{1}{4\pi 10^{-7}}(3.10^{-3}) = 5.74\,10^{-3}m = 5.74\ mm$$

c. Due to the high cost of the permanent magnet, you are asked to minimize the PM volume. You can make changes in your core so that $A_m \neq A_g$ as in Fig. 2. Calculate the A_m and l_m of the magnet so that a minimum amount of PM material is used to get 0.7 T in the air-gap at 180 °C. What is the percentage volumetric reduction in magnet volume compared to part b. (5 points)

For V_m min $\rightarrow$ operate at BH_{max} point

$\Rightarrow B_m = 0.55T$
$H_m = -400\,10^3$ A/m
$A_g = 25\,10^{-6} m^2$

$B_m A_m = B_g A_g$ $\quad A_m = \frac{B_g}{B_m} A_g = 31.81\ mm^2$

$H_m l_m + H_g(g \times 2) = 0$ $\quad l_m = \frac{-B_g}{\mu_0}(2 \times g)\frac{1}{H_m} = 4.18\ mm$

$$\text{Volume reduction} = \frac{V_{m-c} - V_{m-b}}{V_{m-c}} \times 100 = \frac{(31.81 \times 4.18) - (25 \times 5.74)}{(31.81 \times 4.18)} \times 100 = -7.92\%$$

d. The plunger in the air-gap is completely removed and placed back after some time. Graphically show the following operating points of the PM:

- Original operating point (P_0) $\rightarrow$ already calculated
- No plunger (P_1)
- After plunger is placed back (P_2)

on the figure below for part b and part c, separately. You do not have to calculate exact operating points. (10 points)

$$H_m l_m + H_g(2x_g + l_{plunger}) = 0$$

$$B_m = -\mu_0 \left(\frac{A_g}{A_m}\right)\left(\frac{l_m}{2x_g + l_{plunger}}\right) H_m$$

$$A_m B_m = A_g B_g$$

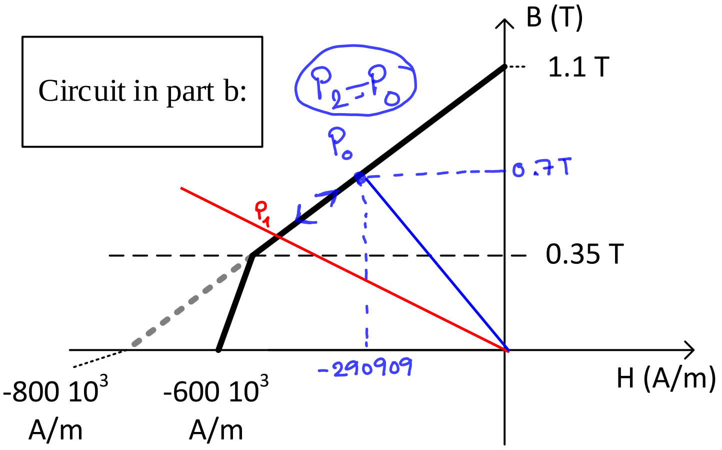
$$H_g = \frac{1}{\mu_0}\left(\frac{A_m}{A_g}\right) B_m$$

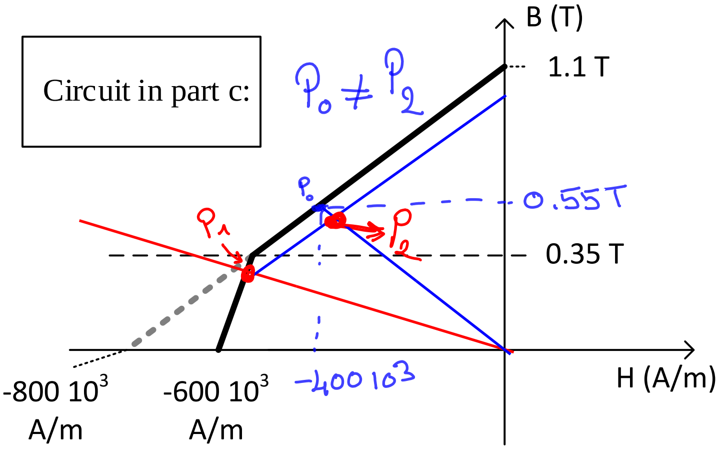
Part b:

$$B_m = -\mu_0 \frac{5.74\,10^{-3}}{3\,10^{-3} + 5\,10^{-3}} H_m = -9.02\,10^{-7} H_m \Rightarrow H_m(0.35T) = -388182\ A/m$$

Part c:

$$B_m = -\mu_0 \left(\frac{25\,10^{-3}}{31.81\,10^{-3}}\right)\left(\frac{4.18\,10^{-3}}{3\,10^{-3} + 5\,10^{-3}}\right) H_m = -5.16\,10^{-7} H_m \Rightarrow H_m(0.35T) = -678258\ A/m$$



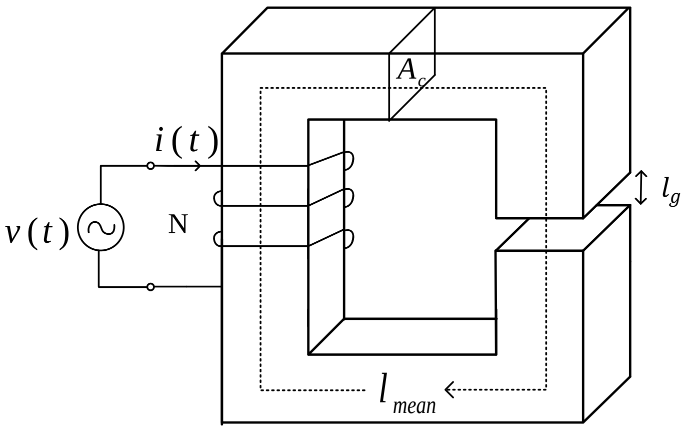


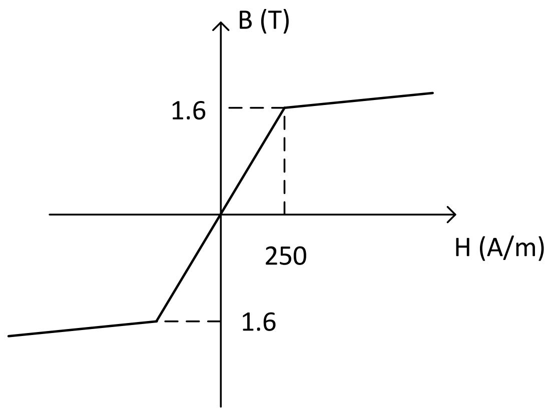
e. Which circuit would you prefer (part b or part c), explain briefly considering the cost and operation of the circuit if plunger is going to be removed and placed back. (5 points)

Part b design is better for the application since PMs will not demagnetize.

Part c design has a lower amount of PM material but it is prone to demagnetization.

Q2) (35 pts, 5 pts each) Consider the singly excited AC system below that has a magnetic core with the given piecewise-linear BH characteristics. Core can be assumed to be perfectly saturated (no magnetic characteristics that is $\mu = \mu_0$) for $B > 1.6\,T$.





$A_c = A_g = 400\,mm^2$

$l_{mean} = 220\,mm$

$l_g = 2\,mm$

$\mu_0 = 4\pi 10^{-7}\,H/m$

$N = 200$

Assumptions: All the leakage and fringing flux components are ignored. Resistance of the coils is ignored.

Part A: Singly excited system

a. Calculate the maximum peak current that can be applied to coil without saturating the core material.

$H_c l_c + H_g l_g = Ni$

$B_c = 1.6T$

$B_c A_c = B_g A_g \Rightarrow B_g = 1.6T$

$i = \left(\frac{B_g}{\mu_0} l_g + 250\, l_c\right) \times \frac{1}{N}$

$= \left(\frac{1.6}{4\pi 10^{-7}} 2 10^{-3} + 250 . 218\, 10^{-3}\right) \times \frac{1}{200}$

$= 13\,A$

b. Draw the φ vs. i characteristics of the magnetic system. Specify critical points n the graph. Find slopes of the lines.

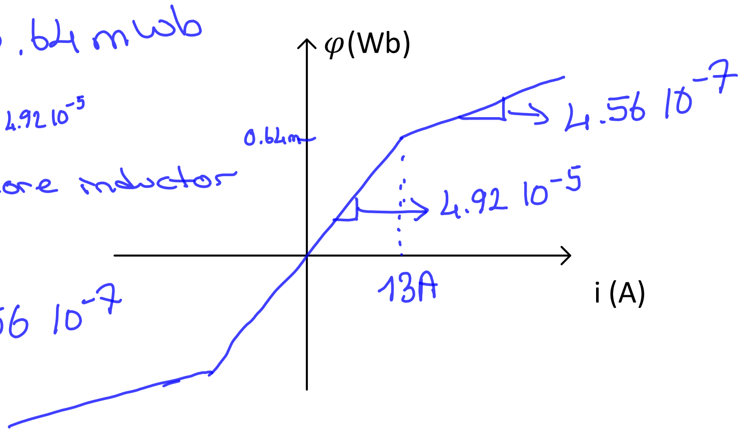
$\varphi = B.A = 1.6 \times 400\, 10^{-6} = 0.64\, mWb$

slope unsaturated $= \frac{0.64\, 10^{-3}}{13} = 4.92\, 10^{-5}$

when core saturates $\rightarrow$ air-core inductor

$\varphi = B.A = \mu_0 H . A = \mu_0 \frac{Ni}{l} . A$

$\varphi = \mu_0 \frac{N . A}{l} i \Rightarrow \frac{\varphi}{i} = 4.56\, 10^{-7}$

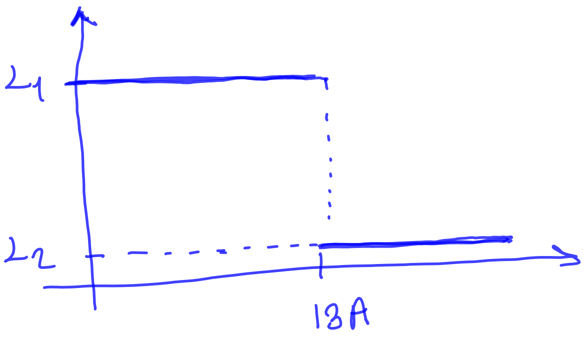


c. Calculate and draw inductance vs. current characteristics of the system both in unsaturated and saturated regions for positive currents.

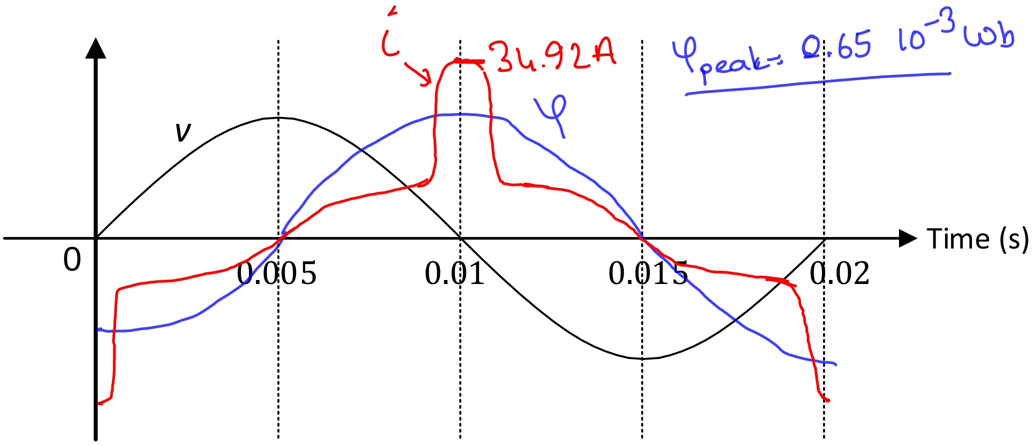
$$L_{diff} = \frac{d\lambda}{di} = N \frac{\Delta \varphi}{\Delta i}$$

$$L_1 = 200 \times 4.92\ 10^{-5} = 9.84\ mH$$

$$L_2 = 200 \times 4.56\ 10^{-7} = 0.912\ \mu H$$



d. For $v(t) = 41\sin(2\pi 50\ t)$, draw flux in the core $\varphi(t)$ and coil current $i(t)$, on the given template below. Specify the **amplitudes** of the waveforms.



$$v(t) = N \frac{d\varphi(t)}{dt} \qquad \varphi(t) = \frac{1}{N}\int v(t)\, d(t)$$

$$\varphi(t) = \frac{-41}{N \cdot 2\pi 50} \cos(2\pi 50 t) = -0.65\ 10^{-3} \cos(2\pi 50 t)$$

$0.65\ 10^{-3} > 0.64\ 10^{-3} \rightarrow$ Distorted chrac. due to saturation

No hysteresis loop $\rightarrow$ $\varphi(t)$ & $i(t)$ inphase.

$i_{peak} \Rightarrow \quad (0.65 - 0.64)\ 10^{-4} = 4.56\ 10^{-7}\ (i_{peak} - 13)$

$$i_{peak} = 34.92\ A$$

e. Design an inductor with the same inductance (unsaturated) and saturation current value as the original design with **no** air-gap. Use the same core material. You are free to change core dimensions and number of turns. There is no single answer to this question, you are free to make assumptions Compare your new design with the original design. Which one is preferable? Please briefly explain.

Note: Make sure that your design makes physically sense.

$W_{mag} = \frac{1}{2} L i^2$ $\quad i_{max} = 13A$

$B_c = 1.6\,T$ & $H_c = 250\,A/m$

$W_{mag} = W_{core} + W_{air}$

$= V_c \frac{1}{2\mu_c} B_c^2 + V_g \frac{1}{2\mu_0} B_c^2 = \left(\frac{V_c}{\mu_c} + \frac{V_g}{\mu_0}\right) \frac{1}{2} B_c^2$

$W_{mag-new} = \left(\frac{V_{c-new}}{\mu_c}\right) \frac{1}{2} B_c^2$

$V_{c-new} \simeq 47\, V_c$

To be able to store same amount of energy, our core must be ~50 times bigger. Assume $l_{c-n} \sim 10 \times l_c$

$A_{c-n} \sim 5 \times A_c$

$H_c l_c = N i \rightarrow N$ should be around 40-50

Select $\underline{N = 50} \Rightarrow \underline{l_c = 2.6\,m}$ (220 mm before)

$L = \frac{N^2}{\mathcal{R}} = \frac{N^2 \mu_c A_c}{l_c}$ $\quad A_c = \frac{9.84\, 10^{-3} \cdot 2.6}{\frac{1.6}{250} \times 2500}$

$\underline{A_c = 1599\,mm^2}$ (400 mm^2 before)

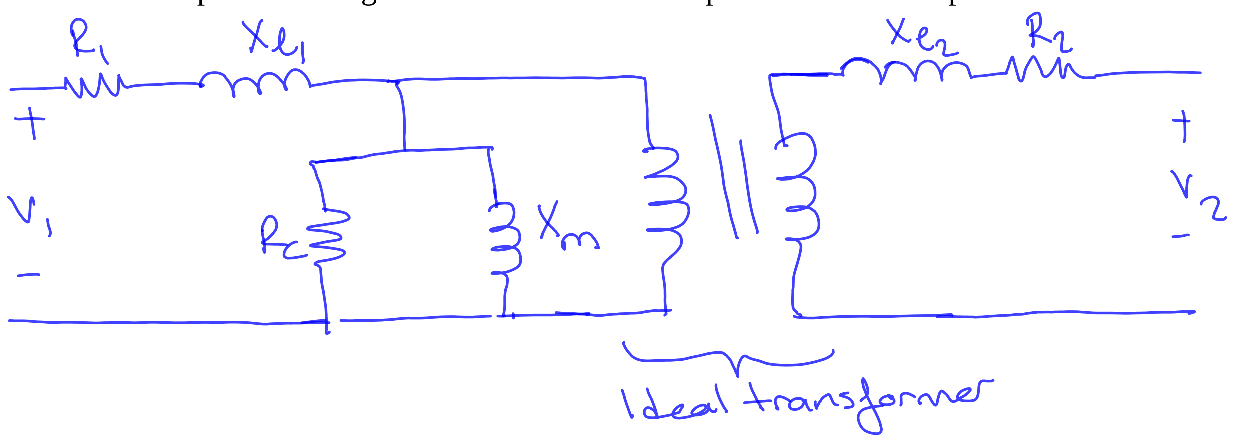
* Core is 47.24 times bigger.

We should use the design with air-gap.

Note: We need to both increase l_c and A_c to make sure the core dimentions are ok.

Part B: Single phase transformer at no-load

f. If we add a secondary coil to the system, we can have a single phase transformer. Draw the equivalent circuit of a single phase transformer with all equivalent circuit components and discuss which components are ignored based on the assumptions in this example.



R_1, R_2 and R_c are ignored since there are no losses in the system.

g. Is it preferred to have an air-gap in a transformer core, explain briefly considering the influence of the air-gap in the equivalent circuit model components.

No. Air-gap will cause a lower X_m, therefore a higher magnetizing current.

This means we store more magnetic energy in the system.

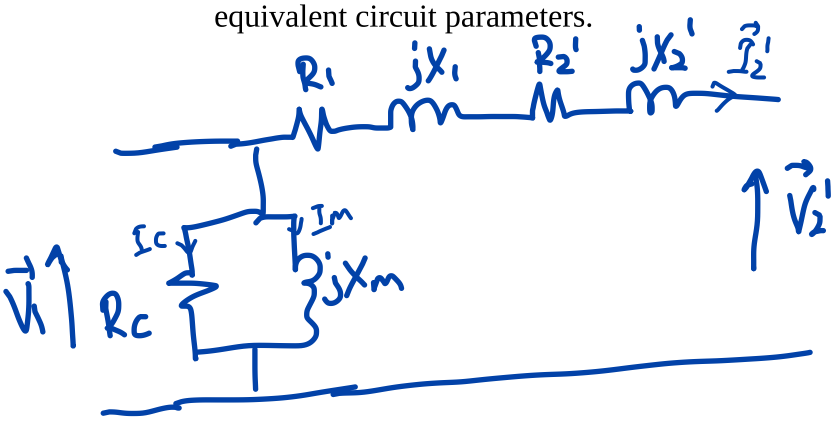
Q3) (35 pts, 5 pts each)

Assume you have a single-phase 1.25MVA, medium voltage(MV, 34.5 kV) to low voltage (LV, 230V) step-down transformer with the following specs:

- R_1=9 Ω
- R_2= 0.0004 Ω
- jX_1= j 36 Ω
- jX_2= j 0.0016 Ω
- R_c= 238 kΩ
- jX_m= j 15 kΩ

Assume the parallel branch is moved to the primary side terminals.

a. Draw the equivalent circuit referred to the primary side (MV). Calculate and label all the equivalent circuit parameters.



$$n = \frac{34.5k}{230} = 150$$

$$R_2' = n^2 . R_2 = 9\Omega$$

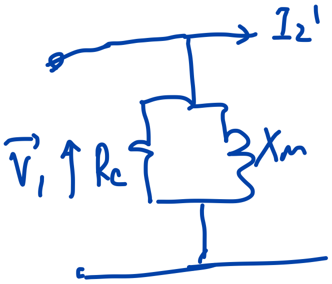
$$jX_2' = j36\Omega$$

$$R_{eq} = R_1 + R_2' = 18\Omega$$

$$jX_{eq} = jX_1 + jX_2' = j72\Omega$$

b. Calculate the real power (P) and total power (S) consumed by this transformer at no load.

No load: $I_2' = 0$



$$P = \frac{V^2}{R_c} = \frac{(34.5k)^2}{238k} = 5kW$$

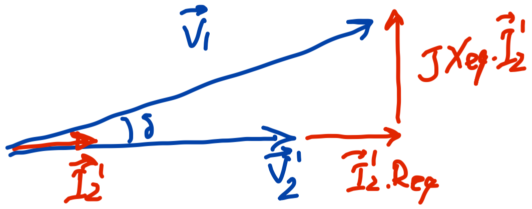
$$Q = \frac{V^2}{jX_m} = 79.35\ kVAr$$

$$S = \sqrt{P^2 + Q^2} = \underline{\underline{79.5kVA}}$$

c. Assume a purely resistive load is connected to the secondary side of the transformer such that, transformer draws 5250 A on the secondary side. Draw the phasor diagram and calculate the actual RMS voltage on the low voltage terminals when the primary voltage is constant at 34.5kV.

$$I_2 = 5250A \Rightarrow I_2' = \frac{5250}{150} = 35A$$

$$\vec{V}_1 = \vec{V}_2' + (R_{eq} + jX_{eq})\vec{I}_2'$$



$$|V_2'| = \sqrt{34.5k^2 - (35.72)^2} - (35.18)$$

$$|V_2'| = 33778\ V$$

$$|V_2| = \frac{|V_2'|}{150} = \underline{\underline{225.2\ V}}$$

d. Calculate the efficiency of the transformer for the operating condition in part (c).

$$P_{copper} = I_2'^2 . R_{eq} = 35^2 . 18 = 22.05kW$$

$$P_{core} = \frac{V_1^2}{R_{eq}} = 5kW$$

$$P_{out} = V_2' . I_2' . \underbrace{\cos(\theta)}_{unity} = 1182kW$$

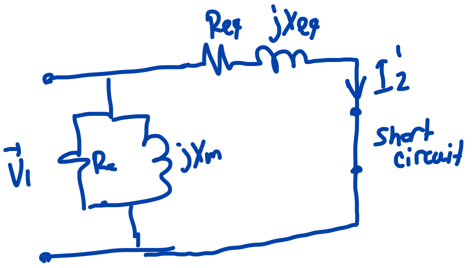
$$\eta = \frac{P_{out}}{P_{out} + P_{losses}}$$

$$\eta = \frac{1182}{1182 + 27.05} = \underline{\underline{97.76\%}}$$

e. Comment on the power factor (leading, lagging, unity) as seen from the primary terminals of the transformer. You don't have to calculate any numerical data, but support your reasoning for full credit.

As it can be seen from the phasor diagram in part (c), there is a lagging phase difference of (δ) between $\vec{V}_1$ and $\vec{I_2'}$. Furthermore, $\vec{I_m}$ is larger compared to $\vec{I_c}$, and making the power factor even more lagging.

f. Assume there is a short-circuit fault at the secondary terminals of the transformer when the primary terminal voltage is 34.5kV. Calculate the short-circuit current on the secondary terminals. Compare the power loss dissipated in the transformer with the loss in part (d), and comment on the consequences of this operating condition.



$|Z_{eq}| = \sqrt{R_{eq}^2 + X_{eq}^2}$
$= 74{,}21\,\Omega$

$I_2' = \dfrac{34.5\,kV}{Z_{eq}} \cong 465\,A \Rightarrow I_2 = 69729\,A$

$P_{copper} = I_2'^2 \cdot R_{eq} = 3.89\,MW!$ $\Rightarrow$ 176 times of the loss at rated conditions.

This will result in very high temperature rise and damage to the transformer, if the fault can not be cleared by automatic protection equipment.

g.

i) List the two testing methods to determine the transformer equivalent circuit parameters, and the required measurements for these tests.	- Open-circuit test - Short-circuit test We need to measure phase current, phase voltage and real-power.
ii) Can the resistance measurements (R_1, R_2) taken by a multimeter can be directly used in the equivalent circuit? Explain.	No, because the multimeter reading will give DC-resistance, but the coil resistance under operation (R_{ac}) will be higher due to skin-depth.
iii) R_{core} is the electrical resistance of the magnetic core. True/False? Explain	No. It is an equivalent resistance we put into the equivalent circuit to simulate the core loss.
iv) If you perform tests at different voltages, how does X_m change with applied voltage? Explain .	X_m simulates the core inductance. If the applied voltage is increased, core starts to saturate, which reduces the L_m, and X_m value.