

1-)

a-) $(n^5 + n^3 + 45)$

$\lim_{n \rightarrow \infty} \frac{n^5 + n^3 + 45}{n^5} = 1 \Rightarrow \Theta(n^5)$

$n^5 + n^3 + 45 \leq n^5 + n^5 + 45n^5$
 $n \geq 1$
 $n=1$ için
 Sağlanıyor

b-) $2n \lg(n+6)^2 + (n+3)^2 \lg n/2$

$= 2n \lg(n+6)^2 + (n+3)^2 \lg n/2$
 $= 4n \lg(n+6) + 2 \lg n/2 (n+3)^2$
 $= 4n \lg(n+6) + 2 \lg \left(\frac{n+3}{2}\right)$
 $= 8 \lg \left(\frac{n^2+6n}{2}\right)$
 $= 8(n+3) \lg \frac{n^2+6n}{2}$
 $= \frac{8n(n+3)}{2} \lg n + 6$
 $= 4n^2 + 12 \lg n + 6$

$n^2(\log n) \in \Theta(n^2(\log n))$

$\log_2 n = a$

$\log_2 n + \log_2 2$

$\log_2 2n$

$\Theta(2 \cdot \log_2 n)$

c-) $(\log_2 n)^2 = \log_2 2n$

2-)

a-) $X(n) = X(n/5) + n$, $n \geq 1$, $X(1) = 1$

$$X(n) = X(n/5) + n$$

$$X(n/5) = X(n/25) + n/5$$

$$X(n/25) = X(n/125) + n/25$$

$$X(n/125) = X(n/5^3) + n/25 + n/125$$

$$X(n) = X(n/5^3) + n/25 + n/5 + n$$

$$X(n) = X(n/5^p) + 5^{1-p} \cdot n + 5^{2-p} \cdot n + 5^{3-p} \cdot n$$

$$n (5^{1-p} + 5^{2-p} + 5^{3-p})$$

$$\frac{n}{5^p} = 1$$

$$n = 5^p$$

$$p = \log_5 n$$

$$5 \cdot 5^{-\log_5 n} + 25 \cdot 5^{-\log_5 n} + 125 \cdot 5^{-\log_5 n}$$

$$n \cdot \left(\frac{5}{n} + \frac{25}{n} + \frac{125}{n} \right)$$

$$1 + n^2 \left(\frac{155}{n} \right) \Rightarrow \boxed{\varepsilon \Theta(n)}$$

b-) $X(n) = 2X(n-3)$, $n \geq 1$, $X(1) = 1$

$$X(n) = 2X(n-3) \rightarrow 2X(n-3)$$

$$X(n-3) = 2X(n-6)$$

$$X(n) = 2 \cdot (2X(n-6)) \rightarrow 4X(n-6)$$

$$X(n) = 4 \cdot X(n-6)$$

$$X(n-6) = 4X(n-12)$$

$$X(n) = 4^2 X(n-12) \rightarrow 16X(n-12)$$

$$X(n-12) = 4^2 X(n-24)$$

$$X(n) = 4^2 4^2 X(n-24)$$

$$\frac{n-2}{3} = \frac{1}{3} \quad p = \log_2 \frac{3n-1}{3}$$

$$2^p \cdot X\left(\frac{n-2^p \cdot 3}{1}\right) = \frac{6n-2}{3} \Rightarrow 2n - \frac{2}{3}$$

$$\boxed{\varepsilon \Theta(2n)}$$

$$c) \quad x(n) = 2x(n-2) - n, \quad x(0) = 0$$

$$x(n) = 2x(n-2) - n$$

$$x(n-2) = 2x(n-4) - (n-2)$$

$$x(n-4) = 2x(n-6) - (n-4)$$

$$x(n-6) = 2x(n-8) - (n-6)$$

$$x(n-2) = 2(2x(n-6) - (n-4)) - (n-2)$$

$$2^2(2x(n-8) - (n-6)) - 2(n-4) - (n-2)$$

$$2^3(2x(n-8) - (n-6)) - 2(n-4) - (n-2)$$

$$T(n) = 2(2^3x(n-8) - 4(n-6) - 2(n-4) - (n-2)) - n$$

$$2^p T(n-2^p) - \sum_{z=1}^{p-1} 2^z (n-2z) \quad p = \frac{n-1}{2}$$

$$\boxed{\mathcal{E} \Leftrightarrow (n, 2^{\frac{n}{2}+1})}$$

$$d) \quad T(n) = 2T(n-1) + 5$$

$$T(n-1) = 2(2(T(n-3)+1)+1)+5$$

$$= 2^2 T(n-3) + 2^2 + 2 + 1$$

$$T(n) = 2 \cdot T(n-p) + 2^p + 2^{p-1} + 2^1 + 1$$

$$2^p T(n-p) + 2^{p-1} - 2 + 1$$

$$n-p=0$$

$$\boxed{p=n}$$

$$= 2^p \cdot 1 + 2^p + 1$$

$$\boxed{\mathcal{E} \Leftrightarrow (2^n)}$$

$$e-1) T(n) = T(n/3) + 1 \quad T(1) = 1$$

$$T(n/3) = T(n/9) + 1$$

$$T(n/9) = T(n/81) + 1$$

$$T(n) = T(n/81) + 3$$

$$n/3^p = 1$$

$$n = 3^p$$

$$p = \log_3 n$$

$$\therefore E \leftrightarrow \log_3 n$$

veya

$$E \leftrightarrow (n)$$